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Retirement income strategies designed in an expected utility framework

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Abstract

Various classes of retirement income strategies are evaluated and their robustness tested in an expected utility framework. Fixed percentage systematic withdrawals from an investment portfolio combined with laddered purchases of immediate life annuities stands out as a superior strategy for retired defined contribution plan participants and IRA holders, yielding better outcomes than alternatives, including longevity insurance. This broad strategy is then applied, step by step, and customized across a wide range of household risk preferences and situations, with due consideration of product costs, taxes, and economic risks and returns.

JEL classifications: G11; G22; D14

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1. Introduction

Products and strategies are emerging in the market to help retirees invest and draw down assets from their 401(k) accounts and IRAs during retirement. Given the competing desires for lifetime income and wealth, for security and flexibility, and for current and future needs, it is appropriate to compare solutions carefully. This article gives a quantitative analysis of different strategies for households, including systematic withdrawals, immediate life annuities, mixed strategies of systematic withdrawals with life annuities, and also the use of

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longevity insurance (advanced-life delayed annuities, ALDA). The ALDA product has been featured in other research, and in a recent amendment to the required minimum distribution (RMD) rules for retirement accounts in the United States.

As realistically as possible, this articles considers fee charges and pricing for various products, income flows from Social Security and pensions, taxes, and random realizations of asset returns, interest rates, and mortality (normal and impaired life). Results are computed from a model using an expected utility framework, which assigns a different utility value to any small change in consumption or final wealth levels. The robustness of strategies is tested across households with varying preferences and circumstances.

A household's chosen strategy is assumed to remain fixed throughout retirement. This essentially acknowledges the complexity of issues facing older retirees, such as their trust in financial institutions and their cognitive agility to make midcourse changes, as well as the governance capacities of financial providers and advisors. The idea of simplifying and easing the path for investors is also found in the fixed strategies used by the popular target-date funds and managed accounts during the accumulation phase. At the same time, it is possible to reoptimize at any time in response to major changes in the retired household's situation, such as the death of a spouse. A more dynamic strategy cognizant of the practical complexities of financial and insurance products and taxes would necessitate a much more complex model which, in turn, would be more opaque to retirees.

Among the many options considered, a new strategy stands out: combining laddered purchases of nominal immediate life annuities (that is, dollar cost averaging) with systematic withdrawals (fixed percentage) from a dynamically changing retirement investment portfolio. This combination strategy outperforms the alternatives using ALDA and inflation-indexed annuities.

2. Evolution of thinking on income strategies

There is growing research literature on retirement income solutions. In a simple loss aversion framework, Pang and Warshawsky (2009) compared products and strategies for producing income and managing wealth in retirement accounts, assuming an initial 50–50 equity-bond allocation. The strategies compared ran the spectrum of liquidity and potential growth.[1] At one end, systematic withdrawals as a fixed percentage of the retirement account provides complete liquidity and growth potential but no guarantees, and significant risk. At the other end, full annuitization using a straight immediate (nominal) life annuity provides no liquidity or growth potential but is fully guaranteed for life in nominal terms. Midspectrum strategies include a mix of systematic withdrawals and gradual annuitization over ten years, a deferred variable annuity with a guaranteed minimum withdrawal benefit rider ("VA+GMWB" with a 5% withdrawal rate), and an immediate variable life annuity invested in bonds and equities.

In general, among the strategies analyzed, the highest real income and lowest chance of income shortfalls are achieved by combining systematic withdrawals and gradual but complete annuitization over ten years—that is, the annuity purchase ladder strategy. Only the purchase of an immediate fixed annuity at retirement, using the entire account balance, achieved a higher real income, at the median outcome, but that strategy leaves no wealth balances ever. The ladder strategy maintains some balances through the first 10 years of retirement. The other strategies (excluding the immediate variable annuity) offer the advantage of significant real balances (i.e., liquidity). The liquidation value in VA + GMWB tends to run to zero in later life, owing to withdrawals and fees. A more aggressive portfolio (70–30 equity-bond split) gives a small relative boost to the VA+GMWB, because the investor takes greater advantage of the insurer's guarantee, but the basic results remain. The relative levels of fees are critical to the analysis. With an overall low level of fees, perhaps owing to institutional pricing, as long as the fees are reduced proportionally across products, the advantage of the annuity ladder strategy still holds.

Warshawsky (2012, Chapter 7) and Warshawsky (2016) are the main studies that develop an analysis of the laddered annuity purchase strategy. Their focus is on optimizing the ladder strategy of immediate annuity purchases, both because it compared well in earlier research with other strategies and because it closely resembles the optimum in the theoretical fully dynamic model of Pang and Warshawsky (2010) described below. The components of strategy design optimization include the withdrawal rate, and the length and extent of annuitization.

Pang and Warshawsky (2010) developed a formal theoretical, somewhat stylized, dynamic model of expected utility maximization in retirement, where the household optimizes consumption and allocates wealth across equities, bonds, and life annuities, with consideration of the receipt of lifetime income flows such as Social Security and defined benefit pensions. In addition to stochastic capital market returns and mortality, households are exposed in this model to uninsured health care cost risks, which increase with age and income decile. Absent a bequest motive, the results indicate that retired households should optimally start annuitizing their wealth around their mid-70s and annuitize fully in their 80s. Retirees continue purchasing annuities throughout their lives as they save some income for sequential purchases of higher yielding life annuities to effectively insure for higher health care expenses later in life. Moreover, with the use of guaranteed annuities, the optimal equity exposure in the remaining portfolio increases with age, reaching nearly 100% for high-income households, until nonannuitized wealth is used up. The consumption level is fairly sustained over the retirement life cycle with the support of annuity income. A modest bequest motive tempers these results, in particular cutting the ultimate use of life annuities, but does not overturn them.

