

# Low- $\beta$ investing with mutual funds

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## Abstract

Contrary to the predictions of CAPM, empirical research has shown that investing in low-beta stocks can improve the mean-variance efficiency of an investor's portfolio. Through forming portfolios of mutual funds based on beta, I examine whether or not mutual fund investors can capitalize on this puzzle. I find that one investing in a portfolio of funds in the top quintile of beta can improve her  $\alpha$  by a statistically significant 2.9% to 4.9% a year, depending on the asset pricing model specification, by holding a portfolio of funds in the bottom quintile of beta instead. © 2014 Academy of Financial Services. All rights reserved.

*JEL classification:* G11; G12; G23

*Keywords:* Mutual fund performance; Low risk stocks; CAPM; Market anomalies; Beta

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## 1. Introduction

The Capital Asset Pricing Model (CAPM) is founded on a simple and intuitive theory of how investors should be compensated for bearing systematic (market) risk, making it is the predominant asset pricing model taught in finance classes and used by practitioners (Association for Financial Professionals (2011), Brotherson, Eades, Harris, and Higgins (2013), Fernández (2013)). Despite the CAPM's theoretical appeal, a trilogy of empirical tests since the creation of the model have consistently shown that the beta-return relationship is flatter than that is predicted by the model.<sup>1</sup> In other words, market participants are undercompensated for bearing incremental market risk. Most perplexingly, some of the more recent

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studies have even revealed a negative and economically significant beta-return relationship (Baker, Bradley, and Wurgler (2011) and Blitz and Van Vliet (2007)). Borrowing constraints, tracking error constraints, irrational investor behavior, and beta estimation risk are some of the explanations that have been espoused for the CAPM's inability to predict returns. Baker, Bradley, and Wurgler (2011), Blitz and Van Vliet (2007), Falkenstein (2010), Fernández (2014), and Hodges, Taylor, and Yoder (2002) provide an excellent discussion of these explanations. At a more fundamental level, Fama and French (2004) attribute the failure of CAPM to a misspecification of the model.

Given that research has consistently shown that investors are undercompensated for bearing market risk, a simple strategy of investing in low-beta stocks can improve the mean-variance efficiency of one's portfolio. However, Domian, Louton, and Racine (2007) show that one must own over 100 stocks to minimize nonsystematic risk. However, according to the Federal Reserve Board's 2013 Survey of Consumer Finances, the median family with financial assets holds only \$21,200 in financial assets. Therefore, it is quite expensive for most individuals to directly own an adequately diversified portfolio of individual stocks, making mutual funds a more attractive candidate for investment. This motivates the purpose of this article, to explore the performance of a strategy of investing in low-beta mutual funds.<sup>2</sup>

To explore the possibility that a low-beta investment strategy can be effectively implemented with mutual funds, I sort funds into portfolios based on quintile rank of beta and compare the performance of the portfolios. The main finding of this empirical study is that each of the portfolios exhibit similar levels of return yet those comprised of lower beta funds are less risky. The practical implication of this study is that a simple strategy of investing in low-beta mutual funds improves the mean-variance efficiency of an investor's portfolio.

## **2. Performance of low-beta funds**

### *2.1. The samples*

To evaluate the performance of low-beta mutual funds, I obtain monthly net-of-expense returns and total net assets (TNA) from Morningstar Direct's survivor-bias-free United States Mutual Funds database on all open-end equity funds classified by Morningstar as having a U.S. broad asset class of "U.S. Stock."<sup>3</sup> The sample excludes funds classified by Morningstar as "Index Funds" or "Enhanced Index Funds." Morningstar Direct is the most complete and timely database offered by Morningstar, Inc., a leading provider of mutual fund data. Monthly returns on share classes are aggregated to the portfolio level by weighting them by their TNA as of the end of the previous month.

I estimate rolling betas for each mutual fund over the prior 60 months using a CAPM regression of the excess returns on each fund against the excess returns on Fama and French's value-weighted portfolio of U.S. stocks. Data on market returns and a one-month Treasury bill rate proxy for the risk-free rate is gathered from Kenneth French's Web site.<sup>4</sup> To better reflect application of the CAPM by practitioners, I also use the 10-year treasury

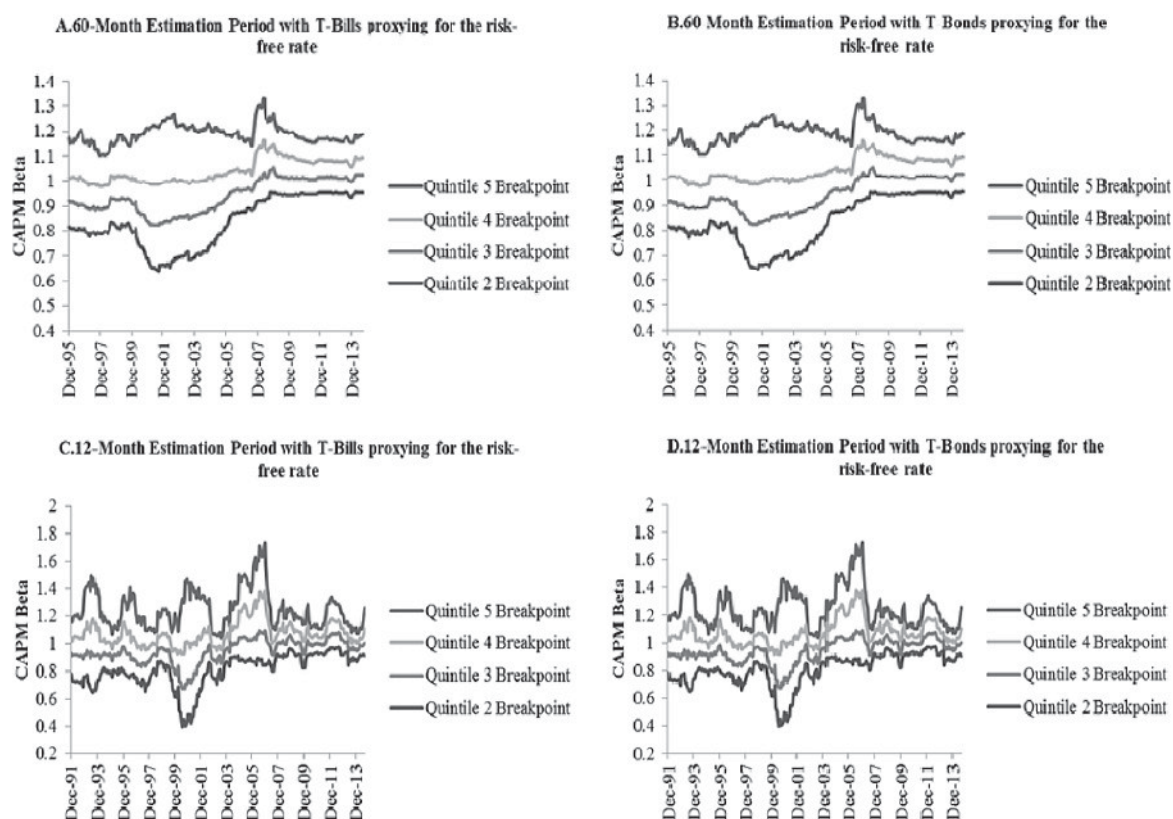


Fig. 1. Time plots of quintile breakpoints of mutual fund betas. Graph A plots the quintile breakpoints of U.S. Stock mutual fund betas derived over a 24–60 month (as available) estimation period using the one-month Treasury bill rate to proxy for the risk-free rate. Graph B does the same but uses the 10-year Treasury note rate to proxy for the risk-free rate. Graph C plots the quintile breakpoints of betas derived over a 12 month estimation period using the one-month Treasury bill rate to proxy for the risk-free rate. Graph D does the same but uses the 10-year Treasury note rate to proxy for the risk-free rate. The returns on the stock market and T-Bill rates from January 1991 through August 2014 are from Kenneth French’s Web site. The 10-year treasury rates are from the St. Louis Fed.

constant maturity rate (not seasonally adjusted) from the Federal Reserve Bank of St. Louis as an alternative proxy for the risk-free rate.<sup>5</sup>

Funds with less than 24 months of returns over the estimation period are discarded. I then sort the funds into five portfolios based on their quintile-rank of beta and compute the TNA-weighted returns on each of the five portfolios over the next month. I then repeat this process in each of the following months to arrive at a time-series of 225 monthly returns on the five beta-sorted portfolios. I also construct a time-series of TNA-weighted returns on a “Universal” portfolio consisting of the all of the funds that comprise the beta-sorted portfolios. To span the spectrum of beta estimation periods that are commonly used by practitioners, I also examine the performance of portfolios based on beta calculated from 12 months of returns.<sup>6</sup> A time plot illustrating the quintile breakpoints of beta is provided in Fig. 1.

The time period of the study was January 1991 through August 2014. January 1991 was chosen as the initial month of the study because the number of share classes with monthly

Table 1 Main results by quintile of beta derived over a 60-month estimation period

