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A marginal cash flow analysis of mortgagors' choices

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Abstract

A great deal of academic research has focused on determinants of the spread between interest rates on conforming versus Jumbo and 15-year versus 30-year mortgages, but much less has been done to help the borrower determine what choice is best for him. We examine these issues from the borrower's frame of reference and find that comparisons of mortgage terms can be facilitated by analyzing the marginal cash flows from one mortgage contract to another. For many borrowers the "conventional wisdom" leads to suboptimal choices; making the better choice can easily produce low-risk double-digit returns. © 2016 Academy of Financial Services. All rights reserved.

JEL classifications: G21; D14

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1. Introduction

A great deal of academic research has focused on the determinants of spreads between various types of mortgages, but many homeowners are guided only by general rules of thumb. This article briefly summarizes the sources of the spreads between different mortgages and then discusses some factors that may help borrowers assess their choices in light of their own personal circumstances. We find that while there is no "one size fits all" optimal selection, the conventional wisdoms often lead to suboptimal choices.

Before analyzing the mortgagor's choices, it is useful to recognize why different types of mortgages feature different rates. In general, for any two different mortgage contracts, the one that imposes more risk on the lender will feature the higher rate. Because long-term securities are more sensitive to interest rate changes than are short-term securities, they are

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more risky to the lender and typically command a premium (as per the Liquidity Preference Hypothesis, e.g., see Copeland, Weston, and Shastri (2005), pp. 262–264). In addition, borrowers have the option to prepay and the option to default on the mortgage. Mortgage interest rates reflect the values of these options (e.g., Kau and Keenan, 1995); starting immediately after the first payment, shorter-term mortgages at all times have a lower outstanding principal than longer-term mortgages, and therefore, the values of both the option to default and the option to refinance are lower for shorter-term mortgages.

Most borrowers will refinance when the terms are sufficiently favorable, and lenders have a good idea of how to value this option. Likelihood of default, however, may be unique to the borrower. Some researchers view default as a cold-blooded decision, with borrowers doing so whenever it is in their best interests (e.g., Kau and Keenan, 1995). In contrast, others (e.g., Brueckner, 2000) suggest that personal characteristics, such as the stability of the borrower's income stream or her reluctance to default on a mortgage, are also important. Without loss of generality, we focus on borrowers of the type Brueckner describes, specifically, ones who know they are less likely to default than the average borrower. Alternately, our analysis can be thought of as an assessment of what benefit the borrower gains if she is willing to reduce the value of her option to default (or to refinance). How can such a borrower signal this characteristic to the lenders and get a better rate than she otherwise would? Certainly just saying "I have a low risk of default" is insufficient because all borrowers can allege that. Instead, the borrower must signal through her choices that the option to default is not worth much to her.

In the insurance literature, there is a similar phenomenon in which insurers try to separate low-risk and high-risk clients from the rest, and then offer each group different rates that better correspond to their risk profiles. Such programs as discounted insurance premia for honors students or nonsmokers are two examples of insurance companies' inferring risk profiles from other personal characteristics. Another method is to offer clients a choice of deductibles. If all rates were actuarially fair conditioned on the population average, then high-risk (low-risk) clients would prefer low (high) deductibles. Insurance companies, knowing this, will adjust the population-wide actuarially fair rates to offer each group a rate that better corresponds to its risk characteristics (as revealed by the choice of deductible). Puelz and Snow (1994) find "strong evidence" that insurance companies offer better rates to drivers who choose high deductibles.

The same principle applies to mortgage markets. *Ceteris paribus*, some types of loans (e.g., longer terms or smaller down payments) are inherently more risky for the lender, in part because of an increase in default risk by the borrower. Borrowers who know that their likelihood of defaulting is much lower than average will prefer to avoid this risk premium, and should, therefore, choose shorter-term mortgages and larger down payments. Dhillon, Shilling, and Sirmans' (1990) finding that wealthier borrowers (who have greater resources and therefore, a presumably lower default risk) are more likely to choose mortgages with shorter maturities can be viewed as corroboration of the first part of this conjecture. Empirical evidence on the second part is mixed, primarily because we cannot perform true experiments, but must rely on observational studies. Lenders can observe part of the borrower's risk profile and insist that high-risk borrowers make larger down payments. Thus, studies of the relation between down payment and default likelihood are complicated by endogeneity problems (as discussed in Harrison, Noordewier, and Yavas (2004).

Many financial transactions generate an economy of scale that translates into better consumer rates for larger amounts. For example, CD rates are typically greater if the customer is willing to commit a larger amount of money. Mortgages have the opposite characteristic: conforming loans are only available up to a specified amount that depends on the average house price in the area, and mortgages above that threshold are classified as Jumbo mortgages and typically charged a higher rate. The reason is that most loans are pooled into diversified portfolios of mortgages and sold, and a larger loan makes it more difficult to diversify the portfolio.

In the remaining sections, we evaluate the borrower's choice between a Jumbo and a conforming mortgage, and that between a 15-year and a 30-year mortgage. Without loss of generality, we assume that interest rates are sufficiently low that the mortgagor's option to refinance has a value sufficiently small that the borrower can ignore it (or, alternatively, that interest rates are very stable, in which case the option to refinance is also small), and similarly we assume that the borrower ignores her option to default (presumably based on personal information that this option will not be, or is at least very unlikely to be, exercised). These assumptions are made for convenience only: even in their absence, our analysis can be viewed as an assessment of the benefit the mortgagor obtains by voluntarily reducing the value of these options. Finally, we assume the borrower swaps mortgages (Jumbo for conforming and 30-year for 15-year) and then we compare marginal cash flows as the mortgagor moves from one type of mortgage to the other.

2. Jumbo versus conforming mortgages

Jumbo rates generally exceed conforming rates. According to the Bloomberg website as of August, 2015, the gap for 30-year mortgages is about 38 basis points (4.24% vs. 3.86%). However, as recently as early March, 2013, the gap was closer to 50 basis points (4.20% vs. 3.70%). These seem typical; Cotterman and Pearce (1996) report that the spread between Jumbo mortgages and conforming loans ranges between 15 and 60 basis points. Thus, an issue that many borrowers face is whether it is worthwhile to try to come up with a larger down payment to avoid a Jumbo mortgage's premium.

The higher rate on a Jumbo mortgage is charged on the entire loan amount, not just on the excess over the threshold. This leads to a very high marginal interest rate when a mortgage exceeds the threshold by only a slight amount. One way to assess this marginal rate is to think of a Jumbo loan in the amount of J dollars as a combination of the maximal amount for a conforming loan, C, plus a marginal loan amount, M, at the marginal rate, as in Fig. 1.