In the face of uncertain health care expenses, it seems logical to expect, and some studies indeed show, immediate life annuities to be a poor investment, owing to their lack of liquidity. However, uninsured health care spending exposure is highest later in life, especially for long-term care needs, precisely when newly purchased immediate life annuities (if available) generate their highest returns, owing to large and growing-with-age mortality credits. Hence, immediate life annuities are a hedge for long-term care and health spending in the absence of complete health and long-term care insurance coverage. Note that this point is orthogonal to the issue of the current health status and, therefore, life expectancy of the potential insured and the appropriateness of the current purchase of immediate life annuities.

Dus, Maurer, and Mitchell (2005, DMM) analyze various strategies, including an immediate real (that is, inflation-indexed) life annuity, delayed annuitization and systematic withdrawals. The withdrawal rate is determined according to a fixed dollar benefit level.

Under a fixed dollar formula, benefits are paid until the plan participant dies or funds are exhausted. Alternate variable formulas are based on fixed percentages or relate to remaining life expectancy. Their evaluative approach is broadly similar to Warshawsky (2012, Chapter 7) using shortfall risk of expected payouts and real balances. If no annuities are available, the optimal strategy is a fixed percentage withdrawal of 7.3%, and the optimal equity share is 75%—remarkably close to the results mentioned above. DMM compare these results to a fixed payment systematic withdrawal equal to the immediate real life annuity payout. Focusing on the expected present value of benefits paid (a concept similar to money's worth in the annuity pricing literature—the larger, the better) as the best single evaluative measure, the fixed percentage approach is shown to be superior to the fixed dollar approach.[2]

DMM find that annuitization is more appealing to older retirees, as compared with systematic withdrawals, another result quite similar to those summarized above. They infer that a strategy of systematic withdrawals followed by full annuitization at age 75 or 85 (delayed annuitization) increases the expected present value of benefits and shrinks the expected present value of shortfall income. This evokes the annuity ladder. Finally, they evaluate the immediate purchase of a delayed annuity (to pay at age 75 or 85) at the beginning of the retirement period—the longevity insurance strategy advocated by some analysts and market participants. DMM find that for retirees who desire any bequests or liquidity, these outcomes are generally inferior to delayed annuitization, particularly for longevity insurance paying at age 85. Moreover, DMM assumed the load on the delayed annuity is the same as for immediate annuities, whereas empirical evidence finds that immediate life annuities have lower loads (see below).

A related article by Horneff, Maurer, Mitchell, and Dus (2008) uses a utility-based framework, with stochastic capital markets (but not inflation) and uncertain lifetimes, to evaluate phased withdrawal plans and fixed payout annuities. They find that the fixed benefit rule performs poorly, running out of funds by age 80, is consistent only with very low levels of risk aversion, and is dominated by other payout rules (the fixed percentage in particular) and by the life annuity.

Horneff et al., also consider combination strategies, in particular the life annuity and the life expectancy withdrawal rule. They find that as risk aversion increases above relatively low levels, the devotion of a significant share of wealth (approaching 90% with no bequest motive) to the immediate life annuity increases welfare significantly. For moderate risk aversion, it is better to delay annuitization until around age 80.

Economists refer to the actual dearth of voluntary annuitization—despite the evidence of economic theory and simulations showing the high utility value of immediate life annuities as insurance against outliving wealth—as the "annuity puzzle." Various explanations have been suggested. Brown and Warshawsky (2004), Dushi and Webb (2004), and Inkmann et al., (2011), among others, show that the extent of annuitization is affected by the availability of annuities in retirement plans, the share of wealth represented by Social Security and defined benefit plans, the load on life annuities arising from adverse selection, extra marketing costs and other factors, levels of financial wealth, life expectancy, education, and bequest motive.

Behavioral biases are now favored as an explanation of the annuity puzzle, prompting calls for new strategies and public policies to overcome cognitive blocks. One such proposed strategy is longevity insurance, a deeply deferred life annuity, also known as an advanced-life delayed annuity (ALDA). An ALDA is purchased at retirement but payouts do not begin until the retiree reaches an advanced age, usually 85. It has been promoted by Milevsky (2005) and Scott (2008), as well as by the prior, Obama, Administration.[3] The ALDA strategy claims to provide liquidity through most of retirement, that is, partial annuitization with longevity risk coverage late in life for a premium that might be perceived as a cheap price. For example, Sexauer et al. (2012) show that in 2010, a 65-year-old retiree would have needed "only" 12% of the portfolio to purchase a deferred (albeit nominal) annuity such that the first payout at age 85 would equal the last payout from a self-amortizing (over 20 years) portfolio of laddered Treasury Inflation Protected Securities (TIPS). Haensly and Pai (2015) have also examined the TIPS/ALDA strategy. It should be noted though that there is no upside investment potential with this strategy and it may not therefore be desirable to many retired investors.

Gong and Webb (2010) compare longevity insurance with complete immediate annuitization, delaying the complete annuitization to a late age, and systematic withdrawals; they do not consider ladders of annuity purchases. They use an expected utility model, but with no risky assets or bequest motive, and assume all annuities are loaded equally. With retirement at age 65 and moderate risk aversion, Gong and Webb find that complete immediate annuitization wins out over longevity insurance (although Gong and Webb say they favor the ALDA).

