FINANCIAL
SERVICES
REVIEW

Financial Services Review 30 (2022) 223–249

Seeking tax alpha in retirement income

James A. DiLellio[a,*], Andreas Simon[b]

[a]*Department of Decision Sciences, Information Systems and Strategy, Graziadio Business School, Pepperdine University, 24255 Pacific Coast Highway, Malibu, CA 90263, USA*
[b]*Leventhal School of Accounting, Marshall School of Business, The University of Southern California, 3660 Trousdale Parkway, Los Angeles, CA 90089-0441, USA*

Abstract

We provide a framework to find an optimal decision for tax-efficient retirement income. By developing a model for income and capital gains tax with stock and bond investments in tax-deferred, tax-exempt, and taxable accounts, we identify three categories of retirees based on their income needs and net worth. We propose and evaluate a simple heuristic to determine the optimal retirement income strategy, quantifying a 0.5% annual return benefit. We call this benefit tax alpha and show its robustness to varying model input parameters. We also suggest approaches for large institutions or FinTech firms to improve their existing financial planning tools.

JEL classifications: G11; H21

Keywords: Retirement income; Longevity risk; Tax efficiency; Asset location

1. Introduction

As the baby boomer generation exits the workforce, their needs for tax-efficient use of their retirement assets grow. According to the St. Louis Fed, approximately 10,000 U.S. baby boomers expect to retire every day from 2019 to 2039.[1] Similar trends are seen in other Western countries, like the United Kingdom, Germany, Australia, and others due to the mid-20th century baby boom.[2] Many of these retirees will have built up substantial assets in tax-deferred accounts, for example, the U.S. 401(k) plans, since their introduction in 1981.[3] Accounts like traditional or rollover U.S. IRAs, 403(b)s, and 457 plans offer similar tax-

*Corresponding author. Tel.: +1-714-403-0085; fax: +1-949-223-2575.
E-mail address: james.dilellio@pepperdine.edu

1057-0810/22/$ – see front matter

deferred investing for non-profit and government employees, and in some cases, have been around for even longer.[4] Consequently, many retirees are now beginning to draw down these assets, either voluntarily or involuntarily, due to required minimum distributions (RMDs).[5]

In addition to tax-deferred accounts, many baby boomers also have tax-exempt accounts, such as the U.S. Roth IRAs, Roth 401(k)s, and Roth 403(b)s. Tax-exempt accounts in the United States have only been around for less time compared with tax-deferred accounts and typically have fewer assets. In the United States, the Roth IRAs also restrict direct contributions for higher wage earners, further limiting the assets in these accounts. Brown et al. (2017) reported tax-exempt retirement income as very beneficial for all wage earners entering retirement as these assets can be used to support higher levels of retirement income in a progressive tax system, like in the United States, Germany, United Kingdom, Canada, Australia, among others.

Moreover, many retirees may have direct ownership in other assets residing in accounts that are taxable each year, such as stocks, bonds, mutual funds, Exchange Traded Funds (ETFs), CDs, and savings accounts. With few exceptions, retirees are also entitled to government benefits, like social security in the United States, or the Superannuation Guarantee in Australia (see Gerrans et al., 2009). Lastly, some retirees may have access to pension plans.

While tax law does vary from country to country, most of the basic concepts of investor taxation (e.g., capital gains, taxes on dividends, and tax-deferred accounts) exist globally in developed countries. Therefore, the problem facing our current and future generations of retirees throughout the world is how they may draw down their assets in the most tax-efficient manner. Improving tax efficiency will extend portfolio longevity and increase the retiree's bequest. The wide variation in retirement assets and retirees living longer, as discussed in Poterba (2014), further complicates this problem. Indeed, Nobel Laureate Bill Sharpe described tax-efficient retirement drawdowns as "one of the most difficult financial problems" he has ever attempted (see Milesvsky, 2020). The paper rises to this challenge, showing how 0.5% of added investment return may be by following optimal decisions in retirement drawdowns.

Addressing tax-efficient retirement drawdowns falls into the category of prescriptive analytics and optimization. Unfortunately, the optimization model is non-linear, constrained, and stochastic. non-linearities arise from a progressive tax system, and constraints occur from income needs, limited (if any) loan options to fund retirement, and RMDs (when applicable). Uncertainty appears in future tax law, investment returns, and inflation in such a model. In this article, we provide a comprehensive mathematical representation of this stochastic optimization problem and provide a simple heuristic to inform future retirement planning. It includes the use of the Sequential Least SQuares Programming (SLSQP) approach, and a heuristic to identify when it can (and cannot) be applied. We also conduct a detailed sensitivity analysis to show how the optimal solution is (or is not) impacted by uncertainty.

2. How this work contributes to the current body of knowledge

The literature in retirement planning generally falls into two categories of analysis using either objective wealth functions or utility functions. We discuss each category below, and how this work contributes and extends previous research in this area.

2.1. Objective wealth functions and tax law models

The first group of research investigates optimization and constraints imposed by federal tax laws. Al Zaman (2008) was the first to introduce a retiree's objective that includes tax rates of their heirs in account selection to fund retirement income. We contribute to this work by including it as part of the optimal decision-making process. We show that the optimal decision can be significantly affected by this parameter. Similarly, recent work by Pfau et al. (2017) analyzed three unique scenarios, including home equity, social security, and other retirement assets in identifying potential enhancements.

Cook et al. (2015) show the significant benefit of Roth conversions in extending portfolio longevity using specific use cases. In the results shown here, we demonstrate a similar increase in longevity by modifying the Common Rule withdrawal strategy to include tax-deferred account withdrawals dependent on the heir's tax rate. The Common Rule withdrawal strategy is often used as the baseline approach used by Coopersmith and Sumutka (2011), Sumutka et al. (2012), and others.

The Common Rule withdrawal strategy is also widely discussed in the popular press by Solin (2010), Rodgers (2009), and Lange (2009), endorsed by large retail investment firms Fidelity (see Fidelity, 2014) and Vanguard (see Vanguard, 2016). The Common Rule withdrawal strategy first applies RMDs to tax-deferred accounts, when applicable, each year. Any unmet income is then met by taxable account funds until exhausted. Next, voluntary tax-deferred account withdrawals are made until these accounts are exhausted. Finally, tax-exempt accounts are used to satisfy retirement income. When tax-exempt funds are exhausted, the retiree has insufficient assets to support their desired retirement income.

However, practitioners like Piper (2013) and Demuth (2016) provide many insights into when and how such an approach is far from optimal. We contribute to this work by showing that the Common Rule strategy provides an important heuristic in seeking a global optimal retirement income decision.

Geisler and Hulse (2018) extended work on the Common Rule by expanding it to include social security benefits and RMDs. They confirmed the persistence of outperformance against the Common Rule withdrawal strategy. The effect of social security on tax-efficient retirement income has also been investigated by Meyer and Reichenstein (2013) and Reichenstein & Meyer (2018), who identify that, for certain lower-income retirees, a spike in marginal tax rates occurs that they call the "tax torpedo." In our work here, we avoid this issue by assuming a retiree's Modified Adjusted Gross Income (MAGI) is high enough that 85% of social security income is taxable as ordinary income, and the remaining 15% is not.[6]

Coopersmith and Sumutka (2011) highlight the benefits of general-purpose optimization routines, as well as the mathematical and implementation challenges it faces due to non-linear constraints imposed by progressive taxes. Their work builds on the pioneering work of linear programming for retirement income planning by Ragsdale et al. (1994). Most recently, Welch (2016, 2017) showed how linear programming models can measure tax-exempt conversion effectiveness and be adapted to upward or downward trending markets. We contribute to this work by using the Sequential Least Squares programming (SLSQP) routine, which is freely available in the SciPy package of the Python 3 open-source programming language.[7]

Coopersmith and Sumutka (2017) evaluated the effectiveness of tax-exempt conversions using a linear programming model when different rates of returns are used. Their article is the first to highlight how different investment returns in tax-deferred and tax-exempt accounts influence the benefit of tax-exempt conversions. DiLellio and Ostrov (2017) show that, under progressive income tax rates, there is a simple geometric representation of account consumption that maximizes wealth transferred to an heir. They also note the existence of multiple optimal solutions for income falling within a given tax bracket, which can confound general-purpose optimization algorithms. This article uses this important insight to develop a "Modified" Common Rule that, with insight into the heir's marginal tax rate, can extend portfolio longevity or increase a bequest.

In DiLellio and Ostrov (2020), the authors extend their previous study by including a taxable account with stock reinvested. They develop a complex algorithm that finds the optimal switching time between tax-exempt and taxable accounts. We extended this work by showing that, under the regime of sufficient but not excessive assets, switching times can also be found using a general-purpose optimization routine, avoiding the complexity of a customized algorithm.

Horan (2006) wrote one of the pioneering articles that quantified, under a progressive tax system, tax-efficient withdrawal strategies. He showed that tax-deferred accounts could take advantage of deductions and lower tax brackets to significantly increase wealth accumulation over the Common Rule withdrawal strategy. We build upon this seminal work by generalizing tax efficiency across three categories of a retiree's net worth, retirement income needs, and retirement horizon. These categories are retirees with insufficient assets, sufficient assets, or excessive assets relative to their income needs and retirement horizon.

2.2. Economic utility functions

Another group of retirement income research uses utility functions. In Brown et al. (2017), the authors find that tax-exempt accounts can be important for high net worth retirees subject to uncertain future investment returns and tax rates. Dammon et al. (2004) conclude that in the presence of liquidity shocks, there is a strong preference for holding taxable bonds in a tax-deferred account and stocks in a taxable account. Fischer and Gallmeyer (2017) reinforce these findings in the context of mutual fund tax efficiency. Garlappi and Huang (2006) predict that the preference to hold bonds in a tax-deferred account can change when tax benefits are more uncertain. Most recently, Kobor and Muralidhar (2020) developed and tested a goal-based, lifetime-income approach that has a better chance of success than a traditional glide-path approach using target-date funds.

Many authors also used life-cycle models that focus on both the accumulation and decumulation phases. Lachance (2013) solves a life-cycle model for both the accumulation and decumulation phases using tax-deferred and tax-exempt accounts. They find that choosing a tax-deferred over a tax-exempt account creates a reduction in wealth, most noticeable for those with higher incomes and pensions, which increases social security taxation. Marekwica et al. (2013) also examine a life-cycle model, including renting versus owning a home. They find that a tax arbitrage exists from mortgage interest payments and investing in tax-deferred accounts. Zhou (2009) developed a life-cycle model and finds that asset location decisions are very sensitive to a progressive tax system. The life-cycle model by Zhou (2012) identified the optimal stock market participation between taxable and tax-deferred accounts, finding a preference for higher tax-deferred account participation earlier in life, and higher participation in taxable accounts later.

3. Model

In this section, we model the various accounts available to retirees, as well as other sources of income. We also describe our tax model and its basis in current tax law in the United States and other developed countries of the world. For details on U.S. rates and income limits, please see details in the Appendix.

