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Portfolio performance with inverse and leveraged ETFs

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Abstract

Turbulent economic and financial times require investors and financial planners to investigate new ways to handle the goal of wealth maximization. This article investigates passive investment strategies that use inverse or leveraged equity exchanged-traded funds (ETFs) in their asset allocation, and quantifies the long-term impact on portfolio performance for the purpose of improving the risk-reward tradeoff. Monte Carlo simulations are used, drawing samples from distributions created by two distinct time periods of historical daily market returns. The findings suggest that, whereas these products are generally not recommended within long-term passive investment strategies, potential diversification benefits exist, dependent on the behavior of equity and debt markets. These findings could materially alter long-term passive portfolio construction methods currently in use by financial planners and individual investors seeking potential diversification benefits using ETFs. © 2014 Academy of Financial Services. All rights reserved.

Jel classification: G1; C6

Keywords: Inverse ETFs; Leveraged ETFs; Diversification; Portfolio; Asset allocation

1. Introduction

The purpose of this article is to provide individual investors and financial planners with guidance on the possible use of inverse and leverage financial instruments to improve the distribution of terminal wealth. The recent economic downturn has called into question the effectiveness of diversification and led some investors to consider alternative investments as part of their overall portfolio. A recent study by Arshanapalli et al. (2010) shows that,

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although more severe than many of the bear markets before it, the recent downturn did benefit from diversification and alternative investments such as gold. So-called "wide diversification" is not entirely new (Mulvey et al., 2007), and suggests that increasing the number of asset classes offers significant benefit. Ammermann et al. (2011) suggests the effectiveness of a style-rotation over sector/industry rotation using exchange-traded funds (ETFs) as a long-flat or synthetic put equity strategy whereas Johnston et al. (2013) suggested portfolio insurance as an alternative. However, the use of inverse and leveraged products as a new asset class in a diversified portfolio has yet to be adequately examined. The results shown here represent a first attempt to quantify their long-term impact for passive investors seeking improved diversification.

Lack of study at a portfolio level does not suggest little is known about the impact of holding these types of investments long-term. To the contrary, a comprehensive study by Cheng and Madhaven (2009) shows how inverse ETFs need to be rebalanced on a daily basis to maintain a constant leverage, and how this can lead to wealth destruction. This wealth destruction occurs largely because of the resulting path dependence on accumulated wealth that can easily diverge from the underlying index over longer holding periods. Additionally, Lu, Wang, and Zhang (2012) show how the longer term performance diverges from the benchmark through periods of up to one year, and caution investors on their use as substitutes for benchmark indices. This wealth destruction is also aggravated by higher volatility, although Trainor and Baryla (2008) notes an interesting corollary that suggests some of these leveraged ETFs can outperform their respective benchmarks in periods of low volatility. Giese (2010) aptly summarizes the benefit of holding leveraged funds in bullish markets, as well as their benefit as an investment product remaining positive, unlike a short position, but highlights these benefits are offset by increased performance volatility.

This article is organized as follows: Section 2 discusses the background and history of inverse and leveraged funds. Section 3 states the research hypothesis, and Section 4 describes the methods used to test our hypothesis. Section 5 presents the terminal wealth distributions for a large number of simulated returns randomly sampled from non-overlapping historical stock and bond time series. Section 6 highlights the relevant statistics from the simulated return distributions and also quantifies risk-adjusted returns using the Sharpe ratio. Section 7 enhances the results from Section 6 by considering other investment options for stock and bond funds. The article concludes with Section 8.

2. Background of inverse and leveraged ETFs

Although the first inverse ETF came into existence in 2007, both inverse and leveraged open-ended mutual funds have been in existence far longer. Since 1997, ProFunds has offered an inverse and leveraged version of the S&P 500 index through two mutual funds. By mid-2010, 150 different inverse and leveraged ETFs were available with a total of $30B of assets under management (see Guedj et al., 2010) Although their track record is brief, the history of these products so far suggests that they are meeting the objectives contained within their offering prospectuses. Specifically for this proposed research article, we assume that they effectively meet their daily objectives of inverse ($-1\times$) or double ($2\times$) daily returns of their underlying index through the use of swaps and other derivative instruments. Unfortunately, the daily releveraging of these products results in potential investment management challenges and risks.[1] Many researchers, including Barnhorst and Cocozza (2010), Cheng and Madhaven (2009), Guedj et al. (2010), Trainor and Baryla (2008), and Lu et al. (2012) have noted that daily rebalancing has tended to reduce their stated investment effectiveness over longer holding periods. Furthermore, the limited history of these products prevents longer term studies of historical returns within a diversified portfolio.

3. Research hypothesis

This research article attempts to uncover whether there are any potential diversification benefits to a passively managed portfolio that holds a portion of its holdings in either inverse or leveraged equity ETFs. To test this hypothesis, three typical long-term investors were considered who seek diversification by holding a broad stock and bond index fund in a tax deferred retirement account that is rebalanced annually. These investors were assumed to be representative of three separate tolerances to risk, and that the risk tolerance can be adjusted based on their portfolio exposure to stocks and bonds. The objective is to determine whether there is any risk-return benefit of these products in a diversified portfolio, or whether, as the previous studies have shown, they should be avoided entirely by long-term passive investors. Perhaps, these products should remain clearly in the hand of speculators and short-term traders. Greater regulatory protections for retail investors may be warranted.

The strategic allocation between stocks and bonds is highly dependent on the time horizon of the investor and their tolerance for risk. This failure to quantify risk in an operational context for the strategic allocation of investment assets has caused the rise of mechanical methods that reflects classical thinking by connecting risk-aversion to one's age. Thus, the "Rule of 120" has appeared in the literature. This suggests that for the stock allocation should be 120 minus the age of the individual. The rule has been modified by others, from about 100 to 130, to accommodate different economic and investment cycles. The remainder of the money is then allocated to fixed interest instruments.[2] The popularity of this approach has motivated many fund providers to offer single funds that automatically follow this approach, naming them either Lifecycle or Target Date Funds (TDFs), although these styles may be too aggressive for risk adverse investors (Pfau, 2010). Table 1 was formatted to reflect the strategic asset allocations used in this study. Additionally, the investor is assumed to annually rebalance their portfolio to these allocations over a 10-year investment period.

In contrast, this study is interested in comparing simulated returns of the above investors versus ones who choose to allocate a small portion of their assets at the beginning of each year of a 10-year investment period to either an inverse or leveraged stock fund. This alternative group of investors is represented by allocations that appear in Table 2, and includes the same stock to bond ratio as established in Table 1. The distinction in Table 2 is that 10% of the investor's retirement account will be allocated towards an inverse or leveraged stock fund. Again, this study assumes that the investor rebalances to these allocations at the beginning of each year.

Table 1 Traditional stock and bond allocations

Risk tolerance	High	Medium	Low
Typical age	25	40	55
Bond allocation	5%	20%	35%
Stock allocation	95%	80%	65%
Stock:bond ratio	19:1	4:1	13:7 (1.86:1)

Notes. This table shows the traditional stock and bond allocations assumed for each simulated 10-year period, rebalanced annually.

4. Research methods

The previous section noted the logic for the strategic asset allocation between stocks and bonds. Although many believe that one must at the same time be specific about the types of equity or fixed income asset chosen, this article suggests that this is more of a tactical asset allocation question. For example, equities could be small cap, mid cap, large cap, or REITs. For fixed income, this could be short-term bonds; intermediate-term bonds; long-term bonds, high yield bonds; inflation-indexed; or money markets and/or T-bills. Further complicating the issue, one could consider for equities international exposure and for bonds the question of corporate versus government bonds further complicated by domestic versus international. Thus, the possible pragmatic tactical choices are large.

To minimize potential problems that could arise from the tactical allocation question, two well-known and large mutual funds were chosen to begin our analysis, representative of rational investment choices and proxies for stocks and bonds. The Vanguard S&P 500 Index Fund (VFINX) was chosen as the proxy to represent domestic equity. This fund attempts to track the performance of the S&P 500, a widely recognized benchmark of the U.S. stock market. This index represents large capitalization firms almost equally weighted between value and growth. The Vanguard Total Bond Market Index Fund (VBMFX) was chosen as the proxy to represent fixed income. This fund attempts to track the Barclays Capital US Aggregate Float Adjusted index. This index is classified as an intermediate-term domestic index. Both funds are no-load and low-fee as well as being passively administrated. The funds are highly popular with Vanguard investors. The S&P fund is second in size whereas the bond fund is third. Vanguard is further known to have very low benchmark tracking errors thereby making these funds valid proxies. For this reason, we also consider other

Table 2 Alternative stock, bond, and inverse or leveraged stock allocation

Risk tolerance	High	Medium	Low
Typical age	25	40	55
Bond allocation	4.5%	18.0%	31.5%
Stock allocation	85.5%	72.0%	58.5%
Alternative stock allocation	10.0%	10.0%	10.0%
Stock:bond ratio	19:1	4:1	13:7 (1.86:1)

Notes. This table shows the alternative stock, bond, and inverse or leveraged stock allocations assumed for each simulated 10-year period, rebalanced annually.