Gong and Webb also calculate loads on commercial annuity products using the standard money's worth methodology of Mitchell et al. (1999). They find that the load is about five percentage points higher on real annuities than on nominal annuities, and that the load is fairly constant across ages 60 through 75 but increases thereafter, again by about five percentage points, based on the best pricing of immediate annuities among three issuers, and a pricing model using Treasury bond yields and general population mortality. A comparison of the loads on longevity insurance (paying at age 85) with immediate annuities, on a nominal basis and issued at age 65, finds the load on the deferred annuity to be about five percentage points higher than for the immediate annuity at one insurance company, and about 20 to 30 percentage points higher at another. Although these findings reflect pricing on one day only (in January 2008), Finkelstein and Poterba (2004) reached broadly similar conclusions based on data from a large United Kingdom insurance company over a 17-year issuance period through 1998. They find that the load is from five to 10 percentage points higher on real than on nominal immediate annuities. We will use these results in our empirical simulations below. Note that even though there is no real ALDA being sold in the market, I do model it in a couple of the simulations below, and make an inference on what its load would be from the above literature.

This literature review has focused on studies bearing directly on retirement income strategies. Other recent studies, however, are also quite relevant to this article because they address directly and empirically parameter values for the bequest motive that are only guessed at in the previous simulation literature. Ameriks et al. (2011), De Nardi et al. (2010), and Lockwood (2012) carefully estimate preference parameters in structurally similar life-cycle expected utility models. They find varying degrees of bequest prevalence and strength of the bequest motive. Below, we use the average and range of their parameter estimates in evaluations of income strategies. Similarly, survey-based formal evidence on the parameters for risk aversion is found in Kimbal et al. (2008). Their risk aversion parameter estimates are measured precisely, based on answers to hypothetical risk situations. By contrast, the time preference parameter is less consistently and more widely measured in a large literature, so we hew more closely here to convention and intuition.

3. A utility-maximizing framework for the evaluation of strategies

3.1. Lifetime utility

I now specify the algorithm or model by which retired households find their optimal strategy. This stochastic simulation optimization model is in the spirit of the theoretical model by Pang and Warshawsky (2010) but is more realistic of product and market conditions. The particular functional forms used here are chosen to (1) enable the use of empirical results in the literature estimating various parameter values (which assumed these functional forms), (2) they are more easily manipulated in stochastic simulation work than other forms, and (3) are fairly common and long-standing in both the theoretical and empirical literatures.

A household is assumed to seek an income and wealth management strategy that maximizes its retirement lifetime utility, which is defined as a function of consumption flows and the bequest amount upon death. Its lifetime utility in retirement is expressed as follows:

$$V_\tau = \sum_{t=0}^{\tau-1}\left[\beta^t h_t u\left(\frac{c_t}{h_t}\right)\right] + \beta^\tau v(b_\tau),$$

where the realized lifetime utility V_τ depends on the survival from $t = 0$ (retirement age 65) through τ (stochastic, maximum age 105), c_t is household consumption and c_t/h_t is on a per capita basis with h_t being the effective number of adults, b_τ is monetary wealth as bequest upon death, and $\beta = 0.97$ is the discount factor (time preference) initially (later we will vary and increase the time preference parameter). Ignoring children, we set h_t to 1 for single retirees and $2^{0.5}$ for couples, taking into account economies of scale in consumption.

The period utility function of consumption takes the constant relative risk aversion (CRRA) form:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where a higher value of σ indicates greater risk aversion. The utility from bequest has a similar functional form, as follows:

$$v(b) = \frac{\eta\left(\kappa + \frac{b}{\eta}\right)^{1-\sigma}}{1-\sigma},$$

where η indicates the strength of the bequest motive and κ imposes a threshold of consumption (in thousands of dollars) above which the bequest motive becomes operative. As the value of κ rises, bequests are increasingly considered a luxury. Using rounded averages of parameter values estimated by De Nardi et al., (2010) and Lockwood (2012),[4] we initially set $\sigma = 4$, $\kappa = 30$, and $\eta = 10$. These parameters depict a modestly conservative attitude of retirees toward risk and a middling motive for a bequest. Bequests are feasible for households, but are not necessarily easy to manage with the consumption threshold of \$30,000 a year ($\kappa = 30$). We consider alternative values later in sensitivity tests. The objective of households is to maximize the expected V_τ over all possible life outcomes of τ. The numerical result of V_τ is a ranking of utility and does not conveniently measure the magnitude of welfare gain or loss of one strategy versus another. Therefore, we calculate the so-called certainty equivalent (CE) consumption that would generate the same level of value V_τ for a life path τ. This constant CE consumption, on a per capita basis, is determined such that:

$$\sum_{t=0}^{\tau}[\beta^t h_t u(CE)] = V_\tau.$$

Further, over N possible life paths (i.e., N series of simulations from $t = 0$ through τ), we calculate the average certainty equivalent consumption (ACE) as:

$$ACE = \sum_{j=1}^{N} \frac{CE_j}{N}.$$

The objective of households is now transformed to search for an income distribution and wealth strategy that achieves the highest ACE. When evaluating strategies, both across broad categories and for specific implementations within a category, the highest ACE should be chosen, for a particular set of preference and economic condition parameters.

3.2. Parameterizations and stochastics

Households (singles or couples) are initially assumed to start with \$250,000 in accounts upon retirement. Household consumption c_t is equal to income that is the sum of systematic withdrawals, annuity payouts, and Social Security benefits. The Social Security benefit is initially set to be $12 \times \$1150$for singles or survivors and $12 \times \$1150 \times 1.5$ for couples (a 50% spousal benefit). These amounts are based on the 50th percentile of the new Social Security awards to retired workers in 2010. The bequest b_τ is set as the fund balance upon death, if greater than zero, plus one month's Social Security benefit.