If the rate on the Jumbo loan is R_J, then R_J is approximately equal to a weighted average of the rate on the conforming loan, R_c, and the rate on the marginal amount, R_m, or $R_J = \frac{C}{C+M} R_c + \frac{M}{C+M} R_m$.[1] Solving for R_m gives us $R_m = R_J + \frac{C}{M}[R_J - R_C]$. Because $R_J > R_c$, the marginal interest rate R_m will always exceed the Jumbo rate. Furthermore, because this second term increases without bound when M approaches zero, the

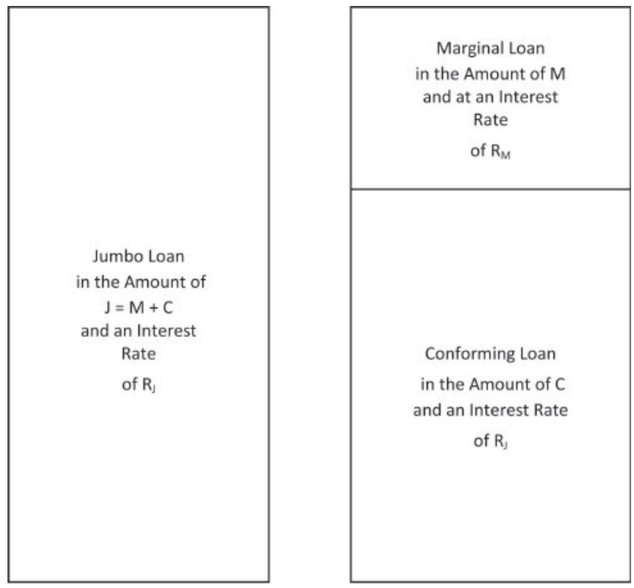


Fig. 1. A Jumbo loan depicted as a maximal conforming loan plus a marginal loan. The figure on the left depicts a Jumbo loan at an interest rate of R_J, while the one on the right portrays a maximal conforming loan in the amount of C and at an interest rate of R_C, plus a marginal loan in the amount of M and at an interest rate of R_M. M is selected as J – C, so both the Jumbo loan and the conforming-marginal combination have the same total value. Because $R_J \cong \frac{C}{C+M} R_C + \frac{M}{C+M} R_M$, solving for the marginal rate R_M gives us $R_M \cong R_J + \frac{C}{M}[R_J - R_C]$.

marginal rate R_m is very high when the threshold is exceeded by only a small amount. Consider, for example, a \$500,000 threshold for a Jumbo loan, and a rate on a Jumbo loan is that is 50 basis points higher than the conforming loan rate of 4.0%. Suppose also that a borrower takes out a mortgage for \$525,000. The interest rate on the marginal \$25,000 in excess of the threshold is about $4.5\% + \frac{500{,}000}{25{,}000}[4.5\% - 4.0\%] = 14.5\%$. This is a very high interest rate for a relatively low-risk endeavor, and strongly suggests that the borrower would be well advised to find some alternate way to come up with the extra \$25,000. For example, Ibbotson and Sinquefield (2013) report that the long-run historical nominal geometric mean return on the S&P 500 has been only about 9.8% from 1926 to 2012, and so by selling \$25,000 worth of stock and adding the proceeds to the down payment, the borrower effectively swaps a risky 9.8% expected return for a very low-risk 14.5% savings. In this rather extreme case, even raiding an IRA (and paying the early withdrawal penalty) would merit serious consideration.

When the marginal loan amount M is larger, the marginal rate drops somewhat. Consider, for example, the same rates as in the previous paragraph, but a loan totaling \$600,000, so that the marginal loan amount M is \$100,000. Now the marginal interest rate is $4.5\% + \frac{500{,}000}{25{,}000}[4.5\% - 4.0\%] = 7.0\%$. Given that this 7.0% is a very low-risk return, finding another source for the marginal \$100,000 merits strong consideration by most, but it is plausible to expect that 7.0% may be below some borrowers' reservation prices[2] and, therefore, acceptable to them.

Table 1 Marginal interest rates R_M for loans exceeding a maximum conforming amount of $500,000 when $R_C = 4\%$

R_J	Total loan						
	$525,000	$550,000	$575,000	$600,000	$650,000	$700,000	$750,000
4.625%	17.125%	10.875%	8.792%	7.750%	6.708%	6.188%	5.875%
4.5%	14.500%	9.500%	7.833%	7.000%	6.167%	5.750%	5.500%
4.375%	11.875%	8.125%	6.875%	6.250%	5.625%	5.313%	5.125%
4.25%	9.250%	6.750%	5.917%	5.500%	5.083%	4.875%	4.750%
4.125%	6.625%	5.375%	4.958%	4.750%	4.542%	4.438%	4.375%

Note. The table features the marginal rate, R_M, which is paid on the amount by which mortgage size exceeds the threshold for a conforming loan. In all cases, the maximum conforming loan size is assumed to be $500,000 and the conforming APR assumed to be 4%. Various possibilities for the Jumbo rate are shown in the different rows, and various sizes in the different columns. For example, in the first entry (R_J = 4.625% and size = $525,000), a borrower who accepts the Jumbo rate on a $525,000 mortgage is effectively taking out a conforming mortgage of $500,000 at 4% and paying a rate of 17.125% on the $25,000 by which his loan exceeds the conforming maximum.

When the marginal amount borrowed is larger, the rate on the marginal amount continues to fall. For example, suppose that the loan is for $1,500,000 total, so that the marginal amount M is $1,000,000. Now the interest rate on M is $4.5\% + \frac{500{,}000}{25{,}000}[4.5\% - 4.0\%] = 4.75\%$. Table 1 shows the marginal rates R_m for loans and spreads of several sizes.

There is no one-size-fits-all when it comes to an individual's reservation price for interest rates. However, Table 1 allows an investor to at least assess the true cost of the marginal loan M. If it is unusually high, she might consider selling other assets to make a larger down payment and bring the loan down to conforming levels, waiting until she has saved a larger down payment, or settling for a less expensive house.