|   | Low    | 2      | 3      | 4      | High   | Universal |
|---|--------|--------|--------|--------|--------|-----------|
| Panel A: T-bills proxy for the risk-free rate |        |        |        |        |        |           |
| Average $R_p - R_f$                           | 6.44%  | 6.74%  | 6.59%  | 7.02%  | 6.27%  | 6.28%     |
| <i>SD</i>                                     | 13.47% | 14.75% | 16.05% | 17.93% | 21.59% | 16.15%    |
| Skewness                                      | -0.82  | -0.78  | -0.74  | -0.66  | -0.49  | -0.76     |
| Kurtosis                                      | 1.85   | 1.68   | 1.43   | 1.25   | 1.11   | 1.29      |
| Sharpe ratio                                  | 0.48   | 0.46   | 0.41   | 0.39   | 0.29   | 0.39      |
| $M^2$ measure                                 | 0.48%  | 0.15%  | -0.59% | -0.91% | -2.52% | -0.95%    |
| Average $R_p - R_m$                           | -0.74% | -0.44% | -0.58% | -0.16% | -0.90% | -0.90%    |
| Tracking error                                | 6.57%  | 4.10%  | 2.23%  | 3.98%  | 8.40%  | 1.84%     |
| Information ratio                             | -0.11  | -0.11  | -0.26  | -0.04  | -0.11  | -0.49     |
| Beta  | 0.77   | 0.89   | 0.99   | 1.10   | 1.27   | 1.00      |
| $\alpha$                                      | 0.92%  | 0.35%  | -0.52% | -0.84% | -2.85% | -0.91%    |
| $t(\alpha)$                                   | 0.72   | 0.40   | -1.00  | -0.98  | -1.69  | -2.11     |
| $R^2$   | 0.84   | 0.94   | 0.98   | 0.96   | 0.89   | 0.99      |
| Average cash holdings                         | 5.62%  | 4.42%  | 3.72%  | 4.18%  | 3.42%  | 4.35%     |
| Average turnover                              | 46.06% | 47.85% | 64.69% | 68.22% | 86.26% | 58.87%    |
| Average expense ratio                         | 0.91%  | 0.87%  | 0.93%  | 1.00%  | 1.11%  | 0.95%     |
| Average idiosyncratic volatility              | 12.11% | 11.29% | 11.47% | 12.85% | 15.67% | 12.42%    |
| Panel B: T-bonds proxy for the risk-free rate |        |        |        |        |        |           |
| Average $R_p - R_f$                           | 4.75%  | 5.04%  | 4.86%  | 5.35%  | 4.59%  | 4.59%     |
| <i>SD</i>                                     | 13.48% | 14.75% | 16.03% | 17.94% | 21.58% | 16.15%    |
| Skewness                                      | -0.82  | -0.77  | -0.74  | -0.65  | -0.50  | -0.76     |
| Kurtosis                                      | 1.87   | 1.67   | 1.44   | 1.23   | 1.11   | 1.30      |
| Sharpe ratio                                  | 0.35   | 0.34   | 0.30   | 0.30   | 0.21   | 0.28      |
| $M^2$ measure                                 | 0.25%  | 0.08%  | -0.54% | -0.62% | -1.99% | -0.84%    |
| Average $R_p - R_m$                           | -0.74% | -0.45% | -0.63% | -0.14% | -0.90% | -0.90%    |
| Tracking error                                | 6.55%  | 4.13%  | 2.22%  | 4.02%  | 8.37%  | 1.84%     |
| Information ratio                             | -0.11  | -0.11  | -0.28  | -0.03  | -0.11  | -0.49     |
| Beta  | 0.77   | 0.89   | 0.99   | 1.10   | 1.27   | 1.00      |
| $\alpha$                                      | 0.52%  | 0.15%  | -0.58% | -0.66% | -2.38% | -0.91%    |
| $t(\alpha)$                                   | 0.41   | 0.18   | -1.12  | -0.76  | -1.43  | -2.11     |
| $R^2$   | 0.84   | 0.94   | 0.98   | 0.96   | 0.89   | 0.99      |
| Average cash holdings                         | 5.63%  | 4.43%  | 3.72%  | 4.18%  | 3.41%  | 4.35%     |
| Average turnover                              | 45.99% | 47.82% | 64.82% | 68.11% | 86.46% | 58.87%    |
| Average expense ratio                         | 0.91%  | 0.87%  | 0.93%  | 1.00%  | 1.11%  | 0.95%     |
| Average idiosyncratic volatility              | 12.12% | 11.30% | 11.47% | 12.84% | 15.67% | 12.43%    |

Panel A displays performance metrics and portfolio characteristics for TNA-weighted portfolios of U.S. Stock mutual funds reconstituted monthly based on quintile ranking of trailing beta derived over a 24–60 month (as available) estimation period. The excess returns on the stock market and the risk-free rate,  $R_f$ , from January 1991 through August 2014 are from Kenneth French's Web site. Returns are annualized through multiplying monthly values by 12. *SDs* are annualized through multiplying monthly values by the square root of 12. Average cash holdings, turnover ratios, expense ratios, and idiosyncratic volatilities of funds that constitute the portfolios are reported as time-series means of the cross-sectional TNA-weighted means. Idiosyncratic volatilities are derived over a 24-month estimation period. Panel B does the same but uses the 10-year Treasury note rate, gathered from the St. Louis Fed, rather than the one-month Treasury bill rate to proxy for  $R_f$ .

TNA data (reported at month-end) from Morningstar Direct increased from 32 to 757 in December 1990.<sup>7</sup> The number of mutual fund portfolios in the Universal portfolio, consisting

of all the funds that comprise the beta-sorted portfolios formed over the 60-month estimation period, increased from 853 to 2,220 over the life of the portfolio. The average number of funds held in the portfolio was 1,993. The initial, final, and average number of funds in the Universal portfolio formed over the 12-month estimation period was 374, 1,987, and 1,612, respectively.

## 2.2. 60-Month estimation period results

Panel A of Table 1 displays the performance and characteristics of portfolios that are constituted based on beta calculated over the 60-month estimation period and the use of the one-month Treasury bill rate as a proxy for the risk-free rate. There is little difference in the average returns across the beta-sorted portfolios yet the betas are monotonically increasing across the portfolios, from 0.77 for the bottom quintile portfolio to 1.27 for the top quintile portfolio. The same pattern is apparent in the annualized *SD* of portfolio returns, which are monotonically increasing across the portfolios from 13.47% to 21.59%. This results in Sharpe ratios and  $M^2$  measures that are globally decreasing across the portfolios, from 0.48 to 0.29 and from 0.48% to  $-2.52\%$ , respectively. These findings imply that mutual fund investors can improve their mean-variance efficiency through investing in low-beta funds. Fig. 2 depicts time plots of wealth, which illustrate the improvements in mean-variance efficiency.

A comparison of the empirical beta-return relationship with that is predicted by the CAPM is illustrated in Fig. 3. The flat empirical relationship results in  $\alpha$ s that are monotonically decreasing across the portfolios. The bottom quintile portfolio outperforms the top quintile portfolio by 3.77% per a year based on  $\alpha$ . Moreover, an independent group *t* test shows that the difference in  $\alpha$ s is statistically significant (*p*-value of 0.07).

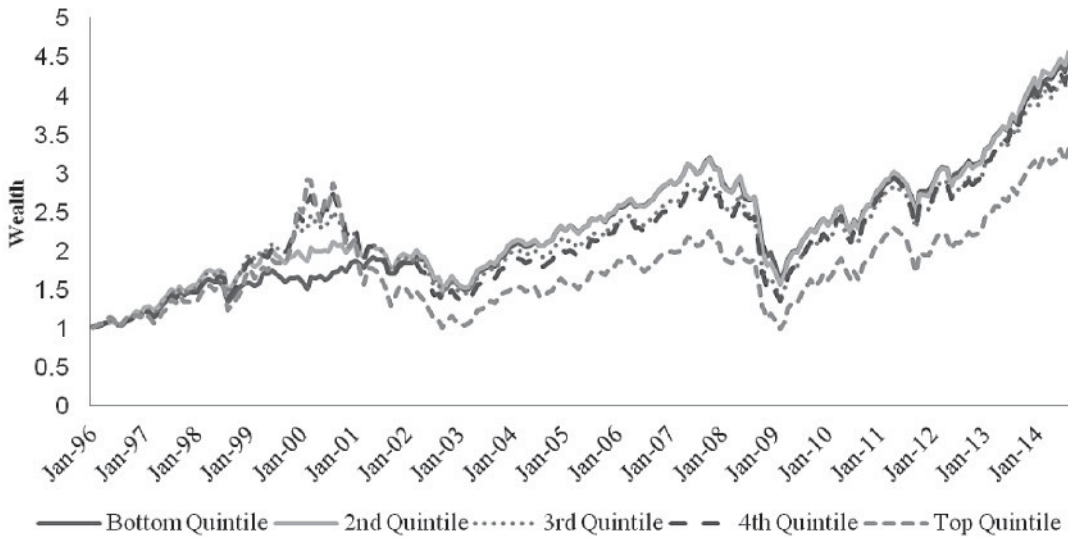
It is important to address the possibility that the betas of the portfolios are driven by cash holdings rather than the betas of stocks held in the constituent funds. If this is the case, then investors seeking to indirectly hold low beta stocks through investing in low beta funds may instead acquire an excessive allocation towards risk-free assets. However, average cash holdings are rather homogeneous among the portfolios, ranging from 3.42% (top quintile) to 5.62% (bottom quintile).<sup>8</sup> Through a “back of the envelope” calculation, one can arrive at what the beta on a portfolio would be if it did not hold any cash. The calculation is as follows:

$$\beta_{i\_no\_cash} = \beta_i / (1 - CASH_i), \quad (1)$$

where  $\beta_{i\_no\_cash}$  denotes what the beta on a portfolio would be if it did not hold any cash,  $\beta_i$  denotes the beta given its actual time-series of returns, and  $CASH_i$  denotes the actual percentage of its assets (in decimal form) that are allocated to cash.

This back of the envelope calculation reveals that the bottom quintile portfolio’s beta would still be lower than that of any other portfolio even if it did not hold any cash ( $0.77/(1 - 0.0562) = 0.82$ ). In summary, the betas of the portfolios are mainly driven by the betas of stocks held by funds in the portfolios rather than cash exposures, assuaging concerns of an undesirable effect on an investor’s allocation to risk-free assets.<sup>9</sup>

Graph A: T-Bills Proxy for the Risk-Free Rate



Graph B: T-Bonds Proxy for the Risk-Free Rate

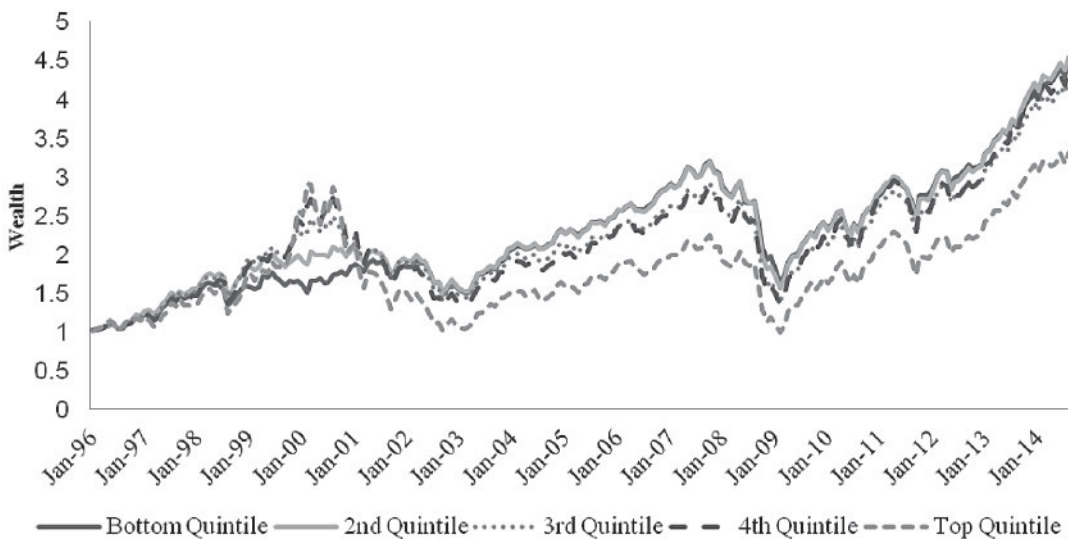
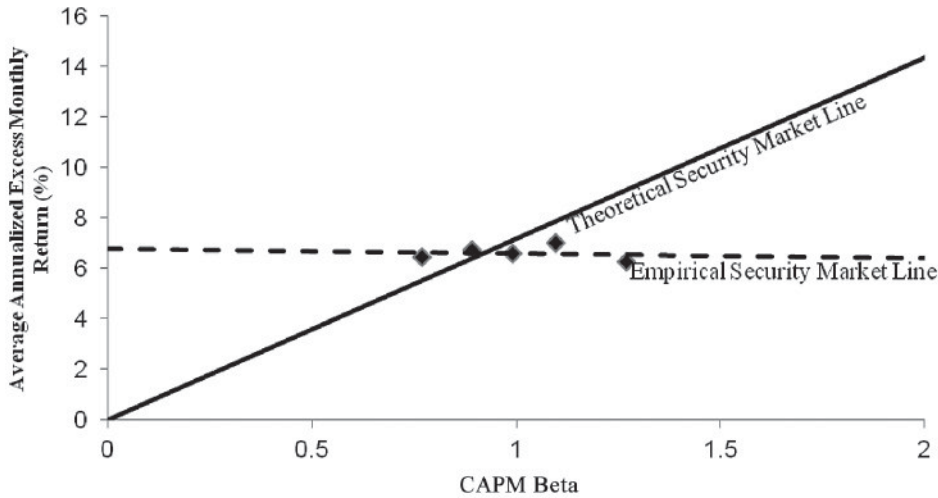


Fig. 2. Wealth: 60-month estimation period results. This figure plots the growth of one dollar invested on January 1, 1996 in TNA-weighted portfolios of U.S. Stock mutual funds reconstituted monthly based on quintile ranking of trailing beta derived over a 24–60 month (as available) estimation period. The returns on the stock market are from Kenneth French’s Web site. The risk-free rate is represented by one-month T-Bills rates, from Kenneth French’s Web site, in Graph A and 10-year treasury rates, from the St. Louis Fed, in Graph B.