Households are initially assumed to have an initial 50–50 equity-bond portfolio mix. This simple allocation, common in many target-date funds at the point of retirement, is chosen so that we can focus on the implications of various withdrawal and annuitization strategies; we leave the analysis of more complex asset allocations, such as international, developing market, real assets, and so on, to future work. As wealth is annuitized, the equity share increases in the remaining assets (up to 100%) so as to maintain roughly the same overall risk exposure.[5] This dynamic asset allocation strategy is an approximation to the optimal one resulting from the theoretical model of Pang and Warshawsky (2010)

Table 1 Summary statistics of simulated annual rates and returns

	Equity return	Bond return	Bond yield	Inflation
Real (%)				
Mean	4.9	2.8	2.5	—
Standard deviation	17.8	9.7	2.4	—
Nominal (%)				
Mean	8.9	6.9	6.5	4.1
Standard deviation	17.3	9.0	2.5	2.7

Source: Author's simulations based on 1962–2011 data.

mentioned above. Although some retirees might initially balk at this high allocation to equities, its riskiness is only apparent and not real because of the substitution of fixed life annuities for bonds; this logic could be explained to retired households by financial advisors.

Variable returns on equities and bonds are proxied by the S&P 500 and the United States 10-year Government Bonds Total Return indexes, respectively. Inflation is measured by the change in the CPI-U index. The stochastic dynamics of asset returns and inflation are modeled as a vector autoregressive (VAR) process, following Campbell and Viceira (2005). The VAR coefficients and variance-covariance matrix, estimated on 1962–2011 quarterly data, are embedded in the simulations to generate a large number of multiyear series of rates and returns. This approach captures the serial correlations among variables and the contemporaneous correlations of market shocks. Summary statistics are reported in Table 1 and simulation details are in the Appendix, section A. Investment management fees are subtracted from returns. We also allow for insurer bankruptcies, with small probabilities, and stochastic partial policyholder recoveries.

The management fee for investments is initially assumed to be a relatively low 25 basis points, which is consistent with a portfolio composed mainly, but not entirely, of indexed equity and bond funds in employer-sponsored retirement accounts or discount IRAs. For annuity purchases before age 75, a 10% load is assumed for nominal immediate life annuities, a 15% load for real immediate life annuities, a 15% load for the nominal ALDA, and a 20% load for the real ALDA (as mentioned above, this latter product does not exist in the market, so it should be regarded as hypothetical here). For annuity purchases after age 75, loads are further assumed to increase linearly by up to 5% until age 85 and flatten off thereafter. These load differentials are consistent with the empirical evidence of age-varying actuarial fairness of annuities reported in Gong and Webb (2010).[6] Single households purchase single life annuities, while married couples purchase joint and survivor (J&S) annuities with 75% survivor benefit. The process of annuitization can be gradual for immediate annuities, taking as long as 30 or more years, although in actual practice, it would be appropriate to limit the process up to age 90, in line with current U.S. market practices for the maximum age for immediate annuity sales. For the ALDA, it is a one-time purchase upon retirement at age 65, with payment commencing at 85—the approach advocated by some analysts, market makers, and policymakers.

The underlying assets for life annuities are assumed to be invested in nominal bonds. The calculation of annuity price/factor uses the 10-year government bond yield, which is stochastic through time, and life tables for annuitants rather than those for the general population, which reflects adverse selection in the voluntary annuity market. Note that this annuity pricing module, including the loads mentioned above, is quite conservative and biases the optimal strategies somewhat away from annuitization. The survivals of households are simulated in the model based on general population mortality rates.

3.3. Implementations and strategies considered

I now consider several broad strategies, including systematic withdrawals, immediate annuities, and deeply deferred annuities, nominal and real. I model retired investors searching among the implementations for the best strategies, first simple ones and then more complex combinations.

3.3.1. Systematic withdrawals: Fixed real dollars or fixed percentage of balances

Retirees can continue to hold investment funds and take periodic systematic withdrawals, which can be constant in real dollars. That is, investors withdraw a certain amount in the first period and adjust the amount for inflation in the following periods. This strategy provides retirees with the same purchasing power over time, but they risk outliving resources at older ages. This approach is related closely to the so-called Bengen rule, discussed below.

Alternatively, withdrawals can be a fixed percentage of the portfolio balance in each period, which will not exhaust the retiree's wealth and implicitly assumes some self-discipline or flexibility on consumption because withdrawals can be very low in adverse investment climates. This approach provides liquidity to investors and bequest potential to their heirs. It allows investors to consume more when funds perform well, but also exposes them to possibly painful declines in consumption when investments fare poorly.[7]

3.3.2. Immediate life annuity: Nominal or real

Life annuities address longevity risk and offer a steady flow of income. Lacking an annuity, retirees' income flow and consumption hinge on how quickly they draw down wealth, how long they live, and their investment outcomes. Retirees who consume too quickly may outlive their financial resources, especially given ever-increasing life expectancy, while the overly cautious may consume well below their means. We consider the most widely available nominal immediate life annuities, whose payouts are constant in nominal terms, as well as real life annuities, whose payouts are indexed to inflation, with extra cost loads, as observed in the market.

3.3.3. Advanced-life delayed annuity

As a relatively recent innovation to insure against longevity, the ALDA purchased at retirement begins payouts at an advanced age, such as 85, to surviving investors. The ALDA premium is typically a fraction of the premium for an immediate annuity with the same payout. The annuity load is nonetheless higher because ALDA providers face higher risk in guaranteeing the interest rate during the interval between purchase and payouts and also possibly from a greater extent of adverse selection among purchasers. Determining the desired level of ALDA payouts and smoothly managing income before benefits begin remains practically challenging for investors. Our simulations assume that investors purchase the ALDA at age 65, make systematic withdrawals before 85, and lower withdrawals by the amount of ALDA payouts at 85. The lifetime income flow can be volatile because of the uncertainty in investment outcomes.