3. 15-Year versus 30-year mortgages (ignoring taxes)

Some borrowers may be constrained by their income because many lenders follow a 28/36 Rule (e.g., Thangavelu, 2015). Under this rule, monthly housing payments should not exceed 28% of gross income, and total debt commitments should not exceed 36% of gross income. If a lender is following this rule in terms of offering a loan, then some borrowers will not have a choice and will have to get a 30-year mortgage. Even for borrowers who are not so constrained, the conventional wisdom is that a 30-year mortgage offers more flexibility than a 15-year mortgage and, therefore, is the wiser choice. For example, Goff and Cox (1998) demonstrate that if the difference in payments is invested in a tax-deferred account like a 401(k), even after adjusting for taxes, mortgagors who live in the house for 30 years will have a higher expected wealth in 30 years.[3] Several other researchers reach similar conclusions.[4] In addition, qualified borrowers who lean towards a 15-year are often steered away with the advice that, if the borrower has extra cash flow during the lifetime of the mortgage, he can always voluntarily pay more towards the principal if he chooses. Certainly borrowers should not accept a rate or term that results in payments with which they are uncomfortable (Grable and Lytton, 1999 present a 20-question survey to assess an individual's degree of risk aversion, and three of the 20 questions pertain to mortgage choices), and if the rates on the two mortgages are the same, then recommending the 30-year is impeccable advice. However, in practice the rates are rarely the same, and the higher rate on the 30-year mortgage is the price of the greater flexibility. *Ceteris paribus*, flexibility is good, but like any other good, it has its reservation price. Borrowers who do not need the flexibility are likely to find that its price is too high.

We begin with an (admittedly unrealistic) extreme example to better identify the tradeoffs involved. Suppose the annual percentage rate (APR) on a 15-year, $500,000 mortgage is 3.6%, while that on an otherwise identical 30-year mortgage is 7.2% (unless otherwise specified, we assume all annual rates are expressed as APRs, i.e., the actual periodic monthly rate is just APR/12). Now monthly payments (throughout the article we consider only interest and principal, and not insurance or property taxes; we consider income taxes and the fact that mortgage interest is tax-deductible in Section 4) on the 15-year mortgage will be $3599.02, and those on the 30-year will be $3393.94. It is difficult to imagine that many borrowers will find the flexibility of having the option to keep the monthly difference of $205.08 for the first 15 years is worth paying the extra $3393.94/month for the last 15 years of the 30-year mortgage. One way of visualizing the true cost of that flexibility is to find the monthly payments of an imaginary 30-year mortgage at the 15-year mortgage APR of 3.6%. Evaluated at the same interest rate as a 15-year mortgage, this hypothetical 30-year mortgage would give the borrower the flexibility to pay $3599.02 – 2273.23 = $1325.79 less per month when compared with the 15-year borrower. However, in exchange for this flexibility to pay $1325.79 less, the 7.2% mortgage rate essentially charges the borrower an additional $3393.94 – 2273.23 = $1120.71/month, leaving the borrower with a net payment that is only $205.08 less than that of the 15-year mortgagor, as depicted in Panel A of Fig. 2. This is clearly a poor proposition for most borrowers.[5] While this example is too extreme to be observed in practice, it establishes that the greater flexibility of the 30-year mortgage has a price, and that price should not be paid if it exceeds the borrower's reservation price. Even with a more realistic example, the flexibility has a nontrivial cost. As depicted in Panel B of Fig. 2, a $500,000, 15-year mortgage with an APR of 3.75% requires monthly payments of $3636.11, while a 30-year mortgage at a monthly rate of 4.50%/month requires payments of $2533.43. Inserting the intermediate step described above finds that a hypothetical 30-year mortgage at 3.75% requires payments of $2315.58, so that in exchange for the flexibility of paying $3636.11 – 2315.58 = $1320.53 less per month, the borrower is paying back $2533.43 – 2315.58 = $217.85. This may be a desirable trade for some, but not for others. The issue is how can we best measure the price of this flexibility so that the borrower can have a solid basis for judgment?

One metric arises from considering the marginal payments of a 30-year mortgagor who swaps that mortgage's cash flows for those of a 15-year mortgage. As in the discussion of Jumbo mortgages, by marginal cash flow we mean any additional cash flows the 15-year borrow pays (or receives) as compared with those of the 30-year borrower. This swap produces a greater cash outflow for the first 15 years in exchange for elimination of all payments after year 15. For example, suppose as above that the APR on a $500,000, 30-year

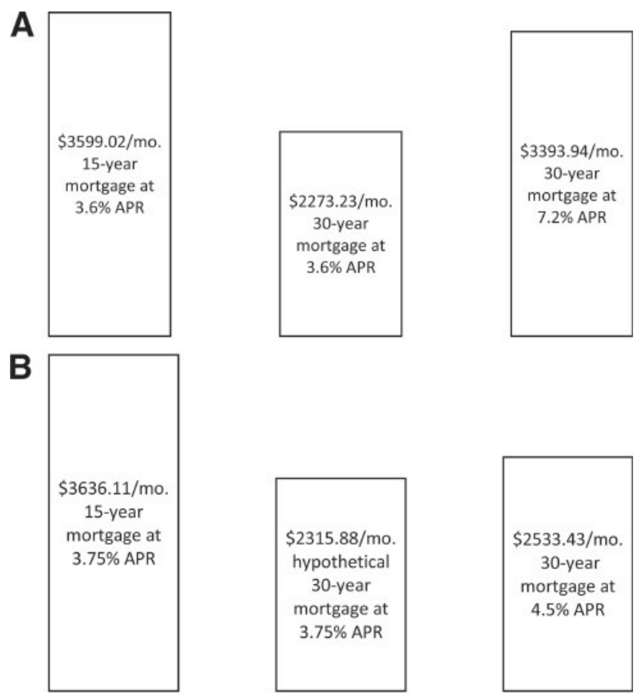


Fig. 2. The price of a 30-year mortgage's greater flexibility. Panel A: Exaggerated example of the price of flexibility. The figure on the left represents the monthly payment for a 15-year, $500,000 mortgage at a 3.6%/year APR, while the one on the right represents that of a 30-year mortgage at an unrealistically high 7.2%/year. The figure in the center represents payments on a hypothetical 30-year mortgage at the 15-year rate of 3.6%. If the mortgagor could secure such a loan, he would have the flexibility to pay $3599.02 – 2273.23 = $1325.79 less per month. However, this flexibility comes with a cost because the actual 30-year rate requires a payment of $3393.94 – 2273.23 = 1120.71 more than the hypothetical mortgage. Thus the 30-year borrower pays 1120.71/month for the option to pay 1325.79 less, and ends up paying only $205.08 per month less than the 15-year borrower. It is difficult to imagine borrowers for whom this $205.08 savings for the first 15 years is worth paying $3393.94/month more for the last 15 years. Panel B: Moderate example of the price of flexibility. This panel makes the same point as Panel A, but with more reasonable values. Here the 30-year borrower is paying back $2533.43 – 2315.88 = $217.85 for the option to pay $3636.11 – 2315.58 = $1320.53 less per month. Certainly some borrowers will find this reasonable, but when the problem is framed this way, many will not.