A related concern is that the funds in the bottom quintile portfolio tend to have high idiosyncratic risk. If this is the case, then mutual fund investors seeking to use a low-beta investment strategy may inadvertently acquire an excessively concentrated portfolio of risky assets. To address this concern, I calculate the average idiosyncratic volatility of the funds in each of the five beta-sorted portfolios. Specifically, I estimate the *SD* of the error term from a CAPM regression of the excess returns on each fund against the excess returns on

Graph A: T-Bills Proxy for the Risk-Free Rate



Graph B: T-Bonds Proxy for the Risk-Free Rate

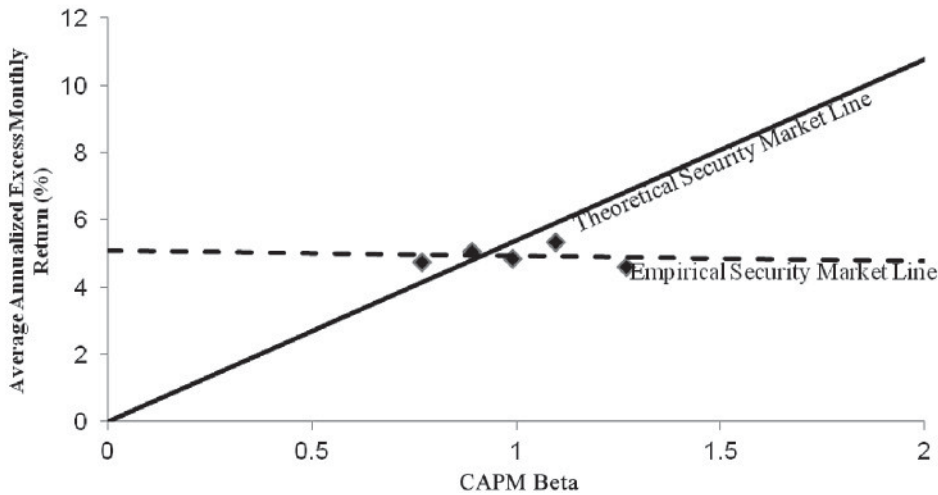


Fig. 3. Empirical versus theoretical security market line: 60-month estimation period results. This figure plots the average excess return and out-of-sample beta of TNA-weighted portfolios of U.S. Stock mutual funds reconstituted monthly based on quintile ranking of trailing beta derived over a 24–60 month (as available) estimation period. The returns on the stock market from January 1991 through August 2014 are from Kenneth French’s Web site. The risk-free rate is represented by one-month T-Bills rates, from Kenneth French’s Web site, in Graph A and 10-year treasury rates, from the St. Louis Fed, in Graph B. The figure contrasts the return-beta relationship with that which would be predicted by CAPM given the average excess return on the stock market.

Fama and French’s value-weighted portfolio of U.S. stocks over the prior 24 months. I do this for each fund in each month. Then for each of the five portfolios I examine the time-series means of the cross-sectional TNA-weighted mean values of idiosyncratic volatility for the constituent funds. Put more formally, it is defined as follows:

$$\text{Average idiosyncratic volatility} = \frac{\sum_{t=1}^T \sum_{i=1}^N \frac{TNA_{i,t}}{\sum_{i=1}^N TNA_{i,t}} \sigma(\varepsilon_i|t)}{T}. \quad (2)$$

The results show considerable homogeneity in the average idiosyncratic volatilities across the five portfolios as they range from 11.29% (quintile 2) to 15.67% (top quintile). Moreover, the average idiosyncratic volatility of the funds in the bottom quintile portfolio (12.11%) is less than that of the TNA-weighted universe (12.42%), assuaging concerns of an undesirable effect on an investor's level of portfolio diversification.

For robustness, I use the 10-year Treasury note rate rather than the one-month Treasury bill rate as a proxy for the risk-free rate in estimating betas of funds and evaluating the performance of beta-sorted portfolios. Because the yield curve is generally upward sloping, the Sharpe ratios across all portfolios are lower than those observed with the one-month T-bill proxy. As displayed in Panel B of Table 1, the results are otherwise rather robust to this long-term bond proxy for the risk-free rate. As displayed in the Appendix Table, the results are also robust to the use of CRSP's Total Return Value-Weighted Index as an alternative proxy for the market return. However, it should be noted that the analysis involving the CRSP index was restricted to January 1991 through December 2013 because of data availability constraints.

### 2.3. 12-Month estimation period results

Panel A of Table 2 conveys the performance and characteristics of portfolios that are constituted based on 12-month betas and the use of the one-month Treasury bill rate as a proxy for the risk-free rate. The results are largely consistent with those of the longer beta estimation period, marked by little differences in average returns but globally increasing betas across the portfolios, ranging from 0.78 (bottom quintile) to 1.28 (top quintile). An illustration of this in mean-beta space is provided in Fig. 4. The annualized *SD* of portfolio returns is also monotonically increasing across the portfolios, from 12.58% to 20.46%. The sharp rise in *SDs* combined with the stable returns across the portfolios results in Sharpe ratios that are monotonically decreasing across the portfolios, from 0.56 to 0.31. The differences in Sharpe ratios are economically meaningful as the  $M^2$  measures decrease from 0.76% to  $-2.93\%$  across the portfolios. Fig. 5 depicts time plots of wealth, which illustrate the improvements in mean-variance efficiency obtained through investing in low-beta funds.

The  $\alpha$ s are monotonically decreasing across the portfolios from 1.13% to  $-3.37\%$  per year. This is because, as illustrated in Fig. 4, the relationship between beta and return is slightly negative. An independent group *t* test reveals that the difference in  $\alpha$ s between the bottom and top quintile portfolios are highly statistically significant (*p*-value of 0.01).

Consistent with the results obtained over the 60-month beta estimation period, the average cash holdings, expense ratio, and (fund-level) idiosyncratic volatility of funds in the bottom quintile portfolio are similar to the general population of funds, represented by the Universal

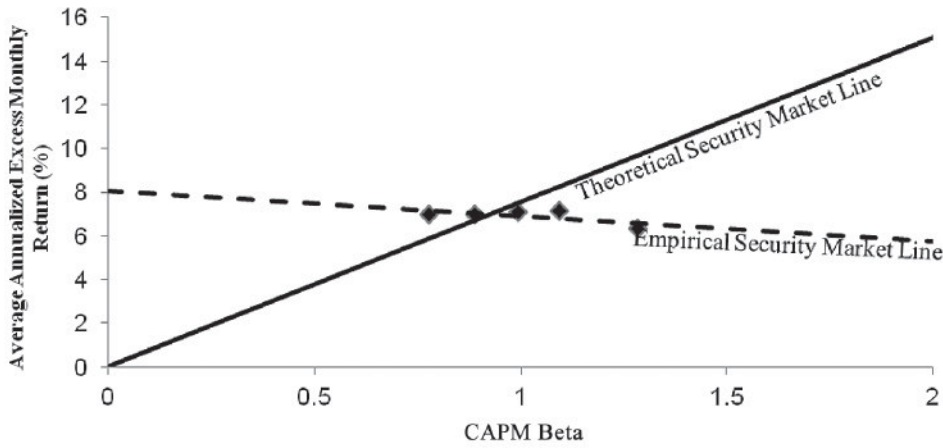


Table 2 Main results by quintile of beta derived over a 12-month estimation period

|   | Low    | 2      | 3      | 4      | High   | Universal |
|---|--------|--------|--------|--------|--------|-----------|
| Panel A: T-bills proxy for the risk-free rate |        |        |        |        |        |           |
| Average $R_p - R_f$                           | 6.99%  | 6.98%  | 7.07%  | 7.14%  | 6.32%  | 6.76%     |
| <i>SD</i>                                     | 12.58% | 13.69% | 14.99% | 16.66% | 20.46% | 15.06%    |
| Skewness                                      | -0.92  | -0.84  | -0.81  | -0.72  | -0.55  | -0.82     |
| Kurtosis                                      | 2.40   | 2.13   | 1.91   | 1.60   | 1.51   | 1.75      |
| Sharpe ratio                                  | 0.56   | 0.51   | 0.47   | 0.43   | 0.31   | 0.45      |
| $M^2$ measure                                 | 0.76%  | 0.07%  | -0.50% | -1.14% | -2.93% | -0.84%    |
| Average $R_p - R_m$                           | -0.55% | -0.56% | -0.47% | -0.40% | -1.22% | -0.78%    |
| Tracking error                                | 5.85%  | 3.74%  | 2.17%  | 3.59%  | 8.33%  | 1.84%     |
| Information ratio                             | -0.09  | -0.15  | -0.22  | -0.11  | -0.15  | -0.42     |
| Beta  | 0.78   | 0.89   | 0.99   | 1.09   | 1.28   | 1.00      |
| $\alpha$                                      | 1.13%  | 0.27%  | -0.42% | -1.11% | -3.37% | -0.79%    |
| $t(\alpha)$                                   | 1.10   | 0.39   | -0.92  | -1.57  | -2.21  | -2.01     |
| $R^2$   | 0.85   | 0.94   | 0.98   | 0.96   | 0.88   | 0.99      |
| Average cash holdings                         | 6.18%  | 4.64%  | 3.96%  | 4.30%  | 3.62%  | 4.68%     |
| Average turnover                              | 50.28% | 56.88% | 62.97% | 72.90% | 84.91% | 62.24%    |
| Average expense ratio                         | 0.93%  | 0.88%  | 0.92%  | 1.00%  | 1.11%  | 0.95%     |
| Average idiosyncratic volatility              | 12.04% | 11.10% | 11.42% | 12.64% | 15.60% | 12.21%    |
| Panel B: T-bonds proxy for the risk-free rate |        |        |        |        |        |           |
| Average $R_p - R_f$                           | 5.16%  | 5.14%  | 5.21%  | 5.36%  | 4.42%  | 4.92%     |
| <i>SD</i>                                     | 12.58% | 13.70% | 14.99% | 16.66% | 20.45% | 15.06%    |
| Skewness                                      | -0.92  | -0.85  | -0.81  | -0.72  | -0.55  | -0.82     |
| Kurtosis                                      | 2.40   | 2.13   | 1.91   | 1.59   | 1.50   | 1.75      |
| Sharpe ratio                                  | 0.41   | 0.38   | 0.35   | 0.32   | 0.22   | 0.33      |
| $M^2$ Measure                                 | 0.53%  | 0.01%  | -0.40% | -0.79% | -2.36% | -0.71%    |
| Average $R_p - R_m$                           | -0.54% | -0.56% | -0.49% | -0.34% | -1.28% | -0.78%    |
| Tracking error                                | 5.85%  | 3.75%  | 2.16%  | 3.59%  | 8.32%  | 1.84%     |
| Information ratio                             | -0.09  | -0.15  | -0.23  | -0.1   | -0.15  | -0.42     |
| Beta  | 0.78   | 0.89   | 0.99   | 1.09   | 1.28   | 1.00      |
| $\alpha$                                      | 0.72%  | 0.07%  | -0.45% | -0.88% | -2.89% | -0.78%    |
| $t(\alpha)$                                   | 0.71   | 0.10   | -0.99  | -1.26  | -1.91  | -2.01     |
| $R^2$   | 0.85   | 0.94   | 0.98   | 0.96   | 0.88   | 0.99      |
| Average cash holdings                         | 6.18%  | 4.67%  | 3.92%  | 4.31%  | 3.62%  | 4.68%     |
| Average turnover                              | 50.29% | 56.93% | 63.14% | 72.67% | 85.02% | 62.24%    |
| Average expense ratio                         | 0.93%  | 0.88%  | 0.92%  | 1.00%  | 1.11%  | 0.95%     |
| Average idiosyncratic volatility              | 12.03% | 11.11% | 11.42% | 12.64% | 15.60% | 12.22%    |