Vanguard stock and bond index funds to enhance our initial findings, based on alternative candidates for an investor's stock and bond allocation.

The inverse and leveraged equity returns were modeled from the daily returns, adjusted for any dividends, of the stock returns, multiplied by negative one (−1) for the inverse stock fund, and positive two (2) for the leveraged fund. Annual fees were assumed at 1%, and were deducted from monthly returns, to include the combined cost of the expense ratio, trading commissions, and a bid-ask spread potentially resulting from lower volume ETFs.[3] Although the value of 1% may appear to be insufficient to cover all investor expenses, it was found by starting with the expense ratio for an existing inverse or leveraged ETF, such as the Proshares Short and Ultra S&P 500 ETFs (symbols SH and SSO), which are both 0.90%. We then applied the bid-ask model developed by DiLellio and Stanley (2011) based on the three-month moving average volume at the end of 2013 for these ETFs, producing a bid-ask spread cost of 0.10% and 0.08%. To account for commissions, we assumed one buy and one sell per year on a portfolio of $100,000 to maintain the 10% allocation to these alternative investments, which produces an annual cost of $20/(10% of $100,000) = 0.2%. Lastly, because we are "replicating" an inverse or leverage ETF using an existing index mutual fund, we wanted to avoid double-counting the expense of managing the fund, assuming that management was sufficiently covered by the 0.90% expense ratio already identified. Thus, we subtract from this total the expense ratio of 0.17% for VFINX, and express the "penalty" as:

Total replication expense (inverse ETF) = 0.90% + 0.10% + 0.20% − 0.17% = 1.03%
Total replication expense (leveraged ETF) = 0.90% + 0.08% + 0.20% − 0.17% = 1.01%

No attempt to model divergence from the daily benchmark was made, which can occur when such a fund trades at either a discount or premium. A recent study of inverse fund performance suggests that such a premium or discount remain fairly small, and quickly revert back to the performance modeled here. Gerasimos (2011) found that in particular, emphasis is given to the ability of these ETFs to meet their daily investment target. In this respect, an average deviation from the daily target amounting to −0.034% is computed. Applying a classification to the deviation from the daily return goal, they found that for about 62% of the examined trading period's duration the return of the average short ETF abstains from its target a maximum rate of 0.5%, either below or above the target.

The inverse and leveraged equity model used herein appears to be a good starting point for this investigation, but does assume certain risks of leveraged and inverse funds are negligible. In fact, investors are exposed to other risks because of the construction of the inverse equity funds. Inverse equity funds replace equity shares with futures and swaps to guarantee the applicable multiples of return. Futures have the benefit of having a clearing corporation stated as the counterparty, which has a very favorable credit risk advantage. On the other hand, swaps clear through banks; this adds another element of credit risk. This is a major area of concern not fully understood. In addition, futures also require standard amounts and fixed times to expiration as well as being marked to the market. Swaps do not. Instead, they are more flexible that accounts for their popularity. Choi and Elston (2009) reported that ProShares Inverse S&P 500 ETF held weightings of 91% in swaps and but 9%

Table 3 Return statistics from daily returns over historical 10-year periods

	Return (annualized)	Standard deviation (annualized)	Total return
Bonds	5.22%	4.39%	72.5%
Stock	3.51%	21.3%	13.9%
Inverse stock	−3.51%	21.3%	−45.5%
Leveraged stock	7.03%	42.6%	−19.5%
"Flat" - January 2, 2001 to December 31, 2010, N = 2,516 samples			
Bonds	7.30%	4.49%	113.33%
Stock	16.27%	14.57%	395.5%
Inverse stock	−16.27%	14.57%	−83.88%
Leveraged stock	32.53%	29.15%	1860.2%
"Rising" - January 2, 1991 to December 29, 2000, N = 2,527 samples			

Notes. This table shows the return statistics from the two 10-year periods observed from adjusted closing prices of stock and bond market proxies. Inverse and leveraged stock return statistics are generated assuming perfect replication of negative and double daily return from the stock return, respectively.

in futures. Further, Cheng and Madhavan (2009) noted that daily return streams from paired leveraged and inversed leveraged ETFs do not net out on a daily basis.

Because of the recent development of inverse and leveraged ETFs, and uncertainty of return distributions of future markets, the Monte Carlo simulation was developed. By randomly drawing from actual frequency distributions observed within over two separate 10 year periods, we used an approach similar to Cheng and Estes (2010) and Ervin et al. (2009). Each frequency distribution was obtained from different, non-overlapping time periods, thereby making them temporally uncorrelated. These time periods were categorized in terms of overall equity market returns as a "flat" from 2001 to 2010, and "rising" from 1991 to 2000. The descriptive statistics over these periods are summarized in Table 3. Annualized values were determined by assuming an average of 20 trading days in a month. Then, annualized returns are found from the average daily returns multiplied by 20*12 and annualized standard deviations were found from daily standard deviations multiplied by $(20*12)^{1/2}$. Daily returns were found as "adjusted closing price at day t" divided by "adjusted closing price at day t-1" minus 1.

A review of Table 3 suggests several important behaviors were observed in the time periods considered. First, bond returns were always positive in a fairly narrow annualized return range of ~5–7%, and mean daily bond returns were not statistically different between the two 10-year sample periods (p-value = 0.28). Bond volatility was also nearly constant, as measured by the annualized standard deviation ranging between 4.39% to 4.49%. The bond returns were in stark contrast to the returns exhibited by stocks over these periods. Annualized stock returns varied significantly, between 3.51% and 16.27%, and the mean daily stock returns were statistically different at a p-value = 0.11. The period from 2001 to 2010 also had a noticeable increase in volatility of ~6% (annualized) versus the previous decade. The inverse stock investment generated a total return that was negative in all cases, which is consistent with the wealth destruction expected analytically from Cheng and Madhavan (2009), because of the negative expected return over long holding periods Conversely, the leveraged stock fund had a large positive total return in the 1991–2000 case, and a negative total return in the 2001–2010 time period.

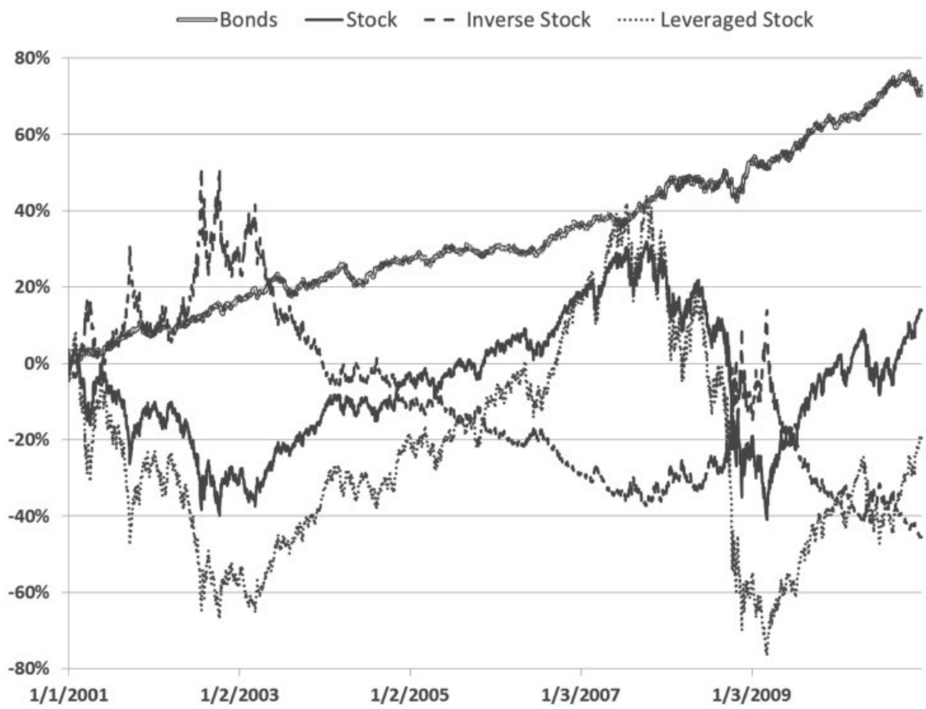

Fig. 1. Cumulative return of bonds, stock, inverse, and leveraged stock funds from January 2001 through December 2010.