mortgage is 4.50%, while that on a 15-year is 3.75%. Payments on the 15-year mortgage would be $3636.11, or $1102.69 more than the $2533.43 payments under the 30-year mortgage. What does the 15-year borrower get for this extra $1102.69/month? He is finished paying for the house at the end of 15 years, while the holder of the 30-year mortgage still has monthly payments of $2533.43 to make for an additional 15 years. Thus, the 15-year mortgagor's investment of the additional $1102.69/month for 15 years has produced a gain (savings) of $2533.43/month for the 15 years after that. The internal rate of return (IRR) of this investment is the value of r satisfying $1102.69(\text{FVIFA}_{180,\ r\%}) = 2533.43(\text{PVIFA}_{180,\ r\%})$.[6] The solution is $r = 0.4632\%$/month, or an APR of 5.56%.

A well-established principle in finance is that, when directly comparing the returns on two investments, it is important to account for risk. For example, it is inappropriate to suggest a CD yielding 3% is a worse investment than a stock index fund producing an expected return of 12% because the index fund has significantly higher risk, and much of that 9% differential is compensation for that risk. Similarly, because leases entail cash outflows that are relatively certain, Ross et al. (2013) point out that the after-tax cost of debt is a more appropriate discount rate when evaluating leases than is the cost of capital. Cheung and Miu (2015) make a similar observation that real estate is comparable to bonds because "bonds and real estate share very similar risk and return characteristics," and Reichenstein (1998) similarly views a mortgage as a short position in bonds. Our example of the marginal cash flows from trading a 30-year mortgage for a 15-year produces cash flows with very low risk, and thus a fair comparison is other investments with low risk. The 5.56% IRR of the previous paragraph is a significantly higher return than other contemporary investments with such low risk (e.g., as of early February, 2014, the yield on 30-year T-Bonds was only about 3.67%) and is earned at only the cost of reducing the values of the mortgagor's options to default or refinance.[7]

Another metric is net present value (NPV); using the 15-year APR of 3.75%, for example, we find the NPV of the marginal cash flows of 15-year mortgage, when compared with those of the 30-year, to be $47,039.79. Given our assumption of no default, the actual value of the house plays no role in the calculations because it is not a marginal effect; whether the borrower takes out a 15-year or a 30-year mortgage affects the principal due at any time, but does not affect the actual value of the house itself.

The previous analysis assumed the mortgage was held for 30 years. What if it is liquidated before then? For example, suppose the house is sold after seven years. Now the 15-year mortgagor still owes $301,169.18, while the 30-year borrower owes $133,962.24 more, or $435,131.42.[8] Thus, compared with the 30-year borrower, the 15-year borrower has paid 1102.69/month more for these first seven years, but owes $133.962.24 less at liquidation. Because both borrowers are liquidating the mortgage, there are no cash flows at all (and, therefore, no marginal cash flows) after year seven. In this case the IRR of the 15-year mortgagor's marginal $1102.69 monthly investment is the value of r satisfying 1102.69($FVIFA_{84,\ r\%}$) = $133,962.24. The solution is 0.84%/month, or an APR of 10.12%, which is significantly greater than the 5.56% APR of the previous example. However, using the same 3.75% as before to find NPV, we now find it to be only $21,721.24, which is lower than the borrower's $47,039.79 NPV in that example. The choice of when to sell the house would probably be exogenous. However, if the mortgagor wanted to rank possible times of sale, the fact that the alternatives have different lifespans would make NPV an inappropriate ranking device. In such cases, finding an annuity equivalent to the NPV (e.g., see Brealey, Myers, and Allen, 2008, pp. 155–160) or even using IRR is a better tool for ranking. The borrower in fact saves more money (NPV) the longer he owns the house, but his marginal NPV, equivalent annuity, and IRR decline when the house is sold at a later date. This can be seen in Table 2 and Fig. 3, which feature several measures of the benefits for various years in which the house is sold or the mortgage otherwise prepaid.

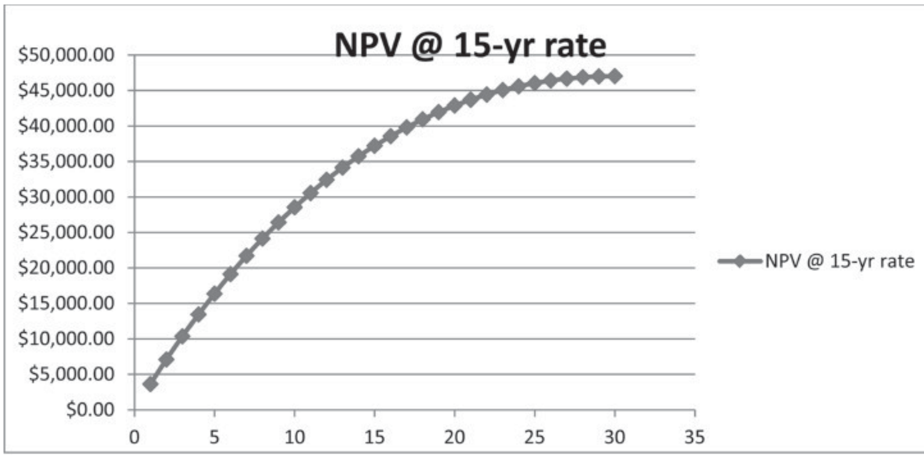
The last entry in Table 2, which a 15-year mortgagor who sells the house in one year earns a return of 56.39% on his marginal investment, is so large that it seems a likely error. It is not. As before, the 15-year borrower pays an extra $3636.11 – $2533.43 = $1102.69 per month more than the 30-year borrower. In exchange, in 12 months the 15-year borrower owes $474,684.48, or $17,249.38 less than the 30-year borrower's $491,933.87. The monthly discount rate that represents the return on the marginal $1102.69 investment is the value of r that solves 1102.69($FVIFA_{12,\ r\%}$) = $17,249.38. This value of r is 4.70% per month, for an APR of 56.39% per year (because here we have monthly compounding, EAR = effective

Table 2 Pre-tax benefits from the marginal investment of a 15-year mortgage's larger payments