There are two methods for defining this optimization problem. The first approach uses power-law utility functions, which may include a coefficient of relative risk aversion, such as those used in Brown et al. (2017), Sialm (2006), and others. Then, future consumption is discounted at some discount factor. These models often include variables for pre-tax income in retirement, taxable accounts, tax-deferred, and tax-exempt retirement accounts. Asset allocation in these models usually consists of a stock portfolio and a riskless bond. Progressive tax brackets may be simplified and future tax rates and investment returns found using a bootstrapping method or a stochastic process model. These models provide strong evidence for optimal decision-making for large portions of the population and help inform policy. However, the use of utility functions prevents their direct application by FinTech companies interested in building software tools needed by the unique circumstances that current and future retirees face.

Cook et al. (2015), Al Zaman (2008), DiLellio and Ostrov (2017, 2020), Coopersmith and Sumutka (2011), Ragsdale et al. (1994), among others, used an alternative optimization approach. These authors impose an objective function to maximize portfolio longevity and/or the size of the bequest. To varying degrees, these authors include additional aspects of their models to better reflect tax law. For instance, many include RMDs, all current tax brackets, qualified versus non-qualified dividends, and the special treatment of capital gains taxes (see Sumutka et al., 2012). We emphasize this latter approach here, as many retirees look to the financial services industry for retirement income planning software. In turn, many in the financial services industry look to academia to advance research that can support the development of algorithms necessary to improve their ongoing practice supporting their clients' needs.

3.1. Decision variables and objective function

Our model definition begins with our *decision variables*. For each year over a fixed retirement horizon of N years, a retiree has to decide how much of their retirement income comes from tax-deferred accounts, tax-exempt accounts, and taxable accounts. We define these decision variables $M_{taxable,t}$, respectively, as $1 \times N$ vectors representing *after-tax income* *sources* for each year $t = 1$ to N.[8] In the following section, we provide details on assumptions on how these after-tax income sources are determined.

Next, we state the optimization problem to maximize our *objective wealth function* for retirees.

$$\text{MAX}\left\{ \gamma\left[(1-\tau_{heir})W_{tax\text{-}deferred,N} + W_{tax\text{-}exempt,N}\right] + W_{taxable,N} \right\} \tag{1}$$

Eq. (1) represents the total wealth transfer to a retiree's non-spouse heirs. Here, τ_{heir} is the marginal tax rate of the retiree's heir, so represents an estimate of the government's share in this account, as described in Reichenstein et al. (2012). The variables $W_{tax\text{-}deferred,N}$ and $W_{tax\text{-}exempt,N}$ are the tax-deferred account and tax-exempt account balances at the retirement horizon N, inflated by a factor $\gamma \geq 1$. This inflation factor represents the additional value in these retirement accounts if the non-spouse heir were to exercise the full stretch provision of 10 years, established by the SECURE Act, as shown in DiLellio and Kinsman (2020). For countries that do not permit stretch provisions in these accounts, $\gamma = 1$. The variable $W_{taxable\,account,N}$ is the taxable account balance, which assumes non-spouse heirs receive a full step-up in cost basis. We also neglect the effect of estate taxes. In the United States, and as reported by Huang and Cho (2017), affects about 0.2% of taxpayers.

We assume the retiree may have other sources of income. This income may include social security or some other lifetime annuity annually adjusted at a rate I_t. This income may also include pension benefits that remain fixed in current-year dollars. We defined these variables as $SS_{t,taxable}$, $SS_{t,tax\,free}$, and P_t, respectively, in retirement year t.

3.2. Constraints

This optimization problem in Eq. (1) is subject to the *inequality constraints* for RMDs and account balances.

$$M_{tax\text{-}deferred,t} \geq RMD_t \text{ for } 1 \leq t \leq N \tag{2}$$

$$e_{tax\text{-}deferreed,t} \geq 0 \text{ for } 1 \leq t \leq N \tag{3}$$

$$e_{tax\text{-}exempt,t} \geq 0 \text{ for } 1 \leq t \leq N \tag{4}$$

$$e_{taxable,t} \geq 0 \text{ for } 1 \leq t \leq N \tag{5}$$

In Eq. (2), we determine RMDs from balances of the previous year's tax-deferred account balances based on the retiree's life expectancy LE_t. If the retiree was born before June 30, 1949, then the retiree's age at the end of the year, a_t, is compared with 70.5 years. Otherwise, the age is compared to 72 years. We obtain life expectancy from the IRS tables, form 590-B, Appendix B.[9] Note that this table does not distinguish between men and women, so our reference to retiree and spouse in the following calculations can be either gender.

$$RMD_t = \begin{cases} 0 \text{ for } a_t < 70.5 \text{ or } 72 \\ \dfrac{e_{tax\text{-}deferred,t-1}}{LE_t} \text{ for } a_t > 70.5 \text{ or } 72 \end{cases} \quad (6)$$

Eqs. (3–5) represent account balances at the end of each retirement year and thus restrict negative balances in any account. We determine account balances each year using the real rate of return of the retiree's investments and deduct any taxes owed. For the tax-deferred account, we find the end-of-the-year account balance in year t, $e_{tax-deferred,t}$, as

$$e_{tax\text{-}deferred,t} = [e_{tax\text{-}deferred,t-1} - (M_{tax\text{-}deferred,t} + T_{tax\text{-}deferred,t})] * (1 + \mu_t). \quad (7)$$

The term μ_t is the time-dependent *real* rate of return of a stock-bond portfolio, and $T_{tax\text{-}deferred,t}$ are the income taxes due from the tax-deferred account distribution. Because there may also be retiree income tax due to their social security and pension benefits received, $T_{tax-deferred,t}$ is found after accounting for this other income. Our model assumes that 85% of social security benefits are taxed as ordinary income, which is an upper bound on the amount of tax that could be due from social security. Otherwise, the value would not solely depend on both tax-deferred account distributions, increasing the complexity of the optimization problem. All examples shown in the following sections satisfy this condition, which is an individual with a modified adjusted gross income of at least \$34,000 or at least \$44,00 for a married couple filing jointly.

For the tax-exempt account, we find the end-of-the-year account balance in year t, $e_{tax\text{-}exempt,t}$, as

$$e_{tax\text{-}exempt,t} = [e_{tax\text{-}exempt,t-1} - M_{tax\text{-}exempt,t}](1 + \mu_t). \quad (8)$$

Note that we assume that the asset allocation in the tax-deferred account is also applied to the tax-exempt account, so the tax-exempt account grows at the same rate μ_t each year. We shall also apply this asset allocation to the taxable account. Therefore, we determine the taxable account end of year balance in year t, $e_{taxable,t}$, as

$$e_{taxable,t} = [e_{taxable,t-1} - (M_{taxable,t} + T_{gains,t})](1 + \mu_t). \quad (9)$$

We model our capital gains tax calculation of $T_{gains,t}$in the following ways. First, we assume all gains are long-term, so we tax these gains at the lower capital gains rate, which is very likely for retirees that are not actively trading assets in their taxable account. Tax brackets used in this paper appear in the Appendix Table A1 and A2, for U.S. taxpayers. However, other developed countries also use a progressive tax system, such as the United Kingdom, Germany, Canada, and Australia, among others. Therefore, while the results that follow will implement only U.S. tax law, we expect these results to be generalizable to any country with a progressive tax system and different rates for capital gains.

We also assume a single cost basis for the taxable account. In fact, a retiree may have purchased stocks or bonds at multiple basis, and could realize additional tax savings if higher basis investments are liquidated first. However, it is also possible that a retiree may not be aware of this fact, and liquidate lower basis investment first, increasing their tax liability. Therefore, the single cost basis represents a weighted average of all cost basis. The framework of the model could be expanded to include multiple cost bases, but the generalization of the findings would be unchanged. Consequently, we do not attempt to reinvest stock dividends and bond coupon payments, but rather use them to satisfy current-year retirement income needs. In the event that tax-deferred account withdrawals, along with other interest income, pensions, and social security exceed the income needed, we transfer these excess amounts to a zero-dividend investment. Using a zero-dividend investment minimizes annual taxes on dividends while maximizing the step-up in cost basis. This approach is similar to the strategy employed by Cook et al. (2015), except we minimize the tax drag of dividends in this separate taxable account, which will be transferred in its entirety to the heirs.

Next, we assume a "glide path" asset allocation of stock and bonds in the taxable account identical to the tax-deferred and tax-exempt accounts, which is consistent with the practice among financial advisors. Additionally, varying asset allocations between accounts can bias optimal drawdowns, delaying account drawdowns with higher stock allocations due to their higher expected returns. The glide path is initially set based on the retiree's age so that stock allocation is equal to 120-age. Thus, a 60-year-old retiree starts with an asset allocation of 60% stock and 40% bonds, with a 1% shift from stocks to bonds each year of retirement. A similar glide path is set for the spouse, in case their age differs. We do not include investor sentiment to market risk, which is likely to affect their asset allocation decisions during retirement. Eqs. (7–9) above can accommodate time variation, so our model's framework is sufficiently flexible to accommodate this effect.

Lastly, we assume stock dividends are all qualified, so they are taxed as a capital gain, like the iShares Core S&P 500 Index ETF (ticker: IVV). Alternatively, we assume bond income is taxed as ordinary income, like the iShares Aggregate Bond Index ETF (ticker AGG). Because return uncertainty is not included in this model, we do not model tax-loss harvesting of either short-term or long-term losses on taxable account investments. However, we do generalize our results using a sensitivity analysis that varies the expected return (among other inputs) to capture the effect of different long-run stock and bond total return performances. We show that the optimization is largely insensitive to stock returns and bond returns.

Fig. 1 provides a graphical representation of these assumptions. This model does include many aspects of the tax systems throughout the world, such as deductions, income tax, and capital gains taxes stack on top of one another. However, other elements, like the U.S. Alternative Minimum Tax (AMT), the United States 3.8% Medicare tax, and local province or state taxes are not included. Sumutka et al. (2012) developed and tested a tax-efficient model that included these additional elements for U.S. retirees. As these taxes are almost always much smaller than those imposed at a national level, we do not expect our findings to change significantly if these country-specific taxes were imposed.

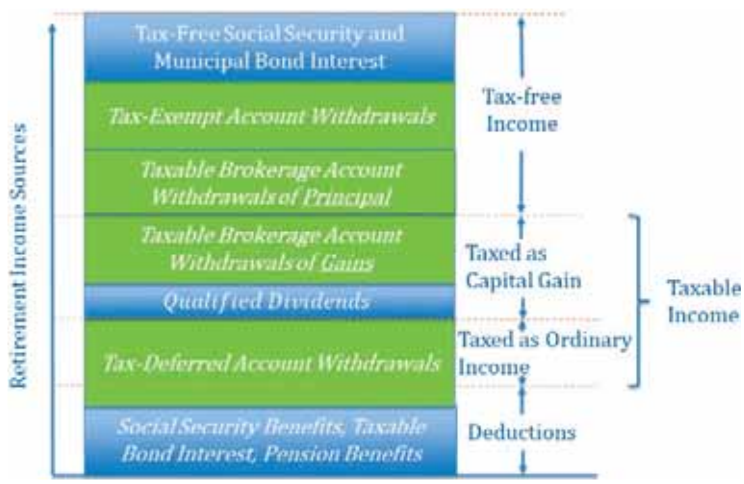

Fig. 1. Conceptual model for retirement income sources and taxation. In this illustration, the retiree's deductions exceed their income from social security, pension, and taxable bond interest.