An important aspect of Table 3 is the distinction between annualized and total returns. The annualized returns for inverse stocks were always "negative" of the stock returns and the leveraged stock was always 2× the stock returns, but the total returns differ. This behavior is because of the path dependency discussed previously, and will be examined through simulating other paths drawn from the daily return distributions that match historical returns. For the historical periods considered here, the time series of cumulative returns are illustrated in Fig. 1 for 2001 to 2010 and Fig. 2 for 1991 to 2000. Note that the extreme difference in scale between Fig. 1 and Fig. 2 were intentional so that it is clear that two very different scenarios of equity returns would be simulated.

To proceed with a simulation of a portfolio of bonds, stocks, and inverse or leveraged stock funds, two alternatives were considered and one selected to simulate future returns. The first was to take a set of historical returns, fit an appropriate distribution to it, and use this distribution to generate random samples. This article chose not to follow this method, because of concern about the behavior at the tails of the distribution not accurately reflecting observed returns. Instead, this article chose an alternate approach that considered a 20-day consecutive return that was available from an observed return history. This method is shown in the Appendix.

The simulation randomly samples over 2,500 monthly returns derived from actual history to create 120 months of returns. The stock, bond and inverse or leveraged monthly returns use the same random number to preserve the historical correlation of returns between each. The same random sample was also used to determine the representative monthly risk-free rate based on T-bills,[5] which is used to determine excess returns and compute the Sharpe ratio for each trial, as described in Sharpe (1994).

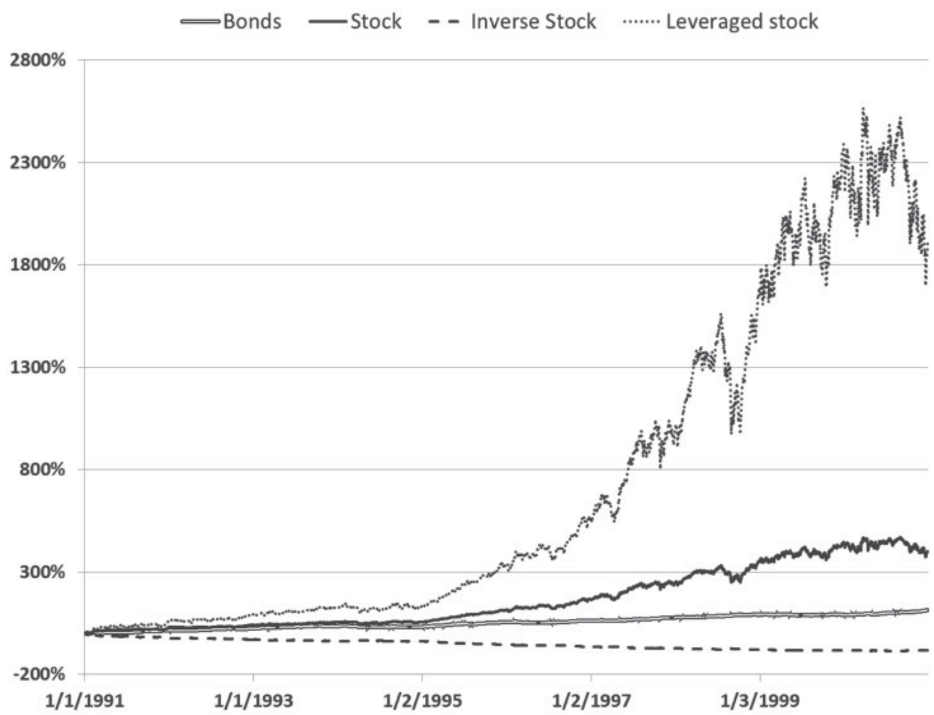

Fig. 2. Cumulative return of bonds, stock, inverse and leveraged stock funds from January 1991 through December 2000.

We based our sampling method assuming that the stock market follows a Markov (e.g., memoryless) process, and is also related to our assertion about efficient markets, that suggests successive price changes are independent (see Fama, 1965). Our specific approach of sampling from a historical distribution is often termed “bootstrapping,” which was originally called computer-intensive methods. Two excellent references for this methodology are Davison and Hinkley (1993) and Efron and Tibshirani (1993).

The rationale for randomly sampling supports the assumption that there are no discernible patterns in the returns of stocks and bonds. Furthermore, the Efficient Market Hypothesis (EMH) states that there are no discernible patterns for returns for stocks and bonds, so consequently, the markets are efficient at all times. Many will disagree with that statement. It is not novel, and it has been a contentious subject for decades. There are a few strong believers on both sides with the vast majority falling somewhere in between. However, the fact remains that passive index investors have the better argument. It can be noted that a large percentage of managers failed to outperform their benchmarks over a longer-time horizon. Jones and Wermers (2011) noted that active returns (adjusted for risk) across managers and time probably average close to zero, net of fees and other expenses, above their benchmark. Their conclusion confirmed what should be expected in a mostly efficient market. In such a market, one should expect fierce competition among active managers which drives average (net) active risk-adjusted returns towards zero. They further state that there exists so-called Superior Active Mangers (SAMS) who should be able to develop better Sharpe ratios. Thus, investors could be rewarded by exposure to such active strategies by SAMS. A similar trend has been observed with enhanced index funds (EIFs), as shown by Chang and Krueger

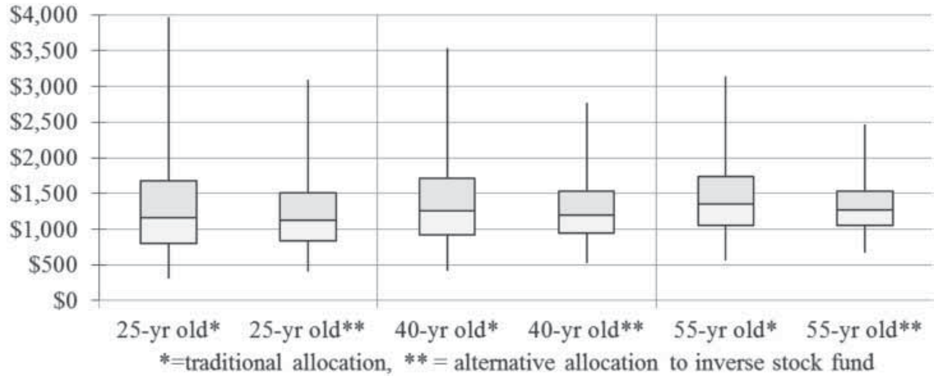

Fig. 3. Terminal wealth distributions for investors using 2001–2010 return distributions. Distribution assumes $1,000 initial wealth, rebalancing each year, and a 1% annual fee incurred by the inverse stock fund.

(2010). Therefore, the controversy will likely continue, but the odds today still favor the Efficient Market Hypothesis.

5. Return distributions of terminal wealth

The distributions of terminal wealth for the three investors classified previously are shown below in Figs. 3 and 4, corresponding to drawing samples from the 2001–2010 and 1991–2000 returns, respectively. Within each figure, two distributions for each investor are shown, corresponding to the investor with and without a 10% exposure to the inverse stock fund. The distributions are represented by boxplots that indicate first percentile (P01), 25th percentile (Q1), 50th percentile (median), 75th percentile (Q3), and 99th percentile (P99) values obtained for terminal wealth, assuming an initial wealth of $1,000.

Referring to the results in Figs. 3 and 4, it appears that for all investors, the distribution of terminal wealth becomes more positively skewed with the addition of the inverse stock fund. The dispersions of the distributions also appear to decrease with the addition of the

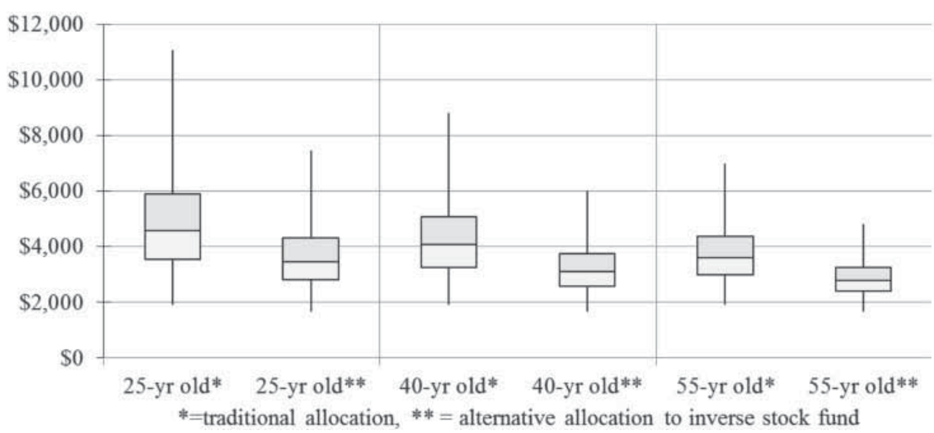

Fig. 4. Terminal wealth distributions for investors using 1991–2000 return distributions. Distribution assumes $1,000 initial wealth, rebalancing each year, and a 1% annual fee for the inverse stock fund.