House sold at month	NPV at 15-year pre-tax rate	Annual change in NPV	Equivalent monthly annuity	IRR (APR)	IRR (EAR)
360	$47,039.79	$40.39	$217.85	5.56%	5.70%
348	$46,999.40	$116.53	$221.74	5.56%	5.70%
336	$46,882.87	$195.02	$225.58	5.56%	5.71%
324	$46,687.85	$275.96	$229.36	5.57%	5.72%
312	$46,411.89	$359.43	$233.09	5.59%	5.73%
300	$46,052.46	$445.55	$236.77	5.60%	5.75%
288	$45,606.91	$534.41	$240.40	5.62%	5.77%
276	$45,072.50	$626.13	$243.97	5.65%	5.80%
264	$44,446.37	$720.82	$247.50	5.69%	5.84%
252	$43,725.55	$818.59	$250.97	5.73%	5.88%
240	$42,906.96	$919.57	$254.39	5.79%	5.94%
228	$41,987.39	$1,023.88	$257.76	5.85%	6.01%
216	$40,963.50	$1,131.66	$261.08	5.94%	6.10%
204	$39,831.85	$1,243.03	$264.35	6.05%	6.22%
192	$38,588.81	$1,358.15	$267.58	6.19%	6.37%
180	$37,230.66	$1,477.16	$270.75	6.38%	6.57%
168	$35,753.50	$1,600.20	$273.87	6.61%	6.81%
156	$34,153.30	$1,727.45	$276.95	6.88%	7.10%
144	$32,425.86	$1,859.05	$279.98	7.19%	7.43%
132	$30,566.81	$1,995.19	$282.96	7.56%	7.83%
120	$28,571.62	$2,136.03	$285.89	8.01%	8.31%
108	$26,435.59	$2,281.77	$288.78	8.55%	8.89%
96	$24,153.82	$2,432.58	$291.62	9.23%	9.64%
84	$21,721.24	$2,588.68	$294.41	10.12%	10.60%
72	$19,132.56	$2,750.26	$297.16	11.30%	11.90%
60	$16,382.30	$2,917.54	$299.86	12.96%	13.75%
48	$13,464.76	$3,090.73	$302.52	15.47%	16.61%
36	$10,374.03	$3,270.07	$305.13	19.71%	21.59%
24	$7,103.96	$3,455.80	$307.70	28.42%	32.42%
12	$3,648.16	$3,648.16	$310.22	56.39%	73.52%

Note. This table shows the financial advantage of the cash flows of a 15-year, $500,000 mortgage at an APR of 3.75% when compared with those of a 30-year mortgage with a 4.5% APR. For example, if the 15-year mortgagor lives in the house for 30 years, then the present value of his marginal cash flows (discounted at the 15-year rate) is $47,039.79, and his rate of return on his marginal payments for the first 180 months is 5.56% APR. If he lives in the house for only, say, 84 months, and then sells, he has a lower but still significant $21,721.24 NPV, but a higher APR of 10.12% on his marginal investment. NPV increases as the house is kept for a longer time, but at a decreasing rate (because the principal is declining). For purposes of comparison, the 1926–2012 geometric mean return on long-term government bonds is 5.7%, with a standard deviation of 9.7%. IRR = internal rate of return; NPV = net present value; APR = annual percentage rate; EAR = effective annual return.

annual return is calculated as $(1 + \text{periodic monthly return})^{12} - 1 = 73.52\%$). Of course, because of transactions costs, few homeowners would purchase if they knew they were going to sell in a year. Nevertheless, if they did sell then, the transactions costs are presumably identical for the 15-year and 30-year borrowers, and so the marginal effect of transactions costs (i.e., any additional cash flows the 15-year borrow pays [or receives] as compared with those of the 30-year borrower) is zero, and the 15-year mortgage still offers a 56.39% return on marginal cash flows relative to the 30-year. Moreover, while the average life of a



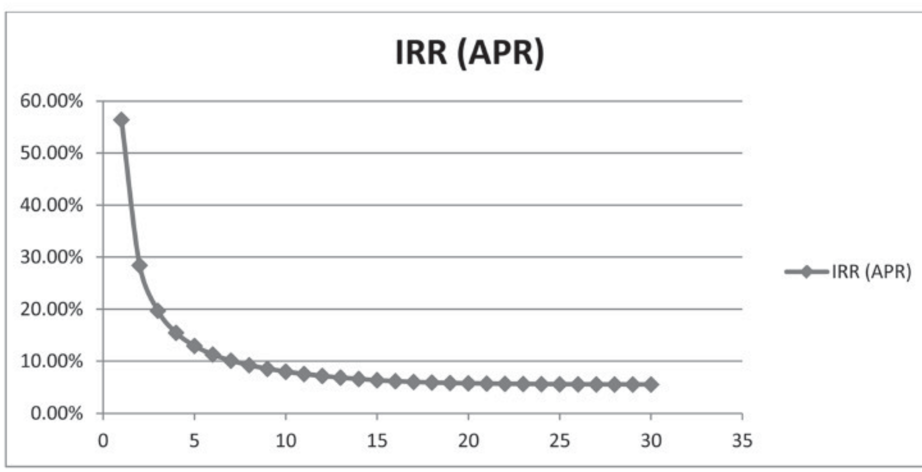


Fig. 3. NPV and IRR of the marginal cash flows created by moving from a 30-year mortgage to a 15-year mortgage (ignoring taxes). The figures above depict the NPV and IRR (expressed as an APR) of the marginal investment (higher payments for 15 years) and marginal benefits (nothing owed after 15 years, and a smaller principal if the house is paid off before 15 years). In both cases the X-axis represents the number of years before the house is sold or both mortgages paid off. Whether measured by NPV or IRR, the financial advantage of the 15-year mortgage is significant. The figures shown are for a $500,000 mortgage, with an APR of 3.75% for a 15-year mortgage and 4.5% for a 30-year mortgage.

mortgage varies over time, it is typically well under 10 years. For example, Freddie Mac's Offering Circular Supplement of June, 2010, suggests a weighted average life of around six and a half years. In this range, the choice of a 15-year mortgage still offers a low-risk, double-digit return when compared with the 30-year mortgage. For purposes of comparison with other low risk investments, we note that Ibbotson and Sinquefield (2013) report that long-term government bonds have a geometric mean return of 5.7% and a standard deviation of returns of 9.7%.