4. Results

The strategies may use nominal or real (inflation-adjusted) product components in their implementations, but reported outcomes are always adjusted for stochastically realized inflations, that is, reported incomes and balances are in real dollars. To initially speed up the simulation computations, which involve searching across a grid of expected utility values to find the optimum, the fixed percentage withdrawal has a one percentage point increment, and the fixed real dollar (annual) withdrawal has a $5,000 increment. The annuitization process can take up to 30 or more years, with a 5-year time period increment, and the purchase can start and end in the range of 0–100% of expected wealth, with a five percentage point increment. Later simulations do use a more refined search grid. I consider both singles and couples as retired households.

4.1. Singles with normal life expectancy

Although the at-least-partial use of annuities is indicated by the literature, and the ACE values (for like preference parameters) reported below are higher than found in strategies using systematic withdrawals alone, I start with systematic withdrawals alone to gain some insight into the best form and level of systematic withdrawals. Among fixed-dollar inflation-indexed withdrawals, the simulations find that a $15,000 withdrawal (that is, 6% of the initial balance, inflation-indexed in subsequent years) generates the highest lifetime utility. Real consumption remains constant, as long as the accounts are not exhausted. If the distribution strategy is based on a fixed percentage of asset balances with a varying dollar amount, the optimal withdrawal is 9% of the portfolio balance each period. Relative to the fixed real-dollar withdrawals, this strategy tends to generate higher consumption in earlier years but lower consumption in later years, owing to a higher volatility of incomes. Overall, the ACE is higher here than with the fixed dollars implementation, indicating the superiority of the fixed percentage approach. See Table A-1 in the Appendix, section B, for the consumption and wealth outcomes for single retirees.

Financial planners often advise retirees to draw down their assets using a 4% rule, as suggested by Bengen (1994). By this rule, households in our model would withdraw $10,000 a year in real dollars, which is lower than the optimal withdrawal amount given above and produces lower expected lifetime utility (by an ACE measure of 23.0, not reported, vs. 25.1 in Table A-1). One important factor is that my analysis includes a safe and inflation-indexed

Table 2 Search for optimal strategies among systematic withdrawals and laddered purchases of immediate life annuities—singles

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Options: Fixed real dollars + real annuity					
Optimal strategy: Withdrawal $10, annuity initial 40% of wealth, ending 45% by 25 years, ACE 25.6					
Income	32.6	29.5	18.6	28.0	4.4
Balance	199.8	89.4	0.0	88.9	72.5
b. Options: Fixed percentage + nominal annuity					
Optimal strategy: Withdrawal 8%, annuity initial 0% of wealth, ending 45% by 20 years, ACE 26.3					
Income	35.6	28.2	21.2	28.4	4.6
Balance	250.0	105.3	0.0	113.2	87.9
c. Options: Fixed real dollars + nominal annuity					
Optimal strategy: Withdrawal $10, annuity initial 30% of wealth, ending 55% by 25 years, ACE 25.8					
Income	33.3	30.1	18.0	28.4	4.9
Balance	219.2	101.3	0.0	99.4	82.1

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

lifetime income flow from Social Security, which supports a higher withdrawal and reduces the risk to income of wealth running out. Scott et al. (2009) argue that the 4% rule is inefficient because it leaves a significant portion of resources unspent. Also, most financial advisors deal with clients with above-average wealth; their spending needs and desires may be relatively modest compared with their assets.

Let individuals now consider adding (laddered) purchases of immediate life annuities (Table 2). In scenario 2a, with both fixed dollar withdrawals and annuity payouts indexed to inflation, singles annuitize 40% of wealth initially and 45% over 25 years.[8] They also simultaneously withdraw $10,000 a year in real terms. This combination foregoes some liquidity (lower balances) but generates a higher and more sustainable income flow than systematic withdrawals alone, implying a greater lifetime utility (ACE of 25.6 vs. 25.1 in Table A-1).

When fixed percentage withdrawals and nominal life annuities are considered in scenario 2b, it is optimal to not annuitize initially (zero percentage), but purchases of immediate annuities reach 45% of wealth by 20 years. The postponed and laddered purchase of nominal annuities helps maintain the real purchasing power of payouts and reduces the timing risk of purchases. For early years in retirement, income is achieved primarily through withdrawals (8% of balance), which are nontrivial amounts given the size of account balances. The ACE is highest here across the strategies/implementations analyzed, at 26.3.

With withdrawals fixed in real dollars along with a ladder of nominal annuities considered in scenario 2c, singles withdraw $10,000 a year and also generate income through significant annuity purchases—initially 30% of wealth and reaching 55% by 25 years. The ACE is lower than in the above "nominal" strategy, because the rigid fixed dollar withdrawal imposes greater risk of running out of funds. Overall, the strategy of fixed percentage withdrawals combined with laddered purchases of nominal immediate annuities wins.[9]

Table 3 Search for optimal strategies among systematic withdrawals and age-85 ALDA—singles

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Options: Fixed real dollars + real ALDA Optimal strategy: Withdrawal $15, ALDA 5% of wealth, ACE 25.2					
Income	28.8	28.8	15.4	27.2	4.3
Balance	295.9	161.4	0.0	151.1	100.6
b. Options: Fixed percentage + nominal ALDA Optimal strategy: Withdrawal 9%, ALDA 0%, ACE 25.7					
Income	36.7	26.7	17.6	27.2	6.4
Balance	254.1	140.3	41.0	146.3	71.4
c. Options: Fixed real dollars + nominal ALDA Optimal strategy: Withdrawal $15, ALDA 5% of wealth, ACE 25.2					
Income	28.8	28.8	15.2	27.2	4.3
Balance	293.7	160.0	0.0	150.7	100.2