Panel A displays performance metrics and portfolio characteristics for TNA-weighted portfolios of U.S. Stock mutual funds reconstituted monthly based on quintile ranking of trailing beta derived over a 12 month estimation period. The excess returns on the stock market and the risk-free rate,  $R_f$ , from January 1991 through August 2014 are from Kenneth French's Web site. Returns are annualized through multiplying monthly values by 12. *SDs* are annualized through multiplying monthly values by the square root of 12. Average cash holdings, turnover ratios, expense ratios, and idiosyncratic volatilities of funds that constitute the portfolios are reported as time-series means of the cross-sectional TNA-weighted means. Idiosyncratic volatilities are derived over a 24-month estimation period. Panel B does the same but uses the 10-year Treasury note rate, gathered from the St. Louis Fed, rather than the one-month Treasury bill rate to proxy for  $R_f$ .

Graph A: T-Bills Proxy for the Risk-Free Rate



Graph B: T-Bonds Proxy for the Risk-Free Rate

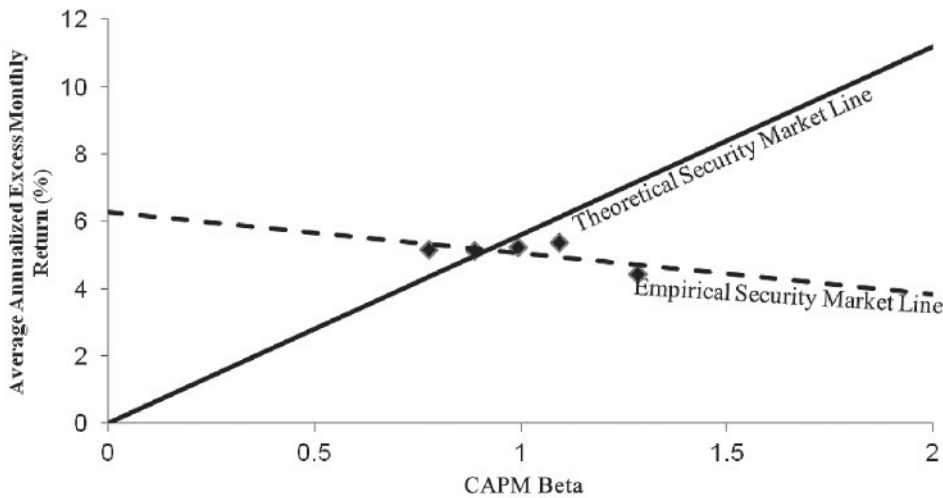
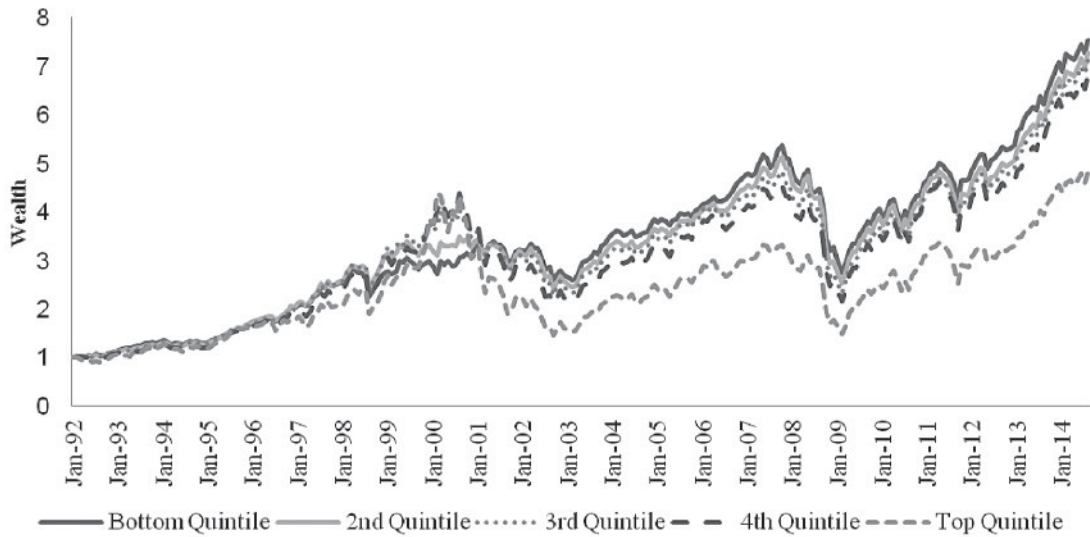


Fig. 4. Empirical versus theoretical security market line: 12-month estimation period results. This figure plots the average excess return and out-of-sample beta of TNA-weighted portfolios of U.S. Stock mutual funds reconstituted monthly based on quintile ranking of trailing beta derived over a 12 month estimation period. The returns on the stock market from January 1991 through August 2014 are from Kenneth French’s Web site. The risk-free rate is represented by one-month T-Bills rates, from Kenneth French’s Web site, in Graph A and 10-year treasury rates, from the St. Louis Fed, in Graph B. Returns are annualized through multiplying monthly values by 12. The figure contrasts the return-beta relationship with that which would be predicted by CAPM given the average excess return on the stock market.

portfolio.<sup>10</sup> Moreover, there is little variation in these characteristics across the beta-sorted portfolios. In summary, the betas of the portfolios are not driven by cash holdings and risk-adjusted performance is monotonically decreasing across the portfolios.

For robustness, I use the 10-year Treasury note rate as the proxy for the risk-free rate. The Sharpe ratios across all portfolios are lower than those observed with the one-month T-bill

Graph A: T-Bills Proxy for the Risk-Free Rate



Graph B: T-Bonds Proxy for the Risk-Free Rate

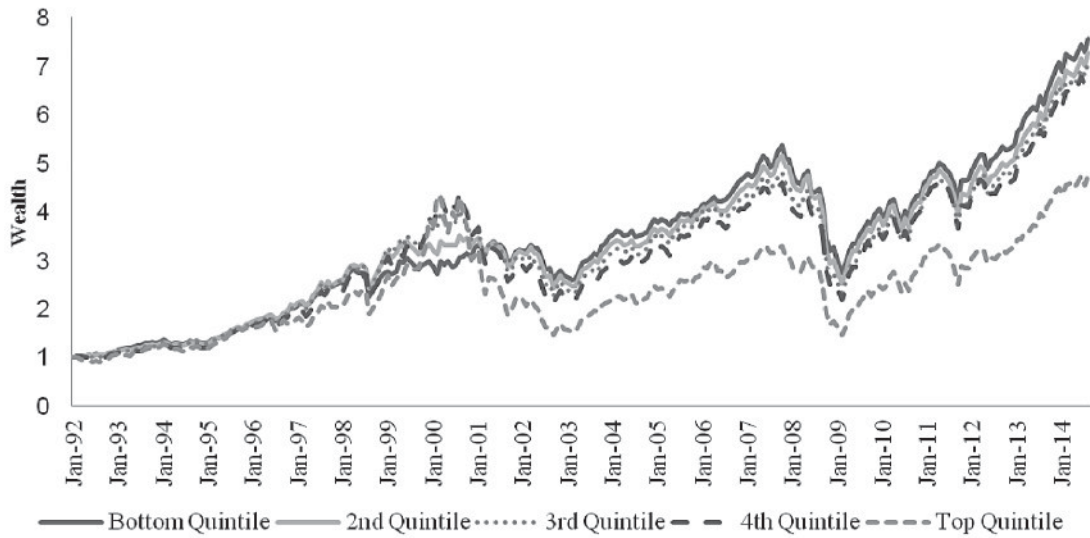


Fig. 5. Wealth: 12-month estimation period results. This figure plots the growth of one dollar invested on January 1, 1992 in TNA-weighted portfolios of U.S. Stock mutual funds reconstituted monthly based on quintile ranking of trailing beta derived over a 12 month estimation period. The returns on the stock market are from Kenneth French’s Web site. The risk-free rate is represented by one-month T-Bills rates, from Kenneth French’s Web site, in Graph A and 10-year treasury rates, from the St. Louis Fed, in Graph B.

proxy. As displayed in Panel B of Table 2, the results are otherwise broadly robust to this long-term bond proxy for the risk-free rate. As displayed in the Appendix Table, the results are also robust to the use of CRSP’s Total Return Value-Weighted Index as the proxy for the market return.

### 3. Persistence in beta exposure

Following much of the prior mutual funds literature, the aforementioned analysis assumes that investors can reconstitute their portfolios of mutual funds every month. However, tax issues and transactions costs likely make such frequent reconstitution activity infeasible. This motivates an analysis of the stability of mutual fund beta exposures over time and also the performance of beta-sorted portfolios that are reconstituted less frequently.

#### 3.1. Stability in rankings

As a “first stab” at addressing persistence in mutual fund beta exposure, I construct two contingency tables of initial and subsequent beta rankings. The height of the bars in Fig. 6 indicate the percentage of funds in quintile rank  $i$  of beta that are ranked in quintile  $j$  of beta 60 months later based on betas calculated over 60-month estimation periods. Fig. 7 conveys the percentage of funds in quintile rank  $i$  of beta that are ranked in quintile  $j$  of beta 12 months later based on betas calculated over 12-month estimation periods. Within Figs. 6 and 7, Table A uses the one-month Treasury bill rate as a proxy for the risk-free rate and Table B uses the 10-year Treasury note rate as a proxy for the risk-free rate.