Finally, the equality constraint for the after-tax retirement income needed, C_t, is

$$(P_t + SS_{t,taxable} + D_t - T) + M_{TDA,t} + \left(D_t^* - T_{qdiv,t} + M_{TA,t}\right) + M_{TEA,t} + SS_{t,tax\ free} = C_t. \tag{10}$$

The first terms represent income taxed as ordinary income, where P_t is the pre-tax pension, $SS_{t,taxable}$ is pre-tax social security, and D_t are pre-tax bond coupon payments from the brokerage account. The ordinary income tax paid on these three income sources is T. The next terms in parentheses are the pre-tax qualified dividends D_t^* from the taxable brokerage account and the taxes paid on the qualified dividends $T_{qdiv,t}$. The term $SS_{t,taxfree}$ is the tax-free portion of social security benefits. Note that in Eq. (10), time-depended retirement income can be accommodated by this model, consistent with retirees differing views of using retirement assets for generating income, supporting health, and intergenerational planning as shown in Chambers et al (2021).

3.3. Withdrawal strategies

We implement three strategies to solve the optimization problem described in the previous section. We find the optimal strategy depends on the relationship between a retiree's net worth (including the present value of annuities) and the retiree's desired retirement income. Fig. 2 presents a summary of this finding that will be supported by results in the next section. In the three categories appearing in Fig. 2, the Common Rule strategy, as described in Coopersmith and Sumutka (2011), provides an important heuristic in selecting an optimal strategy. Here, we define the Common Rule strategy as one that exhausts RMDs first, then taxable account funds, then tax-deferred account funds, then lastly tax-exempt funds.

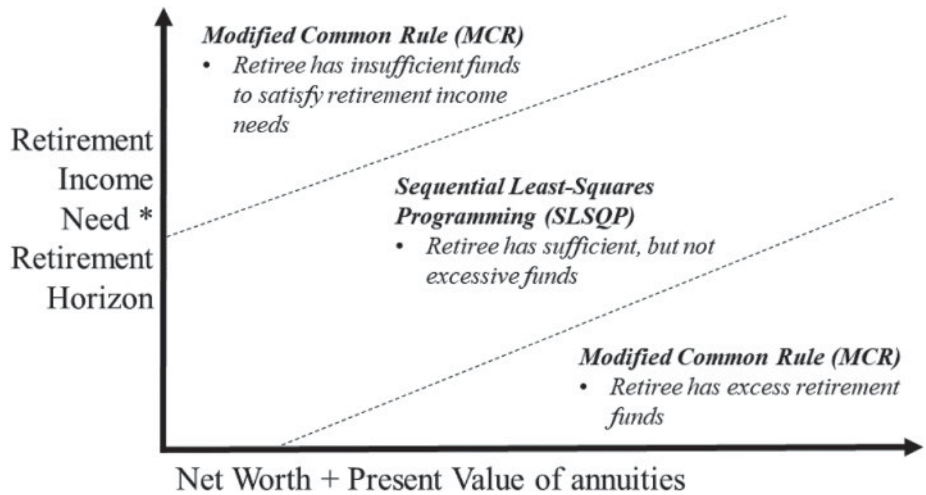

Fig. 2. The optimal withdrawal strategy depends on the relationship between the retirees net worth + present value of annuities and their retirement income needs * their retirement horizon.

Referring to the upper left of Fig. 2, this category corresponds to a situation where the Common Rule strategy exhausts all funds over the retirement horizon. DiLellio and Ostrov (2017) show that portfolio longevity can be extended by more tax-efficient planning that includes knowledge about the heir's tax rate. Cook et al. (2015) show that the most effective approach to possibly extending portfolio longevity is careful Roth conversions during either pre- or early retirement years. In the following section, we show that the Modified Common Rule withdrawal strategy provides a similar benefit of three additional years found in Cook et al. (2015) by first drawing down from the tax-deferred account up to the heir's tax rate. The Modified Common Rule strategy is one that initially withdraws tax-deferred account funds up to the heir's tax bracket, before using taxable account funds. Otherwise, the Modified Common Rule strategy is identical to the Common Rule strategy. We also show that the Modified Common Rule produces 0.35% of additional return over the Common Rule strategy.

The lower left and upper right in Fig. 2 represents the most significant retiree category for increasing a bequest. We identify this category by applying the Common Rule strategy and seeing that there are sufficient, but not excessive funds, in meeting retirement income needs. Put another way, the retiree cannot solely fund their retirement income without voluntary withdrawals from their tax-deferred, tax-exempt, or taxable accounts. In the following section, we show that using the Sequential Least Squares Programming (SLSQP) withdrawal strategy can add the equivalent of 0.54% to the annual return of the Common Rule strategy. The SLSQP withdrawal strategy uses non-linear constrained optimization methods to solve this optimization problem.

The last category appears in the lower right of Fig. 2 when a retiree has some years when RMDs plus other income sources like dividends, pensions, and social security exceed their income needs. In this case, the optimal strategy is again the Modified Common Rule, which takes full advantage of their tax-deferred account up to the tax bracket of their heir. Note that the SLSQP strategy is often not feasible here because the equality constraint in Eq. (10) may not be satisfied in some retirement years. In this last category, we model this so-called "excess income" as a direct contribution to another taxable brokerage account that is invested in a zero-dividend stock, to minimize the corrosive effects of dividends, as suggested by DeMuth (2016). Investing a retiree's excess income in this way maximizes the benefits of the step-up in cost basis realized by a retiree's non-spouse heirs.

4. Results

The following subsections demonstrate the results using the three categories that appear in Fig. 2. Using the Common Rule withdrawal strategy, three outcomes occur.

1. Retiree and spouse *exhaust* all account assets over their retirement horizon.
2. Retiree and spouse have *sufficient* assets to fund their retirement but some of their account assets must be drawn down.
3. Retiree and spouse have an *excess* of assets, generating more income from dividends, interest, coupon payments, and RMDs than is needed over their retirement horizon.

4.1. Retiree and spouse run out of funds over their retirement horizon

Table 1 shows the assumptions that, after following the Common Rule strategy, exhaust all account funds. We use the average spousal age differential of 3 years.[10] These results include the effect of a surviving spouse, who can realize a step-up in cost basis because we assume they reside in a community property state. We also assume these retirees need to begin taking their RMDs in the year they turn 70.5.[11] The implied life expectancy of 95 and 100 years in Table 1 are conservative estimates often used by financial planners. We investigate shorter life expectancies in the following subsections.

We determine the cost basis for stocks and bonds using the nominal return found lower in the table, to be consistent with current assumptions about capital markets. Using the iShares S&P 500 ETF (ticker IVV) as a proxy for stock investing, its lifetime annualized return (from May 15, 2000, to April 27, 2021) was 7.2%. Similarly, the iShares Aggregate Bond Index ETF (ticker AGG) was used as a proxy for bond investing, and its lifetime annual return (from September 22, 2003, to April 27, 2021) was 4.0%. With these returns in mind, a taxable investment in each of these funds 10 years before retirement yields cost bases of 50% for stocks and 68% for bonds. Additionally, we assume a 1% transition from stocks to bonds in retirement, which follows conventional wisdom. To provide support for empirical observations by Gerrans et al. (2009) and investment theory by Samuelson (1969), we include a sensitivity analysis with this input parameter.

We set Social Security to its median value, assume it grows at the rate of inflation appearing lower in Table 1, and begins at the full retirement age of 67.[12] Similarly, we set pension benefits to their median value of $11,000 per year for private and union pensions, but do not adjust its annual payment for inflation.[13] We derive the inflation rate using the long-term average from the Consumer Price Index from 1992 to 2020. However, given how much inflation may change, we include this input as part of the sensitivity analysis explored in the later sections.

Table 1 Input variables for a retiree and spouse that lead to insufficient funds for retirement

Age of retiree, spouse	65, 62
After-tax income needed, per year	\$150k (retiree + spouse), \$140k (surviving spouse)
Retirement time horizon, retiree, and spouse	30 years (retiree), 40 years (spouse)
Community property state?	Yes
Account information	
Retiree and spouse tax-deferred account starting balance	\$800,000 and \$100,000
Retiree and spouse tax-exempt account starting balance	\$400,000 and \$50,000
Taxable account starting balance	\$1,000,000
Cost basis	50% (stocks), 68% (bonds)
Initial asset allocation	60% stocks, 40% taxable bonds
Glide slope transition from stock to bonds	1% per year
Filing status	Married filing jointly, then single
The deduction, tax year	The standard deduction, 2020
Retiree's social security amount and age to start	\$18,500 at age 67
Spouse social security amount and age to start	\$18,500 at age 67
Retiree's pension and start age	\$3,667 at age 65
Spouse's pension and start age	\$3,667 at age 65
Stretch tax-deferred account withdrawals for 10 years	No
Heir's tax rate	25%
Inflation	2.1%
Nominal return, stocks	7.2%
Nominal return, bonds	4%
Stock dividend rate	2%
Bond coupon rate	2.5%

Using the variables in Table 1, the retiree's spouse exhausts all their funds in the 37th year, for portfolio longevity of 36.5 years. The upper pane of Fig. 3 shows how the retiree and spouse use each source of after-tax income to meet the annual income needs using the Common Rule withdrawal strategy. Here, all taxable account funds (orange) are exhausted first, then tax-deferred account funds (blue), and tax-exempt account funds last (green). Also, note that the RMDs (yellow bars overlapping the blue) appear completely over the tax-deferred account bars when representing a binding constraint in years 5 through 17. Lastly, income tax brackets are constant each year, because all the results are in the current year, not future dollars.[14] However, income brackets drop in year 31, when the surviving spouse must file their taxes as "single." The 0% tax bracket corresponds to the income below the standard deduction in the United States or income below the so-called exclusion limit for retirees paying taxes outside the United States.