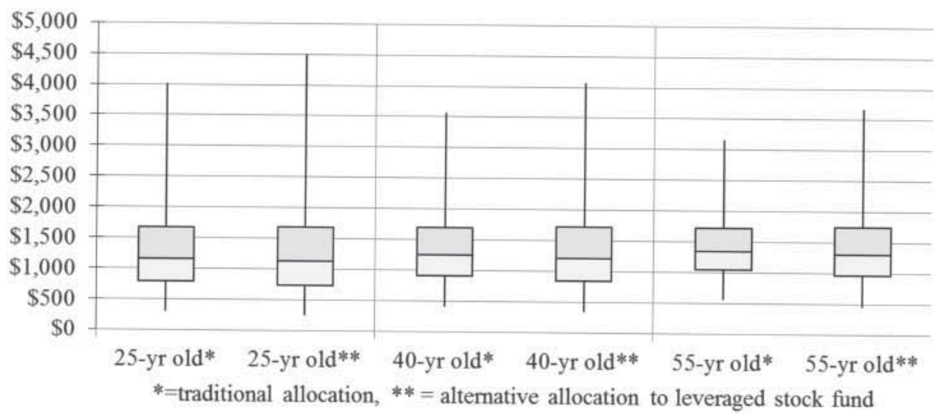

Fig. 5. Terminal wealth distributions for investors using 2001–2010 return distributions. Distribution assumes $1,000 initial wealth, rebalancing each year, and a 1% annual fee for the leveraged stock fund.

inverse stock fund. These two observations translate to what is expected from knowledge of how the inverse stock fund behaves as a hedge against the stock fund. Thus, the average return is decreased and the variance of the return is also decreased. The results from Figs. 3 and 4 are also encouraging because they show that the inclusion of the inverse stock fund appears to be simultaneously reducing volatility and return.

In contrast, Figs. 5 and 6 show the distributions of terminal wealth when an investor includes the leveraged stock fund as part of their asset allocation. Unlike what was seen in Figs. 3 and 4, Figs. 5 and 6 demonstrate the increased volatility that the leveraged fund has on portfolio performance. The results here are less encouraging than what was previously shown, because there does not appear to be a commensurate increase in return for taking the additional risk. This is consistent with Trainor and Baryla's (2008) finding that over one year, a 2× leveraged fund produces 2× the standard deviation of returns, but only increase the return by a factor of 1.4.

Thus, the question remains whether the reduction (increase) in volatility was sufficient enough to justify the reduced (enhanced) returns. Tables 4 and 5 provide an answer this question more completely.

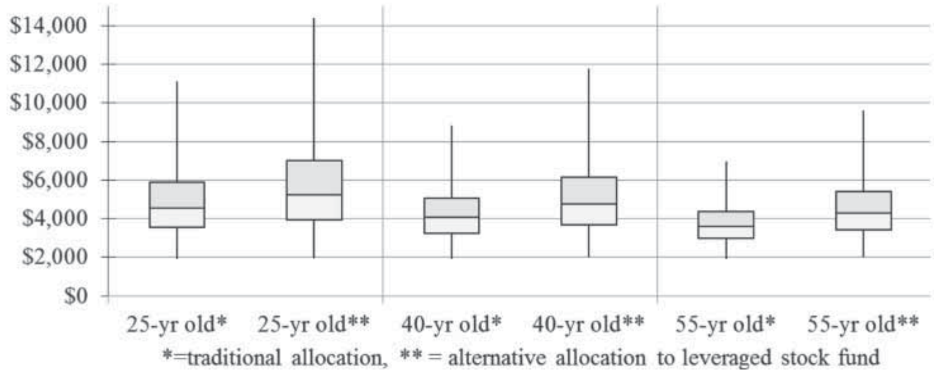

Fig. 6. Terminal wealth distributions for investors using 1991–2000 return distributions. Distribution assumes $1,000 initial wealth, rebalancing each year, and a 1% annual fee for the leveraged stock fund.

6. Return statistics for terminal wealth and Sharpe ratio

To compare the effect of the alternative allocations on terminal wealth and Sharpe ratio statistics found for each of the 100,000 trials, we determined a coefficient of variation (CV) for each. The CV is found from the standard deviation divided by the corresponding mean. We propose that a rational investor prefers any alterative allocation that reduces, but keeps positive, the CV for their terminal wealth (CV-TW) and Sharpe ratio (CV-S). Tables 4 and 5 list these statistics for the simulated periods 2001–2010 and 1991–2000, respectively.

Referring to the upper panel of Table 4 in the row labeled "Terminal Wealth Statistics," smaller positive CV-TW occur in the alternative allocation to the inverse equity fund than the traditional allocation. This reduction is statistically significant at less than a 0.001 level, based on comparing the reduced values (0.458, 0.371, and 0.289) to a 99.9% confidence interval for the CV-TW defined as

$$99.9\% \text{ confidence interval for CV-TW} = \frac{\sigma_{TW}}{\bar{x}_{TW} \pm Z_{99.9\%}\dfrac{\sigma_{TW}}{\sqrt{n}}}.$$

Here σ_{TW} and $\bar{x}_{TW}$ are the standard deviation and mean of terminal wealth, $Z_{99.9\%}$ is the Z-statistic corresponding to a 99.9% interval, and $n = 100{,}000$. This equation follows the form of a coefficient of variation, but the denominator has been changed from the point estimate to an interval estimate, thereby allowing for direct calculation the confidence interval. We propose that CV-TW that fall outside this region are statistically significant at less than a 0.001 level. For the results in Table 4, evaluation this equation yields intervals of [0.573, 0.580], [0.472, 0.476], and [0.378, 0.381] for the 25-year old, 40-year old, and 55-year old investors, respectively. Thus, we can make the claim that the CV-TW has been significantly reduced when the inverse fund is used.

This reduction suggests that although returns are reduced as shown in a reduction in means, risk as measured by the standard deviation, is reduced to a greater degree. Whereas the reduction in CV-TW is encouraging, it is also relevant to note that a similar, but less significant, reduction is possible if the alterative 10% allocation applied to the inverse fund was switched to the risk-free asset. For example, the 25-year old investor's CV-TW reduces from 0.576 to either 0.458 or 0.516 whether their 10% allocation were using inverse funds versus the risk-free assets. Risk free assets also do not reduce the CV-TW as significantly as the inverse funds for 40 and 55-year old investors, with the traditional allocation producing a 0.379 CV-TW for the 55-year old, versus 0.289 and 0.342 for the inverse fund and risk free asset used as the alternative investment. Therefore, the 55-year old investor's CV is reduced by 23.7% when inverse funds are used, versus a 9.8% reduction when risk-free assets are used.

Unfortunately, the benefit of inverse funds discussed above in the context of reducing the CV-TW is not seen in the Sharpe ratio statistics. As can be seen in the row labeled "Sharpe Ratio Statistics" in the upper panel of Table 4, the CV-S is always larger when the alternative allocation with inverse funds is used. Furthermore, and as expected from the Capital Market

Table 4 Statistics using 2001–2010 return distributions for simulation sampling

		Traditional allocation			Alternative allocation with inverse fund			Alternative allocation with risk-free asset		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Inverse fund (−1×)										
Terminal wealth statistics	Standard deviation	$ 770	$ 658	$ 547	$ 566	$ 475	$ 383	$ 680	$ 582	$ 484
	Mean	$1,336	$1,388	$1,442	$1,237	$1,281	$1,326	$1,319	$1,366	$1,415
	Coefficient of variation	0.576	0.474	0.379	0.458	0.371	0.289	0.516	0.426	0.342
Sharpe ratio statistics	Standard deviation	0.093	0.093	0.093	0.093	0.093	0.093	0.093	0.093	0.093
	Mean	0.017	0.028	0.043	0.003	0.012	0.026	0.016	0.027	0.042
	Coefficient of variation	5.352	3.358	2.179	32.536	7.823	3.600	5.749	3.513	2.246
Leveraged fund (2×)										
Terminal wealth statistics	Standard deviation	$ 771	$ 660	$ 548	$ 877	$ 774	$ 671			
	Mean	$1,332	$1,385	$1,439	$1,340	$1,387	$1,436			
	Coefficient of variation	0.579	0.476	0.381	0.655	0.558	0.467			
Sharpe ratio statistics	Standard deviation	0.093	0.093	0.093	0.093	0.093	0.093			
	Mean	0.017	0.027	0.042	0.016	0.024	0.035			
	Coefficient of variation	5.496	3.418	2.206	5.778	3.872	2.681			

Notes. This table shows central tendency and dispersion of terminal wealth and Sharpe ratios for each of the three investors identified. Results were obtained from 100,000 trials, annual rebalancing, and a 1% annual fee (charged monthly) for the simulated inverse or levered stock fund.

