FINANCIAL
SERVICES
REVIEW

Financial Services Review 24 (2015) 359–386

# A new strategy to guarantee retirement income using TIPS and longevity insurance: A second look

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## Abstract

Shankar (2009) proposes a new investment strategy for retirees that bundles Treasury Inflation Protected Securities with a deferred annuity to guarantee real annual withdrawal rates of 5% or more with no risk of financial ruin. This strategy addresses three problems that retirees face: longevity risk, inflation risk, and liquidity risk inherent in the purchase of an immediate annuity. In our article, we evaluate the performance of this proposed strategy under realistic assumptions about costs, security design, and markets. In addition, we evaluate how the bequest motive might affect the choice between Shankar's strategy and an immediate annuity. © 2015 Academy of Financial Services. All rights reserved.

*Jel classification:* D14; G11; G22

*Keywords:* Retirement planning; Withdrawal rates; TIPS; Longevity risk; Financial ruin

## 1. Introduction

An inflation-indexed immediate annuity appears, at first glance, to be an ideal solution to two important financial problems that retirees generally face: maintaining a desired level of real income and avoiding the risk of running out of money, that is, financial ruin. However, Shankar (2009) observes that investors have been extremely reluctant to commit their life savings to immediate annuities. As an alternative, Shankar proposes an investment strategy for retirees that bundles Treasury Inflation Protected Securities (TIPS) with a deferred

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annuity. Shankar's strategy calls for construction of a TIPS ladder for a fixed number of years and purchase of a non-refundable deferred life annuity that begins payments at the end of the life of the ladder. The individual buys the TIPS and the deferred annuity at retirement. The ladder and deferred annuity can be designed so that the real annual payout is the same in each year of retirement.

Shankar (2009) makes the case that this strategy guarantees real annual withdrawal rates, eliminates the risk of financial ruin, and avoids much of the liquidity problem inherent in a non-refundable immediate life annuity. Moreover, he argues that the strategy guarantees real withdrawal rates substantially higher than the 4% rule of thumb that financial advisors often recommend for retirees who hold a conventional portfolio of stocks and bonds.

In this article, we contribute to the literature on personal finance by providing a more nuanced assessment of the performance of Shankar's (2009) proposed strategy. We apply a bootstrap simulation analysis under realistic assumptions about cash management, trading costs, security design, and markets. We construct simulated yield curves for TIPS from historical Treasury yields and simulated expected inflation curves and apply these yield curves to price the TIPS in the ladder rather than use deterministic weighted average real yields as in Shankar. We examine wider ranges of retirement ages (55 to 75 years instead of just 60 or 65 years as in Shankar) and TIPS ladder lengths (10 to 30 years rather than just 15 or 20 years), and we consider deferred annuities that are indexed for inflation (the ideal situation) as well as those that are not (the current situation in practice). We examine the effects of cash management procedures in which payments of TIPS coupons and par are converted to monthly cash payouts via a money market account that earns a nominal T-bill rate. We adjust for mortality credits when valuing the deferred annuity rather than assuming a fixed number of years based on median life expectancy at the end of the TIPS phase as in Shankar's modeling. For each simulated investor, we determine the purchasing power of actual monthly cash payouts and apply stochastic mortality risk to determine time of death. This approach enables us to estimate the expected utility of consumption and bequests from the perspective of the investor's retirement date.

An important question is why investors are reluctant to buy an immediate inflation-indexed annuity with their entire savings. As Shankar (2009) observes, economists offer a variety of explanations, including unattractive design of annuity products, existence of programs such as Social Security that perform essentially the same function, and behavioral explanations. We apply expected utility analysis to explore how the strength of the bequest motive affects a rational investor's choice between a non-refundable immediate life annuity and a TIPS ladder bundled with a non-refundable deferred life annuity.

Our results are in close agreement with Shankar (2009) for the specific scenarios that he presents in detail. We show that, under a wider range of retirement ages and ladder periods, a TIPS ladder/inflation-indexed deferred annuity strategy provides a real payout rate that is modestly less than that from a corresponding immediate annuity but without the irreversible commitment while still avoiding risk of ruin. We also confirm that the target real payout rate is strongly determined by market real yields at time of retirement, where the target rate is the rate calculated in the construction of the TIPS ladder/deferred annuity strategy.

On the other hand, we identify circumstances under which Shankar's (2009) optimistic conclusions about his proposed strategy do not hold. First, because the target real payout rate is determined by market yields and expected inflation at time of retirement, our simulation suggests that investors at retirement may see low target real payout rates when market real yields are low. The same problem arises (and for the same reason) with immediate annuities; Shankar's proposed strategy does not eliminate this drawback. We explore which combinations of retirement age and TIPS ladder period offer investors the best odds of an attractive real payout rate regardless of market conditions.

Second, Shankar's (2009) strategy does not guarantee a fixed real payout rate. When the deferred annuity is indexed for inflation, we show that the *realized* real payout rate can dip modestly. During the TIPS payout phase, these dips are rare and transient but may be as much as 0.5%. During the deferred annuity payout phase, the retiree may face fluctuations (as much as 0.25%) when the annuity is indexed for inflation with a lag. On the other hand, when the deferred annuity is not indexed for inflation, investors who survive to the deferred annuity phase have a good chance of experiencing significantly lower *realized* real payout rates late in life when they are least able to redress the situation. This point is important as long as a market for inflation-indexed deferred annuities does not exist.

Third, we identify circumstances under which the allocation to the deferred annuity premium could be much greater than the 20 to 30% in the scenarios that Shankar (2009) discusses. We explore circumstances under which this allocation is likely to be low and circumstances under which it is likely to be high.

## 2. Literature review

The literature on sustainable withdrawal rates (i.e., rates that do not lead to financial ruin) has two threads. In the earlier thread, the authors focus on passive strategies in which the asset allocation and withdrawal rate are selected and fixed at retirement. For example, Cooley, Hubbard, and Walz (2003) examine risk of ruin for fixed withdrawal rates from an all equity portfolio. Milevsky & Robinson (2005) study risk of ruin when mortality risk and rates of return are stochastic. Ervin, Faulk, and Smolira (2009) evaluate the effects of different combinations of asset allocation, lengths of savings and retirement periods, savings rate, and availability of social security income in scenarios where individuals set their withdrawal rates to smooth lifetime income. In the later thread, the authors consider dynamic strategies in which various features are adjusted as circumstances warrant. For example, Stout (2008) and Spitzer (2008) examine flexible withdrawal rates, while Gupta, Pavlik, and Synn (2012) evaluate semi-passive balanced fund portfolios.

Neither passive nor dynamic strategies proposed in the literature guarantee a stream of retirement income that is independent of future portfolio returns and is guaranteed for life. Shankar's (2009) proposed strategy potentially can do both. Little attention has been given in the literature to bundling bond ladders and deferred annuities. Shankar presents the earliest systematic analysis and description of strategies that combine a TIPS ladder with a deferred life annuity. (A more recent article by Sexauer, Peskin, and Cassidy (2012) proposes the same strategy.) The key to the strategy is that the investor can design a "buy and forget" portfolio by allocating savings at retirement between a suitably designed TIPS ladder and a deferred annuity in a manner that, in principle, locks in a real payout rate over the investor's remaining life. Therefore, it offers the advantages of a non-refundable immediate, inflation-indexed life annuity without the associated liquidity problem.

Bernheim (1991) concludes that bequest motives are strong for a large segment of the population. Thus, the bequest motive might play a significant role in explaining why less than two percentage of retirees annuitize their retirement savings, as Shankar (2009) notes. We examine the bequest motive in an attempt to ascertain the circumstances under which an investor prefers an immediate inflation-indexed annuity to a strategy composed of a TIPS ladder and a deferred annuity.

## 3. Strategy design

Our analytical model is analogous to the one that Shankar (2009) presents. However, the details of our approach are sufficiently different that we provide a complete description.

### 3.1. Key similarities and differences

As in Shankar (2009), we assume that the investor uses all of savings at retirement to purchase the TIPS for the ladder and to pay a lump-sum premium for the deferred annuity. Another similarity is that strategies with an inflation-indexed deferred annuity are designed so that the annual inflation-indexed payout is approximately the same each year, whether the payout is from the TIPS ladder in the first phase of retirement or from the deferred annuity in the second phase of retirement. In addition, if the deferred annuity is not indexed, then the ladder is designed so that the fixed nominal value of the annual annuity payment equals the level annual payout from the TIPS ladder expressed in dollars at retirement.

One important difference is that we apply pension annuity factors that are discrete-time versions of the formulas developed by Milevsky (2006). These formulas correctly account for the mortality risk. Furthermore, in our analysis we apply inflation-indexed yields based on TIPS (rather than real yields) when we calculate the number of TIPS to use and the cost of the deferred annuity. This approach is more consistent with the pricing of TIPS and inflation-indexed deferred annuities, especially if the insurance company were to invest the premium in TIPS to match its future real cash outflows to the promised annuity payments. Finally, we work with monthly rather than annual payouts. This approach facilitates the simulation of cash flows from TIPS as they occur throughout the year.

### 3.2. Pension annuity factors

The immediate pension annuity factor (IPAF) is the present value of a one-dollar real immediate annuity where the present value calculation takes mortality credits into account. (See section 10.6 in Milevsky (2006) for a good explanation of what mortality credits are and how annuities work.) We apply the IPAF to calculate payments from the immediate, inflation-indexed annuities that we compare to the TIPS ladder/deferred annuity strategies. Please see Appendix A for how we adapt Milevsky's IPAF formula for our analysis.

We also use the IPAF to calculate the deferred pension annuity factor. The deferred pension annuity factor (DPAF$^*$) is the present value of a one-dollar real deferred annuity where the present value calculation takes mortality credits into account. To determine the monthly real dollar annuity payment, we calculate the DPAF$^*$, assuming discrete, real monthly payouts. In this article, we apply a discrete version of Milevsky's (2006) Eq. (6.14). Please see Appendix A for an explanation of how we adapt Milevsky's DPAF$^*$ formula for our analysis.

### 3.3. Construction of the tips ladder

The investor's task is to construct a TIPS ladder and purchase an inflation-indexed deferred annuity whose first payment is one month after the end of the life of the ladder. In our article, as in Shankar (2009), we design the ladder and the annuity so that the target real annual payout is the same from each.

The monthly inflation-indexed deferred annuity payment (in terms of dollars at retirement) is

$$P^*_{DA} = \frac{(1 - w_{TL})W(1 - f_{DA})}{DPAF*}, \tag{1}$$

where DPAF$^*$ is calculated at time of retirement, $W$ is savings at retirement, $w_{TL}$ is the proportion of savings used to construct the TIPS ladder, and $f_{DA}$ is the percentage of the premium paid as fees to the insurance company. (We assume that this fee is 2%.)