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

In Table 3, an ALDA is considered instead of laddered purchases of immediate annuities. In scenarios 3a and 3c, the retiree would optimally withdraw $15,000 systematically, inflation-adjusted, and use 5% of initial wealth to purchase an ALDA, with payouts indexed to inflation or nominal, respectively. The simulated level of demand for the ALDA seems trivial. Recall, however, that the ALDA payout commences 20 years after purchase. The ages covered by an ALDA have higher mortality rates than earlier years, which actuarially reduces the cost for any life annuity payout. Further, with discounting for time (interest) for the 20-year waiting period, the ALDA premium is substantially lower. Discounting would lower the premium by about 65% for a real ALDA and 85% for a nominal ALDA, assuming the average interest and inflation rates reported in Table 1. Put differently, the age-85 payout from an ALDA that is purchased with 5% of wealth is equivalent to the payout from an immediate annuity that is purchased with 15–30% of initial wealth, depending on the contract terms.

Nonetheless, the ALDA has significant shortcomings. First, achieving a smooth connection between withdrawals and commencement of payout is difficult. Consumption may experience cliff changes by the time the ALDA begins payouts because of declines in wealth. Notably in scenario 3b, the ALDA loses its appeal entirely when a fixed percentage withdrawal is used. And second, households are likely to be better off using immediate annuities rather than an ALDA, as indicated by higher ACEs in Table 2 versus Table 3, in part owing to the lower loads on immediate annuities. More important, the immediate annuities perform better in managing a steady income flow and avoiding extremely low incomes.

4.2. Singles with a short life expectancy

I examine now how the strategies/implementations would vary for single retirees with an impaired life expectancy. I model impaired mortality as equivalent to those with a spinal cord injury, based on the estimates of Strauss et al. (2005). Their life expectancy is about seven

Table 4 Search for optimal strategies among systematic withdrawals and immediate annuities—singles with an impaired life expectancy

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Options: Fixed real dollars + real annuities					
Optimal strategy: Withdrawal $15, annuity, initial and ending, 25% of wealth, ACE 27.2					
Income	35.7	34.0	18.7	32.6	4.6
Balance	220.5	138.8	0.0	126.0	72.0
b. Options: Fixed percentage + nominal annuity					
Optimal strategy: Withdrawal 11%, annuity, initial 0% of wealth, ending 30% by 20 years, ACE 27.9					
Income	41.3	32.5	23.3	32.8	6.3
Balance	250.0	138.5	6.8	137.7	79.6

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

years shorter than that for the general population. As shown in Table 4, these households would make more aggressive withdrawals—$5,000 more a year as a fixed dollar withdrawal strategy or three percentage points higher as a fixed percentage strategy (compared with cases 2a and 2b above, respectively). Being less likely to reach advanced ages, these retirees would also generally reduce their purchases of immediate life annuities. Here mortality is simulated based on the impaired population life table but assumes that annuity pricing uses the regular annuitant life table, because annuity underwriting is uncommon and expensive in the United States.

4.3. Couples

Tables 5 and 6 report the results for two-person (same age) retired households when both annuities and systematic withdrawals are considered. The optimal strategies for couples are

Table 5 Search for optimal strategies among systematic withdrawals and laddered purchases of immediate life annuities—couples

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Options: Fixed real dollars + real annuity					
Optimal strategy: Withdrawal $10, annuity initial 35% of wealth, ending 45% by 25 years, ACE 23.5					
Income	37.8	33.3	17.4	30.4	6.7
Balance	213.6	87.3	0.0	90.5	80.0
b. Options: Fixed percentage + nominal annuity					
Optimal strategy: Withdrawal 8%, annuity initial 0% of wealth, ending 50% by 25 years, ACE 24.3					
Income	41.3	31.3	19.9	31.1	7.0
Balance	250.0	88.9	0.0	102.6	87.2
c. Options: Fixed real dollars + nominal annuity					
Optimal strategy: Withdrawal $10, annuity initial 25% of wealth, ending 65% by 25 years, ACE 23.8					
Income	39.3	34.2	17.2	31.0	7.4
Balance	223.3	77.3	0.0	88.8	86.1

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

Table 6 Search for optimal strategies among systematic withdrawals and age-85 ALDA—couples

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Options: Fixed real dollars + real ALDA Optimal strategy: Withdrawal $15, ALDA 5% of wealth, ACE 23.6					
Income	35.7	35.7	16.2	30.7	6.5
Balance	296.5	142.1	0.0	139.4	105.5
b. Options: Fixed percentage + nominal ALDA Optimal strategy: Withdrawal 9%, ALDA 0%, ACE 23.9					
Income	43.1	30.0	17.5	30.2	8.5
Balance	250.3	126.8	37.0	136.8	71.6
c. Options: Fixed real dollars + nominal ALDA Optimal strategy: Withdrawal $15, ALDA 5% of wealth, ACE 23.7					
Income	35.7	35.7	16.1	30.7	6.6
Balance	296.3	142.8	0.0	139.6	104.4

Source: Authors' simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

similar to those for singles. Using systematic withdrawals alone, it is optimal for couples to withdraw $15,000 in real dollars, or alternatively 9% of balances each year (Table A-2 in Appendix, section B). When the option of laddered purchases of immediate life annuities is considered, couples should eventually annuitize from 45 to 65% of wealth, varying with the combinations of nominal and real products (Table 5). Initial annuitization is generally lower for couples than for singles. Also note that the ACE is lower for couples than for singles because the same resources have to be shared for a couple even when there are some economies of scale in living. For retired couples, the optimal strategy is a program of systematic withdrawals of 8% of balances combined with a laddered purchase of immediate nominal life annuities over 25 years, rising from zero percentage to 50% of wealth. When an ALDA is considered, a small fraction (5%) of wealth on ALDA purchase is optimal and improves welfare relative to the systematic withdrawals alone, according to the measure of ACE (Table 6). Overall, households benefit from adding annuities to their retirement portfolios. Immediate life annuities again serve better than ALDAs.