Table 5 Statistics using 1991–2000 return distributions for simulation sampling

		Traditional allocation			Alternative allocation with inverse fund			Alternative allocation with risk-free asset		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Inverse fund ($-1\times$)										
Terminal wealth statistics	Standard deviation	$1,932	$1,456	$1,071	$1,219	$ 915	$ 668	$1,565	$1,194	$ 889
	Mean	$4,898	$4,293	$3,756	$3,661	$3,240	$2,863	$4,399	$3,900	$3,453
	Coefficient of variation	0.394	0.339	0.285	0.333	0.283	0.233	0.356	0.306	0.257
Sharpe ratio statistics	Standard deviation	0.091	0.091	0.092	0.092	0.092	0.092	0.091	0.091	0.091
	Mean	0.279	0.286	0.295	0.251	0.254	0.256	0.279	0.286	0.295
	Coefficient of variation	0.328	0.320	0.311	0.367	0.363	0.361	0.327	0.319	0.310
Leveraged fund ($2\times$)										
	Standard deviation	$1,929	$1,454	$1,070	$2,588	$2,049	$1,598			
	Mean	$4,896	$4,290	$3,753	$5,765	$5,130	$4,558			
	Coefficient of variation	0.394	0.339	0.285	0.449	0.399	0.351			
Sharpe ratio statistics	Standard deviation	0.091	0.091	0.091	0.091	0.091	0.091			
	Mean	0.278	0.286	0.294	0.283	0.290	0.297			
	Coefficient of variation	0.328	0.320	0.311	0.322	0.314	0.306			

Notes. This table shows central tendency and dispersion of terminal wealth and Sharpe ratios for each of the three investors identified. Results were obtained from 100,000 trials, annual rebalancing, and a 1% annual fee (charged monthly) for the simulated inverse or levered stock fund.

Line, the use of the risk-free asset in the alternative allocation has no effect on the Sharpe ratio mean or standard deviation.[6]

As shown in the lower panel of Table 4, there is no diversification benefit from using a leveraged stock fund. Here, we reran the 100,000 trials in our simulation, which produced a small differences in the fourth significant digit, between the lower and upper panels for the "Traditional allocation." In all cases, the CV-TW and CV-S increase when the leveraged stock fund is included as the alternative asset. This result is not surprising, because this time period sampled (2001–2010) did not have rising equity prices.

For completeness, we also tested levels of statistical significance between the top and bottom panels in Table 4 using the confidence intervals estimated above. Recalling our 99% confidence intervals cited previously for the traditional allocation, we note that the CV-TW and CV-S in the bottom panel labeled "Traditional allocation" all fall within these intervals, which is expected since it represents simply another random draw of 100,000 trials in the simulation.

The results so far have suggested that the inverse stock fund may be a useful addition to a diversified portfolio of stocks and bonds, because the CV-TW was consistently reduced, to varying degrees. This benefit *always exceeded* the benefit an investor could realize if a similar allocation was made to a risk-free asset. However, one may argue the contrary if attention is solely given to the Sharpe ratio statistics.[7] Additionally, the use of the leveraged stock in the alternative allocation does not improve diversification in this "flat" equity case from 2001 to 2010.

Table 5 provides the alternative perspective on inverse and leveraged stock funds as an alternative investment in "rising" equity markets, such as those seen that in the period from 1991 to 2000. Intuitively, one may suspect that the use of an inverse stock fund in rising equity markets should be avoided. Indeed, the upper panel of Table 5 in the row labeled "Terminal Wealth Statistics" shows that the mean terminal wealth is significantly reduced, but so is the standard deviation. In fact, the CV-TW is still significantly reduced, but to less of a degree then what was seen in Table 4. These reduced values all fall below the 99.9% confidence intervals for CV-TW in Table 5, which are [0.393, 0.396], [0.338, 0.340], and [0.284, 0.286] for the 25-year old, 40-year old, and 55-year old investors, respectively. These findings suggest that the CV-TW can be improved when long-term equity prices are either rising or flat.

Unfortunately, the benefits of allocations with inverse stock funds still do not reduce the CV-S. Referring to the upper panel of Table 5 in the row labeled "Sharpe Ratio Statistics," we see the same relationships observed in Table 4. That is, the CV-S does not improve with the inclusion of the inverse fund as an alternative asset.

Lastly, and somewhat surprisingly over a period of rising equity prices, there is a no diversification benefit in the CV-TW from using a leveraged ETF, as shown in the lower panel of Table 5. In all cases, the CV-TW increases, suggesting that the leveraged stock fund provides no risk-return benefit. This finding may be counterintuitive, when considering the positive upward trend of equities in the 1991–2000 time period, but is partly reconciled when reviewing the lower panel of Table 5 labeled "Sharpe Ratio Statistics." Here, the CV-S shows a marginal risk-adjusted improvement from the use of the leveraged stock fund in a rising equity market, because these values all fall slightly below the 99.9% confidence intervals for the CV-S in Table 5 of [0.325, 0.327], [0.317, 0.319], and [0.311, 0.313]. Thus, we see that there is a benefit to an alternative allocation using leveraged equity funds, provided an investor relies on Sharpe ratio, not terminal wealth statistic, in rising equity markets.

Again for completeness, we also tested levels of statistical significance between the top and bottom panels in Table 5 using the confidence intervals estimated above. Recalling our 99% confidence intervals cited previously for the traditional allocation, we again note that the CV-TW and CV-S in the bottom panel labeled "Traditional allocation" all fall within these intervals. Recall that this is expected since it represents another random draw of 100,000 trials in the simulation.

7. Enhancing the study by utilizing other stock and bond funds

One criticism that could be raised by the results shown so far is that it only incorporated one specific stock index fund (VFINX) and one specific bond index fund (VBMFX). Thus, the results shown above may not be generally applicable to other funds utilized by individual investors and financial planners, even if these stock and bond funds are major players in their respective investment categories. Therefore, to strengthen the findings from the previous section, we identified additional stock and bond index funds to see if the results shown previously still hold. For stock funds, we identified four potential stock indices to compliment and extend the findings previously shown that were based on the S&P 500 index fund, which is a large capitalization stock fund. The four candidate stock funds represented mid and small capitalization stocks, along with the Nasdaq and Dow Jones industrials indices. Although many other stock index funds could be considered, such as international developed and emerging markets, we chose these because of their wide familiarity to individual investors and financial planners who we believe would treat them as likely candidates for their stock investment. We also identified two additional bond funds to extend the midterm maturity high quality corporate and U.S. government bond fund examined previously. These new bond funds provide short and long-term maturities. In terms of an expanded set of bond funds, although we recognized that there are potentially many other bond funds from which to select, we believe these carry risks that may prevent many bond investors from considering them as their primary bond investment. These alternatives included junk, international, and emerging market bonds. Lastly, we chose not to include municipal bonds because we are interested in investments in a tax deferred or tax exempt account, such as an IRA or 401k. A summary of the original stock and bond funds, which appears on the first row, along with the expanded set of funds to be analyzed, appear in Tables 6 and 7.

Including these additional stock and bond index funds represented a challenge, as our simulation approach required daily returns starting no later than January 1, 1991. For some of these funds, the inception dates noted in Tables 6 and 7 are many years after this date. Consequently, the updated simulation results include the small and midcap results in both 10-year time periods (1991–2000 and 2001–2010), but our Nasdaq, short term, and long term bond index mutual funds could only support simulated results for the 10-year time period of 2001–2010 because of their later inception date. We also did not to include the Dow

Table 6 Previously assumed stock fund and additional stock index funds utilized to enhance the study

Index	Stock fund selected	Symbol	Inception date
S&P 500	Vanguard 500 Index Mutual Fund	VFINX	August 31, 1976
Mid-caps	Vanguard Extended Market Index Fund	VEXMX	December 21, 1987
Small-caps	Vanguard Small Cap Index, Investor Class	NAESX	October 10, 1960
Nasdaq	USAA NASDAQ-100 Index	USNQX	November 9, 2000

Notes. The first row of this table lists the fund used in the previous sections. The later rows show the additional funds utilized in this section.

Industrials Index as no appropriate index fund could be found. We did consider the use of the Dow 30 ETF index fund (SPDR Dow Jones Industrial Average Index ETF, symbol DIA), which became available on January 13, 1998. However, its high degree of similarity measured by correlation from 2001 to 2010 daily returns against VFINX was 0.96, suggesting it would generate results that would be very similar to those already provided.

Our modeling approach to develop the inverse and leveraged ETF returns followed the approach used in Section 4. Tables 8 and 9 summarize the total replication expense, which in some cases, differed from our previous assumption of a 1% cost. Thus, the results in this section were found by using the total replication expense determined in the last column of these tables.