Let the TIPS ladder be an N-year ladder designed to support consumption for the first N years in retirement. For the first year, set aside sufficient funds from the allocation to the TIPS ladder to cover consumption in the first year. For each remaining year, identify a coupon TIPS in the market that matures before but as close as possible to the start of that year. For example, for year two in the ladder, find a coupon TIPS that matures as late in the first year as possible; for year three, find a coupon TIPS that matures as late in the second year as possible; and so forth. Choosing the maturities in this fashion minimizes interest rate risk and inflation risk when the ladder has an annual structure as in Shankar (2009). The par from a maturing TIPS is deposited in a cash account that earns the nominal return on one-month Treasury bills rolled over month to month.

Let $P^*_{TL}$ be the monthly payout, expressed in retirement month dollars, to be supported by the TIPS ladder. Then $P^*_{TL} = \frac{A}{12}$, where $A$ is the annual inflation-indexed payout. We design the ladder and annuity so that $P^*_{TL} = P^*_{DA}$. The ideal annual payout is

$$A = \frac{W(1 - f_{DA})}{(DPAF*/12) + (1 - f_{DA})((1 + f_B)\boldsymbol{p}^T\sum^{-1}\mathbf{1} + 1)}. \tag{2}$$

Please see Appendix B for details about the solution, including how we calculate the number of TIPS of each maturity to buy.

Shankar (2009, p. 67) observes that commercially available deferred annuities are not indexed for inflation. Hence, we also examine strategies where the deferred annuity is not indexed. The construction of the TIPS ladder and its associated deferred annuity follow the same steps as when the annuity is indexed. In particular, the nominal deferred annuity payout is set equal to the TIPS ladder payout in dollars at retirement. However, because the *nominal* deferred annuity payout is fixed, its real value tends to decline over time.

### 3.4. Expected utility

In addition to evaluating risk of financial ruin, we also estimate expected utility to evaluate investor preferences. The general form of the constant relative risk aversion (CRRA) utility function in this study is

$$U(C;\,\eta,\,\chi) = \frac{exp\left((1-\eta)ln\left(\frac{C}{\chi}\right)\right) - 1}{1-\eta}, \quad C > 0, \tag{3}$$

where $\eta$ is the risk aversion level, $\eta \neq 1$, $\chi > 0$ is a scaling factor, $C$ is the cash flow (either consumption in the given period or the bequest at death) and

$$U(C;\,1,\,\chi) = ln\left(\frac{C}{\chi}\right), \quad C > 0, \tag{4}$$

when the risk aversion level $\eta = 1$. In all cases, the absolute risk-aversion is

$$A(C) = -\frac{U''(C)}{U'(C)} = \frac{\eta}{C}. \tag{5}$$

Note that the absolute risk-aversion is scale independent. In the simulation, we use a scale factor $\chi_C = \$1{,}000$ when evaluating utility of consumption and $\chi_B = \$10{,}000$ when evaluating utility of bequests. This adjustment compresses the numerical range of the utility function and makes the results more numerically manageable. (As a result, the strength of the bequest motive, D, is implicitly conditional on the scale factors. However, we apply the same scaling factors throughout, so ordering based on expected utility is consistent.)

When $\eta \geq 1$, a numerical problem arises in the utility of consumption and utility of bequest functions defined in Eqs. (3) through (5). As C approaches 0, the utility approaches minus infinity. We mitigate this problem by replacing very low or zero dollar payouts and bequests with de minimus levels. Please see Appendix C for details.

In the simulation, we estimate expected utility for a given strategy by averaging the realized utility for all simulated investors who use the strategy. We calculate the realized utility for a given simulated investor as

$$\begin{aligned} U(\boldsymbol{C}(\omega),\,B(\omega);\,\eta,\,D) = (1-D)\sum_{t=1}^{T(\omega)-1} \delta^{-t}U(C_t(\omega);\,\eta,\,\chi_C) \\ + D\delta^{-T(\omega)}U(B(\omega);\,\eta,\,\chi_B), \end{aligned} \tag{6}$$

where $D$ is the strength of the bequest motive, $0 \leq D \leq 1$; $\delta$ is the one-month time discount factor for utility; $T(\omega)$ is time of death in months after retirement for simulated life $\omega$; $C_t(\omega)$ is the consumption in month $t$ after retirement, expressed in dollars at retirement (and $\boldsymbol{C}(\omega)$ is a vector of these values); and $B(\omega)$ is the bequest at time $T(\omega)$ in retirement year dollars.

## 4. Simulation methods

### 4.1. Markets in the simulation

Treasury auction schedules are determined by the financing needs of the federal government. Hence, new maturities are not offered every month. The auction schedule in this simulation is similar to that followed by the U.S. Treasury for most of 2010–2013. Specifically, 5-year TIPS are auctioned in April, August, and December; 10-year TIPS are auctioned in January, March, May, July, September, and November; and 30-year TIPS are auctioned each February, June, and October. Thus, in any given month, an investor at retirement must go to the secondary market to buy TIPS with suitable maturities for the ladder. Small-scale purchases by individuals through Treasury Direct generally incur no purchase fees. In the simulation, investors incur transactions costs only for TIPS purchased in the secondary market. We adopt the simplifying assumption that all auctions, purchases, coupon dates, and maturity dates are end-of-month.

TIPS are indexed to the non-seasonally adjusted Consumer Price Index for All Urban Consumers (CPI-U). The Treasury sets the reference price level for TIPS up for auction in a given calendar month as the CPI-U for the third preceding calendar month. For example, if the auction month is July, then the reference price level for July 1 is the April CPI-U. If the auction date is midmonth, then the official indexation lag is two and a half months. If the auction date is end-of-month, then the official indexation lag is three months. In the simulation, we set the indexation lag at three months, because we assume all auctions are end-of-month.

Trading in the secondary market exposes individual investors to market friction in the form of lot size restrictions and trading costs. In the secondary market the retail investor pays dealer markups, commissions, and bid-ask spreads that may be as much as 2% (e.g., see Aschkenasy (2005), Bullock (2005)). We assume that investors incur transactions costs of 2% of the bond price for TIPS bought or sold in the secondary market, but we simplify by ignoring lot size restrictions and assuming that TIPS may be traded in any multiple of $100 par (indexed for inflation after the auction date).

### 4.2. Construction of inflation-indexed zero-coupon yield curves

Inflation-indexed TIPS have been issued only since 1997. To create simulated histories that are more representative of historical yield and inflation environments in the United States, in each simulated month we construct an inflation-indexed zero-coupon Treasury yield curve.

Before the simulation begins, we construct historical nominal yield curves. Our data sources for 1926–2012 are the daily constant maturity rate (CMT) database compiled by the

Board of Governors of the Federal Reserve System (2013), the FRED Economic Data web site at the Federal Reserve Bank of St. Louis (2013), and Morningstar (2013). The construction proceeds in three stages.

In the first stage, we construct a discrete nominal Treasury yield curve at the end of each historical month with bond equivalent yields (assuming semiannual interest) at monthly maturity intervals from one month to six months and then at six-month intervals from month six out to 30 years. While spline interpolation is preferred (and is the approach currently applied by the U.S. Treasury to construct its yield curves), we apply it only for historical months February 1977 and later. For earlier historical months, the number of knot points determined by available yield data are too small for spline interpolation to work well. Hence we apply linear interpolation to construct yield curves for earlier historical months.

In the second stage, we convert the historical discrete nominal yield curve to a yield curve for nominal Treasury zero rates by applying an iterative method to the coupon bond yields each month. (See, e.g., Hull [2011, pp. 86–88] for an illustration.) In the process, we also convert discretely compounded yields to continuously compounded, annualized yields.

In the third stage, we apply a natural cubic spline to interpolate for maturities in monthly intervals between the six-month maturities on the nominal zero-coupon yield curve. The end result is a historical nominal zero-coupon Treasury yield curve for each historical month with yields at monthly intervals along the curve.

We adapt models for expected inflation that we then apply to construct historical yields and prices for inflation-indexed bonds in the simulation. The simulation applies Fama's (1975) model for one-month estimates of expected inflation. For time horizons of one year and longer, the simulation uses a modified version of Kothari and Shanken's (2004) model. In their model, the predictor variables are a short-term yield, a yield spread, a realized Treasury bill real return, and inflation over the most recent month.

A crucial task at the start of each time step in the simulation is construction of an expected inflation curve. At each simulated month, the simulator constructs an expected inflation curve with a maturity spectrum from one month to 30 years in one-month intervals. Using the historical data drawn for the simulated month and the model coefficients estimated at the start of the investor's life, the simulator calculates the expected inflation for one-month, one-year, two-year, and three-year horizons. For horizons less than three years, the simulator applies cubic splines to interpolate between the one-month, one-year, two-year, and three-year estimates of expected inflation. For longer horizons, the simulator sets expected inflation equal to the three-year estimate. The simulator uses the expected inflation curve to determine the zero-coupon inflation-indexed TIPS yield curve and to carry out bond pricing and indexing of nominal cash flows as needed for the current time step.

The inflation-indexed zero-coupon Treasury yield curve is determined in the following way. Let $y_t(m)$ represent the annualized yield at time $t$ on a nominal zero-coupon Treasury bond that matures in $m$ months. Let $y_t^*(m)$ represent the annualized yield at time $t$ on an inflation-indexed zero-coupon Treasury bond that matures in $m$ months. A model for the relation between these two yields at $t$ is

$$y_t(m) = [E_t\{\tilde{r}_t(m)\} + RPR_t(m)] + [E_t\{\tilde{h}_t(m)\} + RPI_t(m)], \tag{7a}$$

$$y_t^*(m) = [E_t\{\tilde{r}_t(m)\} + RPR_t(m)] + RPL_t(m), \tag{7b}$$

where

- $E_t\{\tilde{r}_t(m)\}$ = expected real yield at time $t$ on a zero-coupon Treasury bond that matures in $m$ months,
- $RPR_t(m)$ = risk premium at time $t$ for uncertainty about future real yield over $t$ to $t + m$,
- $E_t\{\tilde{h}_t(m)\}$ = expected inflation at time $t$ for the period from $t$ to $t + m$,
- $RPI_t(m)$ = risk premium at time $t$ for uncertainty about inflation over $t$ to $t + m$, and
- $RPL_t(m)$ = risk premium at time $t$ for liquidity of the inflation-indexed bond relative to liquidity of the nominal Treasury bond, where both bonds mature in $m$ months.