The lower pane of Fig. 3 shows the Modified Common Rule withdrawal strategy increasing portfolio longevity to 39.7 years or about 3.2 years. Put another way, the Common Rule withdrawal strategy would need to increase its investment returns by 0.35% per year to achieve the same portfolio longevity.[15] Recall that the Modified Common Rule strategy first uses tax-deferred account assets up to the top of the income bracket associated with greater efficiency relative to the heir. In this case, the heir's marginal tax rate is 25%, so tax-efficient tax-deferred account withdrawals can occur up to the top of the 24% bracket, but not the bracket above it corresponding to the 32% income tax rate. The most noticeable difference between the upper and lower panes in Fig. 3 is the RMDs are no longer binding during any

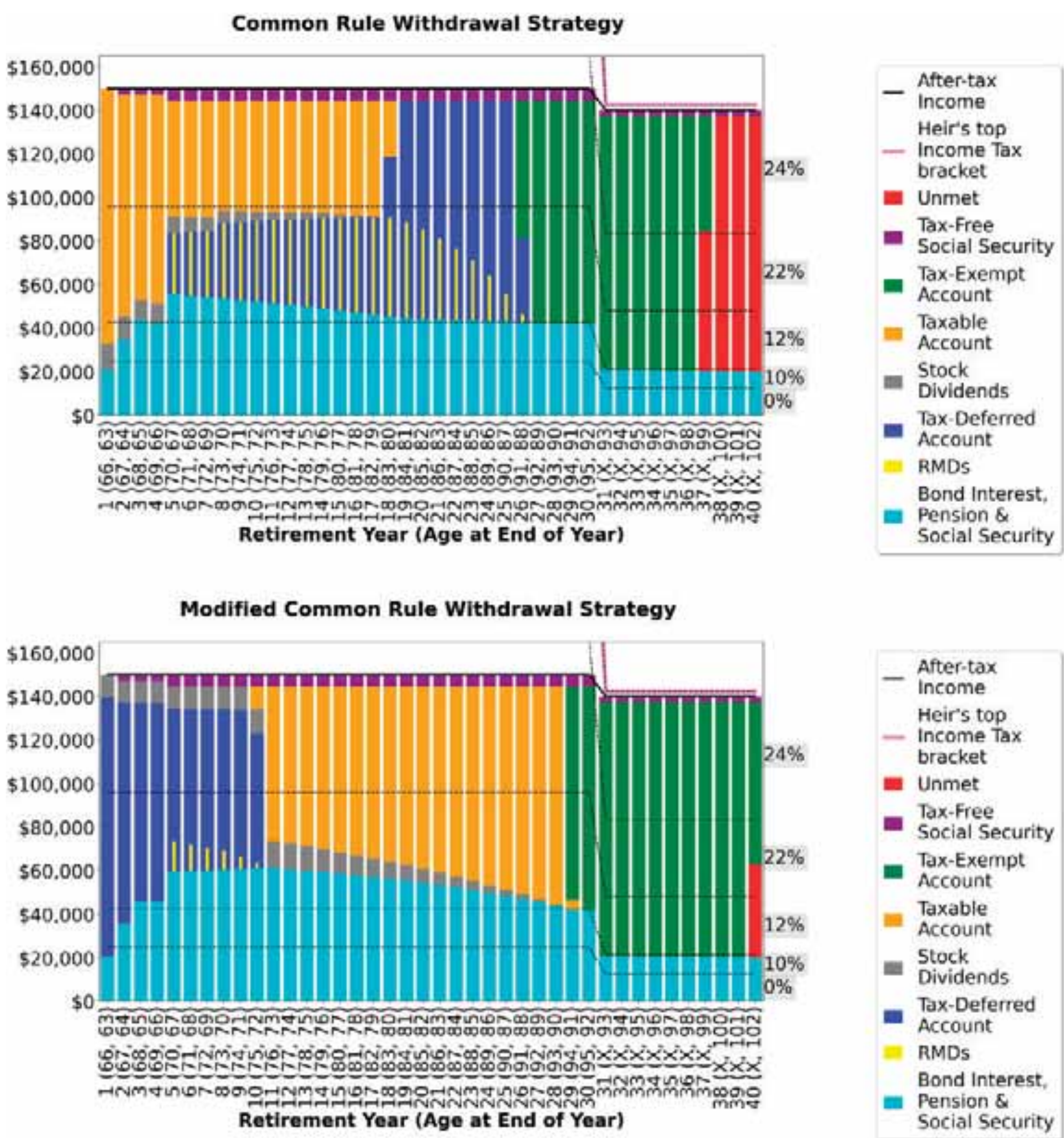

Fig. 3. Meeting after-tax income using the Common Rule withdrawal strategy (upper pane) and Modified Common Rule withdrawal strategy (lower pane) using variables in Table 1. The retiree and spouse exhaust all their accounts.

year. There is also a longer duration of stock dividends (gray) and bond interest (light blue) contributing to retirement income needs because the taxable account assets are not used up as quickly in the lower pane of Fig. 3.

Fig. 4 shows how up to four different taxes are paid each year of retirement: two sources of income taxes and two sources of capital gains taxes. In the early retirement years, using the Common Rule strategy in the upper pane, the retiree pays very little tax, because the income tax from ordinary income barely exceeds the standard deduction, and no tax-deferred account withdrawals occur yet. Additionally, the retiree and spouse can realize gains by selling stock and bond shares in their taxable brokerage account that, with an initial fraction of their cost basis shown in Table 1, does not trigger any long-term capital gains taxes. This

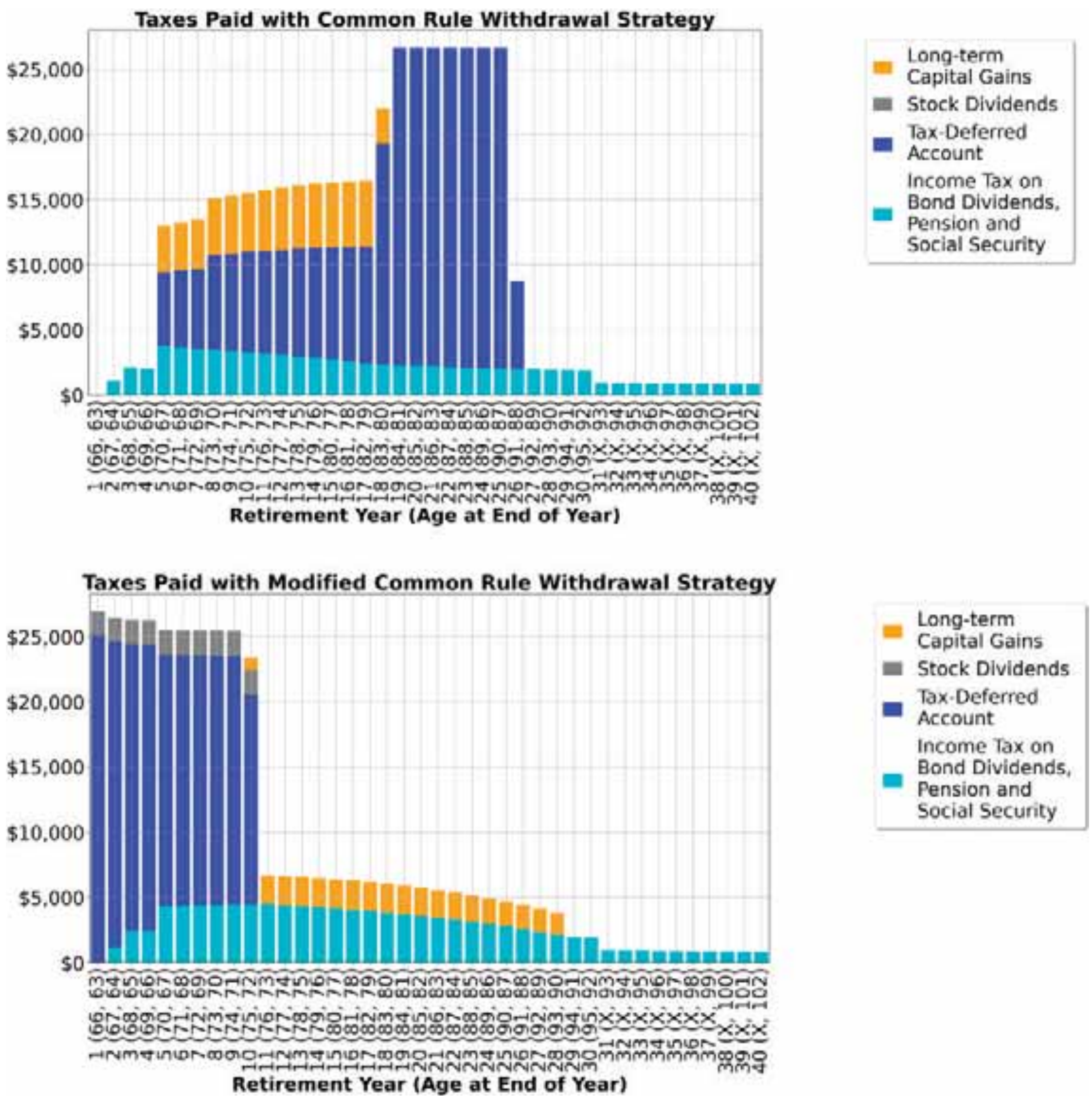

Fig. 4. Taxes paid with the Common Rule and Modified Common Rule withdrawal strategies using conditions in Table 1. The retiree and spouse exhaust all their accounts.

small capital gains tax liability is thanks to the fairly large 0% capital gains bracket that is mostly available due to limited ordinary income.

In the years 5 to 17, RMDs initiate and are a binding constraint. Also, long-term capital gains are realized above the 0% capital gain rate, so taxes become more significant. Once the retiree exhausts the taxable brokerage account in year 18, retirement income shifts to withdrawals entirely from the tax-deferred account, significantly increasing income taxes paid in years 19–25. The retiree exhausts the tax-deferred account in year 26. In the later years, taxable income barely exceeds the standard deduction, thanks to the funds from the tax-exempt account, so minimal taxes are due.

The lower pane of Fig. 4 shows how shifting taxes to earlier years increased the portfolio longevity. In years 1–10, the retiree uses tax-deferred funds instead of taxable brokerage account funds. The Modified Common Rule withdrawal strategy identifies their use because they are more efficient than the possible transfer to the heir, whose tax rate is assumed at 25% in Table 1. Then, in the later years, the retiree pays some long-term capital gains taxes, along with a small amount of income tax. After year 31, the taxes paid are identical to the Common Rule withdrawal strategy.

Therefore, a key insight here is that portfolio longevity increases if one defies conventional wisdom to always defer taxes until future years. These results demonstrate that withdrawing from the tax-deferred account earlier than the RMDs suggest can provide real benefits. In this case, the longevity increased by about 3 years. Or, in terms of investment returns, provides a tax alpha of about 0.35%. In the sensitivity section that follows, we show that these results are mostly generalizable across several varying conditions changed from Table 1.

4.2. Retiree and spouse have sufficient funds for their retirement

In this case, the retiree has a sufficient, but not excessive, net worth. Recall, sufficient retirement funds mean assets must be drawn down at some point in retirement. Excessive means that interest, coupons, and dividends, along with RMDs, exceed the retiree's income needs at some point during retirement. Therefore, there is an opportunity to increase their bequest. The retiree and spouse's assumptions are identical to the values appearing in Table 1, except we reduced the retirement horizons to 15 years (retiree) and 20 years (spouse). Table 2 summarizes the final account values under three withdrawal strategies. It shows that the increase in wealth transfer to non-spouse heirs is 22% larger with the SLSQP withdrawal strategy over Common Rule. For the Common Rule strategy to produce the same wealth transfer amount as the SLSQP strategy, asset returns would need to increase by 0.54% per year.