4.4. Other household preferences and situations using the combined withdrawal-laddered strategy

I now consider alternative preferences and household situations. Given the prior results indicating the overall superiority of combinations of fixed percentage withdrawals with laddered purchases of nominal life annuities, we search for optimal strategies just within that class. Table 7 shows the specific optimal strategies and range of results in several cases.

With greater wealth (here $600,000), there is more room for the operation of the bequest motive (the minimum consumption threshold is easily met), and therefore the withdrawal rate is lower as is the ultimate extent of annuitization (compare Table 7a with Table 2b).

Table 7 Search for optimal strategies among fixed percentage systematic withdrawals and laddered purchases of nominal immediate life annuities—alternative situations and preferences

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Situation: Initial wealth is $600,000					
Optimal strategy: Withdrawal 7%, annuity initial 0% of wealth, ending 20% by 20 years, ACE 41.49					
Income	61.92	45.39	30.04	45.74	
Balance	638.12	359.16	75.66	363.56	
b. Situation: As in a. above, but with higher risk aversion $\sigma = 6$					
Optimal strategy: Withdrawal 7%, annuity initial 10% of wealth, ending 30% by 20 years, ACE 43.40					
Income	62.78	46.14	30.79	46.51	
Balance	579.88	319.14	57.62	322.98	
c. Situation: Initial wealth is $1,000,000, high risk aversion $\sigma = 7.6$ stronger bequest motive $\eta = 20$ more forward-looking, patient $\beta = 1.10$, refined search grids					
Optimal strategy: Withdrawal 4%, annuity initial 7.5% of wealth, ending 20% by 21 years, ACE 49.1					
Income	79.6	55.9	39.5	56.9	12.5
Balance	1314.7	806.5	348.2	806.4	301.4
d. Situation: As in a. above, except couple both age 62, initial wealth is $150,000, and $\beta = 1.0$					
Optimal strategy: Withdrawal 6%, annuity initial 20% of wealth, ending 75% by 30 years, ACE 19.76					
Income	31.69	26.56	17.36	25.31	4.83
Balance	127.57	48.77	0.0	54.32	45.98

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

Hence, the balance holdings across possible outcomes are all proportionately higher here. If risk aversion is higher ($\sigma = 6$ vs. 4 in the baseline), the single-member household would boost eventual annuitization by 10 percentage points across the life cycle (compare Table 7b with Table 7a) owing to its greater preference for security, in particular about income; indeed income outcomes are uniformly larger and balances lower. In situation 7c, I change a number of preferences and situations simultaneously: the household is wealthier ($1,000,000), more risk averse ($\sigma = 7.6$), more desirous of leaving a bequest ($\eta = 20$ vs. 10 in the baseline), more forward-looking/patient ($\beta = 1.10$), while our search grid across solutions to find the optimal strategy is more refined both for withdrawal rates, and the extent and timing of annuitization. These changes drop the fixed percentage withdrawal rate substantially, while the annuitization pattern is largely unchanged. Wealth outcomes are uniformly and relatively higher here because the optimal strategy consistent with the preferences and situation gives more emphasis to retaining balances.

In the final frame of Table 7, I move in the opposite direction—preferences remain the same, but wealth is lower and retirement for the couple is at age 62. The optimal strategy places a higher emphasis on income, as evident in the much greater extent of annuitization, albeit the purchase laddering takes place over a longer horizon because the younger couple has a much higher probability of at least one member surviving to old age. The longer horizon for the household also explains the lower rate of withdrawals. The specific optimal strategies vary widely with household preferences and demographic situations.

Table 8 Search for optimal strategies among fixed percentage systematic withdrawals and laddered purchases of nominal immediate life annuities—alternative situations and preferences, with additional consideration of taxes and minimum distribution requirements

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Situation: Panel 2b					
Optimal strategy: Withdrawal 9%, annuity initial 0% of wealth, ending 55% by 20 years, ACE 23.2					
After-tax income	31.5	24.8	17.4	24.4	4.6
Tax-deferred balance	250.0	80.9	0.0	97.5	88.9
Taxable balance					
At age 95	0.5	0.0	0.0	0.1	0.5
b. Situation: Panel 5b					
Optimal strategy: Withdrawal 8%, annuity initial 0% of wealth, ending 60% by 25 years, ACE 21.03					
After-tax income	35.2	26.6	16.7	26.5	6.0
Tax-deferred balance	250.0	77.8	0.0	95.8	88.2
Taxable balance					
At age 95	0.1	0.0	0.0	0.1	0.9
c. Situation: Panel 7b					
Optimal strategy: Withdrawal 7%, annuity initial 15% of wealth, ending 40% by 20 years, ACE 38.3					
After-tax income	54.2	40.1	26.9	40.4	8.8
Tax-deferred balance	545.9	285.9	25.2	290.1	172.8
Taxable balance					
At age 95	40.7	9.9	0.0	13.8	14.0
d. Situation: Panel 7c					
Optimal strategy: Withdrawal 5%, annuity initial 5% of wealth, ending 20% by 30 years, ACE 44.30					
After-tax income	70.3	50.6	33.7	50.8	11.4
Tax-deferred balance	1195.5	746.6	250.7	734.5	296.8
Taxable balance					
At age 95	331.8	138.0	42.6	154.9	94.5