We simplify by assuming away convexity issues and relative uncertainty of tax burdens on nominal versus indexed bonds. This model is consistent with the decomposition of bond yields in the literature (e.g., Barnes, Bodie, Triest, and Wang (2010); Bekaert and Wang (2010); Kothari and Shanken (2004)). The real rate of return is the sum of the expected real yield and the real yield risk premium. Thus, the inflation-indexed yield in Eq. (7b) equals the real rate of return plus a relative liquidity risk premium.

Solve Eq. (7a) for the real rate of return and substitute the result into Eq. (7b):

$$y_t^*(m) = [y_t(m) - E_t\{\tilde{h}_t(m)\} - RPI_t(m)] + RPL_t(m). \tag{8}$$

This equation serves as a template for constructing the inflation-indexed yield curve at each time step in the simulation, given yields $y_t(m)$ on the nominal zero-coupon Treasury yield curve and the expected inflation rates $E_t\{\tilde{h}_t(m)\}$.

Eq. (8) requires two risk premiums: the liquidity risk premium for inflation-indexed Treasury bonds, and the inflation risk premium for a nominal Treasury bond with the same maturity as the corresponding indexed bond. Evidence in the literature indicates that zero is a reasonable estimate of the liquidity risk premium (e.g., D'Amico, Kim, and Wei (2008) and Christensen and Gillan (2011)). We model the inflation risk premium based on average risk premiums for 1990–2007 estimated by D'Amico, Kim, and Wei (2008, Figs. 2c and Fig. 3c). They report an average inflation risk premium of about 0.25% for one-year maturities and about 0.75% for 10-year maturities. We apply linear interpolation to assign risk premiums as a function of maturity from one to 10 years. For maturities less than one year, we interpolate between a zero risk premium at maturity and 0.25% risk premium with one year to maturity. For maturities greater than 10 years, we assign a risk premium of 0.75%.

To estimate the price of a TIPS in the simulation, we use the inflation-indexed zero-coupon Treasury yield curve constructed in each simulated month and apply a pricing model based on Evans (1998) to expected coupons and par.

### *4.3. Outline of the bootstrap simulation procedures*

We initialize the simulator for each strategy and scenario with a different seed for the random number generator. Hence, simulation results that we report for each strategy and scenario are statistically independent. Each time that we run the simulator, it performs 10,000 statistically independent replications, where each corresponds to the simulated lifetime of an investor. At the start of each replication, a simulation time clock initializes.

In each simulated lifetime, the stationary block bootstrap works as follows. At the initial time step, the simulator draws a historical month at random from 1926 to 2012 along with the corresponding zero-coupon nominal Treasury yield curve and inflation rate. Each month has an equal probability that it will be drawn. At the next time step, with probability $1/\gamma$, the simulator draws the next month at random; otherwise, it draws the next consecutive historical month. (If the previous historical month is December 2012, then the simulator draws January 1926.) The expected length of a sequence of consecutive historical months is $\gamma$. (The results that we report are for $\gamma = 60$. Test runs for $\gamma = 48$ and $\gamma = 72$ indicate the simulation results are robust to choice of $\gamma$.)

The simulation has three phases for each simulated investor: prehistory, market generation, and payout. In the prehistory phase, the simulator generates a time series of yields to begin estimating expected inflation as well as to estimate components of the bond pricing function. The simulator draws a stationary block bootstrap sample of 1,200 historical months with their respective nominal Treasury yield curves and inflation rates. Then the simulator estimates the coefficients of the model for expected inflation. We make the simplifying assumption that the inflation model coefficients are stationary. Hence, these coefficients apply throughout the remainder of the simulated investor's lifetime. (An alternative approach is to re-estimate the coefficients for the expected inflation curve in each simulated month based on the simulated historical data to date in the simulated history. However, the expected inflation curves are unrealistically volatile from one month to the next. Moreover, the end results are generally similar to those when the expected inflation model coefficients are estimated once per lifetime, and the simulation runs an order of magnitude longer when the coefficients are re-estimated each month.)

The purpose of the market generation phase is to create a simulated secondary market in TIPS. At each time step, information about outstanding securities in the secondary market is updated and new coupon TIPS may be issued, depending on the auction schedule. To determine the coupon rate for a newly issued bond, the simulator first calculates the coupon rate as if the bond were priced at par, where price equals the sum of the discounted real coupon payments and par, where discounting is back to the auction date. (We assume no arbitrage opportunities; hence the auction yield is the same as the market yield.) Then, following the Treasury custom at auctions for new issues, the simulator rounds the coupon rate down to the nearest one eighth of one percentage (with a floor of 0.125%) and prices the bond accordingly. The market is saturated after 360 months.

The payout phase assumes that the investor has accumulated $500,000 in savings and is about to retire. In all cases, on retirement the investor uses all savings either to purchase an immediate, inflation-indexed annuity or to construct a TIPS ladder and buy a deferred annuity (that may or may not be indexed, depending on the scenario). The simulator uses mortality data to determine when the investor dies. We construct a unisex mortality table based on the Society of Actuaries RP 2014 Mortality Tables (2014) for healthy annuitants.

## 5. Results

### 5.1. Payout rate

A key point to keep in mind is that the target real payout rate determined in the construction of the TIPS ladder/deferred annuity strategy (described earlier in Section 3.3) is not necessarily the realized real payout rate. Practical matters, such as cash management during the TIPS payout phase and how (if at all) the deferred annuity is indexed for inflation, can cause the realized rate to deviate from the target rate.

We examine three general questions. First, what is the target level of the annual real payout rate, given age at retirement and length of the TIPS ladder? Second, what is the relation between the target real payout rate and market yields at retirement? Finally, how much uncertainty is there about the realized real payout rate over retirement, given market yields at retirement and choice of strategy? Alternatively, does the strategy assure that a retiree will receive a fixed real payout rate?

#### 5.1.1. Target level of the real payout rate

The target level of the real payout rate from a TIPS ladder/inflation-indexed deferred annuity strategy is close to but lower than the level of the target real payout from an inflation-indexed immediate annuity, conditional on the retirement age. Please see the first three columns in Table 1. The distribution of the payout rates across all simulation histories (hence, across the distribution of market yield curves at retirement) is similar, because both strategies are indexed for inflation. The payout rates on the TIPS ladder/deferred annuity strategy, however, are lower than for the corresponding immediate annuity, because the immediate annuity investment returns are boosted by the mortality credits whereas TIPS returns are not.

For strategies with 10-year ladders and inflation-indexed deferred annuities, the payout distribution across all simulation histories is the closest to that for an inflation-indexed immediate annuity. For example, among investors who retire at age 65 and opt for a 10-year TIPS ladder, the median real payout rate is 5.45% versus a median of 5.65% for immediate annuities; the 5th-percentile rate is 4.66% versus 4.80%; and the 95th-percentile is 8.31% versus 8.80%. Given the retirement age, the distribution of the real payout rates shift downward as the length of the TIPS ladder increases, as illustrated by the 5th-percentile, median, and 95th-percentile statistics. The reason is that the longer the TIPS ladder, the shorter the deferred annuity payout phase, and hence the greater the proportion of the investment that is not benefiting from mortality credits. Put another way, the shorter the ladder, the more the TIPS ladder/deferred annuity strategy resembles an immediate annuity.

Table 1 also shows that, given the length of the TIPS ladder, the later in life that the investor retires, the greater the real payout rate. The same pattern holds for immediate annuities for the same reason: a given level of retirement wealth needs to support fewer years of payouts.

When the deferred annuity is not indexed for inflation, the target real payout rate from the TIPS ladder/deferred annuity strategy is substantially higher than the corresponding

Table 1 Distribution of target real annual payout rates (%) by strategy across simulated investor histories

| Retirement age (years) | Strategy | Deferred (or immediate) annuity indexed for inflation | | | Deferred annuity not indexed for inflation | | |
|---|---|---|---|---|---|---|---|
| | | 5%-tile | Median | 95%-tile | 5%-tile | Median | 95%-tile |
| 55 | Immediate annuity | 3.44 | 4.29 | 7.40 | | | |
| 55 | 10-year ladder & DA | 3.40 | 4.24 | 7.20 | 4.39 | 5.66 | 9.43 |
| | 15-year ladder & DA | 3.35 | 4.17 | 7.04 | 4.15 | 5.24 | 8.41 |
| | 20-year ladder & DA | 3.28 | 4.09 | 6.85 | 3.89 | 4.82 | 7.54 |
| | 25-year ladder & DA | 3.15 | 3.92 | 6.70 | 3.56 | 4.36 | 7.03 |
| | 30-year ladder & DA | 2.99 | 3.72 | 6.38 | 3.21 | 3.94 | 6.55 |
| 60 | Immediate annuity | 4.01 | 4.87 | 7.91 | | | |
| 60 | 10-year ladder & DA | 3.95 | 4.81 | 7.61 | 4.86 | 6.10 | 9.65 |
| | 15-year ladder & DA | 3.86 | 4.63 | 7.42 | 4.55 | 5.57 | 8.54 |
| | 20-year ladder & DA | 3.69 | 4.43 | 6.99 | 4.16 | 5.03 | 7.70 |
| | 25-year ladder & DA | 3.47 | 4.24 | 6.93 | 3.73 | 4.48 | 7.07 |
| 65 | Immediate annuity | 4.80 | 5.65 | 8.80 | | | |
| 65 | 10-year ladder & DA | 4.66 | 5.45 | 8.31 | 5.47 | 6.70 | 10.18 |
| | 15-year ladder & DA | 4.46 | 5.20 | 7.90 | 5.02 | 5.98 | 8.63 |
| | 20-year ladder & DA | 4.14 | 4.86 | 7.46 | 4.45 | 5.28 | 7.80 |
| 70 | Immediate annuity | 5.86 | 6.72 | 9.76 | | | |
| 70 | 10-year ladder & DA | 5.57 | 6.34 | 9.03 | 6.24 | 7.37 | 10.47 |
| | 15-year ladder & DA | 5.12 | 5.83 | 8.36 | 5.52 | 6.38 | 8.98 |
| 75 | Immediate annuity | 7.41 | 8.24 | 11.43 | | | |
| 75 | 10-year ladder & DA | 6.68 | 7.40 | 10.05 | 7.17 | 8.22 | 11.07 |

The payout rates are real annual payouts as a percentage of total funds at retirement. For strategies with a deferred annuity (DA), the annuity payments begin one month after the TIPS ladder phase. When the deferred annuity is indexed for inflation, it is indexed in the same manner as TIPS, that is, indexed to the CPI-U with a three-month lag.

strategy with an indexed deferred annuity and may even exceed that from an inflation-indexed immediate annuity. Please see the last three columns in Table 1. For example, among investors who retire at age 65 and opt for a 10-year TIPS ladder, the median target real payout rate is 6.70% versus a median of 5.65% for immediate annuities; the 5th-percentile rate is 5.47% versus 4.80%; and the 95th-percentile is 10.18% versus 8.80%.