The drawdown decisions for the Common Rule and SLSQP withdrawal strategies appear in Fig. 5 upper and lower panes. There are several unique differences in these decisions. First, for the SLSQP withdrawal strategy, tax-deferred account withdrawals (in blue) occur much earlier, so RMDs are no longer binding constraints. Also, tax-deferred account withdrawals occur throughout retirement, helping to prevent a higher level of income tax in the later years, especially for the surviving spouse filing as single in years 16–20. Lastly, the retiree uses the taxable brokerage account much more sparingly so that the heir can realize the step-up in cost basis from this account.

Table 2 Account Balances at the end of retirement for common rule, modified common rule, and SLSQP withdrawals when the retiree and spouse have sufficient, but not excessive, funds for retirement

Strategy	Tax-deferred account, retiree	Tax-exempt account, retiree	Taxable brokerage balance	Tax-deferred account, spouse	Tax-exempt account balance, spouse	Wealth transfer to heirs
Common rule	$461,937	$798,355	$0	$57,742	$99,794	$1,287,908
Modified common rule	$0	$798,355	$632,418	$0	$99,794	$1,530,567
SLSQP	$20,186	$265,305	$1,255,685	$2,523	$33,163	$1,571,185

Note: SLSQP = Sequential Least Squares Programming.

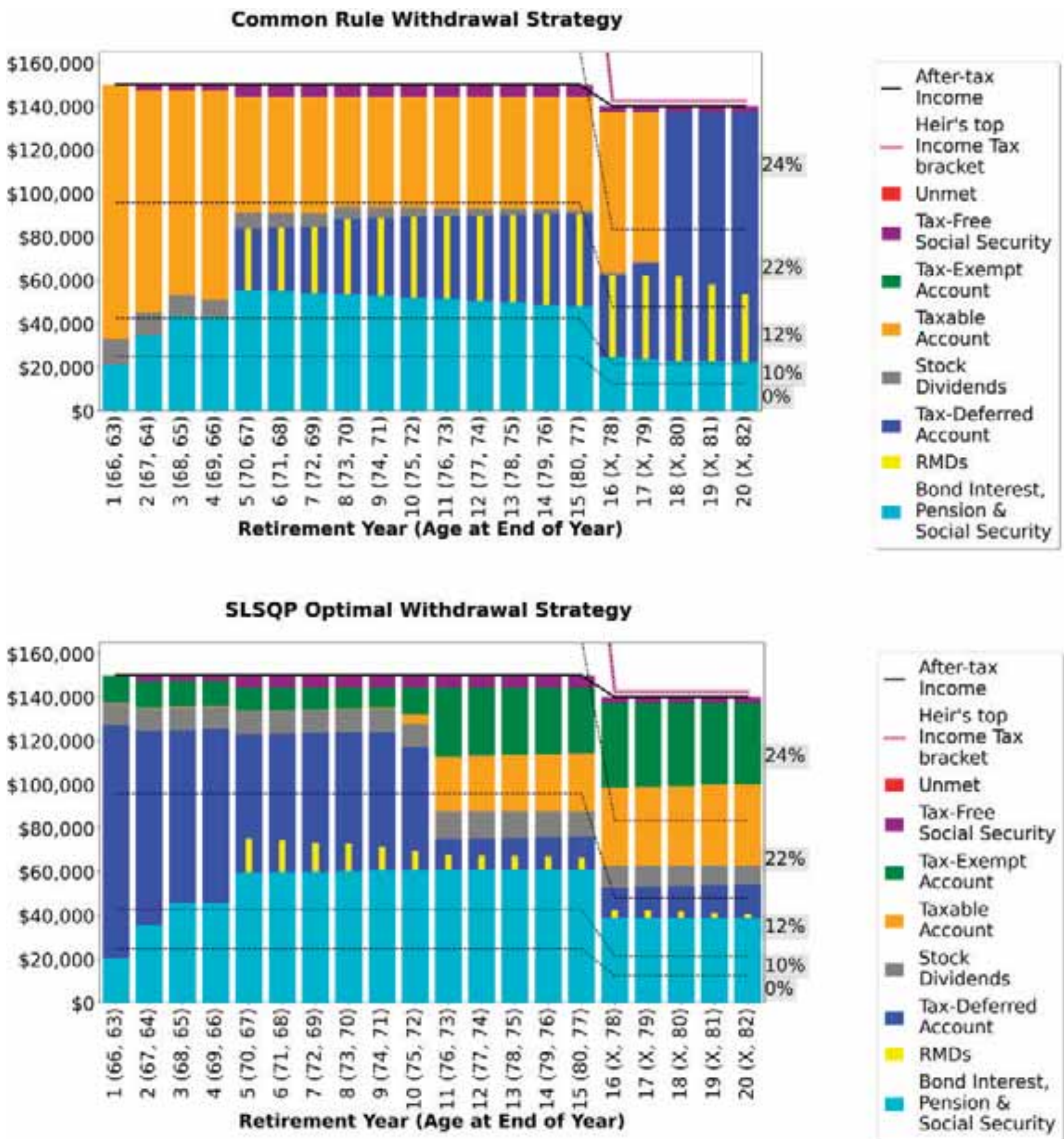

Fig. 5. Meeting after-tax income using the Common Rule and Sequential Least Squares Programming (SLSQP) withdrawal strategies and conditions in Table 1 with a shorter retirement horizon. The retiree and their spouse have sufficient, but not excessive, funds for retirement.

The upper and lower panes of Fig. 6 show the significant difference in taxes paid between the Common Rule and SLSQP withdrawal strategies. The most obvious difference is that the retiree pays taxes much earlier in the SLSQP versus the Common Rule strategy. However, notice that the total taxes paid by the SLSQP strategy are in the low $20,000 range for the first seven years, then dropping to between about $5,000 and $10,000 for the remaining retirement horizon.

The Common Rule strategy follows a more “conventional wisdom” recommended by many CPAs that often suggest delaying taxes for as long as possible. Under the Common Rule withdrawal strategy, the retiree pays very little tax in years 1–4. Like in the previous

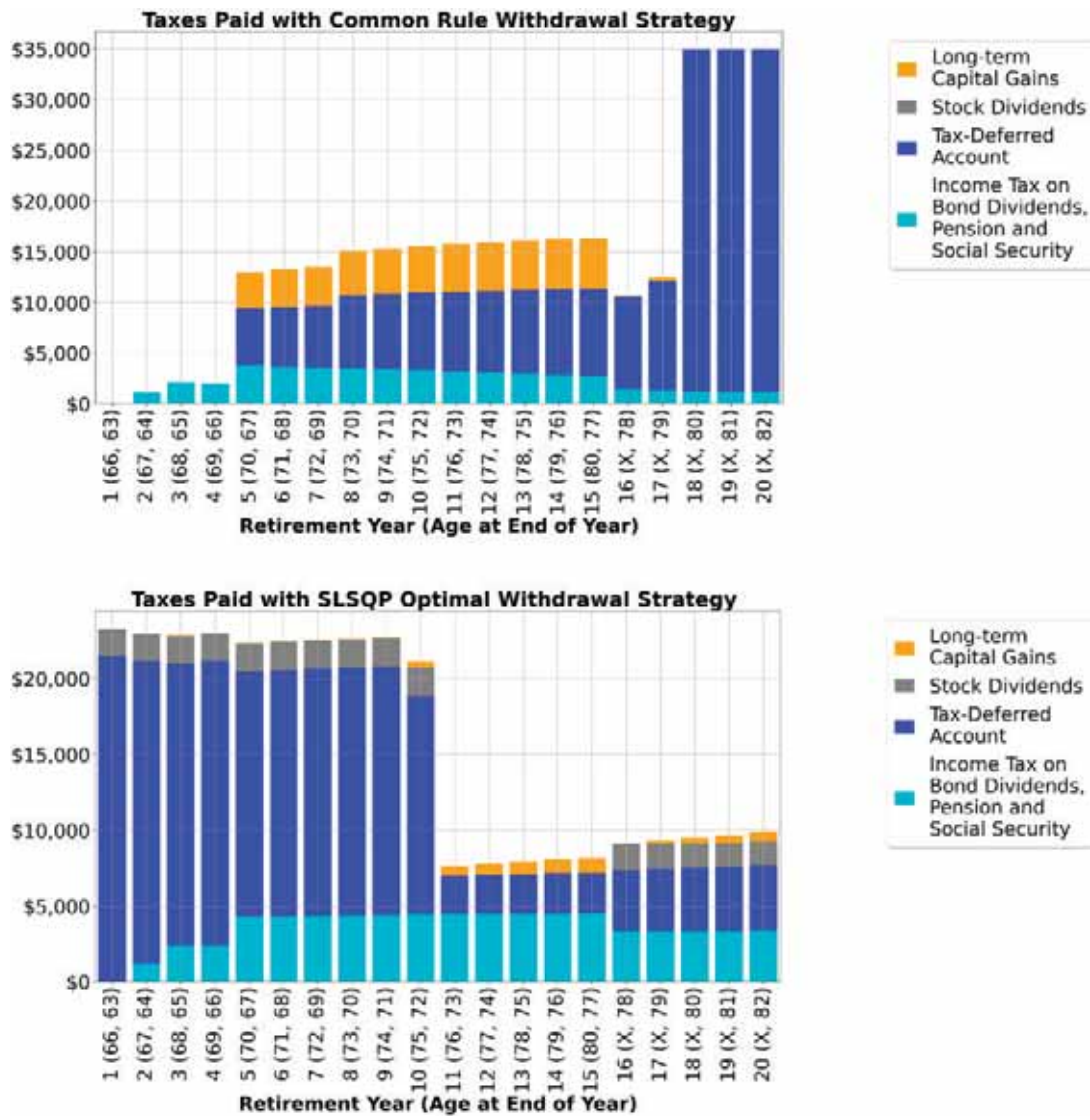

Fig. 6. Taxes paid with the Common Rule withdrawal sequence and Sequential Least Squares Programming (SLSQP) withdrawal strategies and conditions in Table 1 with a shorter retirement horizon. The retiree and their spouses have sufficient, but not excessive, funds for retirement.

subsection, low taxes are due to the initial fraction of the taxable account cost basis shown in Table 1 not triggering any long-term capital gains taxes. In years 5–15, there is a gradual increase in taxes from both the RMDs and long-term capital gains. Years 16–17 see a decrease in taxes because the taxable brokerage account offsets the higher tax rates of the surviving spouse. The final five years 18–20 see a significant tax increase from withdrawals from the tax-deferred account and the surviving spouse filing taxes as single. Therefore, in these final years, the surviving spouse pays nearly $35,000 of income tax to maintain their desired consumption of $140,000 per year. These later years show how sensitive the surviving spouse is to losing the larger standard deduction by no longer being able to file their taxes as married filing jointly.