The tables show that there is considerable persistence in beta exposure. For example, 51% of funds that rank in the lowest quintile of beta are subsequently ranked in that same quintile 60 months later. Moreover, 45% of funds in the lowest quintile of beta that do change rank transition to the second quintile of beta. The contingency tables show similar persistence within the other initial quintile ranks of beta as well.

#### 3.2. Time plots of beta ranking

To gain deeper insight into how mutual funds' beta exposures change over time I examine the percentage of funds initially ranked in quintile  $i$  of beta that are subsequently ranked in quintile  $j$  in each month from the 12th to the 60th after initial ranking based on betas calculated over 12-month estimation periods. I display the event time plots for each quintile  $i$  in separate graphs.

The graphs displayed in Fig. 8, pertaining to the use of the one-month Treasury bill rate as a proxy for the risk-free rate, further illustrate that the beta exposures of mutual funds are rather stable over time. For example, of the funds initially ranked in the lowest quintile of beta, 48% remained in that quintile 12 months later and 39% remained in it 60 months later. Moreover, of the funds initially ranked in the lowest quintile that transitioned to another quintile, 45% transitioned to the second quintile 12 months later and 35% transitioned to the second quintile 60 months later. The graphs displayed in Fig. 9, pertaining to the use of the 10-year Treasury note rate as a proxy for the risk-free rate, further illustrate that the beta exposures of mutual funds are rather stable over time.

Table A: T-Bills Proxy for the Risk-Free Rate

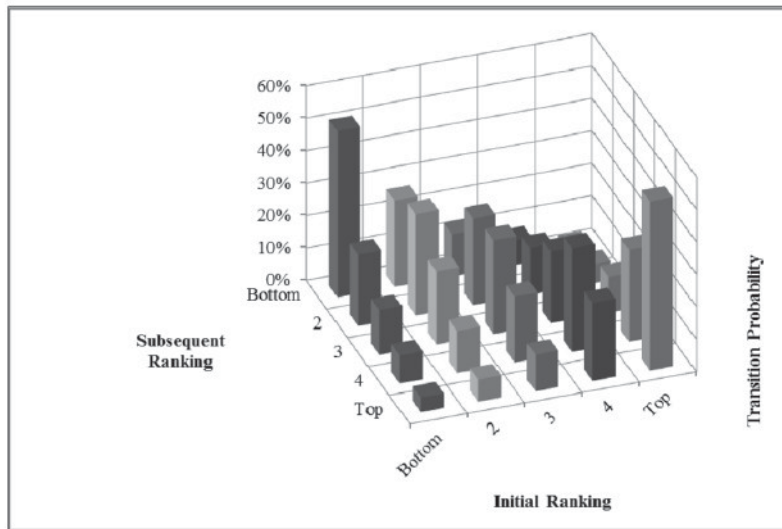


Table B: T-Bonds Proxy for the Risk-Free Rate

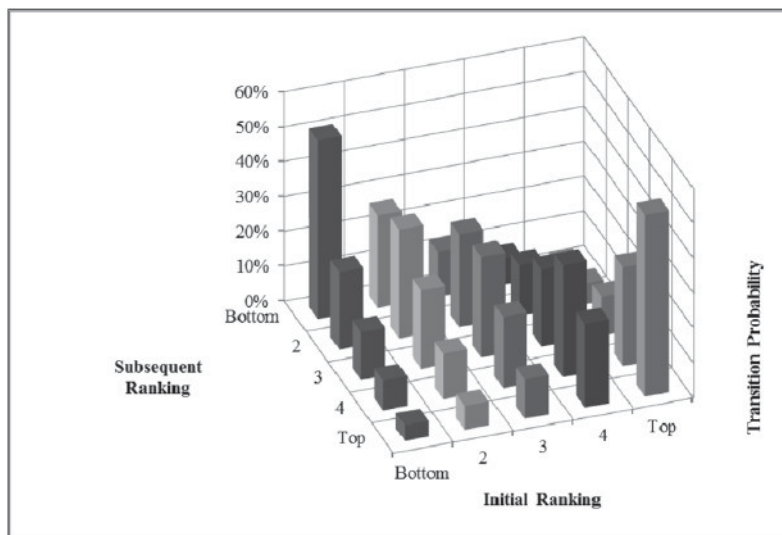
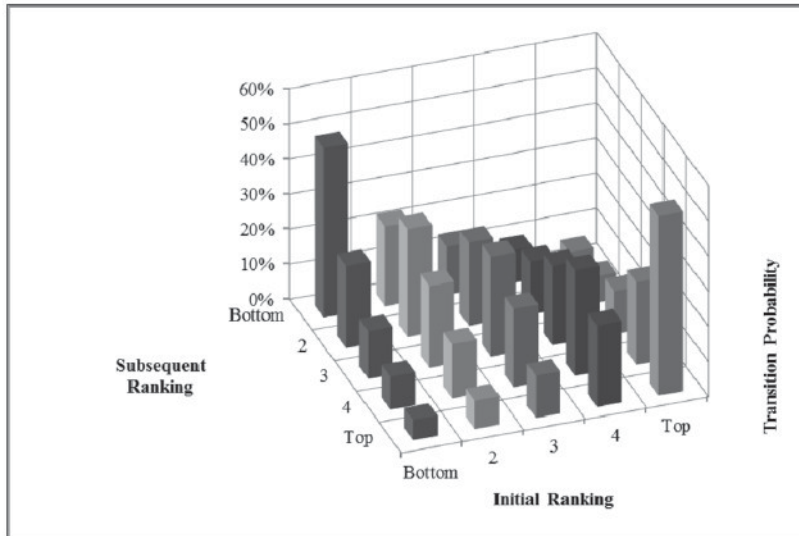


Fig. 6. Contingency tables of beta rankings: 60-month evaluation interval. The bars in the tables indicate the percentage of U.S. Stock mutual funds ranked in quintile  $i$  that are ranked in quintile  $j$  60 months later based on betas derived over a 24–60 month (as available) estimation period. The returns on the stock market from January 1991 through August 2014 are from Kenneth French’s Web site. Table A uses the one-month T-Bill rate, from Kenneth French’s Web site, to proxy for the risk-free rate and Table B uses the 10-year treasury rate, from the St. Louis Fed, to proxy for the risk-free rate.

### 3.3. Performance of portfolios with alternative reconstitution frequencies

The beta exposures of mutual funds tend to be rather stable over time. This suggests that the frequency at which mutual fund investors reconstitute their portfolios has little impact on the performance of a low-beta investment strategy. To examine this possibility, I construct

*Table A: T-Bills Proxy for the Risk-Free Rate*



*Table B: T-Bonds Proxy for the Risk-Free Rate*

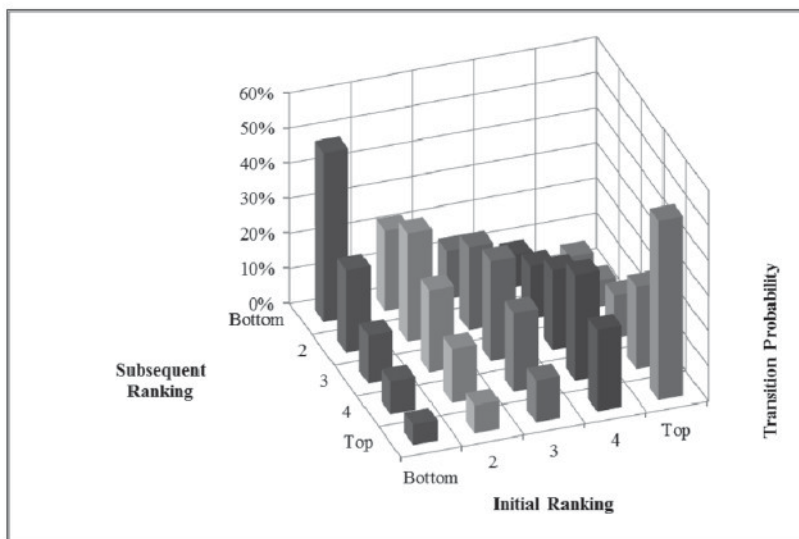


Fig. 7. Contingency tables of beta rankings: 12-month evaluation interval. The bars in the tables indicate the percentage of U.S. Stock mutual funds ranked in quintile *i* that are ranked in quintile *j* 12 months later based on betas derived over a 12 month estimation period. The returns on the stock market from January 1991 through August 2014 are from Kenneth French’s Web site. Table A uses the one-month T-Bill rate, from Kenneth French’s Web site, to proxy for the risk-free rate and Table B uses the 10-year treasury rate, from the St. Louis Fed, to proxy for the risk-free rate.

beta-sorted portfolios of mutual funds that are reconstituted at various frequencies, ranging from once a month to once every five years.

The graphs in Fig. 10 display the beta, average return, Sharpe ratio, and  $\alpha$  of portfolios constituted based on betas derived over a 60-month estimation period using one-month Treasury bill rates to proxy for the risk-free rate. The frequency of portfolio reconstitution ranges from once every month to once every 60 months. Graph A illustrates that betas