However, the target real payout rate holds only during the TIPS ladder phase. The nominal value of the deferred annuity payout is set equal (by construction) to the target real dollar payout. However, the purchasing power of these nominal payouts is eroded by inflation over the investor's retirement. Hence, the realized real payout rate during the deferred annuity payout phase can be substantially lower than the target rate for the strategy.

The distribution of the target real payout rate for a TIPS ladder/non-indexed deferred annuity strategy is higher than that for a corresponding strategy with an inflation-indexed deferred annuity, because the non-indexed annuity is less expensive than its indexed counterpart. Hence, for a given level of wealth at retirement and a given length ladder, more of that wealth can be invested in the TIPS ladder. Because the realized real payout rate during the TIPS ladder phase usually is the target rate by design, this rate is higher when a greater proportion of wealth at retirement is dedicated to the ladder.

Table 2 Linear regression of target real annual payout rate on the value-weighted real yield of the TIPS ladder at retirement

| Retirement age (years) | Length of TIPS ladder (years) | Deferred annuity indexed for inflation | | | Deferred annuity not indexed for inflation | | |
|---|---|---|---|---|---|---|---|
| | | Intercept estimate | Slope estimate | $R^2$ | Intercept estimate | Slope estimate | $R^2$ |
| 55 | 10 | 3.54% | 0.654 | 89.6% | 4.90% | 0.755 | 70.2% |
| | 15 | 3.39% | 0.647 | 94.3% | 4.40% | 0.680 | 82.3% |
| | 20 | 3.22% | 0.647 | 97.1% | 3.90% | 0.644 | 90.7% |
| | 25 | 3.02% | 0.646 | 98.1% | 3.44% | 0.634 | 95.1% |
| | 30 | 2.81% | 0.651 | 98.3% | 3.02% | 0.640 | 96.8% |
| 60 | 10 | 4.07% | 0.644 | 90.4% | 5.32% | 0.748 | 72.9% |
| | 15 | 3.87% | 0.636 | 95.2% | 4.74% | 0.658 | 85.5% |
| | 20 | 3.62% | 0.631 | 97.7% | 4.14% | 0.628 | 93.5% |
| | 25 | 3.33% | 0.631 | 98.2% | 3.60% | 0.222 | 96.7% |
| 65 | 10 | 4.74% | 0.638 | 92.0% | 5.90% | 0.713 | 76.7% |
| | 15 | 4.44% | 0.620 | 96.3% | 5.15% | 0.626 | 88.8% |
| | 20 | 4.05% | 0.615 | 98.2% | 4.39% | 0.608 | 95.7% |
| 70 | 10 | 5.62% | 0.622 | 98.5% | 6.60% | 0.678 | 80.4% |
| | 15 | 5.10% | 0.598 | 97.3% | 5.59% | 0.607 | 93.5% |
| 75 | 10 | 6.71% | 0.606 | 95.9% | 7.46% | 0.639 | 86.2% |

The payout rates are real annual payouts as a percentage of total funds at retirement. The deferred annuity payments begin one month after the TIPS ladder phase. When the deferred annuity is indexed for inflation, it is indexed in the same manner as TIPS, that is, indexed to the CPI-U with a three-month lag. All coefficient estimates are significantly different from zero at the one percentage level.

### *5.1.2. Influence of market yields*

Market yields at retirement determine the target payout rate in a complex fashion by Eq. (2). (In the notation of this equation, the annual real payout rate is A/W.) Yields at retirement enter into the calculation through the deferred pension annuity factor and the market prices of the TIPS in the ladder.

If the deferred annuity is indexed for inflation, then the inflation-indexed zero-coupon Treasury yield curve at retirement determines both $\mathrm{DPAF}^*$, the deferred pension annuity factor, and the vector of TIPS prices, $\mathbf{p}$. Both are in the denominator of Eq. (2). The higher the yields, the smaller the value of $\mathrm{DPAF}^*$ (see Eq. [A1]), hence the greater the payout rate. Furthermore, the higher the yields, the lower the TIPS prices (all else equal), hence the greater the payout rate. Intuitively, if the inflation-indexed zero-coupon Treasury yield curve shifts upward (loosely speaking, if real yields rise), then the present value of the inflation-indexed payments from the TIPS and deferred annuity is smaller. Equivalently, a given level of wealth at retirement can purchase a stream of larger inflation-indexed payouts.

In Table 2, we present results from linear regressions of the target real payout rate on the value-weighted real yield of the TIPS ladder at retirement, conditional on retirement age and length of the TIPS ladder; see the first three columns for versions of the strategy that use an inflation-indexed deferred annuity. Goodness of fit as measured by $R^2$ is 90% or better. The value-weighted real yield on the TIPS ladder is a proxy for the yield curve. The longer the TIPS ladder, the better this average yield represents the entire yield curve, and the tighter the relation between the payout rate and this average yield. The slope estimate is approximately the same (about 0.6) across retirement ages and ladder lengths in the simulation. Hence, each increase of 1% in the value-weighted real yield of the TIPS ladder translates into about a 0.6% increase in the strategy payout rate, on average.

When the deferred annuity is not indexed for inflation, the target payout rate is determined by Eq. (2) where $DPAF^*$ is replaced by DPAF, the deferred pension annuity factor calculated with yields from the nominal zero-coupon Treasury yield curve. This curve is related to the inflation-indexed version; please see Eq. (7). Ignoring the risk premiums, the nominal yield at each point on the yield curve equals the inflation-indexed yield plus expected inflation for that maturity. Hence, the effect of the inflation-indexed Treasury yields on the ideal payout rate is muddied by the expected inflation curve. Nonetheless, the positive relation between the market yields at retirement (represented by the value-weighted real yield on the TIPS ladder) and the target real payout rate still is strong. See the last column in Table 2.

Given the investor's wealth at retirement (W), the deferred annuity fee ($f_{DA}$), and the secondary market trading fee for TIPS ($f_B$), the target annuity payout for the TIPS ladder/deferred annuity strategy is completely determined by the inflation-indexed zero-coupon Treasury yield curve at retirement (along with the expected inflation curve at retirement, if the annuity is not indexed), conditional on the TIPS currently available in the market and their real coupon rates. This last factor is captured by $\Sigma$ in Eq. (2). In the simulation (as in actual markets), the coupon rates and maturities of available TIPS change over time. Hence, to be precise, the distribution of the target real payout rates in our simulation is determined not only by the market yields at retirement but also by the specific characteristics of the TIPS available at retirement.

### 5.1.3. Comparison with Shankar

The simulation results concerning target real payout rates are close agreement with the deterministic calculations reported by Shankar (2009). Shankar carries out this analysis for assumed real returns of 0%, 1%, 2%, and 3% for TIPS and the calculation of the deferred annuity premium. Shankar's results are for strategies with inflation-indexed deferred annuities. In Table 3, we compare the payout rates reported in Shankar's Table 2 with our simulation results.

The level of the target real payout rates in the simulation is about the same as in Shankar (2009) for comparable versions of the TIPS ladder/deferred annuity strategy and market real yields at retirement. For example, consider an investor who retires at age 65, set up a 20-year TIPS ladder, and buys the corresponding inflation-indexed deferred annuity. The statistics in the "left tail" column of Table 3 correspond to real market yields at retirement of about 0%. The average real payout rate in Shankar is 4.47%, while the corresponding rate from the simulation (based on the 5th-percentile statistic) is 4.14%. The statistics in the "middle" column correspond to real yields of about 1%. The average payout rate in Shankar is 4.97%, while the corresponding rate from the simulation (based on the median) is 4.86%.

Shankar (2009, p. 54) claims that his proposed strategy ". . . would allow retirees to enjoy real withdrawal rates substantially higher than the 4% . . ." suggested by the current con-

Table 3 Comparison of target real annual payout rates: Simulation vs. Shankar (2009)

| Retirement age (years) | Length of TIPS ladder (years) | Gender | Source | Target real payout rate in retirement (%)* | | |
|---|---|---|---|---|---|---|
| | | | | Left tail[†] | Middle[‡] | Right tail[§] |
| 60 | 20 | Male | Shankar | 4.23 | 4.73 | 5.54 |
| | | Female | Shankar | 3.86 | 4.37 | 5.20 |
| | | Unisex | Simulation | 3.69 (5%-tile) | 4.43 (M) | 5.15 (75%-tile) |
| | | | | 3.62 (E0) | 4.25 (E1) | 5.20 (E2.5) |
| 65 | 15 | Male | Shankar | 5.28 | 5.78 | 6.57 |
| | | Female | Shankar | 4.73 | 5.23 | 6.04 |
| | | Unisex | Simulation | 4.46 (5%-tile) | 5.20 (M) | 5.93 (75%-tile) |
| | | | | 4.44 (E0) | 5.06 (E1) | 5.99 (E2.5) |
| 65 | 20 | Male | Shankar | 4.59 | 5.09 | 5.88 |
| | | Female | Shankar | 4.35 | 4.85 | 5.66 |
| | | Unisex | Simulation | 4.14 (5%-tile) | 4.86 (M) | 5.53 (75%-tile) |
| | | | | 4.05 (E0) | 4.66 (E1) | 5.59 (E2.5) |

All strategies in this table consist of a TIPS ladder and an inflation-indexed deferred annuity. The deferred annuity payments begin one month after the TIPS ladder phase. The strategies are designed so that the target real annual payout rate is the same during the TIPS ladder phase and the deferred annuity phase.

* Shankar's annual real payout rates in retirement are based on a deterministic analysis for assumed real returns of 0%, 1%, 2%, and 3% for TIPS and the calculation of the deferred annuity premium. Results are from his Table 2. The rates from our simulation are target real payout rates for individual investors, and the statistics are based on 10,000 replications for each strategy.

[†] Payout rates from Shankar correspond to calculations based an assumed real return of 0%. Target payout rates from our simulation either are the 5%-tile statistics or the expected target payout based on linear regression analysis (E0), given a 0% value-weighted average real yield on the TIPS ladder. (Note: the 5%-tile of the value-weighted average real yield on the TIPS ladder is about 0.09% for 15-year ladders and about 0.14% for 20-year ladders in the simulation.)