Comparing the upper and lower pane of Fig. 6 indicates that tax minimization may not occur. This finding is consistent with Sumutka et al. (2012), who noted that "… several common tax minimization and estate planning strategies do not produce optimal results." These results provide further evidence that simply minimizing taxes paid does not lead to a larger bequest to the retiree's heir.

4.3. Retiree and spouse have a large excess of funds

In this last case, the retiree and spouse have a large net worth with more income from dividends, interest, coupon payments, and RMDs than is needed at some point over their retirement horizon. Here, the Modified Common Rule provides a solution marginally better than the Common Rule.

In this next example, we begin by using the data from Table 1. Like in the previous subsection, we reduce the retirement horizon to 15 years for the retiree and 20 years for the spouse. We also increase the retiree's tax-deferred account to start at $2.7M, and consumption while married filing jointly to $180,000 per year. The account values at the end of retirement using the Common Rule and Modified Common Rule withdrawal strategies appear below in Table 3.

Reviewing the results from Table 3, we see that the Modified Common Rule strategy increases Common Rule wealth transfer by about 7%. Increasing the stock and bond returns by 0.24% with the Common Rule strategy produces the same wealth as the Modified Common Rule strategy.

This example is also one that highlights the limitations of general-purpose constrained optimization programs like SLSQP. If we apply the SLSQP withdrawal strategy here, it does find an optimal solution that satisfies the equality constraint in Eq. (10). However, the total wealth transfer to heirs is $3,764,923, which is less than found with the Modified Common Rule. Thus, the SLSQP strategy appears to have gotten "stuck" in a local maximum, so it did not yield a global maximum. And, if the SLSQP withdrawal strategy could not find a solution, which may occur when the equality constraint in Eq. (10) is unmet, this general-purpose optimization algorithm could spend 10–15 minutes and report back that no solution could be found. The deployment of any enterprise application that consumes precious computational resources while not providing actionable insights must be avoided. Thus, we have

Table 3 Account Balances at the end of retirement for common rule and modified common rule withdrawals

Strategy	Tax-deferred account, retiree	Tax-exempt account, retiree	Taxable brokerage balance	Tax-deferred account, spouse	Tax-exempt account balance, spouse	Excess income account balance	Wealth transfer to heirs
Common Rule	$2,501,404	$798,355	$617,383	$92,645	$99,794	$73,004	$3,534,072
Modified Common Rule	$1,594,256	$798,355	$1,629,133	$59,047	$99,794	$0	$3,767,258

Note: Retiree and spouse have excessive amounts of retirement assets.

evidence that there is a need for a more customized optimization algorithm for this category of retirees, as we demonstrate with the Modified Common Rule strategy.

Fig. 7 presents the retirement income sources used to satisfy the retiree and spouse's income needs, then later the surviving spouse's income needs. The upper panel shows the results using the Common Rule withdrawal strategy, where RMDs are a binding constraint in years 5–20. The RMDs produce excess income as mentioned previously. To minimize the effect of taxes, income over $140,000 in years 14–20 is reinvested in a zero-dividend stock mutual fund or ETF. We assume this zero-dividend investment made in these later years has a nominal return equal to the stock investment in the retiree's other investment accounts.[16]

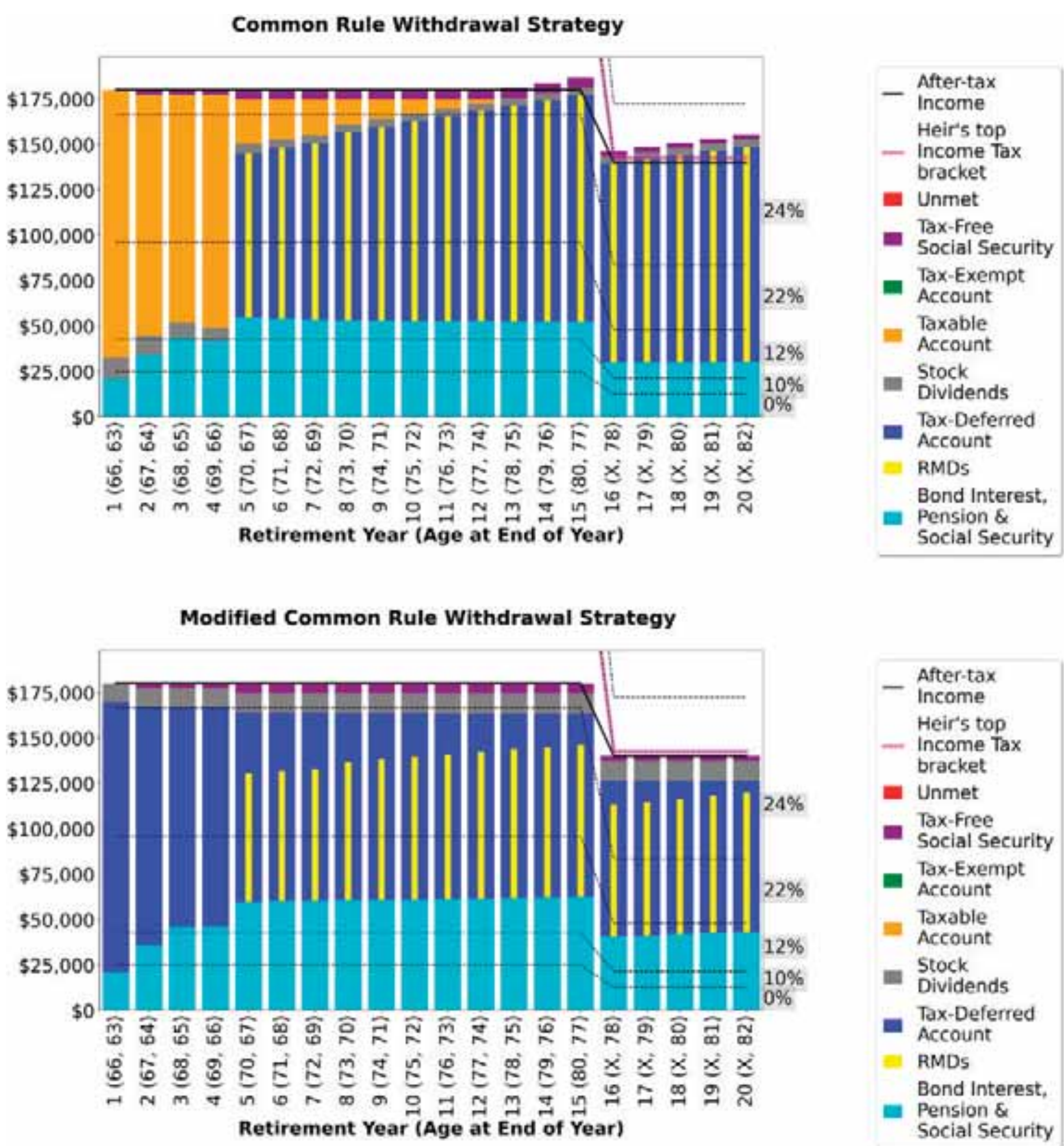

Fig. 7. Meeting after-tax income using the Common Rule and Modified Common Rule withdrawal strategies and conditions in Table 1 with shorter retirement horizon, higher tax-deferred account starting account balances, and higher retiree and spouse income needs. Retirees and spouses have excessive amounts of retirement assets.

The lower panel of Fig. 7 shows drawdowns with the Modified Common Rule, where all RMDs from the tax-deferred account are kept as a non-binding constraint. It also prevents excess income from occurring in years 14–20 for the surviving spouse.

Fig. 8 shows the taxes paid corresponding to the retirement income decisions shown in Fig. 7. Similar to what appeared in Fig. 6, the optimal decision increases taxes paid in earlier retirement years. However, in later years, taxes paid with the Modified Common Rule fall below those paid with the Common Rule. Thus, we see another example where the benefit of the step-up in cost basis provided by the taxable brokerage account outweighs the additional and earlier taxes paid by the retiree.

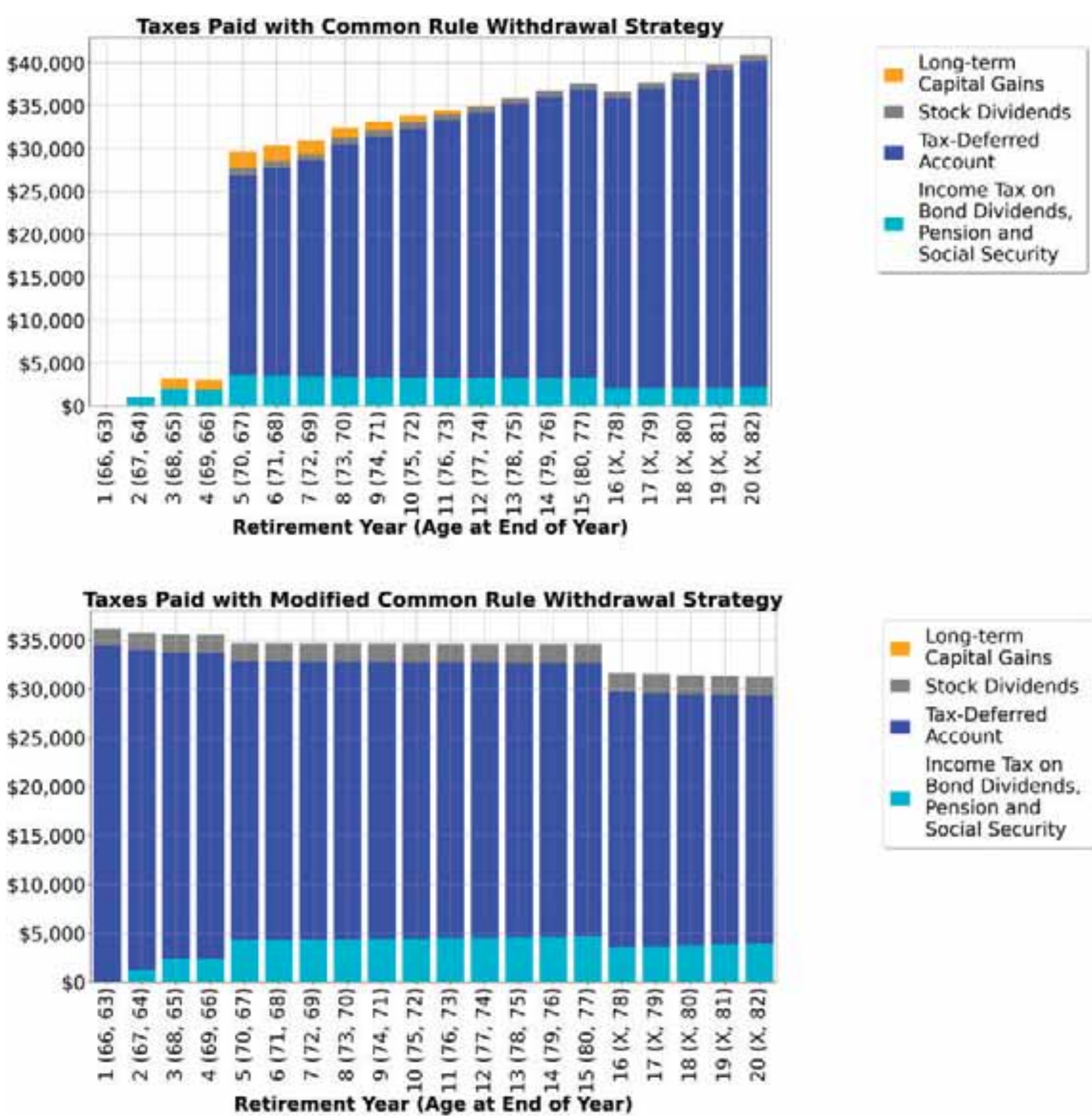

Fig. 8. Taxes paid with the Common Rule withdrawal sequence and Modified Common Rule withdrawal strategies and conditions in Table 1 with shorter retirement horizon, higher tax-deferred account starting account balances, and higher retiree and spouse income needs. Retirees and spouses have excessive amounts of retirement assets.