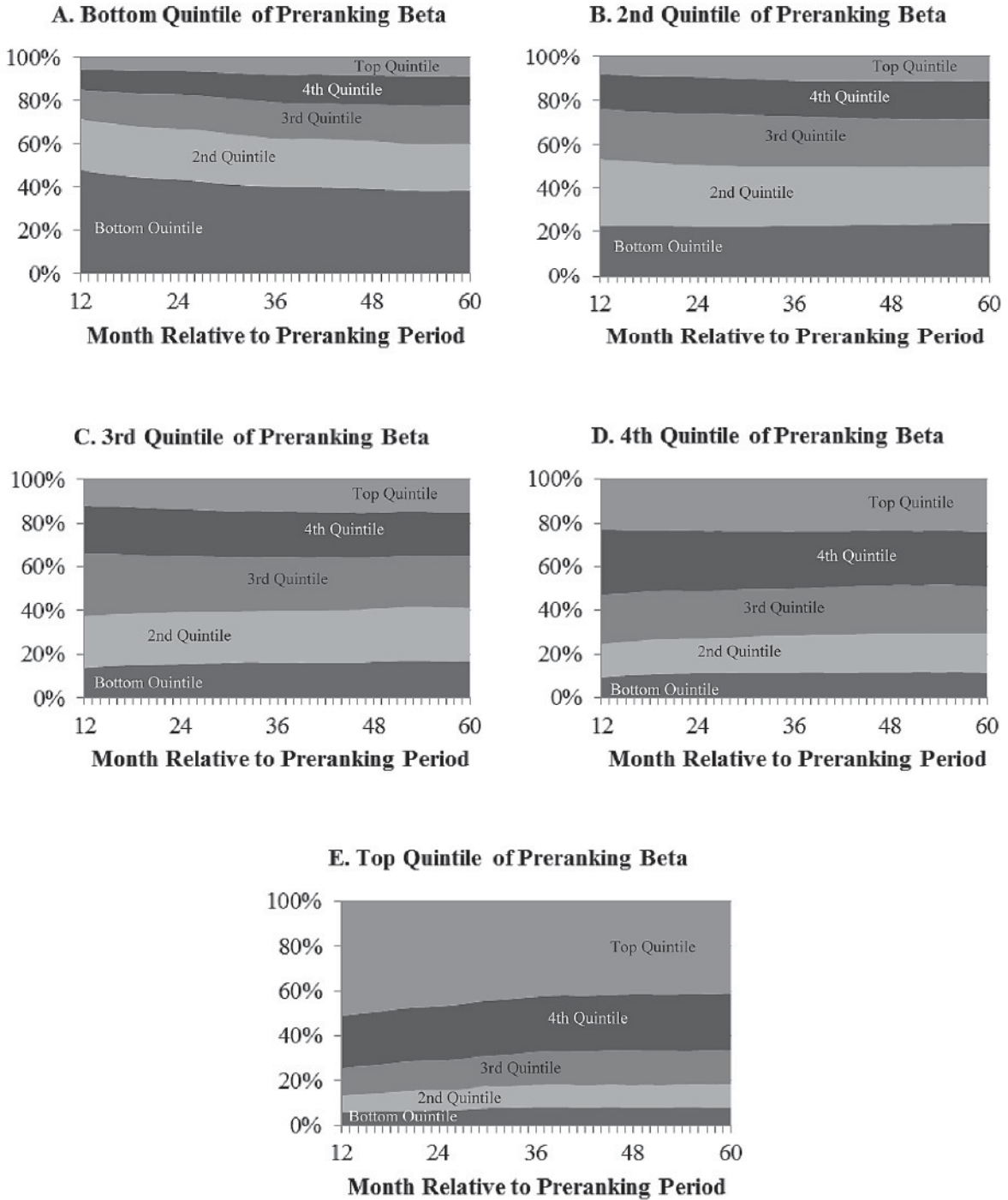


Fig. 8. Time plots of post-ranking beta quintiles by pre-ranking beta quintile: T-bill proxy for the risk-free rate. These graphs plot the percentage of U.S. Stock mutual funds in each month from  $t_{12}$  through  $t_{60}$  that are ranked in each quintile of trailing beta derived over a 12-month estimation period. Graphs A, B, C, D, and E pertain to funds in the bottom, second, third, fourth, and top quintile of beta in  $t_0$ , respectively. The excess returns on the stock market from January 1991 through August 2014 are from Kenneth French’s Web site.

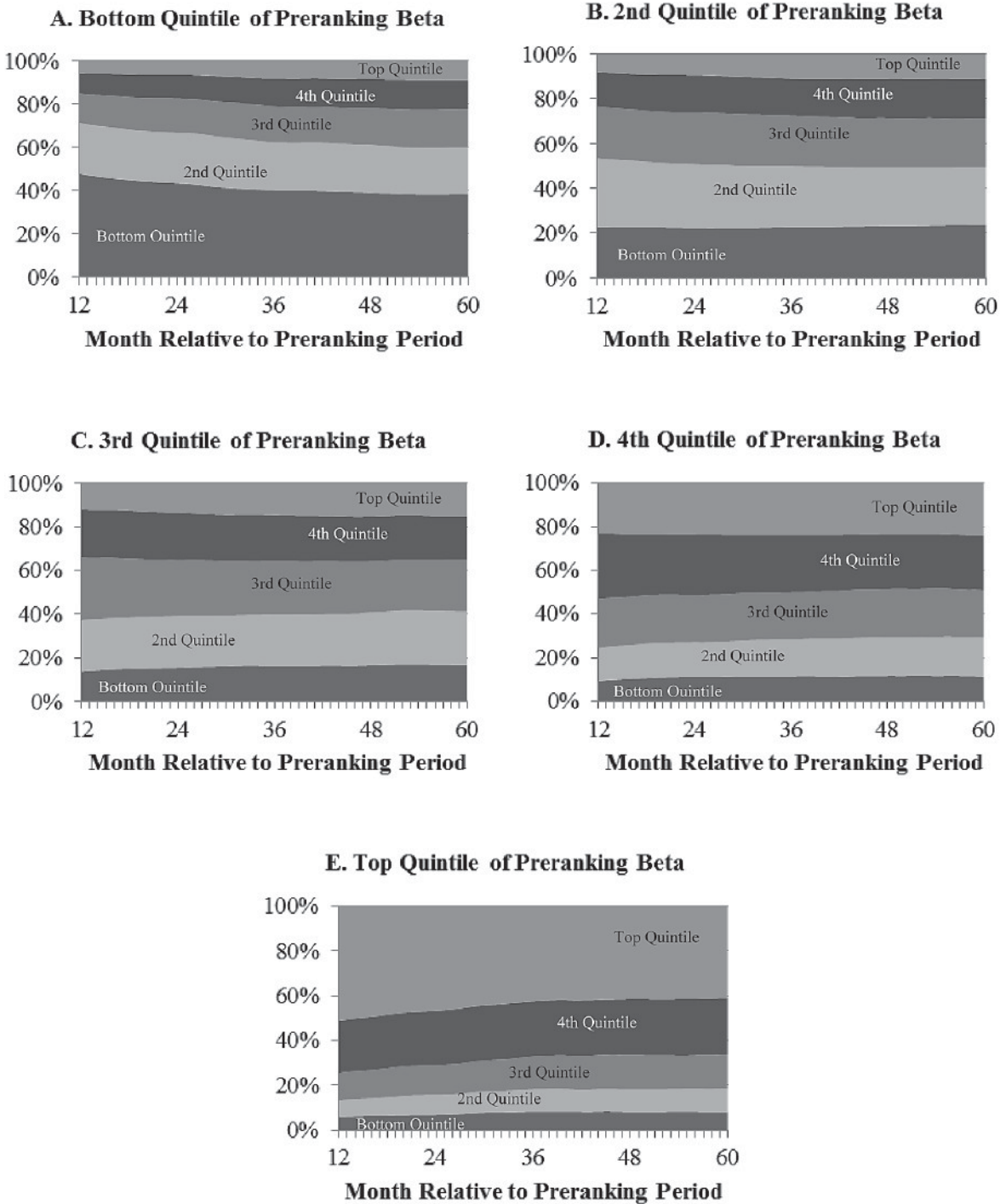


Fig. 9. Time plots of post-ranking beta quintiles by pre-ranking beta quintiles: T-bond proxy for the risk-free rate. These graphs plot the percentage of U.S. Stock mutual funds in each month from  $t_{12}$  through  $t_{60}$  that are ranked in each quintile of trailing beta derived over a 12-month estimation period. Graphs A, B, C, D, and E pertain to funds in the bottom, second, third, fourth, and top quintile of beta in  $t_0$ , respectively. The returns on the stock market from January 1991 through August 2014 are from Kenneth French's Web site. The commensurate rates on 10-year Treasury notes are from the St. Louis Fed.



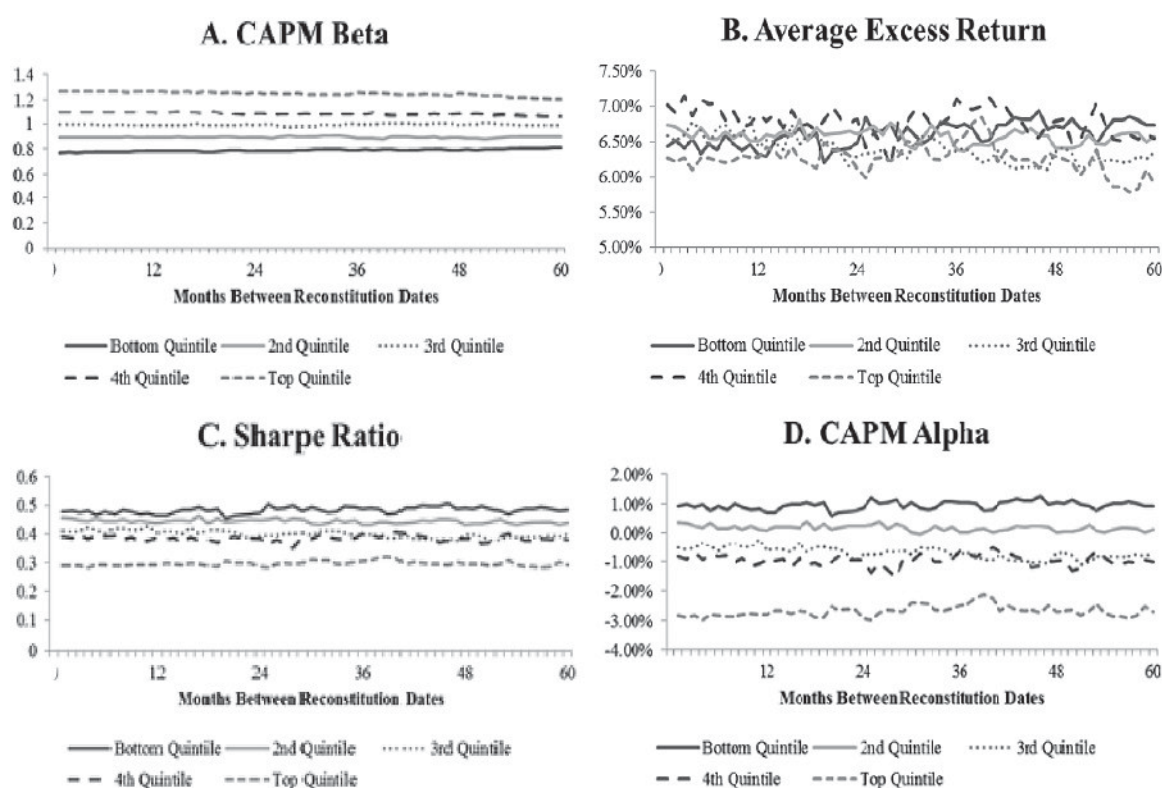


Fig. 10. Returns on beta-sorted portfolios reconstituted at low frequencies: 60-month estimation period results with T-bill proxy for the risk-free rate. These graphics display selected risk and performance metrics for TNA-weighted portfolios of U.S. Stock mutual funds constituted based on quintile ranking of trailing beta derived over a 24–60 month (as available) estimation period. The period between reconstitution dates ranges from 1 to 60 months. The excess returns on the stock market from January 1991 through August 2014 are from Kenneth French's Web site.

converge towards unity as the length of time between reconstitution dates expands. However, the differences in the betas on each of the portfolios across reconstitution frequency specifications are rather modest. For example, the beta of the bottom quintile portfolio that is reconstituted once every 60 months (0.81) is still lower than that of the second quintile portfolio that is reconstituted once every month (0.89). Moreover, the betas are monotonically increasing across the portfolios, regardless of the frequency of portfolio reconstitution. Unsurprisingly, there is little difference in the average return on each of the beta-sorted portfolios across all reconstitution frequency specifications.