[‡] Payout rates from Shankar correspond to calculations based an assumed real return of 1%. Target payout rates from our simulation either are the median statistics (M) or the expected target payout based on linear regression analysis (E1), given a 1% value-weighted average real yield on the TIPS ladder. (*Note:* the median of the value-weighted average real yield on the TIPS ladder is about 1.15% for 15-year ladders and about 1.32% for 20-year ladders in the simulation.)

[§] Payout rates from Shankar correspond to the simple average of the results based assumed real returns of 2% and 3%. Target payout rates from our simulation either are the 75%-tile statistics or the expected target payout based on linear regression analysis (E2.5), given a 2.5% value-weighted average real yield on the TIPS ladder. (Note: the 75%-tile of the value-weighted average real yield on the TIPS ladder is about 2.47% for 15-year ladders and about 2.55% for 20-year ladders in the simulation.)

census in the literature. We find that this claim is true for many but not all circumstances. Specifically, the 5th-percentile statistic for the target real payout rate (assuming an inflation-indexed deferred annuity) is less than 4% when the retirement age is 55 or 60; the median target real payout rate is less than 4% for early retirement (age 55) with long TIPS ladders (25 years or longer). Please see the first two columns in Table 1. Our linear regression results in Table 2 show that these lower real payout rates occur when market real yields are low. Nonetheless, if an investor waits at least until age 65 to retire, then the TIPS ladder/inflation-indexed deferred annuity strategy is highly likely to have a target real payout rate exceeding 4%, provided that the value-weighted real yield on the TIPS ladder is at least 0%.

#### *5.1.4. Shortfall risk*

An important claim in Shankar (2009) as well as in Sexauer, Peskin, and Cassidy (2012) is that the TIPS ladder/deferred annuity strategy guarantees a fixed real payout rate. We analyze this claim by examining shortfall risk over retirement relative to the target real annual payout rate. Specifically, we examine the difference between the target real annual payout rate and the lowest realized 12-month real payout rate.

*5.1.4.1. Shortfall risk when the deferred annuity is indexed.* First, consider TIPS ladder/deferred annuity strategies in which the annuity is indexed for inflation. We find that the shortfall risk is likely to be small. Please see the first three columns in Panel A of Table 4. The median shortfall in the real payout rate is on the order of about 0.1% across all strategies, and the 95th-percentile shortfall is about 0.75% or less. In real dollar terms, if the ideal payout ratio is 4% (corresponding to annual real payout of $20,000 when wealth at retirement is $500,000), then a shortfall of 0.75% is about 19% of the annual payout (or $3,750 in purchasing power).

Over the TIPS ladder phase, however, we find the shortfall is a rare, transient event. Depending on the retirement age and length of the ladder, the odds of a shortfall are about 5% to 15%. For almost all simulated investors who experience a shortfall, the shortfall occurs only once (specifically, in only one ladder year). For example, among investors who retire at age 65 and elect a 20-year TIPS ladder, 86.78% experience no shortfall during the ladder period (i.e., the realized real payout rate equals the target rate); 12.74% experience one month with no payout; and 0.48% experience two months with no payout.

These relatively small fluctuations are a consequence of the cash management strategy in the simulation. Specifically, during the life of the ladder, the TIPS for a given annual rung in the ladder are selected to mature shortly before that year. The proceeds are held in a cash account that earns a nominal one-month nominal T-bill return. Hence, fluctuations in the T-bill rate relative to the realized inflation rate occasionally cause the retiree to come up short at the end of a year in the TIPS ladder phase.

Over the deferred annuity payout period, fluctuations in the real payout rate are much more common but also are small and on the order of 0.25% or less. Please see the last three columns of Panel A in Table 4. In the simulation, when deferred annuities are inflation-indexed, they are indexed to the CPI-U with a three-month lag. We calculate the real value of each realized payout in terms of retirement date dollars. As a consequence, the indexation lag induces relatively small fluctuations in the real payout rate during the deferred annuity payout phase. In some cases, the fluctuations may even be in favor of the retiree; see the 5th-percentile column. When this shortfall statistic is negative, it means that the realized real payout rate in all rolling 12-month periods exceeded the target payout rate for at least five percent of investors.

These fluctuations also affect inflation-indexed immediate annuities. In our simulations, the median range for immediate annuity payouts (measured as the difference between the highest and lowest 12-month rolling realized real payout rates) is about 0.25% and the 95th-percentile range is about 0.5% or less. In short, any annuity that is indexed with a lag will not perfectly track changes in purchasing power as measured by changes in the CPI-U.

Table 4 Shortfall risk: Target real annual payout rate minus lowest realized rolling 12-month real payout rate (%)

| Retirement age (years) | TIPS ladder (years) | Over retirement | | | Over life of TIPS ladder | | | Over deferred annuity payout | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 5%-tile | Median | 95%-tile | 5%-tile | Median | 95%-tile | 5%-tile | Median | 95%-tile |
| Panel A: Strategies with inflation-indexed deferred annuity | | | | | | | | | | |
| 55 | 10 | 0.00 | 0.11 | 0.42 | 0.00 | 0.00 | 0.40 | −0.03 | 0.09 | 0.22 |
| | 15 | 0.00 | 0.10 | 0.61 | 0.00 | 0.00 | 0.56 | −0.04 | 0.08 | 0.20 |
| | 20 | 0.00 | 0.08 | 0.60 | 0.00 | 0.00 | 0.56 | −0.05 | 0.06 | 0.19 |
| | 25 | 0.00 | 0.05 | 0.57 | 0.00 | 0.00 | 0.53 | −0.07 | 0.05 | 0.18 |
| | 30 | 0.00 | 0.00 | 0.50 | 0.00 | 0.00 | 0.47 | −0.08 | 0.04 | 0.17 |
| 60 | 10 | 0.00 | 0.12 | 0.47 | 0.00 | 0.00 | 0.44 | −0.05 | 0.09 | 0.23 |
| | 15 | 0.00 | 0.09 | 0.67 | 0.00 | 0.00 | 0.61 | −0.05 | 0.07 | 0.21 |
| | 20 | 0.00 | 0.05 | 0.63 | 0.00 | 0.00 | 0.58 | −0.07 | 0.06 | 0.20 |
| | 25 | 0.00 | 0.01 | 0.57 | 0.00 | 0.00 | 0.54 | −0.09 | 0.04 | 0.18 |
| 65 | 10 | 0.00 | 0.12 | 0.53 | 0.00 | 0.00 | 0.50 | −0.06 | 0.08 | 0.25 |
| | 15 | 0.00 | 0.07 | 0.73 | 0.00 | 0.00 | 0.67 | −0.08 | 0.07 | 0.23 |
| | 20 | 0.00 | 0.01 | 0.64 | 0.00 | 0.00 | 0.59 | −0.10 | 0.05 | 0.20 |
| 70 | 10 | 0.00 | 0.11 | 0.61 | 0.00 | 0.00 | 0.57 | −0.09 | 0.08 | 0.28 |
| | 15 | 0.00 | 0.02 | 0.75 | 0.00 | 0.00 | 0.68 | −0.12 | 0.06 | 0.24 |
| 75 | 10 | 0.00 | 0.08 | 0.67 | 0.00 | 0.00 | 0.64 | −0.13 | 0.07 | 0.30 |
| Panel B: Strategies with deferred annuity that is not indexed for inflation | | | | | | | | | | |
| 55 | 10 | 0.00 | 3.20 | 6.15 | 0.00 | 0.00 | 0.48 | 1.12 | 3.33 | 6.22 |
| | 15 | 0.00 | 2.98 | 5.44 | 0.00 | 0.00 | 0.66 | 1.18 | 3.17 | 5.56 |
| | 20 | 0.00 | 2.65 | 4.87 | 0.00 | 0.00 | 0.62 | 1.08 | 2.98 | 5.07 |
| | 25 | 0.00 | 2.22 | 4.46 | 0.00 | 0.00 | 0.56 | 1.12 | 2.78 | 4.82 |
| | 30 | 0.00 | 0.70 | 3.97 | 0.00 | 0.00 | 0.49 | 0.97 | 2.60 | 4.51 |
| 60 | 10 | 0.00 | 3.10 | 5.99 | 0.00 | 0.00 | 0.52 | 0.83 | 3.29 | 6.08 |
| | 15 | 0.00 | 2.79 | 5.24 | 0.00 | 0.00 | 0.70 | 0.94 | 3.10 | 5.36 |
| | 20 | 0.00 | 2.36 | 4.68 | 0.00 | 0.00 | 0.62 | 0.89 | 2.95 | 5.02 |
| | 25 | 0.00 | 1.04 | 4.16 | 0.00 | 0.00 | 0.55 | 0.99 | 2.78 | 4.68 |
| 65 | 10 | 0.00 | 3.01 | 5.93 | 0.00 | 0.00 | 0.57 | 0.75 | 3.33 | 6.10 |
| | 15 | 0.00 | 2.50 | 5.05 | 0.00 | 0.00 | 0.72 | 0.69 | 3.10 | 5.29 |
| | 20 | 0.00 | 1.38 | 4.44 | 0.00 | 0.00 | 0.63 | 0.66 | 2.92 | 4.83 |
| 70 | 10 | 0.00 | 2.73 | 5.72 | 0.00 | 0.00 | 0.63 | 0.51 | 3.31 | 5.94 |
| | 15 | 0.00 | 1.74 | 4.87 | 0.00 | 0.00 | 0.74 | 0.50 | 3.10 | 5.26 |
| 75 | 10 | 0.00 | 2.07 | 5.60 | 0.00 | 0.00 | 0.69 | 0.12 | 3.25 | 5.96 |

All strategies consist of a TIPS ladder and a deferred annuity. Annuity payments begin one month after the TIPS ladder phase. When the annuity is indexed for inflation, it is indexed in the same manner as TIPS, that is, to the CPI-U with a three-month lag. Payout rates are real annual payouts as a percentage of total funds at retirement. The statistics in this table are conditional on the retiree collecting at least 12 payments in the respective period.

*5.1.4.2. Shortfall risk when the deferred annuity is not indexed.* The story is much worse when the deferred annuity is not indexed for inflation. The shortfall over retirement can be large. Please see Panel B in Table 4. Over the TIPS ladder phase, the shortfall risk is low, and magnitude of the shortfall is about the same as for corresponding scenarios when the deferred annuity is indexed. However, if the retiree survives to the deferred annuity phase, then the shortfall can be dramatic. The median shortfall is on the order of about half of the median target real payout ratio.

This shortfall is not transient. Although deflationary periods can occur in the simulation as over the historical period from which our expected inflation curves are estimated, the general trend in purchasing power in the United States over the last century has been downward. Hence, over the TIPS ladder phase of 10 to 30 years, purchasing power of the nominal deferred annuity payment is likely to be reduced significantly. This trend generally continues during the annuity payout. Hence, the shortfall statistics in Panel B of Table 4 represent shortfall at or near the end of the retiree's life.