Table 4 Portfolio longevity sensitivity (years) to heir's tax rate

τ_{heir}	Common Rule	Modified Common Rule	Improvement vs. Common Rule	Equivalent tax alpha
0%	36.5	36.8	0.3	0.03%
15%	36.5	37.5	1.0	0.11%
25%	36.5	39.7	3.2	0.35%
35%	36.5	39.7	3.2	0.35%

Note: Base case results use inputs from Table 1 with τ_{heir} = 25%. Retiree and spouse have insufficient funds for retirement.

5. Sensitivity analysis of the heir's tax rate

We begin by revisiting the results from the previous subsection to see how portfolio longevity increases with the Modified Common Rule strategy and changes with the heir's tax rate. Recall that the Modified Common Rule strategy is identical to the Common Rule, except it draws down the tax-deferred account initially up to the heir's tax bracket. Also, note that the Common Rule strategy is not affected by changing the heir's tax rate.

The results in Table 4 suggest that drawing down the tax-deferred account faster is more efficient for the retiree than the heir to pay income taxes on these distributions. However, because the retiree exhausts all of their accounts, delaying this drawdown when the retiree has an heir at a lower tax rate provides little benefit. This is primarily because RMDs for these distributions are less tax-efficient than could be realized by an heir with a lower marginal tax rate. In the extreme case when the heir is a qualifying charitable organization with a 0% tax rate, the Modified Common Rule strategy provides only a fraction of a year of additional longevity.

We next examined how the effect changing the heir's tax rate can have on the results in Tables 2 and 3. Table 5 summarizes the results that vary the heir's tax rate away from the 25% rate used to produce Table 2 when the retiree and spouse have sufficient retirement assets. We also applied the Modified Common Rule withdrawal strategy to each of these examples, and we found the SLSQP strategy always provided a higher wealth transfer to the heir. The primary observation here is that the SLSQP withdrawal strategy provides the greatest improvement when the heir's tax rate is higher.

Table 6 provides a similar sensitivity analysis, using conditions used to create Table 3 when the retiree and spouse have excessive retirement assets. We also applied the SLSQP withdrawal strategy but found that the Modified Common Rule strategy was superior in all cases we examined. Also, the SLSQP had very long run times in each case in Table 6.

Table 5 Wealth transfer sensitivity to heir's tax rate

τ_{heir}	Common Rule	SLSQP	$ Improvement vs. Common Rule	Equivalent tax alpha
0%	$1,417,828	$1,544,800	$126,972	0.21%
15%	$1,339,876	$1,524,163	$184,287	0.33%
25%	$1,287,908	$1,571,185	$283,277	0.54%
35%	$1,235,940	$1,568,913	$332,973	0.69%

Note: Base case results use inputs from Table 1, but shortened retiree's horizon to 15 years and spouse's horizon to 20 years. Retiree and spouse have sufficient retirement assets. SLSQP = Sequential Least Squares Programming.

Table 6 Wealth transfer sensitivity to heir's tax rate

τ_{heir}	Common Rule	Modified Common Rule	$ Improvement vs. Common Rule	Equivalent tax alpha
0%	$4,182,585	$4,183,990	$1,405	< 0.01%
15%	$3,793,477	$3,935,081	$141,604	0.14%
25%	$3,534,072	$3,767,259	$233,187	0.23%
35%	$3,274,667	$3,601,928	$327,261	0.34%

Note: Base case results use inputs from Table 1, but shortened retiree's horizon to 15 years and spouse's horizon to 20 years, as well as increasing the retiree's tax-deferred account to start at $2.7M, and increasing consumption while married filing jointly to $180,000 per year. Retiree and spouse have excessive retirement assets.

Similar to Table 4, Table 6 shows that the Modified Common Rule withdrawal strategy provides the greatest improvement when the heir's tax rate is higher.

Tables 4, 5, and 6 indicate that the Common Rule withdrawal strategy can provide an important *diagnostic*. Recall the results from Table 5 were based on the Common Rule that produced no excess income, or involuntary income above the income need. In all situations where this occurred, the SLSQP strategy always produced superior results. Similarly, Tables 4 and 6 showed that the Modified Common Rule strategy was superior in the presence of insufficient or excess income. And, in the case of Table 6, provided significantly shorter computational run times and guaranteed a solution that satisfied all constraints.

5.1. Sensitivity analysis

To understand the effect of selected parameters, we recalculated the results from the previous sections, using high and low values that appear below in Table 7. We changed each value throughout the forecast, except for the values in the second to the last row. In the row labeled "Percent Increase to Tax Rates," we assumed it begins in 2026, which is the year the 2017 Tax Cuts and Jobs Act (TCJA) expires. We chose the high and low values based on ranges observed from historical averages. For an excellent history of income tax rates from 1916 to 1999 (see Sialm (2006).

Fig. 9 shows how the tax alpha of 0.35% changes using the Modified Common Rule strategy changes. The tax alpha is most sensitive to increasing tax rates, exceeding 0.50% if tax rates increase in the future. Increasing the allocation towards bonds also increases tax alpha, as there is a greater opportunity for tax efficiency. Interestingly, there appears to be little sensitivity to stock and bond rate of returns, indicating the robustness of tax alpha to long-term market return trends.

Fig. 10 shows the sensitivity of the 0.54% tax alpha when the retiree and their spouse shorten their retirement horizon to 15 and 20 years. Tax alpha is the most sensitive future

Table 7 Baseline, low and high values for sensitivity analysis

	Low value	Baseline	High value
Asset allocation to stocks (bonds)	50% (50%)	60% (40%)	70% (30%)
Stock rate of return	6%	7.2%	8.4%
Bond rate of return	3%	4%	5%
Inflation rate	1.5%	2.1%	4.2%
Percent change to tax rates starting in 2026	–40%	0%	+40%
Asset allocation glideslope	0%	1%	2%

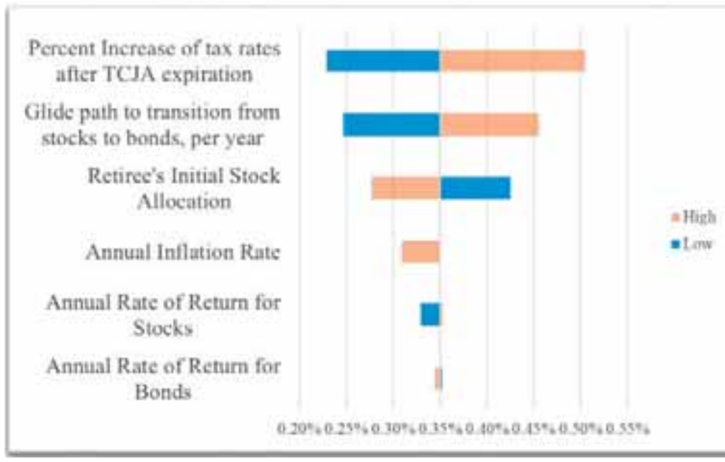

Fig. 9. Sensitivity analysis for retirees with insufficient retirement funds.

tax rate, as well as to glide path. It is also very sensitive to the retiree's initial stock allocation, and least sensitive to stock and bond returns.

Finally, Fig. 11 shows that tax alpha for retirees with excessive retirement income is most sensitive to inflation, followed by a glide path, and the tax increases after the TCJA expires. Tax alpha is least sensitive to stock and bond returns.

5.2. Implications to practice

Large financial institutions, like Fidelity (2014) and Vanguard (2016), currently provide retirement income planning tools that rely solely on the Common Rule withdrawal strategy. Our findings suggest that their enterprise applications should not be entirely discarded due to their heuristic benefits. Instead, results from the Common Rule strategy can be used to guide the next level of optimization that has a substantial benefit to tax-efficient retirement plans. Some FinTech firms, including Personal Capital, Betterment, and others, are beginning to move in this direction. We hope this article's insights provide the financial services industry with guidance to enhance and improve outcomes for retirement income plans.

Our findings also challenge the conventional wisdom advocated by many CPAs to defer taxes for as long as possible. We show the value of strategically paying some taxes earlier to avoid large taxes later due to RMDs or switching tax filing from married to single. Lastly, we show that tax alpha or the additional pre-tax return realized by an optimal strategy is usually most

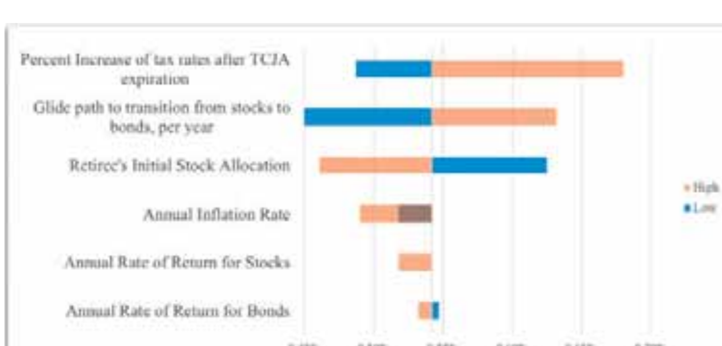

Fig. 10. Sensitivity analysis for retirees with sufficient retirement funds.

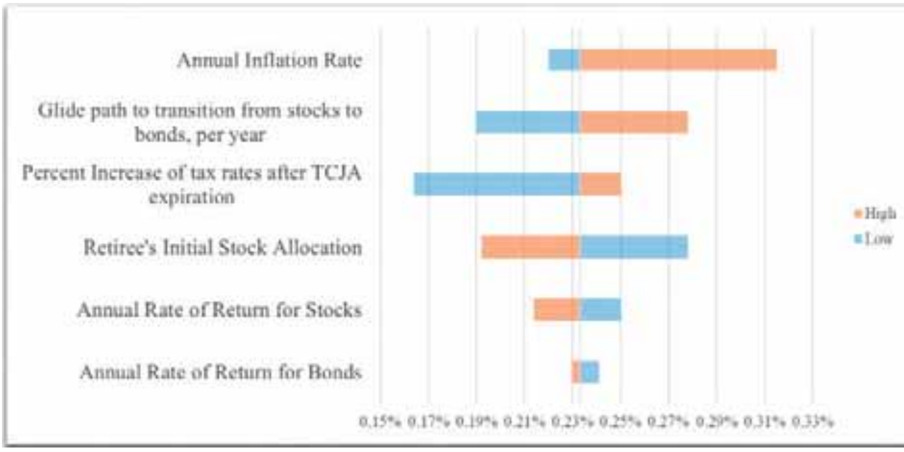

Fig. 11. Sensitivity analysis for retirees with excess retirement funds.

sensitive to future tax rates, and least sensitive to the rate of return for the stock market. This latter finding suggests simulation of stock returns is likely unnecessary to quantify the tax alpha.