To begin understanding the potential of the expanded set of funds, descriptive statistics are once again found and appear in Table 10. The top panel of Table 10 shows the return statistics for the period of "flat" equity prices of 2001–2010, while the bottom is for "rising" equity prices of 1991–2000. Over the period of 2001–2010, longer term bonds provided a higher return but also a higher volatility. Midcaps and Small-cap funds also generated higher returns than the S&P 500 large-cap fund (reported in Table 3 at 3.51%) with a marginally higher volatility, whereas the Nasdaq fund returned nearly the same as the large-cap fund, but at nearly three times the volatility. The results over the period of 1991–2000, a period of rising equity prices, showed less difference between holding a large-cap stock versus holding either small or midcap stocks. Annualized returns between large, mid, and small cap stocks are all within 1% of each other from 1991 to 2000, and annualized volatilities are within 2% of each other.

When considering the effect of holding the inverse funds over the both 10-year time periods, the annualized returns were all negative and slightly larger in magnitude than their long-index counterparts. Similarly, the annualized returns of holding the leveraged funds were all slightly less than double. These results are consistent with the higher expenses

Table 7 Previously assumed bond fund and additional bond index funds utilized to enhance the study

Index	Bond index fund	Symbol	Inception date
Total bond market	Vanguard Total Bond Market Index	VBMFX	December 10, 1986
Short-term bond market	Vanguard Short-Term Bond Index	VBISX	February 28, 1994
Long-term bond market	Vanguard Long-Term Bond Index	VBLTX	February 28, 1994

Notes. The first row of this table lists the fund used in the previous sections. The later rows show the additional funds utilized in this section.

Table 8 Selected ProShares short (−1×) ETF products

ETF symbol	Fund name	Expense ratio	Estimated bid-ask spread	Commissions	Long index expense ratio	Total replication expense
SH	Short S&P 500	0.90%	0.10%	0.2%	−0.17%	1.03%
MYY	Short Mid-cap400	0.95%	0.37%	0.2%	−0.24%	1.28%
RWM	Short Russell 2000	0.95%	0.13%	0.2%	−0.24%	1.04%
PSQ	Short QQQ	0.95%	0.19%	0.2%	−0.64%	0.7%

associated with these funds, as well as the effect volatility can have on expected returns of these types of investments over longer periods (see Lu et al., 2012). Similarly, the leveraged funds over each of these two time periods were also consistent with the longer period expectation of providing slightly less than two-times the annualized return of their long-index counterpart, while doubling the volatility.

Another notable result from Table 10 is that positive annualized returns generally yielded positive total returns and negative annualized returns produced negative total returns. However, because the annualized returns were obtained from arithmetically averaged daily returns, there is a notable exception to this relationship when the average daily returns is very small, as in the case of Nasdaq that had a negative cumulative return because of the dot-com crash in the early 2000s. Lastly, although there was statistical significance between the S&P 500 returns between 1991 and 2000 and 2001–2010 at the 0.11 level, there was no statistical difference in the mid and small-cap returns (p-values = 0.41 and 0.53, respectively).

7.1. Results for midcaps, small caps, and Nasdaq index funds

Tables 11 and 12 provide results for terminal wealth and Sharpe ratio statistics obtained from a 100,000 trials using the midcap fund identified in Table 6, and simulating the inverse and leveraged version of it. During the period of "flat" equity prices from 2001 to 2010 shown in Table 11, the use of the inverse midcap fund, shown in the upper panel, is consistent with the findings in the previous section when using large cap funds were assumed. Specifically, the CV-TW is reduced below the 99.9% confidence intervals of [0.725, 0.736], [0.596, 0.603], and [0.476, 0.481] for the 25-year old, 40-year old, and 55-year old investors, when the inverse midcap fund is used, indicating statistical signifi-

Table 9 Selected ProShares leveraged (2×) ETF products

ETF Symbol	Fund name	Expense ratio	Estimated bid-ask spread	Commissions	Long index expense ratio	Total replication expense (annual)
SSO	Ultra S&P 500	0.90%	0.08%	0.2%	−0.17%	1.01%
MVV	Ultra MidCap400	0.95%	0.16%	0.2%	−0.24%	1.07%
UWM	Ultra Russell 2000	0.95%	0.15%	0.2%	−0.24%	1.06%
QLD	Ultra QQQ	0.95%	0.11%	0.2%	−0.64%	0.62%

Table 10 Return statistics from daily returns over historical 10-year periods

	Return (annualized)	Standard deviation (annualized)	Total return
Bonds (short term)	4.27%	2.88%	55.8%
Bonds (long term)	7.00%	9.88%	97.9%
Mid-cap	8.31%	23.5%	78.9%
Inverse mid-cap	−9.59%	23.5%	−72.6%
Leveraged mid-cap	15.6%	47.0%	59.4%
Small-cap	9.68%	24.6%	100.9%
Inverse small-cap	−10.7%	24.6%	−76.3%
Leveraged small-cap	18.30%	49.1%	90.7%
Nasdaq	3.55%	30.2%	−9.76%
Inverse Nasdaq	−4.25%	30.2%	−60.4%
Leveraged Nasdaq	6.48%	60.3%	−70.4%
"Flat" - January 2, 2001 to December 31, 2010, N = 2515 samples			
Mid-cap	15.6%	16.1%	349.4%
Inverse mid-cap	−16.9%	16.1%	−85.2%
Leveraged mid-cap	30.1%	32.3%	1267.6%
Small-cap	15.3%	14.6%	346.7%
Inverse small-cap	−16.3%	14.6%	−84.0%
Leveraged small-cap	29.5%	29.2%	1322.0%
"Rising" - January 2, 1991 to December 29, 2000, N = 2527 samples			

Notes. This table shows the return statistics from the two 10-year periods observed from adjusted closing prices of stock and bond market proxies. Inverse and leveraged stock return statistics are generated assuming perfect replication of negative and double daily return from the stock return, respectively, along with incurring an average daily expense as shown in Tables 8 and 9.

cance at less than 0.001 level. The use of the risk free asset also produces a CV-TW that is lower than this interval, but is less statistically significant. Also like shown previously, CV-S does not corroborate this finding, as the use of leveraged midcap fund in this period of "flat" equity prices does not reduce the CV-S relative to the traditional allocation case.

Table 12 corresponds to a period of "rising" equity prices from 1991 to 2000, and results are similar to the previous results when the large cap fund. Again, CV-TW is significantly reduced when the inverse fund is used instead of the traditional allocation, since each investor's CV-TW is outside the 99.9% confidence interval obtained for the traditional allocation of [0.565, 0.571], [0.476, 0.481], and [0.390, 0.393]. A similar, but less statistically significant reduction occurs when the risk-free asset is used in the alternative allocation. The CV-S is not reduced when the inverse fund is used, which is consistent with previous results. As seen previously, this period of rising equity prices does not show any benefit of the use of the leveraged fund based on the CV-TW. Again, the use of the leveraged fund provides an improvement to the CV-S over the traditional allocation, based on the values of 0.474, 0.459, and 0.441 all lower than the lower bound on the 99.9% confidence interval for CV-S of [0.487, 0.492], [0.469, 0.473], and [0.442, 0.447].

Tables 13 and 14 provide results for terminal wealth and Sharpe ratio statistics obtained from a 100,000 trials using a small-cap fund. The period of "flat" equity prices from 2001 to 2010 shown in Table 13 indicates the previous results hold, where the use of the inverse fund reduces the CV-TW below the 99.9% confidence interval for the traditional allocation of [0.747, 0.759], [0.617, 0.625], and [0.494, 0.499] for the 25-year old, 40-year old, and

Table 11 Statistics using 2001–2010 return distributions for simulation sampling, mid-cap index fund (VEXMX)

		Traditional allocation			Alternative allocation with inverse (−1×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$1,597	$1,260	$ 967	$1,086	$ 850	$ 639
	Mean	$2,187	$2,102	$2,021	$1,857	$1,791	$1,727
	CV-TW	0.730	0.600	0.478	0.585	0.474	0.370
Sharpe ratio statistics	Standard deviation	0.095	0.095	0.096	0.095	0.096	0.096
	Mean	0.084	0.092	0.104	0.073	0.080	0.091
	CV-S	1.129	1.032	0.918	1.310	1.195	1.054
		Alternative allocation with risk-free asset			Alternative allocation with leveraged (2×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$1,349	$1,073	$ 829	$1,949	$1,602	$1,296
	Mean	$2,067	$1,994	$1,924	$2,332	$2,252	$2,174
	CV-TW	0.653	0.538	0.431	0.836	0.712	0.596
Sharpe ratio statistics	Standard deviation	0.096	0.096	0.096	0.095	0.095	0.095
	Mean	0.084	0.092	0.104	0.084	0.090	0.099
	CV-S	1.140	1.041	0.925	1.131	1.053	0.964

Notes. This table shows central tendency and dispersion of terminal wealth and Sharpe ratios for each of the three investors identified. Results were obtained from 100,000 trials, annual rebalancing, and an annual fee as shown in the last column of Tables 8 and 9 for the simulated inverse or leveraged stock fund.