### *5.2. Initial allocation and reluctance to annuitize*

The earlier the age when the deferred annuity payout begins and the shorter the life of the TIPS ladder, the higher the allocation at retirement to the deferred annuity premium. This relation generally holds true whether the deferred annuity is indexed for inflation or not. Please see Panel A of Table 5. These results are consistent with common financial sense. The earlier the age when a deferred annuity begins its payout, the more payments the investor is likely to collect. Hence the annuity is more expensive. Furthermore, conditional on age of retirement, the shorter the life of the TIPS ladder, the longer the period in retirement that the investor expects to require payouts from the deferred annuity. Hence, the deferred annuity is more expensive. Thus, the allocation at retirement to the deferred annuity premium is greater. See Panel B in Table 5.

We also examine the relation between the allocation to the deferred annuity and the value-weighted real yield on the TIPS ladder at retirement. The latter serves as proxy for market yields at retirement. Conditional on age of retirement and length of the TIPS ladder in years, the allocation to the deferred annuity generally declines the higher the market yields. For example, see Fig. 1. Again, this relation makes financial sense. The lower the discount rate applied to real dollar payouts from the deferred annuity, the greater the present value of the deferred annuity. The distribution of market yield curves at retirement across simulated retiree histories accounts for the range of initial allocations to the deferred annuity, given retirement age and length of the TIPS ladder. Please see the 5th- and 95th-percentile statistics in Table 5.

An important claim in Shankar (2009) is that a TIPS ladder/deferred annuity strategy significantly reduces the liquidity problem present when buying a non-refundable immediate life annuity. Shankar (p. 58) states, " . . . the IPRA strategy allows the bulk of the retirement portfolio to be held in TIPS and requires only a small fraction of the retirement savings to be used to pay the non-refundable longevity policy premium; this mitigates the 'large irreversible commitment' inherent in buying an immediate lifetime annuity with the entire retirement savings." However, we determine that for shorter ladders and low real market yields at retirement, the investor may need to allocate 50% or more of savings at retirement to the deferred annuity premium. Please see Table 5.

Nevertheless, the simulation results show that the investor can keep the allocation to the deferred annuity low regardless of market real yields at retirement if the investor chooses a long TIPS ladder. The earlier that the investor retires, the longer the TIPS ladder needs to be. For example, an investor who wants to cap the allocation to a non-indexed deferred annuity to about 25%, regardless of market rates at retirement, needs a 20-year ladder if retiring at age 55 but only a 15-year ladder if retiring at age 65.

Table 5 Initial asset allocation for strategies with TIPS ladder and deferred annuity

| Retirement age (yrs.) | TIPS ladder (yrs.) | Allocation to inflation-indexed deferred annuity in Shankar | Age when annuity payout begins (years) | Allocation to inflation-indexed deferred annuity in the simulation | | | Allocation to non-indexed deferred annuity in the simulation | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 5%-tile | Median | 95%-tile | 5%-tile | Median | 95%-tile |
| Panel A: Scenarios ranked by median allocation to the inflation-indexed deferred annuity in the simulation | | | | | | | | | |
| 55 | 30 | | 85 | 0.037 | 0.084 | 0.117 | 0.007 | 0.034 | 0.062 |
| 60 | 25 | | 85 | 0.051 | 0.104 | 0.140 | 0.013 | 0.048 | 0.082 |
| 65 | 20 | Male: 0.073; Female: 0.116 | 85 | 0.075 | 0.135 | 0.172 | 0.023 | 0.073 | 0.112 |
| 55 | 25 | | 80 | 0.081 | 0.165 | 0.220 | 0.020 | 0.077 | 0.131 |
| 70 | 15 | | 85 | 0.120 | 0.188 | 0.228 | 0.048 | 0.118 | 0.165 |
| 60 | 20 | Male: 0.138; Female: 0.203 | 80 | 0.119 | 0.207 | 0.264 | 0.035 | 0.114 | 0.173 |
| 75 | 10 | | 85 | 0.207 | 0.291 | 0.327 | 0.115 | 0.216 | 0.271 |
| 55 | 20 | | 75 | 0.153 | 0.275 | 0.347 | 0.046 | 0.152 | 0.231 |
| 65 | 15 | Male: 0.190; Female: 0.267 | 80 | 0.173 | 0.276 | 0.332 | 0.071 | 0.177 | 0.243 |
| 60 | 15 | | 75 | 0.222 | 0.354 | 0.424 | 0.090 | 0.229 | 0.313 |
| 70 | 10 | | 80 | 0.285 | 0.393 | 0.442 | 0.158 | 0.295 | 0.367 |
| 55 | 15 | | 70 | 0.264 | 0.419 | 0.497 | 0.105 | 0.274 | 0.372 |
| 65 | 10 | | 75 | 0.344 | 0.477 | 0.534 | 0.188 | 0.358 | 0.445 |
| 60 | 10 | | 70 | 0.397 | 0.540 | 0.605 | 0.219 | 0.414 | 0.507 |
| 55 | 10 | | 65 | 0.432 | 0.592 | 0.660 | 0.236 | 0.456 | 0.555 |
| Panel B: Scenarios ranked by age at retirement and length of the TIPS ladder | | | | | | | | | |
| 55 | 10 | | 65 | 0.432 | 0.592 | 0.660 | 0.236 | 0.456 | 0.555 |
| | 15 | | 70 | 0.264 | 0.419 | 0.497 | 0.105 | 0.274 | 0.372 |
| | 20 | | 75 | 0.153 | 0.275 | 0.347 | 0.046 | 0.152 | 0.231 |
| | 25 | | 80 | 0.081 | 0.165 | 0.220 | 0.020 | 0.077 | 0.131 |
| | 30 | | 85 | 0.037 | 0.084 | 0.117 | 0.007 | 0.034 | 0.062 |
| 60 | 10 | | 70 | 0.397 | 0.540 | 0.605 | 0.219 | 0.414 | 0.507 |
| | 15 | | 75 | 0.222 | 0.354 | 0.424 | 0.090 | 0.229 | 0.313 |
| | 20 | Male: 0.138; Female: 0.203 | 80 | 0.119 | 0.207 | 0.264 | 0.035 | 0.114 | 0.173 |
| | 25 | | 85 | 0.051 | 0.104 | 0.140 | 0.013 | 0.048 | 0.082 |
| 65 | 10 | | 75 | 0.344 | 0.477 | 0.534 | 0.188 | 0.358 | 0.445 |
| | 15 | Male: 0.190; Female: 0.267 | 80 | 0.173 | 0.276 | 0.332 | 0.071 | 0.177 | 0.243 |
| | 20 | Male: 0.073; Female: 0.116 | 85 | 0.075 | 0.135 | 0.172 | 0.023 | 0.073 | 0.112 |
| 70 | 10 | | 80 | 0.285 | 0.393 | 0.442 | 0.158 | 0.295 | 0.367 |
| | 15 | | 85 | 0.120 | 0.188 | 0.228 | 0.048 | 0.118 | 0.165 |
| 75 | 10 | | 85 | 0.207 | 0.291 | 0.327 | 0.115 | 0.216 | 0.271 |

Simulation results are based on unisex mortality risk, whereas Shankar (2009) evaluates scenarios separately for male and female mortality risk. Results from Shankar's Table 2 are based on a deterministic analysis for an assumed real return of 1% for TIPS and the calculation of the deferred annuity premium. By comparison, the median value-weighted average yield on TIPS ladders in the simulation is about 1.15% for 15-year ladders and about 1.32% for 20-year ladders.

### *5.3. The bequest motive and preferences for retirement strategies*

To evaluate the effect of the strength of the bequest motive, D, on reluctance to commit all savings at retirement to an immediate annuity, we analyze expected utility at several

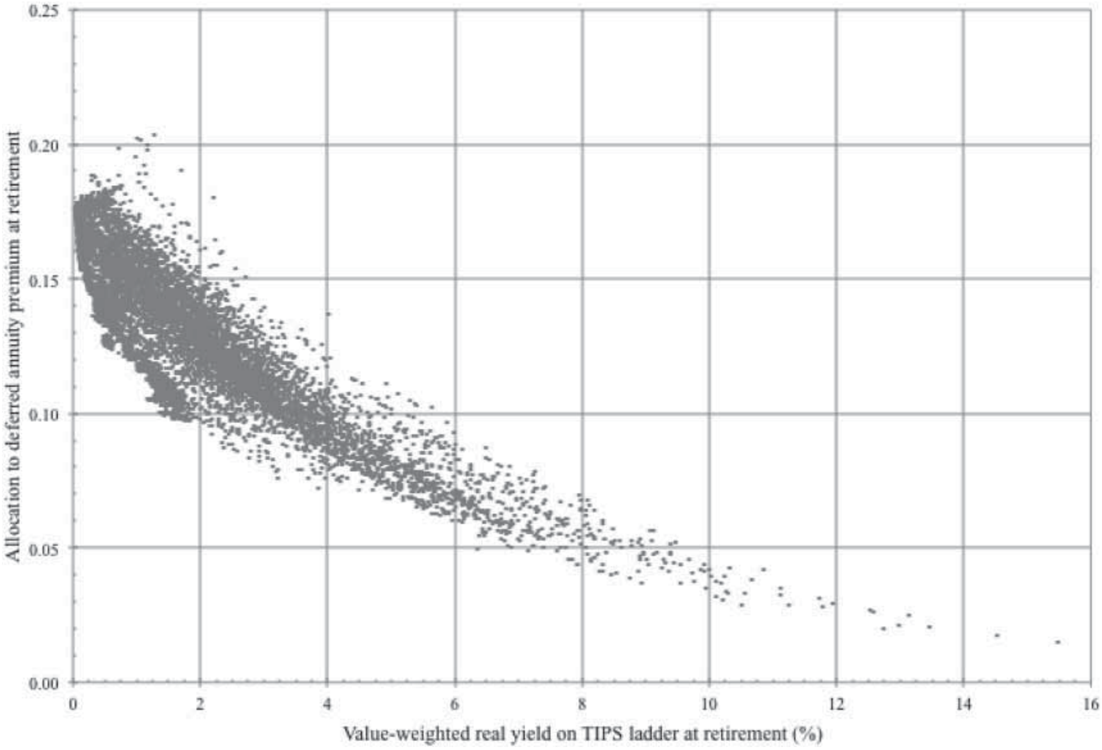

Fig. 1. This figure plots the allocation to the inflation-indexed deferred annuity for each simulated investor against the market value-weighted real yield on the TIPS ladder; both values are at the time of retirement. For this figure, all investors retire at age 65 and survive at least 12 months in retirement. At retirement, each investor buys the TIPS for a 20-year ladder and pays a premium on a deferred annuity that begins payouts one month after the 20-year ladder phase ends.

levels of D ranging from zero (consumption has utility but bequests do not) to one (bequests have utility but consumption does not). When $D = 0$, we hypothesize that the investor will always prefer the immediate annuity to a strategy of a TIPS ladder combined with a deferred annuity. At the other extreme, when $D = 1$, we hypothesize that the investor will always prefer a strategy of a TIPS ladder combined with a deferred annuity over an immediate annuity.