The result that the advantage of the 15-year mortgage is more pronounced when the time the house is sold is shorter has interesting implications. Specifically, younger buyers are more likely to move (not only because they are more likely to have children and have other motivations to trade up to larger houses, but also because they are more mobile and likely to move because of their job). Thus, while young buyers may place the greatest value on the flexibility a 30-year mortgage has to offer, they might also be the ones to stand to gain the most from the marginal investment required of a 15-year mortgage. Moreover, as Samuelson (1994) and Bodie, Merton, and Samuelson (1992) point out, younger investors may find it optimal to take more risks because, if the outcomes are unfavorable, they still have time to make the required adjustments later in life. If any group were to find the higher costs of a 30-year mortgage acceptable, it is likely to be this group, as they want to invest any additional money in higher-risk ventures as Goff and Cox (1998) suggest.[9] On the other hand, this age group may also earn the highest returns from a 15-year mortgage because of the greater likelihood they will sell the house relatively early, which realizes the highest return on the marginal cash flows. The situation appears clearer for older homebuyers who will want to reduce their risks as they get closer to the end of their working years, and for such individuals a 15-year mortgage is more likely to be the optimal choice. Whether the extra costs of the 30-year mortgage are acceptable depends on such characteristics as the borrower's age, the stability of her cash flows, her degree of risk aversion, her overall portfolio, and her progress towards retirement goals.

It is easy to say that the \$1102.69 is a good investment, but just where is that money to be found? One possibility is that it can take the place of some alternate investments, for example, any part of an investment portfolio that would otherwise be allocated to low-risk investments. The borrower who sells the house after seven years, for example, will be better placed at that time if he has chosen the 15-year mortgage, as he has effectively invested the \$1102.69 at an APR of 10.12%, which is equivalent to an EAR of 10.60%. This is considerably higher than the expected return of a low-risk bond portfolio, and even higher than the historical geometric mean return of 9.8% from investing in equity. Consider, for example, a younger homebuyer who knew a move was likely after about seven years in his current location. Because of its substantially lower risk, the riskless EAR of 10.6% on the marginal investment in a 15-year mortgage would seem to dominate the risky 9.8% he would expect to earn if the same marginal cash flows were invested in an all-equity portfolio.

4. 15-Year versus 30-year mortgages (considering taxes and different levels of interest rates)

In a different setting, Fortin et al. (2007) find that rules of thumb also do not work very well in a refinancing context. Specifically, they found that, when considering taxes and time value of money, the breakeven point for refinancing was 35% to 40% more distant than otherwise estimated. However, in their framework, refinancing required fees that provided no tax shield, and which were used to reduce interest expense, which reduces the tax benefits. In our framework, interest for either mortgage is tax deductible, so taxes do not affect the result in such a lopsided way, as can be seen in Table 3. Although all calculations were made with after-tax cash flows and after-tax discount rates, all rates in the tables are reported on a pretax basis to facilitate comparisons, with the pretax rate = after-tax rate/(1-tax rate). Because interest constitutes a greater proportion of the payments for the 30-year mortgage than for the 15-year, the 30-year has a comparative advantage in this regard when taxes are considered. However, it is also true that more interest is paid, even on an after-tax basis. The

Table 3 After-tax benefits from the marginal investment of a 15-year mortgage's larger payments

House sold at month	NPV at 15-year after-tax rate	Annual change in NPV	Equivalent monthly annuity	IRR (APR)	IRR (EAR)
360	$35,427.62	$38.89	$140.21	5.38%	5.52%
348	$35,388.73	$110.97	$143.30	5.38%	5.52%
336	$35,277.76	$183.49	$146.33	5.39%	5.52%
324	$35,094.27	$256.48	$149.29	5.40%	5.53%
312	$34,837.79	$329.99	$152.19	5.41%	5.55%
300	$34,507.79	$404.05	$155.03	5.43%	5.56%
288	$34,103.74	$478.71	$157.80	5.45%	5.59%
276	$33,625.03	$554.00	$160.53	5.48%	5.62%
264	$33,071.03	$629.97	$163.19	5.51%	5.65%
252	$32,441.06	$706.65	$165.80	5.56%	5.70%
240	$31,734.42	$784.09	$168.35	5.61%	5.75%
228	$30,950.33	$862.33	$170.86	5.67%	5.82%
216	$30,088.00	$941.41	$173.30	5.76%	5.91%
204	$29,146.59	$1,021.39	$175.70	5.86%	6.02%
192	$28,125.20	$1,102.29	$178.05	6.00%	6.17%
180	$27,022.91	$1,184.18	$180.34	6.18%	6.36%
168	$25,838.73	$1,267.09	$182.59	6.40%	6.60%
156	$24,571.64	$1,351.07	$184.80	6.67%	6.87%
144	$23,220.57	$1,436.17	$186.95	6.97%	7.20%
132	$21,784.41	$1,522.43	$189.06	7.33%	7.58%
120	$20,261.98	$1,609.91	$191.12	7.76%	8.04%
108	$18,652.06	$1,698.66	$193.15	8.29%	8.61%
96	$16,953.40	$1,788.73	$195.12	8.95%	9.33%
84	$15,164.68	$1,880.16	$197.06	9.81%	10.26%
72	$13,284.52	$1,973.02	$198.96	10.95%	11.52%
60	$11,311.50	$2,067.35	$200.81	12.56%	13.31%
48	$9,244.16	$2,163.21	$202.63	14.99%	16.07%
36	$7,080.95	$2,260.66	$204.40	19.10%	20.86%
24	$4,820.29	$2,359.75	$206.14	27.51%	31.26%
12	$2,460.54	$2,460.54	$207.85	54.44%	70.31%

Note. This table shows the financial advantage of after-tax cash flows of a 15-year, $500,000 mortgage at an APR of 3.75% when compared with those of a 30-year mortgage with a 4.5% APR, assuming a 33% tax rate for the mortgagor. NPV is calculated using the after-tax 15-year rate [=(pre-tax rate)(1–.33)]. IRRs are calculated on an after-tax basis, but expressed here on a pre-tax basis, that is, as (after-tax IRR)/(1-tax rate). For example, if the 15-year mortgagor lives in the house for 84 months, and then sells, he has a $15,164.68 NPV and a pre-tax IRR (expressed as an APR) of 9.81% on his marginal investment. NPV increases as the house is kept for a longer time, but at a decreasing rate (because the principal is declining). IRR = internal rate of return; NPV = net present value; APR = annual percentage rate; EAR = effective annual return.

values in Table 3 are less than their analogs from Table 2, but the difference is not large. For example, in Table 2, the IRR of the marginal cash flows of a mortgagor who used a 15-year mortgage, but kept the house for 30 years, was 5.56%, but the corresponding entry in Table 3 falls to only 5.38%. One reason the drop between after-tax returns is so small (compared with the large difference in Fortin et al.) is that the main benefit of the 15-year mortgage is that the principal is paid down faster, and this is not subject to taxes.