55-year old, respectively. A similar, but less significant reduction occurs if the risk-free asset is used. Furthermore, these results are still not corroborated by looking at the CV-S value. There is also still no evidence that the leveraged fund provides any benefit during “flat” equity prices, as both the CV-TW and CV-S show increases.

In the period of “rising” equity prices from 1991 to 2000 shown in Table 14, the results for the terminal wealth statistics are similar to before, with the inverse of the small cap fund significantly reducing the CV-TW relative to the traditional allocation’s 99.9% confidence intervals of [0.555, 0.561], [0.467, 0.472], and [0.382, 0.385]. This is also true, but less significant, when the CV-TW under the risk-free asset is evaluated. And again, like the large and mid caps, the CV-S does not show any improvement when the inverse small cap fund is used. The use of the leveraged small cap index fund produces results similar to before, where a significant improvement in CV-S occurs against the traditional 99.9% confidence intervals of [0.482, 0.487], [0.463, 0.467], and [0.439, 0.443].

Table 15 provides the last extension to alternative equity funds and the use of inverse and leveraged funds from them in an alternative allocation. Here, which covers the period of “flat” equity prices from 2001 to 2010, all the findings shown previously continue to hold. The CV-TW, with the use of the inverse Nasdaq index fund, is still reduced when compared to the traditional allocation, based on 99.9% confidence intervals of [0.928, 0.946], [0.745, 0.757], and [0.583, 0.590], and the reduction is more significant than if the risk-free asset

Table 12 Statistics using 1991–2000 return distributions for simulation sampling, mid-cap index fund (VEXMX)

		Traditional allocation			Alternative allocation with inverse (−1×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$2,705	$2,004	$1,441	$1,705	$1,259	$ 896
	Mean	$4,762	$4,191	$3,683	$3,597	$3,196	$2,836
	CV-TW	0.568	0.478	0.391	0.474	0.394	0.316
Sharpe ratio statistics	Standard deviation	0.097	0.097	0.096	0.097	0.097	0.096
	Mean	0.198	0.206	0.216	0.179	0.183	0.190
	CV-S	0.488	0.470	0.447	0.541	0.527	0.508
		Alternative allocation with risk-free asset			Alternative allocation with leveraged (2×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$2,181	$1,641	$1,198	$3,672	$2,868	$2,203
	Mean	$4,284	$3,814	$3,391	$5,654	$5,051	$4,506
	CV-TW	0.509	0.430	0.353	0.649	0.568	0.489
Sharpe ratio statistics	Standard deviation	0.097	0.097	0.097	0.096	0.096	0.096
	Mean	0.198	0.205	0.216	0.203	0.210	0.218
	CV-S	0.490	0.472	0.450	0.474	0.459	0.441

Notes. This table shows central tendency and dispersion of terminal wealth and Sharpe ratios for each of the three investors identified. Results were obtained from 100,000 trials, annual rebalancing, and an annual fee as shown in the last column of Tables 8 and 9 for the simulated inverse or leveraged stock fund.

were used. Unfortunately, the Sharpe ratio statistics do not support a reduction in CV-S when the inverse fund is used. Further, the use of the leveraged Nasdaq index fund provides no benefit in terms of CV-TW or CV-S, which is similar to the results shown for other equity funds during the period of "flat" equity prices from 2001 to 2010.

7.2. Results for short-term and long term bond funds

To complete our evaluation of alternative funds, we considered two variations of bond holdings beyond the VTBMI fund. These two funds represented shorter and longer maturity bond indices, and we evaluated each as its own case when the VTSMI fund was used along with an alternative allocation to either inverse of the stock fund, 2× leverage of the stock fund, or a risk-free asset alternative. The results for this variation in bond funds appear in Tables 16 and 17, and confirm what was seen previously.

Once again, use of the inverse stock fund provides a diversification benefit by reducing the CV-TW beyond the 99.9% confidence intervals of [0.578, 0.585], [0.474, 0.479], and [0.377, 0.380] for the short-term bond fund index in Table 16, with a similar, but less significant reduction in the CV-TW when the risk-free asset is used. Again, there appears to be no benefit in terms of CV for the Sharpe ratios. Lastly, the use of the leveraged fund does not provide any benefit in terms of either terminal wealth or Sharpe ratio, which is again consistent with the 2001–2010 period with "flat" equity prices already reported.

Table 13. Statistics using 2001–2010 return distributions for simulation sampling, small-cap index fund (NAESX)

		Traditional allocation			Alternative allocation with inverse (−1×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$1,912	$1,480	$1,112	$1,277	$ 981	$ 725
	Mean	$2,538	$2,384	$2,239	$2,100	$1,984	$1,872
	CV-TW	0.753	0.621	0.497	0.608	0.495	0.387
Sharpe ratio statistics	Standard deviation	0.096	0.096	0.096	0.096	0.096	0.096
	Mean	0.102	0.110	0.121	0.091	0.098	0.108
	CV-S	0.939	0.874	0.795	1.056	0.984	0.892
		Alternative allocation with risk-free asset			Alternative allocation with leveraged (2×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$1,594	$1,245	$ 944	$2,374	$1,918	$1,524
	Mean	$2,370	$2,239	$2,115	$2,738	$2,592	$2,452
	CV-TW	0.673	0.556	0.446	0.867	0.740	0.621
Sharpe ratio statistics	Standard deviation	0.096	0.096	0.097	0.096	0.096	0.096
	Mean	0.102	0.110	0.121	0.101	0.107	0.115
	CV-S	0.941	0.876	0.797	0.945	0.893	0.832

Notes. This table shows central tendency and dispersion of terminal wealth and Sharpe ratios for each of the three investors identified. Results were obtained from 100,000 trials, annual rebalancing, and an annual fee as shown in the last column of Tables 8 and 9 for the simulated inverse or leveraged stock fund.

Similar findings appear in Table 17, where the 99.9% confidence intervals for CV-TW are [0.578, 0.585], [0.479, 0.484], and [0.395, 0.398] for the 25-year old, 40-year old, and 55-year old investors, respectively. There is a significant decrease in the CV-TW when the inverse fund is used, and to a lesser degree when the risk-free asset is included. No benefits could be found in terms of the CV from the Sharpe ratio statistics.

8. Conclusions

This article investigated the risks and possible opportunities of a "120 – age" annual reallocation strategy of stocks and bonds, but also included a small allocation of inverse or leveraged stock fund. The assessment was based on three different risk aversion levels, and simulated inverse and leveraged equity fund returns that included expenses and fees. We also included several variations of alternative stock and bond funds that might be selected by individual investors and financial planners in their asset allocation decision. Terminal wealth and Sharpe ratio statistics were obtained by Monte Carlo simulation that selected from return histories using two unique historical periods, representing both "flat" and "rising" cases of long-term equity returns, rather than assuming prices followed standard geometric Brownian motion.

Table 14 Statistics using 1991–2000 return distributions for simulation sampling, small-cap index fund (NAESX)

		Traditional allocation			Alternative allocation with inverse (−1×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standarddeviation	$2,556	$1,902	$1,373	$1,626	$1,205	$ 860
	Mean	$4,579	$4,053	$3,582	$3,494	$3,121	$2,785
	CV-TW	0.558	0.469	0.383	0.465	0.386	0.309
Sharpe ratio statistics	Standard deviation	0.093	0.093	0.093	0.094	0.094	0.094
	Mean	0.192	0.200	0.211	0.174	0.179	0.186
	CV-S	0.486	0.466	0.441	0.541	0.524	0.502
		Alternative allocation with risk-free asset			Alternative allocation with leveraged (2×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$2,076	$1,565	$1,145	$3,495	$2,741	$2,114
	Mean	$4,125	$3,692	$3,300	$5,424	$4,870	$4,367
	CV-TW	0.503	0.424	0.347	0.644	0.563	0.484
Sharpe ratio statistics	Standard deviation	0.094	0.094	0.093	0.094	0.094	0.093
	Mean	0.192	0.200	0.211	0.198	0.204	0.213
	CV-S	0.489	0.469	0.444	0.475	0.458	0.438

Notes. This table shows central tendency and dispersion of terminal wealth and Sharpe ratios for each of the three investors identified. Results were obtained from 100,000 trials, annual rebalancing, and an annual fee as shown in the last column of Tables 8 and 9 for the simulated inverse or leveraged stock fund.