The simulation results are consistent with both hypotheses. At the extremes for D, the expected utilities of the inflation-indexed immediate annuity and the TIPS ladder/inflation-indexed deferred annuity strategy are significantly different and differ in the predicted direction. These results hold for all variations that we examined of the TIPS ladder/deferred annuity strategy, where annuities (immediate and deferred) are indexed to the CPI-U in the same manner as TIPS, and for all levels of risk aversion from 0.5 to 5.0 in our utility model.

For intermediate values of the bequest motive, D, the general pattern is that the higher the level of risk aversion, the lower the cross-over value of D at which the investor switches from preference for an inflation-indexed immediate annuity to preference for a TIPS ladder/inflation-indexed deferred annuity strategy. Please see Table 6. The expected utility of a bequest contributes more to total expected utility in two ways. First, as D increases, greater weight is given to it; see Eq. (6). Second, for a given retirement age and length of the TIPS ladder, the expected utility of a bequest grows exponentially as the risk aversion increases.

Table 6 Preference for immediate annuity vs. TIPS ladder/deferred annuity strategy in terms of strength of bequest motive (D); cross-over value for D where preference switches from the immediate annuity to the TIPS ladder/deferred annuity

| Retirement age (years) | Length of TIPS ladder (years) | Level of risk aversion ($\eta$) | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0.5 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 |
| 55 | 10 | 0.95 | 0.95 | 0.90 | 0.65 | 0.65 | 0.55 |
| 55 | 15 | 0.95 | 0.95 | 0.85 | 0.65 | 0.65 | 0.65 |
| 55 | 20 | 0.95 | 0.95 | 0.85 | 0.65 | 0.55 | 0.55 |
| 55 | 25 | 0.95 | 0.95 | 0.85 | 0.65 | 0.45 | 0.45 |
| 55 | 30 | 0.95 | 0.95 | 0.85 | 0.55 | 0.35 | 0.35 |
| 60 | 10 | 0.95 | 0.95 | 0.75 | 0.55 | 0.55 | 0.55 |
| 60 | 15 | 0.95 | 0.95 | 0.85 | 0.55 | 0.55 | 0.55 |
| 60 | 20 | 0.95 | 0.95 | 0.85 | 0.55 | 0.45 | 0.45 |
| 60 | 25 | 0.95 | 0.95 | 0.85 | 0.45 | 0.35 | 0.35 |
| 65 | 10 | 0.95 | 0.95 | 0.75 | 0.45 | 0.45 | 0.35 |
| 65 | 15 | 0.95 | 0.95 | 0.75 | 0.45 | 0.35 | 0.35 |
| 65 | 20 | 0.95 | 0.95 | 0.85 | 0.50 | 0.25 | 0.25 |
| 70 | 10 | 0.95 | 0.95 | 0.65 | 0.40 | 0.25 | 0.25 |
| 70 | 15 | 0.95 | 0.95 | 0.80 | 0.40 | 0.25 | 0.25 |
| 75 | 10 | 0.95 | 0.95 | 0.70 | 0.30 | 0.15 | 0.15 |

The comparison is between inflation-indexed immediate annuities and strategies with a fixed-life TIPS ladder and an inflation-indexed deferred annuity that begins payouts after the ladder phase. All annuities are indexed to the CPI-U with a three-month lag, analogous to TIPS. We compare the expected utilities for the immediate annuity and the ladder/deferred annuity strategies for D, the value of the strength of the bequest motive, where $D = 0.0, 0.1, 0.2, \ldots, 1.0$. If $D = 0$, then investors gain utility only from consumption and not from bequests; if $D = 1$, then investors gain all utility from bequests and none from consumption. The cross-over point is the midpoint between the highest value of D for which the investor prefers the immediate annuity to the ladder/deferred annuity based on expected utility (i.e., the expected utility is significantly higher for the immediate annuity) and the lowest value of D for which the investor prefers the ladder/deferred annuity. To test for significance, we calculate the version of the Wilcoxon statistic for the difference in expected values, where average rank replaces tied ranks. We define the difference as significant if the two-sided $p$-value is less than 0.10.

Please see column 6 in Table 7. That is, the bequest is relatively more valuable the greater the level of risk aversion under the utility models in our analysis.

In addition, for risk aversion levels greater than one, the crossover value of D tends to fall the older the investor is when retiring, all else equal (specifically, for ladders of the same length). For example, compare strategies with 10-year TIPS ladders in Table 6. This result is consistent with the fact than an older retiree has a shorter life expectancy and thus is more likely to be able to leave a bequest in the form of the balance of the TIPS ladder at death.

The fact that most investors in the real world are reluctant to commit all of their savings at retirement to immediate annuities is consistent with Bernheim's (1991) conclusion that bequest motives are strong for a large segment of the population. Our results in Table 6 illustrate a more subtle point. The higher the level of risk aversion and the older the investor at retirement, the weaker the bequest motive needs to be for the investor to prefer an investment strategy that avoids committing all of the investor's savings to a non-refundable immediate life annuity.

Given the current unavailability of inflation-indexed deferred annuities, an interesting question is whether the same observations hold for TIPS ladder/deferred annuity strategies

Table 7 Expected utilities at different levels of risk aversion; retirement at age 65

| Risk aversion ($\eta$) | Immediate inflation-indexed annuity | | 20-year TIPS ladder with inflation-indexed deferred annuity | | |
|---|---|---|---|---|---|
| | Expected utility of consumption | De minimus expected utility of bequest* | Expected utility of consumption | Expected utility of bequest | Absolute expected utility of bequest† |
| 0.5 | 205.52 | −0.76 | 164.04 | 1.56 | 2.32 |
| 1 | 160.57 | −1.27 | 132.93 | 0.23 | 1.50 |
| 2 | 104.57 | −4.98 | 91.09 | −1.99 | 2.99 |
| 3 | 73.38 | −27.41 | 61.67 | −12.24 | 15.16 |
| 4 | 54.53 | −184.37 | 13.25 | −83.14 | 101.23 |
| 5 | 40.76 | −1384.03 | −244.33 | −624.42 | 759.61 |

* The immediate annuity strategy uses all cash at retirement to purchase an irrevocable immediate life annuity. Hence, no cash is available for a bequest. However, when we calculate utilities, we set de minimus values on monthly consumption and the bequest at death to avoid extreme negative values when the consumption or bequest are tiny. Thus, the "expected utility of bequest" for an immediate annuity is the average of the utility of the de minimus bequest discounted back to retirement at the inflation rates for each investor's history.

† The absolute expected utility of bequest is the expected utility of bequest minus the de minimus value that corresponds to no bequest.

when the deferred annuity is not indexed. Not surprisingly, the crossover point for D frequently is higher, because the real payout ratio falls over the course of the deferred annuity phase. However, the general pattern illustrated in Table 6 still holds, because the TIPS ladder is the source of funds for a bequest. While the non-indexed deferred annuity has a lower real payout than either the TIPS ladder or a corresponding inflation-indexed deferred annuity, this payout is in the late phase of retirement, and the retiree might not even survive to that phase.

## 6. Conclusions

The ideal solution for a retiree is a source of income that presents no risk of financial ruin, protects against loss of purchasing power, shields against longevity risk, and provides a target standard of living. In theory, a strategy that combines a fixed-term TIPS ladder with an inflation-indexed deferred annuity addresses the first three concerns.

We show that Shankar's (2009) claims about the advantages of a TIPS ladder/deferred strategy are valid under many circumstances but not all. The strategy performs best if the deferred annuity is indexed for inflation. Because the target real payout rate is determined by the market real yields at retirement, some combinations of retirement age and length of the TIPS ladder have payout rates less than 4% when yields are very low. To assure a real payout rate significantly above 4%, we show that the investor should wait to retire until age 65 or later. We also find that when we make realistic assumptions about cash management methods during the TIPS ladder phase and the indexation lag of the deferred annuity, the realized payout rate for an individual investor can fluctuate over retirement. However, these fluctuations usually are small enough not be catastrophic, rarely occur during the TIPS ladder phase, and may be above as well as below the target payout rate during the deferred annuity phase (and, in any case, are no worse than for a retiree with an inflation-indexed immediate annuity).

A major advantage that a TIPS ladder/deferred annuity strategy has over a non-refundable immediate life annuity is that only part of the investor's wealth at retirement is irrevocably allocated to an annuity. Because of investors' reluctance to annuitize, this feature is attractive. However, if market real yields are low at retirement, then the allocation to the deferred annuity premium could be much greater than 30%. For example, if an investor retires at age 65, sets up a 10-year TIPS ladder, and buys a non-indexed deferred annuity, then the allocation to the deferred annuity might be 19% or lower (if market real yields are high) or 44% or higher (if market real yields are low). Nonetheless, the investor has an excellent chance of keeping the deferred annuity premium below 30% of wealth at retirement if the investor builds a sufficiently long ladder.

On the other hand, if the deferred annuity is not indexed for inflation, then the realized real payout rate over the deferred annuity phase can decline substantially at a point in life when the retiree is least able to compensate. Unfortunately, as Shankar concedes, a market does not yet exist for inflation-indexed deferred annuities. Moreover, the difficulty in long-term forecasting of inflation seems likely to continue to deter insurance companies from offering such a product.

Finally, we explore the relation between the strength of the bequest motive and preference for an inflation-indexed immediate annuity versus a TIPS ladder/deferred annuity strategy. Our results are consistent with evidence in the literature that the bequest motive is strong. If investors make choices based on expected utility, then they prefer an immediate annuity when strength of the bequest motive is low, and they prefer the TIPS ladder/deferred annuity when strength of the bequest motive is high. Moreover, the greater the investor's risk aversion, the more likely the investor prefers the TIPS ladder/deferred annuity for a given level of strength of the bequest motive. These results suggest that the TIPS ladder/deferred annuity strategy may have considerable appeal to individual investors who are reluctant to annuitize their life's savings.