It is appropriate to compare the riskless returns we have found so far with contemporary riskless returns of the same type (taxable or tax-exempt). By analogy, whether a starting salary of $50,000/year for an entry-level engineering position is better than average depends

Table 4 A comparison of the IRRs Earned from a 30-year mortgage to 15-year mortgage swap compared with rates on comparable T-Bonds (pre-tax) or Municipal Bonds (after-tax)

Ignoring taxes		
Years	EAR of 15-year to 30-year marginal cash flows	EAR of comparable T-Bond
2	32.42%	0.25%
5	13.75%	0.89%
10	8.31%	2.05%
30	5.70%	3.27%

Considering taxes			
Years	Taxable equivalent EAR of 15-year to 30 year marginal cash flows	EAR of comparable Municipal Bond	Taxable equivalent EAR of comparable Municipal Bond
1	70.31%	0.24%	0.36%
2	31.26%	0.35%	0.52%
5	13.31%	0.82%	1.23%
10	8.04%	1.85%	2.76%
30	5.52%	2.98%	4.45%

Note. This table compares returns from the mortgage swap (from Tables 2 and 3) with those from T-Bonds (in the before-tax case) and Municipal Bonds (in the after tax case). Bond returns were taken from the Bloomberg website in early March, 2013. In all cases, annual returns are expressed as effective annual returns (EAR) to facilitate comparison. The taxable equivalent EAR of the mortgage swap and the Municipal Bonds is found by taking the specified EAR of after-tax cash flows and dividing by (1-t).

on whether we are talking about a position in 1975, 1995, or 2015. In our case, for purposes of comparison, Bloomberg reported the 30-yield municipal bond yield to be 2.96%, with a taxable equivalent yield of 2.98%/(1−0.33) = 4.45%, as of early March, 2013. A comparison of the rates earned by swapping a 15-year mortgage for a 30-year and the rates offered by T-Bonds and Municipal bonds of comparable maturities is shown in Table 4.

Next we examine what happens for different levels of interest rates. Clearly the strategy of swapping a 30-year for a 15-year mortgage will be more beneficial when the spread between the two rates is larger, so we do not analyze different spreads. However, the question remains whether the strategy becomes more profitable or less profitable when mortgage rates increase. We find NPV declines a little for larger mortgage rates, but this should be no surprise—*ceteris paribus*, larger discount rates necessarily cause present values to fall. We also find IRR rises as the mortgage rates rise. For example, assuming the house is sold in 30 or more years, we find that on a pretax basis the gap between the IRR earned on the swap and the APR on the 30-year mortgage ranges from 98 to 125 basis points as the 30-year rate assumed values between 3.50% and 6.50%. In contrast with NPV, the IRRs are increasing in the interest rates, even when the spread between 15-year and 30-year rates is held constant. Table 5 summarizes the results for other rates and years until the house is sold.

Finally, while the 15-year mortgage offers large benefits relative to the 30-year in today's interest rate environment, whether this is generally true remains to be seen. Table 6 uses the Freddie Mac database to compare the two mortgages if initiated at various times. Because the 15-year database goes back only to September, 1991, and interest rates do not change

Table 5 A comparison of NPV and IRR earned from a 30-year for 15-year swap for various levels of mortgage rates

House sold in	No taxes NPV	No taxes IRR (APR)	Tax rate = 33% NPV	Tax rate = 33% IRR (APR)
15 year rate = 2.75%, 30-year rate = 3.5%				
1 year	$3,662.60	53.57%	$2,465.93	51.79%
2 years	$7,155.87	26.57%	$4,839.26	25.75%
5 years	$16,659.71	11.64%	$11,412.36	11.30%
10 years	$29,470.17	6.86%	$20,593.15	6.66%
30 years	$49,974.76	4.48%	$36,556.52	4.35%
15 year rate = 3.75%, 30-year rate = 4.5%				
1 year	$3,648.16	56.39%	$2,460.54	54.44%
2 years	$7,103.96	28.42%	$4,820.29	27.51%
5 years	$16,382.30	12.96%	$11,311.50	12.56%
10 years	$28,571.62	8.01%	$20,261.98	7.76%
30 years	$47,039.79	5.56%	$35,427.62	5.38%
15 year rate = 4.75%, 30-year rate = 5.5%				
1 year	$3,633.12	59.67%	$2,454.71	57.51%
2 years	$7,049.87	30.47%	$4,799.58	29.45%
5 years	$16,096.28	14.34%	$11,201.28	13.88%
10 years	$27,664.15	9.18%	$19,903.38	8.89%
30 years	$44,226.88	6.65%	$34,239.17	6.42%
15 year rate = 5.75%, 30-year rate = 6.5%				
1 year	$3,617.55	63.45%	$2,448.47	61.02%
2 years	$6,993.91	32.74%	$4,777.30	31.59%
5 years	$15,803.66	15.81%	$11,082.90	15.27%
10 years	$26,755.14	10.40%	$19,522.02	10.04%
30 years	$41,550.23	7.75%	$33,009.72	7.47%

Note. Values for NPV and IRR are calculated in the same fashion as for Tables 2 and 3, but for different levels of interest rates. NPV declines when rates are higher because higher rates cause present to fall, but IRR increases as rates increase. IRR = internal rate of return; NPV = net present value; APR = annual percentage rate.

dramatically from month to month, we compare the two mortgages every three years, starting in January, 1992 and ending in January, 2013. Table 6 features the results.

As in Table 5, the advantage of a 15-year mortgage over a 30-year is generally greater when the overall rates are greater; here we see this is true, even when the gap between rates is smaller. For example, January, 2013 features the lowest rates, but the largest spread between 15-year and 30-year rates at 71 basis points. Assuming the mortgage is liquidated in seven years, it also has the second lowest IRR of any of the eight times examined. Conversely, the January, 1992 and January, 1995 rates are the largest of the set, and offer the largest IRRs for a mortgage liquidated in seven years, even though their spreads between the 15-year and 30-year rates are only about average. Because our analysis has ignored the option to refinance, it is very likely that this increase in IRR is because of the refinancing

Table 6 A comparison of 15-year and 30-year mortgages initiated in January, 1992 through January, 2013