The Sharpe ratios of the beta-sorted portfolios are rather stable across reconstitution frequency specifications, as illustrated in Graph C, and do not exhibit any relationship with the reconstitution frequency. For example, the Sharpe ratio of the bottom quintile portfolio reconstituted once every 60 months (0.48) is identical to that of one that is reconstituted once every month. The frequency of portfolio reconstitution also has little impact on the  $\alpha$ . For example, the annualized  $\alpha$  of the bottom quintile portfolio reconstituted every 60 months (0.92%) is the same as that of one that is reconstituted every month. Similarly, the differential in annualized  $\alpha$ s between the bottom and top quintile portfolios reconstituted once every 60 months (3.65%) is very close to that which is observed when the portfolios are reconstituted

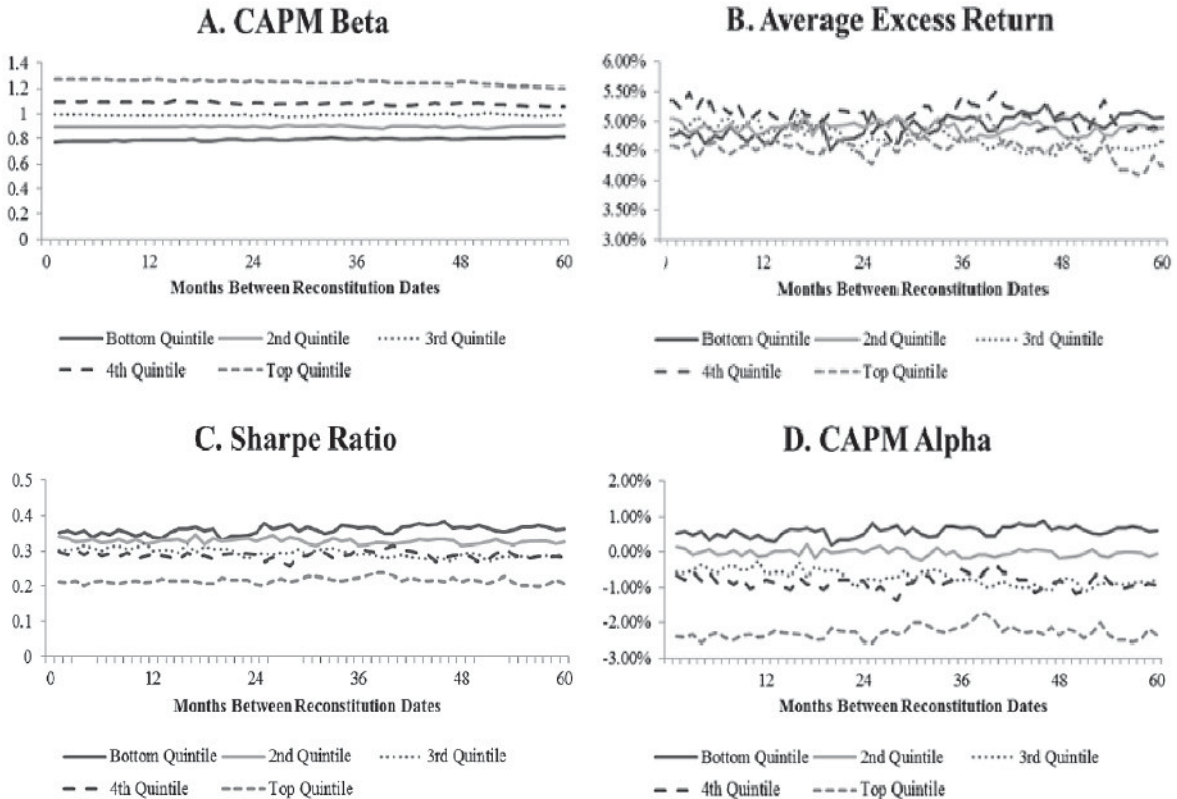


Fig. 11. Returns on beta-sorted portfolios reconstituted at low frequencies: 60-month estimation period results with T-bond proxy for the risk-free rate. These graphics display selected risk and performance metrics for TNA-weighted portfolios of U.S. Stock mutual funds constituted based on quintile ranking of trailing beta derived over a 24–60 month (as available) estimation period. The period between reconstitution dates ranges from 1 to 60 months. The returns on the stock market from January 1991 through August 2014 are from Kenneth French’s Web site. The commensurate rates on 10-year Treasury notes are from the St. Louis Fed.

every month (3.77%). The betas and performance of the portfolios are also stable across reconstitution frequency specifications when 10-year Treasury note rates rather than one-month Treasury bill rates are used to proxy for the risk-free rate, as shown in Fig. 11.

The graphs in Fig. 12 illustrate the betas and performance of portfolios constituted based on betas derived over a 12-month estimation period using one-month Treasury bill rates to proxy for the risk-free rate. As was observed through the use of the 60-month beta estimation period, there is a trend of convergence towards unity in the betas of the portfolios as the time interval between reconstitution dates expands, as illustrated in Graph A. However, the trend towards convergence is subtle. For example, the beta of the bottom quintile portfolio reconstituted once every 60 months (0.81) is only 4% greater than one that is reconstituted once a month (0.78).

In contrast to the 60-month estimation period specification, there is slightly greater variation in the performance of the portfolios across reconstitution frequencies when the portfolios are formed based on betas derived over a 12-month estimation period. This is illustrated in Graphs C and D. However, the performance of the bottom quintile portfolio does not deteriorate as the frequency of reconstitution activity decreases. The results are

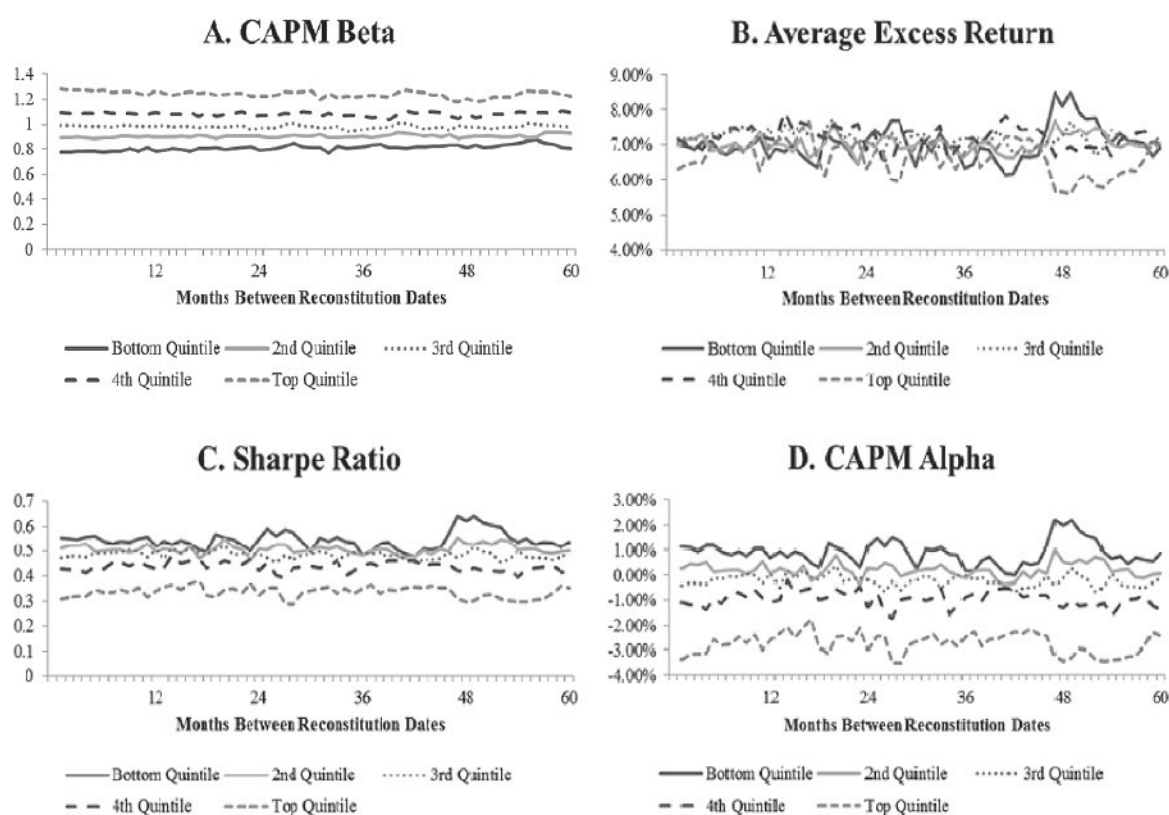


Fig. 12. Returns on beta-sorted portfolios reconstituted at low frequencies: 12-month estimation period results with T-bill proxy for the risk-free rate. These graphics display selected risk and performance metrics for TNA-weighted portfolios of U.S. Stock mutual funds constituted based on quintile ranking of trailing beta derived over a 12 month estimation period. The period between reconstitution dates ranges from 1 to 60 months. The excess returns on the stock market from January 1991 through August 2014 are from Kenneth French's Web site.

robust to the use of the 10-year Treasury note rate as an alternative proxy for the risk-free rate, shown in Fig. 13.

#### 4. Conclusion

Prior research has shown that the beta-return relationship is flatter than that which is predicted by CAPM, which implies that mean-variance efficiency can be improved through investing in low-beta stocks. This article explores how investors can use mutual funds to effectively implement a low-risk investing strategy.

Through constructing portfolios of domestic equity mutual funds that are reconstituted each month based on quintile rank of beta, I find that investors can decrease their risk without compromising returns through owning low-beta mutual funds. I also find that mutual fund beta exposures are considerably stable over time, suggesting that it may not be necessary for one to engage in frequent portfolio reconstitution activity to benefit from investing in low-beta funds. To test this possibility, I examine the performance of beta-sorted portfolios