## Appendix A: Pension annuity factors

To determine the monthly real dollar annuity payment, first calculate the immediate pension annuity factor (IPAF), assuming discrete, real monthly payouts. This article applies a discrete version of Milevsky's (2006) Eq. (6.3). In the discrete version, the stochastic present value of a pension annuity is

$$a_x(\tilde{D}_x) = \sum_{i=1}^{\tilde{D}_x} exp\left( - y^*_{t(x)}(i)\frac{i}{12}\right), \tag{A1}$$

where $y^*_{t(x)}(i)$ is the annualized inflation-indexed yield at time *t(x)* on a zero-coupon, default-risk free TIPS that matures in *i* months; *t(x)* is the date on which the investor is age *x* (in months); and $\tilde{D}_x$ is the random number of months until death, given that the investor is alive at age *x*. Mortality index tables generally are constructed so that

$$P\{\tilde{D}_x > i | current\ age\ x\} = MI(x+i)/MI(x), \tag{A2}$$

where *MI(x)* is the mortality index value at age *x* (in months). Thus,

$$P\{\tilde{D}_x \leq i | current\ age\ x\} = 1 - MI(x + i)/MI(x). \tag{A3}$$

When $\tilde{D}_x$ is a discrete, integer-valued random variable,

$$P\{\tilde{D}_x = d | current\ age\ x\}$$

$$= P\{\tilde{D}_x \leq d | current\ age\ x\} - P\{\tilde{D}_x \leq d - 1 | current\ age\ x\}. \tag{A4}$$

Thus,

$$P(\tilde{D}_x = d | current\ age\ x\} = [MI(x + d - 1) - MI(x + d)]/MI(x). \tag{A5}$$

The IPAF is defined as

$$\bar{a}_x = \sum_{d=1}^{D(x)} a_x(d)\, P\{\tilde{D}_x = d | current\ age\ x\}, \tag{A6}$$

where *D(x)* is the maximum possible months until death in the mortality table, given that the investor is alive at age *x*.

The deferred pension annuity factor (DPAF$^*$) is the present value of a one-dollar real deferred annuity where the present value calculation takes mortality credits into account. To determine the monthly real dollar annuity payment, first calculate the DPAF$^*$, assuming discrete, real monthly payouts. This article applies a discrete version of Milevsky's (2006) Eq. (6.14). In the discrete version, the stochastic present value of a deferred pension annuity is

$$\bar{a}_{x,u} = \bar{a}_{x+u} P\{\tilde{D}_x > u | current\ age\ x\} exp\left( - y^*_{t(x)}(u)\frac{u}{12}\right), \tag{A7}$$

where the first payment of the deferred annuity is $u + 1$ months after the investor turns age *x* (in months); $\bar{a}_{x+u}$ is defined by Eq. (A6), where $x + u$ replaces *x*; and the probability is defined by Eq. (A2). We assume that the deferred annuity is indexed to inflation in the same manner as TIPS. In particular, the inflation index is the Consumer Price Index for All Urban Consumers (CPI-U), non-seasonally adjusted, and the annuity is indexed with the same lag as TIPS.

We also analyze the strategy when the deferred annuity is not indexed. In that case, we carry out the calculations for the deferred pension annuity factor, DPAF, with $y_{t(x)}(i)$, the annualized nominal yield at time *t(x)* on a zero-coupon, default-risk free Treasury bond that matures in *i* months. The fixed nominal value of the deferred annuity payment is set equal to the time of retirement dollar value of the level inflation-indexed payouts from the TIPS ladder.

## Appendix B: Construction of the TIPS ladder

Let $\Sigma_{i,n}$ be the sum of the inflation-indexed cash flows to be paid in year *i* by the bond that matures in ladder year *n*. $\Sigma_{i,n}$ is expressed in retirement date dollars. When $i < n$, $\Sigma_{i,n}$ is the sum of the two coupon payments to be paid in ladder year *i*. When $i = n$, $\Sigma_{i,n}$ is the sum of the one or two coupon payments to be paid in ladder year *n* and the par to be paid at maturity. When $i > n$, $\Sigma_{i,n} = 0$. Thus, in year *k*, the total of the payments from the TIPS is

$$A_k = \sum_{j=k}^{N-1} \Sigma_{k,j} n_j, \quad k = 1, 2, \ldots, N-1, \tag{B1}$$

where $n_j$ is the number of TIPS that mature in ladder year $j$, $N$ is number of years in the ladder, and $A_k$ is expressed in terms of retirement date dollars. (Keep in mind that TIPS payments in ladder year $k$ finance consumption in ladder year $k + 1$.)

Consistent with the financial planning paradigm of consumption smoothing (see, e.g., Kotlikoff (2007)), we impose the constraint that $A_k = A$ for all years $k$. For convenience, we express the system of equations in (B1) in matrix notation:

$$\Sigma \mathbf{n} = A\mathbf{1}, \tag{B2}$$

where $\Sigma$ is an N-1 by N-1 upper triangular array. By design, this array is non-singular. Hence we can solve for the number of bonds of each maturity:

$$\mathbf{n} = A\Sigma^{-1}\mathbf{1}. \tag{B3}$$

Total cost of the ladder is

$$w_{TL}W = A(1 + f_B)\mathbf{p}^T\Sigma^{-1}\mathbf{1} + A, \tag{B4}$$

where $w_{TL}$ is the allocation of wealth $W$ at retirement to the TIPS ladder; $f_B$ is the transactions cost paid to buy TIPS, expressed as a percentage of price; $\mathbf{p}$ is a vector of prices for the TIPS maturing in years $k = 1, 2, \ldots$, N - 1 of the ladder (where price is dollars per \$100 par); and the second term, $A$, on the right-hand side is the cash set aside for consumption in the first year of the ladder.

At this stage in the calculation, we simplify by assuming that the transactions cost, $f_B$, is the same rate for all TIPS. In the simulation, once the investor has determined how many of each TIPS issue to buy, the investor then pays transactions fees that depend on the venue: 2% of the market price if purchased in the secondary market, and no fees if purchased at an auction.

Let $P^*_{TL}$ be the monthly payout, expressed in retirement month dollars, to be supported by the TIPS ladder. Then $P^*_{TL} = \frac{A}{12}$. We design the ladder and annuity so that $P^*_{TL} = P^*_{DA}$, where

$$P^*_{DA} = \frac{(1 - w_{TL})W(1 - f_{DA})}{DPAF^*}, \tag{B5}$$

and $f_{DA}$ is the percentage of the total annuity premium paid as a transactions fee to the issuer.

Hence,

$$\frac{A}{12} = \frac{(1 - w_{TL})W(1 - f_{DA})}{DPAF^*}. \tag{B6}$$

Solve for $w_{TL}$ in Eq. (B4) and substitute into Eq. (B6):

$$\frac{A}{12} = \frac{(1 - ((1 + f_B)\mathbf{p}^T\textstyle\sum^{-1}\mathbf{1}+1)(\mathrm{A/W}))\mathrm{W}(1-\mathrm{f}_{\mathrm{DA}})}{\mathrm{DPAF}^*}. \tag{B7}$$

After solving for A and rearranging terms, we have

$$A = \frac{W(1 - f_{DA})}{(DPAF^*/12) + (1 - f_{DA})((1 + f_B)\mathbf{p}^T\textstyle\sum^{-1}\mathbf{1} + 1)}. \tag{B8}$$

To solve for the number of TIPS for each maturity, substitute the solution from Eq. (B8) into Eq. (B3). In the simulation, we round down to the nearest whole number of TIPS, then calculate the actual cost of constructing the ladder, assuming that an amount of cash equal to $A$ is set aside for the consumption in the first year. We then recalculate the actual weight on the ladder, $w_{TL}$, by dividing actual cost by $W$. Substituting the actual weight back into Eq. (B5) gives the actual monthly deferred annuity payout, $P^*_{DA}$, in retirement year dollars.

Because the investor cannot purchase fractional TIPS, the target real payout from the TIPS ladder and the target real payout from the deferred annuity will differ. However, because we assume that the investor's life savings is \$500,000, this difference is modest and does not materially affect conclusions about the strategy.

## Appendix C: Utility functions

When $\eta \geq 1$, a numerical problem arises in the utility of consumption and utility of bequest functions defined in Eqs. (3) through (5). As C approaches 0, the utility approaches minus infinity. In this study, consumption in a given month might be zero, and the bequest might be small or zero. It might seem plausible simply to omit calculation of the utility for that specific month (or for the bequest). However, doing so implicitly sets the utility equal to zero. At a month when $C < \chi_C$ or $B < \chi_B$, the utility is negative. In that instance, not calculating utility is equivalent to setting a floor on consumption or a floor on bequests that equals the scale factor. On the other hand, if the analysis allows low or zero values for consumption or the bequest, then the utility calculation may explode or produce an undefined result.

Our solution is to set a floor lower than the cutoff by an order of magnitude. Specifically, the cutoff for consumption is $\theta_{\mathrm{C}} = \$100$, and the cutoff for bequests is $\theta_{\mathrm{B}} = \$1{,}000$. That is, the utility function for consumption in period t is

$$U(C;\, \eta,\, \chi) = \begin{cases} \dfrac{exp\left((1 - \eta)ln\left(\dfrac{C}{\chi_C}\right)\right) - 1}{1 - \eta}, & C \geq \theta_C \\[2ex] \dfrac{exp\left((1 - \eta)ln\left(\dfrac{\theta_C}{\chi_C}\right)\right) - 1}{1 - \eta}, & C < \theta_C \end{cases} \tag{C1}$$

when risk aversion $\eta \neq 1$, and

$$U(C;\,\eta,\,\chi) = \begin{cases} ln\left(\dfrac{C}{\chi_C}\right), & C \geq \theta_C \\ ln\left(\dfrac{\theta_C}{\chi_C}\right), & C < \theta_C \end{cases} \tag{C2}$$

when $\eta = 1$. The analogous definitions apply to utility of bequests.

In effect, we implicitly assume a de minimis level of monthly consumption and bequest at death. A consequence of defining utility in this manner is that realized utility for a given simulated investor is biased upward if any payout in retirement or the bequest falls below the corresponding cutoffs. The magnitude of the bias depends on frequency and magnitude of events where $C_t < \theta_C$ and/or the bequest $B < \theta_B$.

When we compare estimates of expected utility for different strategies and scenarios, the range of expected utilities may be compressed because of this bias. Thus, tests for significance of differences in expected utility might not be significant when, in fact, they are, that is, the bias is toward the null hypothesis of no difference. Hence, the tests are more conservative in rejecting the null hypothesis of no difference in expected utility.

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