Mortgages initiated in	15-year		30-year		IRR if mortgage liquidated in			
	APR	Points	APR	Points	1 year	7 years	15 years	30 years
January, 1992	8.01%	1.7	8.43%	1.8	43.38%	12.10%	9.58%	9.06%
January, 1995	8.80%	1.8	9.15%	1.8	38.87%	12.26%	10.12%	9.67%
January, 1998	6.58%	1.4	6.99%	1.4	37.75%	10.23%	8.00%	7.53%
January, 2001	6.64%	0.9	7.03%	0.9	36.36%	10.12%	8.00%	7.54%
January, 2004	5.02%	0.7	5.71%	0.7	54.09%	10.83%	7.33%	6.57%
January, 2007	5.97%	0.4	6.22%	0.4	24.36%	8.13%	6.82%	6.53%
January, 2010	4.44%	0.6	5.03%	0.7	54.33%	9.39%	6.41%	5.76%
January, 2013	2.70%	0.7	3.41%	0.7	49.07%	8.27%	4.94%	4.21%

Note. Specified IRRs are for the marginal cash flows incurred by a 15-year mortgagor relative to those of a 30-year mortgagor. APR and points are taken from the Freddie Mac website. The borrower is assumed to have a 33% marginal tax rate. IRRs are expressed as APRs and on a pre-tax basis, that is, pre-tax IRR = (after-tax IRR)/(1-tax rate). IRR = internal rate of return; APR = annual percentage rate.

option's greater value when interest rates are higher. The 15-year mortgage reduces the value of this option relative to the 30-year, and in exchange 15-year borrowers are compensated with a higher IRR.

5. Conclusions

We have shown that Jumbo loans and 30-year mortgages can be substantially more expensive than conforming loans and 15-year mortgages. What appears to be only a slight gap in the mortgage rates produces large net present values and internal rates of return of marginal cash flows, that is, cash flows of conforming loans compared with those of Jumbo loans, and of 15-year mortgages relative to those of 30-year mortgages. The "conventional wisdom" often steers homeowners towards 30-year mortgages because of their "greater flexibility," but that extra flexibility comes with a steep cost, particularly for homeowners who will liquidate the mortgage before maturity. We find, for example, that even on an after-tax basis, 15-year borrowers who will liquidate their mortgages at the median term of about seven years will earn a riskless internal rate of return of about 10% on their marginal cash flows relative to their 30-year borrower counterparts.

Notes

1 This is just an approximation in a multi-period context because the loans are paid down at different rates and so the weights change over time. For example, a 30-year, \$100,000 mortgage at a rate of 0.40%/month requires payments of \$524.67/month, but at 0.80%/month would require payments of \$848.16/month. However, the two loans combined do not have an average rate of exactly 0.60%/month, but rather a monthly rate satisfying $(524.67+848.16)(\text{PVIFA}_{360,r\%})=200{,}000$, which is closer to 0.61%/ month. [Throughout the paper we use $\text{PVIFA}_{n,r\%}$ to denote the present value of an

annuity of \$1 per period at a discount rate of r% per peiord. This can be calculated as $\text{PVIFA}_{n,\ r\%} = \frac{1}{r}\left[1 - \frac{1}{(1+r)^n}\right]$, where r is expressed as a decimal, not a percent.] For a further discussion of this issue, and for conditions under which the relationship holds exactly, see Miles and Ezzell (1980).

2 A reservation price is generally defined to be the greatest price an individual is willing to pay for something, or the lowest price he will accept if he is the seller. In this context, it refers to the highest rate the mortgagor is willing to pay for a loan.

3 There are several differences in the approach taken here and that taken by Goff and Cox. First, while their assumptions of a 0.5% gap (7.5% vs. 8.0%) between 15-year and 30-year rates was conservative for its time—the Freddie Mac survey reports an average gap of only 0.36% (6.78% vs. 7.14%) in April, 1988—it is rather small by today's standards. For example, the Freddie Mac survey indicates that as of January, 2014, the average gap was 95 basis points (3.48% vs. 4.43%). Second, their framework mixes the fairly riskless cash flows from a mortgage with the relatively riskier cash flows from investing in equity (and while it is true that both their investors ultimately invest some marginal cash flows in equity, the 30-year borrower is doing so for a longer time and, therefore, is subject to more risk), while this article compares the riskless cash flows with each other by focusing on IRR of the marginal cash flows. Finally, while their analysis centered on a borrower who will keep the house for at least 30 years, we consider a number of possible dates on which the mortgage is prepaid. Nevertheless, their article presents a number of conditions that favor the 30-year mortgage.

4 For example, while Amromin, Huang, and Sialm (2007) focus on prepayment of existing mortgages rather than the choice between mortgages, they do classify "taking out a mortgage with a maturity shorter than the standard 30 years" as a form of "mortgage prepayment," which they conclude is inferior to investing the marginal proceeds in a tax-deferred account. We find that, even when taxes are considered, the lower rate on a 15-year mortgage typically makes it a better choice.

5 More extreme theoretical examples are possible. For example, suppose the APR on the 30-year were 8.4%. Now the monthly payments of \$3812.30 on the 30-year mortgage would *exceed* those on the 15-year. Clearly this is impossible in any reasonable kind of equilibrium, so that the rate on the 15-year mortgage establishes an upper bound on that of the 30-year.

6 $\text{FVIFA}_{n,\ r\%} = \frac{1}{r}[(1 + r)^n - 1]$ denotes the future value of an annuity of \$1 per year, and $\text{FVIF}_{n,\ r\%} = (1 + r)^n$ denotes the future value of a dollar now compounded for n periods at r% per period. Because we wanted the IRRs for many different sets of cash flows, we used the IRR function in MS Excel to generate the answer to this (and all the IRR entries in Tables 2, 3, 5, and 6), but the cash flow registers on almost all financial calculators can also find the value of r = IRR.

7 The procedure by which this gain is obtained is very similar to riding the yield curve (e.g., Bieri and Chincarini, 2005), with a couple of exceptions. Riding the yield curve

involves buying a long-term bond (and sometimes hedging it with sale—or short-sale—of a short-term bond) to earn higher returns. Here the mortgagor is a net borrower (or seller of a bond), and so eschews the longer-term mortgage in favor of one with a shorter term, thus earning savings from the difference in rates. However, riding the yield curve, even in its hedged form, incurs some risk, while the process described here is riskless from the borrower's perspective.

8 The amount the 15-year mortgagor still owes at the end of 84 months can be calculated as 3636.11($PVIFA_{96,\ 0.3125\%}$) or, equivalently, as 500,000($FVIF_{84,\ 0.3125\%}$) – 3636.11($FVIFA_{84,\ 0.3125\%}$). The calculation for the 30-year mortgagor is analogous, but with a periodic monthly rate of 0.375%.

9 In addition, they are probably the age bracket that faces the greatest uncertainty in their future cash-flow stream, as well as least likely to be able to afford the higher payments of a 15-year mortgage. Goff and Cox (1998) identify other advantages of the 30-year relative to the 15-year mortgage.

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