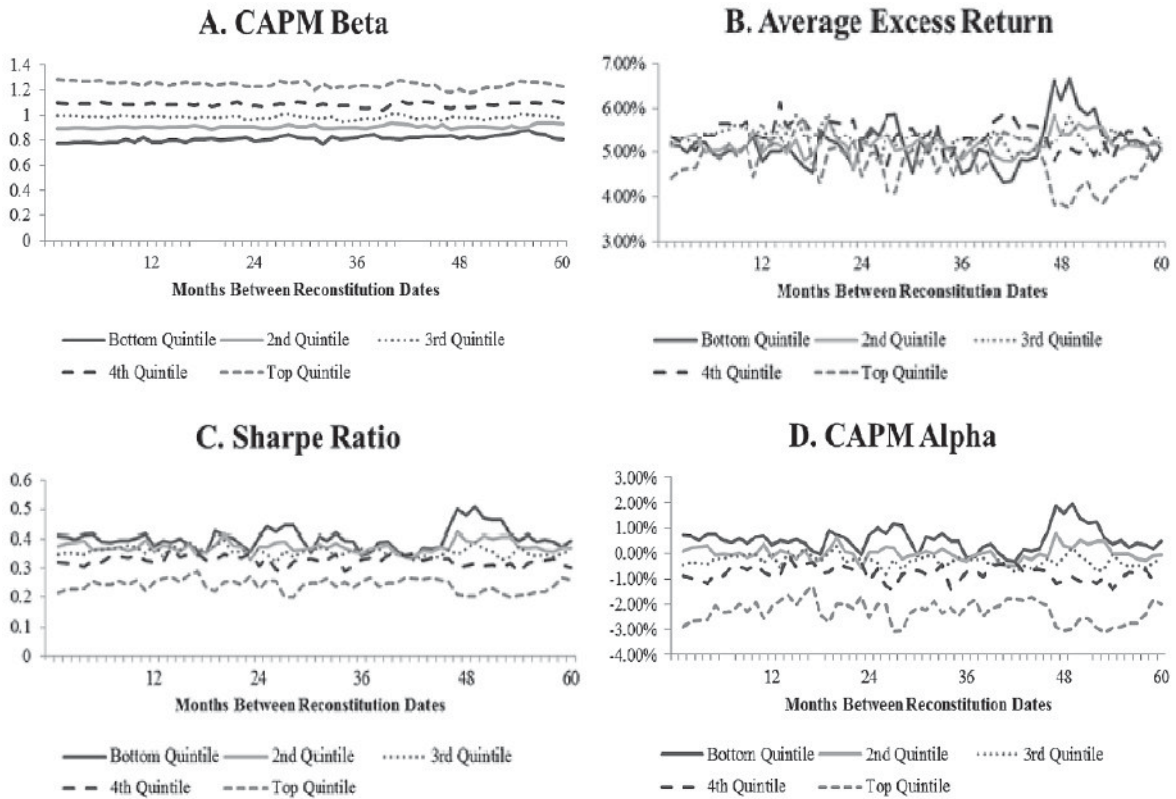


Fig. 13. Returns on beta-sorted portfolios reconstituted at low frequencies: 12-month estimation period results with T-bond proxy for the risk-free rate. These graphics display selected risk and performance metrics for TNA-weighted portfolios of U.S. Stock mutual funds constituted based on quintile ranking of trailing beta derived over a 12 month estimation period. The period between reconstitution dates ranges from 1 to 60 months. The returns on the stock market from January 1991 through August 2014 are from Kenneth French’s Web site. The commensurate rates on 10-year Treasury notes are from the St. Louis Fed.

of funds that are reconstituted at alternative frequencies ranging from bi-monthly to once every five years. The performance of the portfolios formed based on betas derived over a 12-month estimation period does vary somewhat across the reconstitution frequency specifications. However, the performance of the bottom quintile portfolio is not diminishing in the length of time between reconstitution dates and it typically dominates that of its counterparts across reconstitution frequencies.

The central implication of this study is that through tilting their portfolios towards low-beta mutual funds, investors can reduce their risk without compromising return, regardless of how frequently they trade. However, I make no statement on if and when the low-beta puzzle will cease to exist.

**Notes**

- 1 See, for example, Black (1972), Blume and Friend (1973), Fama and French (2004), Fama and MacBeth (1972), Frazzini and Pedersen (2014), Lakonishok, Shleifer, and Vishny (1994), Pinfold, Wilson, and Li (2001), and Stambaugh (1982).

- 2 Karceski (2002) found that mutual funds tend to overweight stocks in the top and bottom deciles of beta, which further motivates the purpose of this study.
- 3 Other asset classes are Balanced, Commodities, International Stock, Money Market, Municipal Bond, Sector Stock, and Taxable Bond. Morningstar does not assign funds to multiple asset classes.
- 4 Details on the construction of the variables gathered from Kenneth French's Web site can be found at [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data\\_Library/f-f\\_factors.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html). I am grateful to Kenneth French for providing this data.
- 5 In Brotherson, Eades, Harris, and Higgins (2013) survey of corporations and financial advisers, 52% of corporations and 73% of financial advisers used 10-year treasuries to represent the risk-free rate.
- 6 In Jacobs and Shivdasani's (2012) analysis of a survey of financial executives conducted by the Association for Financial Professionals, 98% of respondents reported that they calculated betas over a 1 (29%), 2 (13%), 3 (15%), or 5 (41%) year period.
- 7 A time plot of the number of share classes in each month with TNA data is available from the author upon request.
- 8 Monthly cash holdings are reported in Morningstar Direct based on feedback from surveys it conducts. Based on a conversation with a representative at Morningstar, if a fund fails to respond to a survey with its cash holdings data it is reported as having zero cash holdings. Therefore, fund-months with zero cash holdings are not included in the calculation of average cash holdings.
- 9 It is also interesting to note that the average portfolio turnover ratio of constituent funds is increasing across the portfolios and that the average expense ratio of the bottom quintile portfolio (0.91%) is similar to that of the universe of all funds (0.95%). These statistics are based on annual year-end values because of a lack of availability of monthly data from Morningstar.
- 10 Idiosyncratic volatilities were estimated over a 24-month period. Similar results, available from the author upon request, were generated through the use of a 12-month estimation period.

Appendix Table Main results by quintile of beta with the CRSP VW Index proxy for the market

|  | Low    | 2      | 3      | 4      | High   | Universal |
|--|--------|--------|--------|--------|--------|-----------|
| Panel A: 60-month beta estimation period results |        |        |        |        |        |           |
| Average $R_p - R_f$                              | 6.20%  | 6.54%  | 6.45%  | 6.86%  | 6.20%  | 6.10%     |
| <i>SD</i>  | 13.63% | 14.91% | 16.20% | 18.16% | 21.89% | 16.34%    |
| Skewness   | -0.82  | -0.76  | -0.76  | -0.65  | -0.50  | -0.76     |
| Kurtosis   | 1.79   | 1.56   | 1.46   | 1.17   | 1.07   | 1.23      |
| Sharpe ratio                                     | 0.45   | 0.44   | 0.40   | 0.38   | 0.28   | 0.37      |
| $M^2$ measure                                    | 0.51%  | 0.25%  | -0.41% | -0.75% | -2.30% | -0.82%    |
| Average $R_p - R_m$                              | -0.74% | -0.40% | -0.48% | -0.08% | -0.73% | -0.83%    |
| Tracking error                                   | 6.75%  | 4.12%  | 1.96%  | 3.76%  | 8.32%  | 1.49%     |
| Information ratio                                | -0.11  | -0.10  | -0.25  | -0.02  | -0.09  | -0.56     |
| Beta   | 0.76   | 0.88   | 0.98   | 1.09   | 1.27   | 0.99      |
| $\alpha$   | 0.91%  | 0.42%  | -0.36% | -0.69% | -2.57% | -0.79%    |
| $t(\alpha)$                                      | 0.70   | 0.48   | -0.78  | -0.84  | -1.52  | -2.24     |
| $R^2$  | 0.84   | 0.94   | 0.99   | 0.96   | 0.90   | 0.99      |
| Average cash holdings                            | 5.70%  | 4.45%  | 3.76%  | 4.31%  | 3.49%  | 4.40%     |
| Average turnover                                 | 45.85% | 47.85% | 63.96% | 68.24% | 87.15% | 58.87%    |
| Average expense ratio                            | 0.91%  | 0.87%  | 0.92%  | 1.00%  | 1.11%  | 0.95%     |
| Average idiosyncratic volatility                 | 12.18% | 11.27% | 11.43% | 12.87% | 15.71% | 12.44%    |
| Panel B: 12-month beta estimation period results |        |        |        |        |        |           |
| Average $R_p - R_f$                              | 7.01%  | 6.79%  | 6.99%  | 6.86%  | 6.32%  | 6.63%     |
| <i>SD</i>  | 12.63% | 13.87% | 15.06% | 16.85% | 20.68% | 15.20%    |
| Skewness   | -0.93  | -0.86  | -0.79  | -0.73  | -0.53  | -0.82     |
| Kurtosis   | 2.39   | 2.21   | 1.71   | 1.61   | 1.44   | 1.71      |
| Sharpe ratio                                     | 0.56   | 0.49   | 0.46   | 0.41   | 0.31   | 0.44      |
| $M^2$ measure                                    | 1.15%  | 0.16%  | -0.23% | -1.09% | -2.64% | -0.65%    |
| Average $R_p - R_m$                              | -0.27% | -0.48% | -0.29% | -0.42% | -0.96% | -0.64%    |
| Tracking error                                   | 6.02%  | 3.78%  | 1.97%  | 3.36%  | 8.30%  | 1.55%     |
| Information ratio                                | -0.04  | -0.13  | -0.15  | -0.12  | -0.12  | -0.42     |
| Beta   | 0.77   | 0.89   | 0.98   | 1.09   | 1.28   | 1.00      |
| $\alpha$   | 1.43%  | 0.35%  | -0.17% | -1.08% | -2.99% | -0.61%    |
| $t(\alpha)$                                      | 1.37   | 0.48   | -0.39  | -1.63  | -1.94  | -1.84     |
| $R^2$  | 0.85   | 0.94   | 0.98   | 0.97   | 0.88   | 0.99      |
| Average cash holdings                            | 6.13%  | 4.72%  | 4.03%  | 4.40%  | 3.72%  | 4.74%     |
| Average turnover                                 | 50.01% | 56.76% | 62.74% | 73.02% | 85.30% | 62.24%    |
| Average expense ratio                            | 0.93%  | 0.88%  | 0.92%  | 1.01%  | 1.11%  | 0.95%     |
| Average idiosyncratic volatility                 | 12.05% | 11.06% | 11.44% | 12.64% | 15.63% | 12.22%    |

This table displays performance metrics and portfolio characteristics for TNA-weighted portfolios of U.S. Stock mutual funds reconstituted monthly based on quintile ranking of trailing beta. Panel A displays the results for portfolios reconstituted based on beta derived over a 24–60 month (as available) estimation period. Panel B displays the results for portfolios reconstituted based on beta derived over a 12 month estimation period. The returns on the stock market,  $R_m$ , from January 1991 through December 2013, are represented by CRSP's Total Return Value-Weighted Index. The risk-free rate,  $R_f$ , is from Kenneth French's Web site. Returns are annualized through multiplying monthly values by 12. *SDs* are annualized through multiplying monthly values by the square root of 12. Average cash holdings, turnover ratios, expense ratios, and idiosyncratic volatilities of funds that constitute the portfolios are reported as time-series means of the cross-sectional TNA-weighted means. Idiosyncratic volatilities are derived over a 24-month estimation period.

## Acknowledgments

I am grateful for the insightful comments received from presentations at The American College, the Villanova School of Business, the National Graduate Institute for Policy Studies (Japan), Texas Tech University, the 2012 annual meeting of the Academy of Financial Services, the 2012 annual meeting of the Southern Finance Association, and the IV World Finance Conference. I thank Melissa Cenneno and Matthew Hobson for their excellent research assistance and the New York Life Insurance Company for their financial support. Wharton Research Data Services (WRDS) was used in preparing this article. This service and the data available thereon constitute valuable intellectual property and trade secrets of WRDS and/or its third-party suppliers. A prior version of this article was titled “Capitalizing on the Greatest Anomaly in Finance with Mutual Funds.” Based upon the suggestion of a referee, the title was changed to something less radical.

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