The findings, developed under various stock and bond alternatives, showed that under both “flat” and “rising” return histories, annual rebalancing to include a 10% allocation towards an inverse stock fund provides a diversification benefit by reducing the CV of terminal wealth. The diversification benefit is strengthened when stock returns are “flat.” Further, it always exceeded the benefit of simply using a risk-free asset. Unfortunately, the benefit observed by measuring the CV of terminal wealth was not corroborated with Sharpe ratio statistics. Additionally, leveraged ETFs were never found to provide a risk-reward benefit preferred by rational investors interested in terminal wealth, but Sharpe ratio statistics did show a benefit in rising equity markets. These results, and the potential diversification benefits of inverse and leveraged ETFs, call into question the current recommendation that they are always detrimental to long-term investors, and only beneficial for short-term trading. Although these findings suggest that additional analytical and empirical studies may be warranted to better assess their impact on asset allocation decisions made by individual investors and financial planners, both must consider the addition of inverse or leverage stock funds to improve wealth maximization under appropriate risk-reward tradeoff. The source of this somewhat unexpected behavior likely lies in a return history that is not normally distributed in the observed distribution’s tails, and thus may influence long-range plans that are important to individual investors and financial planners.

Future work in this area can cover several areas. First, a more comprehensive set of cases

Table 15 Statistics using 2001–2010 return distributions for simulation sampling, Nasdaq index fund (USNQX)

		Traditional allocation			Alternative allocation with inverse (−1×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$1,280	$1,063	$ 859	$ 923	$ 758	$ 598
	Mean	$1,366	$1,415	$1,465	$1,269	$1,310	$1,351
	CV-TW	0.937	0.752	0.587	0.728	0.579	0.443
Sharpe ratio statistics	Standard deviation	0.092	0.092	0.092	0.093	0.093	0.092
	Mean	0.014	0.021	0.031	0.004	0.010	0.019
	CV-S	6.572	4.451	3.004	22.837	9.266	4.783
		Alternative allocation with risk-free asset			Alternative allocation with leveraged (2×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$1,135	$ 945	$ 764	$1,536	$1,310	$1,101
	Mean	$1,363	$1,404	$1,446	$1,387	$1,429	$1,473
	CV-TW	0.833	0.673	0.528	1.107	0.916	0.748
Sharpe ratio statistics	Standard deviation	0.093	0.093	0.092	0.092	0.092	0.092
	Mean	0.014	0.021	0.031	0.014	0.019	0.026
	CV-S	6.558	4.442	3.000	6.827	4.960	3.604

Notes. This table shows central tendency and dispersion of terminal wealth and Sharpe ratios for each of the three investors identified. Results were obtained from 100,000 trials, annual rebalancing, and an annual fee as shown in the last column of Tables 8 and 9 for the simulated inverse or leveraged stock fund.

could be considered. For example, evaluating the effects of "3×" leveraged and "−2×/−3×" inverse ETFs is suggested to see if the observations found using "−1×" inverse and "2×" leveraged ETFs considered here persist. Like Giese (2010), we also observe that the optimal leverage strongly depends on the prevailing market conditions, which were not exhaustively considered here. Further, alternative simulation approaches could be used, such as simulating the price path of stocks and bonds with variations in their stochastic parameters. Lastly, other measures of diversification could be examined, such as the diversification effect as studied by Hight (2010).

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Table 16 Statistics using 2001–2010 return distributions for simulation sampling, short-term bond index fund (VBISX)

		Traditional allocation			Alternative allocation with inverse (−1×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$ 770	$ 647	$ 527	$ 565	$ 467	$ 369
	Mean	$1,324	$1,358	$1,392	$1,227	$1,256	$1,284
	CV-TW	0.581	0.477	0.379	0.461	0.372	0.287
Sharpe ratio statistics	Standard deviation	0.093	0.093	0.093	0.093	0.093	0.093
	Mean	0.016	0.023	0.034	0.001	0.007	0.015
	CV-S	5.852	4.013	2.737	76.871	13.945	6.084
		Alternative allocation with risk-free asset			Alternative allocation with leveraged (2×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$ 676	$ 570	$ 465	$ 873	$ 760	$ 649
	Mean	$1,314	$1,344	$1,375	$1,329	$1,360	$1,391
	CV-TW	0.515	0.424	0.338	0.657	0.559	0.466
Sharpe ratio statistics	Standard deviation	0.093	0.093	0.093	0.093	0.093	0.093
	Mean	0.016	0.023	0.034	0.015	0.020	0.028
	CV-S	5.951	4.050	2.750	6.296	4.573	3.329

Notes. This table shows central tendency and dispersion of terminal wealth and Sharpe ratios for each of the three investors identified. Results were obtained from 100,000 trials, annual rebalancing, and an annual fee as shown in the last column of Tables 8 and 9 for the simulated inverse or leveraged stock fund.

Notes

1 Fidelity noted that because of rebalancing and other risks, leverage and inverse leveraged ETFs are intended as short term trading vehicles for sophisticated investors actively monitoring their portfolios on a daily basis. *Source:* http://personal.fidelity.com/research/etf/content/leveraged_etn_etf.shtml.

2 The U.S. Securities and Exchange Commission (2011), while not stating the Rule of 120, implied it indirectly in their suggestion to for investors to consider Target Date Funds (TDF) in 'Beginners' guide to asset allocation, diversification, and rebalancing." *Source:* http://www.sec.gov/investor/pubs/assetallocation.htm.

3 For example, see http://www.proshares.com/funds/sh.html.

4 The correlation between stocks and bonds were as follows: (1991–2000, 2001-2010) = 0.047 and −0.255.

5 The authors wish to thank Ken French from making daily risk-free rates available at his Web site. *Source:* http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

6 The small variation seen in the Sharpe ratio means at the third decimal place is because of limiting the number of trials to 100,000.

7 An interesting but problematic issue occurs when simulating time periods that have "falling" equity prices, such as observed over 1999 to 2008. This case was not reported

Table 17 Statistics using 2001–2010 return distributions for simulation sampling, long-term bond index fund (VBLTX)

		Traditional allocation			Alternative allocation with inverse (−1×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$ 781	$ 690	$ 605	$ 573	$ 497	$ 426
	Mean	$1,343	$1,432	$1,526	$1,243	$1,317	$1,395
	CV-TW	0.581	0.482	0.397	0.461	0.377	0.306
Sharpe ratio statistics	Standard deviation	0.093	0.093	0.094	0.093	0.093	0.094
	Mean	0.018	0.034	0.055	0.004	0.019	0.042
	CV-S	5.096	2.745	1.694	23.859	4.827	2.263
		Alternative allocation with risk-free asset			Alternative allocation with leveraged (2×) fund		
		25-year old	40-year old	55-year old	25-year old	40-year old	55-year old
Terminal wealth statistics	Standard deviation	$ 686	$ 606	$ 530	$ 882	$ 801	$ 722
	Mean	$1,331	$1,410	$1,493	$1,349	$1,429	$1,514
	CV-TW	0.515	0.430	0.355	0.654	0.560	0.477
Sharpe ratio statistics	Standard deviation	0.093	0.093	0.094	0.093	0.093	0.093
	Mean	0.018	0.034	0.056	0.017	0.029	0.045
	CV-S	5.084	2.742	1.693	5.420	3.162	2.060

Notes. This table shows central tendency and dispersion of terminal wealth and Sharpe ratios for each of the three investors identified. Results were obtained from 100,000 trials, annual rebalancing, and an annual fee as shown in the last column of Tables 8 and 9 for the simulated inverse or leveraged stock fund.

because it violated our desire to use temporally uncorrelated historical periods. Nevertheless, when the simulation drew from this time period, a negative mean Sharpe ratio was found, making interpreting a CV of Sharpe ratio statistics problematic because both a small standard deviation or large negative mean of the Sharpe ratio could lead to a large negative CV of the Sharpe ratio. Thus, the negative CV value for the Sharpe ratio statistics for the "falling" equity price case are not as easily reconcilable to preference for rational investors.

8 This fund was acquired by Vanguard. Its actual inception date is in 1960, but began operating as a passive low-cost index fund in 1989. *Source:* http://socialize.morningstar.com/NewSocialize/forums/p/94965/94965.aspx#94965.

Appendix

Step	Simulation process
1	Assign a numerical index of 1 to n for each of the daily returns observed in the historical 10-year period.
2	For each day, determine the cumulative return up to and including the previous 20 days, so that the accumulated returns from these days are representative of the distribution of monthly returns.
3	Generate 120 random numbers ranging from 1 to n.
4	Select returns from the distribution of representative monthly returns determined in Step 2 using the random numbers found in Step 3, and generate 10 years of monthly returns. Use the same set of random numbers for each trial to select monthly returns from each of investment category, ensuring that the historical correlation among assets is preserved.[4]
5	Repeat for 100,000 trials, collecting terminal wealth and Sharpe ratio for each trial.
6	Determine mean, standard deviation, and coefficient of variation in terminal wealth and Sharpe ratios found from each of the 100,000 trials generated in Step 5.

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