6. Conclusions

In this article, we developed a framework for finding the optimal decision in tax-efficient retirement income. We showed that optimizing a retirement income plan that begins with using the Common Rule withdrawal strategy can offer important insights into the next stage of this optimization problem. We identified three categories of retirement income plans and showed how there is no "one size fits all" solution to the problem. We provide model-driven evidence that general-purpose optimization routines can provide significant improvements over the Common Rule in one category, while not in all. We provide a simple heuristic to show how alternative optimization methods perform and include a sensitivity analysis to generalize the results. Contrary to conventional wisdom, we also show that paying income taxes earlier due to tax-deferred account distributions can be more tax-efficient, contributing about 0.5% of additional annual returns. We also demonstrated how these three categories of tax-efficient withdrawal strategies performed under differing assumptions on investment returns, inflation, and tax rates. We closed on the implications these findings have on large financial institutions and smaller FinTech startups in the financial planning space, as well as insights for CPAs involved in multiyear tax planning.

Notes

1 https://www.stlouisfed.org/on-the-economy/2019/may/how-many-people-will-be-retiring-in-the-years-to-come
2 https://en.wikipedia.org/wiki/Mid-20th_century_baby_boom
3 http://benna401k.com/401k-history.html
4 https://www.newretirement.com/retirement/what-is-a-403b/

5 In 2019, the SECURE Act delayed the onset of RMDs from 70.5 to 72 for individuals born after June 30, 1949.
6 In the United States, 85% of social security is taxable income if MAGI exceeds $34,000 (single) or $44,000 (married filing jointly).
7 https://docs.scipy.org/doc/scipy/reference/optimize.minimize-slsqp.html
8 After-tax drawdowns are more helpful in supporting after-tax income needs, so all decision variables are after-tax values.
9 https://www.irs.gov/pub/irs-pdf/p590b.pdf
10 https://www.rgj.com/story/money/business/2018/11/28/retirement-planning-challenges-age-gap-relationships/2142317002/
11 Changing their RMDs to the year they turn 72 does not have a meaningful effect on the results. (longevity = 33.5 with Common Rule and 37.4 with Modified Common Rule)
12 https://www.aarp.org/retirement/social-security/questions-answers/how-much-social-security-will-i-get.html
13 http://www.pensionrights.org/publications/statistic/income-pensions/
14 This approach to keeping income brackets fixed each year means they are constant in real dollars. It is also consistent with the long-standing practice at the IRS annually increasing brackets by the rate of inflation.
15 We found the value of 0.35% by re-running this case with the Common Rule withdrawal strategy and gradually increasing the stock and bond returns until the portfolio longevity was 39.7 years.
16 The largest zero-dividend ETF is First Trust Dow Jones Internet Index ETF, ticker: FDN, with over $10B in management.

Appendix

Income and capital gains tax rates in the United States and other developed countries

Table A1 U.S. income tax rates and brackets, 2020 tax year

Income tax rate	Income limit, single	Income limit, married filing jointly
10%	$9,875	$19,750
12%	$40,125	$80,250
22%	$85,525	$171,050
24%	$163,300	$326,600
32%	$207,350	$414,700
35%	$518,400	$622,050
37%	None	None

Table A2 U.S. capital gains tax rates and brackets, 2020 tax year

Capital gains tax rate	Income limit, single	Income limit, married filing jointly
0%	$40,000	$80,000
15%	$441,450	$496,600
20%	None	None

Acknowledgments

This research was supported by the Julian Virtue Professorship Endowment at the Graziadio Business School, Pepperdine University. We are also grateful for our research assistants on this project, Airika Corley and Sophia Venegas.

References

Brown, D. C., Cederburg, S., & O'Doherty, M. S. (2017). Tax uncertainty and retirement savings diversification. *Journal of Financial Economics*, *126*, 689-712.

Chambers, B., Walker, R., Feng, J., & Gu, Y. (2021). The silver tsunami: An enquiry into the financial needs, preferences and behaviours of retirees. *Accounting & Finance*, *61*, 645-687.

Cook, K. A., Meyer, W., & Reichenstein, W. (2015). Tax-efficient withdrawal strategies. *Financial Analysts Journal*, *71*, 16-29.

Coopersmith, L. W., & Sumutka, A. R. (2011). Tax-efficient retirement withdrawal planning using a linear programming model. *Journal of Financial Planning*, *24*, 50-59.

Coopersmith, L. W., & Sumutka, A. R. (2017). The impact of rates of return on Roth conversion decisions and retiree savings wealth. *Journal of Personal Finance*, *16*, 29-39.

Dammon, R. M., Spatt, C., & Zhang, H. H. (2004). Optimal asset location and allocation with taxable and tax-deferred investing. *The Journal of Finance*, *59*, 999-1037.

DeMuth, P. (2016). *The Overtaxed Investor: Slash Your Tax Bill & Be A Tax Alpha Dog*. Los Angeles, CA: Alpha Dog Press.

DiLellio, J. A., & Ostrov, D. N. (2017). Optimal strategies for traditional versus Roth IRA/401(k) consumption during retirement. *Decision Sciences*, *48*, 356-384.

DiLellio, J. A., & Ostrov, D. N. (2020). Toward constructing tax efficient withdrawal strategies for retirees with traditional 401(k)/IRAs, Roth 401(k)/IRAs, and taxable accounts. *Financial Services Review*, *28*, 67-95.

DiLellio, J. A., & Kinsman, M. (2020). The SECURE Act and your retirement objectives. *Graziadio Business Review*, *23*.

Fidelity. (2014). *Detailed Methodology: Fidelity Retirement Income Planner*. Available at http://personal.fidelity.com/planning/retirement/pdf/rip_methodology.pdf.

Fischer, M., & Gallmeyer, M. (2017). Taxable and tax-deferred investing with the limited use of losses. *Review of Finance*, *21*, 1847-1873.

Garlappi, L., & Huang, J. (2006). Are stocks desirable in tax-deferred accounts? *Journal of Public Economics*, *90*, 2257-2283.

Geisler, G., & Hulse, D. (2018). The effects of social security benefits and RMDs on tax-efficient withdrawal strategies. *Journal of Financial Planning*, *31*, 36-47.

Gerrans, P., Clark-Murphy, M., & Speelman, C. (2009). Asset allocation and age effects in retirement savings choices. *Accounting & Finance*, *50*, 301-319.

Kobor, A., & Muralidhar, A. (2020). Targeting retirement security with a dynamic asset allocation strategy. *Financial Analysts Journal*, *76*, 38-55.

Lange, J. (2009). *Retire Secure!* Hoboken, NJ: Wiley.

Lachance, M. (2013). Roth versus traditional accounts in a life-cycle model with tax risk. *Journal of Pension Economics and Finance*, *12*, 28-61.

Marekwica, M., Schaefer, A., & Sebastian, S. (2013). Life cycle asset allocation in the presence of housing and tax-deferred investing. *Journal of Economic Dynamics and Control*, *37*, 1110-1125.

Horan, S. M. (2006). Withdrawal location with progressive tax rates. *Financial Analysts Journal*, *62*, 77-87.

Huang, C. C., & Cho, C. (2017). Ten facts you should know about the Federal Estate Tax. *Center on Budget and Policy Priorities*, 1-7.

Meyer, W., & Reichenstein, W. (2013). The tax torpedo: Coordinating social security with a withdrawal strategy to minimize taxes. *Retirement Management Journal*, *3*, 25-32.

Milesvsky, M. (2020). *Retirement Income Recipes in R.* Switzerland: Springer.

Piper, M. (2013). *Can I Retire*? St. Louis, MO: Simple Subjects, LLC.

Reichenstein, W., & Meyer, W. (2018). Understanding the tax torpedo and its implications for various retirees. *Journal of Financial Planning*, *31*, 38-45.

Reichenstein, W., Horan, S. M., & Jennings, W. W. (2012). Two key concepts for wealth management and beyond. *Financial Analysts Journal*, *68*, 14-14.

Rodgers, R. (2009). *The New Three-Legged Stool.* Hoboken, NJ: Wiley.

Sialm, C. (2006). Stochastic taxation and asset pricing in dynamic general equilibrium. *Journal of Economic Dynamics and Control*, *30*, 511-540.

Solin, D. R. (2010). *The Smartest Retirement Book You'll Ever Read.* New York, NY: Perigee Trade.

Sumutka, A. R., Sumutka, A. M., & Coopersmith, L. W. (2012). Tax-efficient retirement withdrawal planning using a comprehensive tax model. *Journal of Financial Planning*, *25*, 41-52.

Poterba, J. M. (2014). Retirement security in an aging population. *American Economic Review*, *104*, 1-30.

Pfau, W., Tomlinson, J., & Vernon, S. (2017). *Optimizing Retirement Income by Integrating Retirement Plans, IRAs, and Home Equity.* Schaumburg, IL: Society of Actuaries. Available at https://www.soa.org/493461/globalassets/assets/files/resources/research-report/2017/2017-optimize-retire-income-plans-iras-equity.pdf

Ragsdale, C. T., Seila, A. F., & Little, P. L. (1994). An optimization model for scheduling withdrawals from tax deferred retirement accounts. *Financial Services Review*, *3*, 93-108.

Samuelson, P. A. (1969). Lifetime portfolio selection by dynamic stochastic programming. *The Review of Economics and Statistics*, *51*, 239-246.

Vanguard. (2016). *How to Turn Retirement Savings Into Retirement Income*. Valley Forge, PA: The Vanguard Group, Inc. Available at https://retirementplans.vanguard.com/segment/pdfs/RetIncShortBRO.pdf

Welch, J. S. (2016). Measuring the financial consequences of IRA to Roth IRA conversions. *Journal of Personal Finance*, *15*, 47-55.

Welch, J. S. (2017). A 3-step procedure for computing sustainable retirement savings withdrawals. *Journal of Financial Planning*, *30*, 45-55.

Zaman, A. A. (2008). Reconsidering IRA and Roth IRA Keeping bequests and other options in mind. *Journal of Wealth Management*, *11*, 82-92.

Zhou, J. (2009). The asset location puzzle: Taxes matter. *Journal of Economic Dynamics and Control*, *33*, 955-969.

Zhou, J. (2012). Life-cycle stock market participation in taxable and tax-deferred accounts. *Journal of Economic Dynamics and Control*, *36*, 1814-1829.