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

4.5. Add consideration of household taxes

I now add consideration of personal taxes on the retired household to the model. In particular, I add parameters for the payment of taxes (aggregating federal, state and local government impositions) on income and bequests made at the average effective rates of the household. For most households, estate tax rates are low or zero, but income tax rates, even for middle-class retirees, can be 15% or higher. I also add to the model the minimum distribution requirements on tax-deferred account balances which can force distributions in excess of optimal levels determined, particularly at older ages; for example, beyond the ages of 90 and older, distributions of 10% and more are required. If the legally required distribution in a year is higher than the optimal, then we place the distribution in excess of the optimal in a taxable investment account that will eventually be used for liquidity or bequest purposes, that is, to support consumption if all other retirement wealth has been used up, but otherwise held in reserve.

I illustrate in Table 8 the impact of taxes on optimal strategies for four cases shown above, in panels 2b, 5b, 7b, and 7c. I assume a 15% effective income tax rate and a zero effective

Table 9 Search for optimal strategies among fixed percentage systematic withdrawals and laddered purchases of nominal immediate life annuities—alternative situations and preferences, with additional consideration of taxes and minimum distribution requirements, and defined benefit pensions added

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Situation: Panel 7c (8d)					
Optimal strategy: Withdrawal 4.5%, annuity initial 0% of wealth, ending 15% by 24 years, ACE 49.0					
After-tax income	76.4	56.8	39.1	56.8	11.5
Tax-deferred balance	1301.8	813.4	276.5	798.5	316.9
Taxable balance					
At Age 95	527.1	202.2	67.7	238.6	149.9
b. Situation: Panel 7d					
Optimal strategy: Withdrawal 4.0%, annuity initial 0% of wealth, ending 80% by 27 years, ACE 21.9					
After-tax income	35.3	29.9	18.9	28.4	5.6
Tax-deferred balance	167.3	73.6	0.0	75.7	60.2
Taxable balance					
At Age 95	31.6	1.3	0.0	4.6	9.6

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

estate tax rate. In general, because a significant portion of retirement resources are sent to the government, to produce income levels (and utility) roughly comparable (but inevitably a bit lower) to the situation before the consideration of taxes, higher withdrawals and greater annuity purchases must be made. The remaining balances will be lower. In addition, with the addition of minimum distribution requirements, for higher wealth levels or favorable investment outcomes, some tax-favored retirement assets will eventually be placed in taxable investment accounts, particularly at older ages.

4.6. Add pensions

Finally, I add defined benefit pensions to the model. The pension benefits can be either indexed to inflation or not, and can be either for the individual alone or for both members of the couple (assumed as a joint-and-two-thirds-to-survivor annuity). Here I also only use a refined search grid to find the optimal strategy and limit the laddering of annuities up to age 90. With the addition of pension income but no subtraction of other retirement resources, we naturally will expect a higher ACE, everything else equal. Because the pension is paid as a life annuity, one would also expect less need for purchasing immediate life annuities as well as lower withdrawals. These are indeed the model outcomes shown below in Table 9 when I add pensions (at the same level as Social Security, but unindexed and split among both members of the couple), on top of taxation and minimum distribution requirements, to cases 7c (8d) and 7d.

4.7. The full model treatment of three disparate example cases

Here I show three example cases, with divergent preferences and household demographic and economic situations, their optimal strategies determined by the full model, and the range

Table 10 Description of three different retired households

Household	Age(s)	Wealth	Equity allocation	Investment expense	Tax rate	Health status	Social security	Pension
A	68, 63	$400K	70%	77bps	20%	Good	$18K	$12K both
B	70	$1.5M	75%	47bps	25%	Poor	$18K	$12K
C	66, 64	$250K	40%	33bps	10%	Good	$15K	$6K both, COLA

Household	β	η	σ
A	1.075	30	7.6
B	1.075	30	5.1
C	1.15	3	10.4

Source: Author.

of possible outcomes over the households' retirement lifetimes. Table 10 gives the full specifications of the cases in terms of preference parameters, demographic situation, wealth, and so on for each household.

Now I show in Table 11 the optimal strategies and range of outcomes for the three households. The first household has fairly moderate retirement means, but a large bequest motive, and is somewhat risk averse. Their solution is a quite modest withdrawal rate, and significant annuitization over time, which produces fairly steady income and, relative to their means, significant asset holdings, both in the tax-deferred and taxable

Table 11 Search for optimal strategies among fixed percentage systematic withdrawals and laddered purchases of nominal immediate life annuities—three households, full model

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Household: 10A					
Optimal strategy: Withdrawal 4%, annuity initial 0% of wealth, ending 35% by 21 years, ACE 27.6					
After-tax income	48.3	38.8	26.1	37.9	7.2
Tax-deferred balance	505.5	257.1	20.4	258.2	157.7
Taxable balance					
At age 95	211.8	64.3	12.1	103.1	94.0
b. Household: 10B					
Optimal strategy: Withdrawal 6%, no annuity, ACE 43.4					
After-tax income	107.4	82.7	49.7	80.3	17.9
Tax-deferred balance	1927.5	1354.7	658.3	1313.8	393.8
Taxable balance					
At age 95	250.0	124.5	NA	154.2	67.6
c. Household: 10C					
Optimal strategy: Withdrawal 4%, annuity initial 0% of wealth, ending 80% by 24 years, ACE 27.4					
After-tax income	41.8	34.6	24.8	33.7	5.3
Tax-deferred balance	265.5	119.7	0.0	122.5	96.0
Taxable balance					
At age 95	43.8	4.2	0.1	10.9	19.7

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

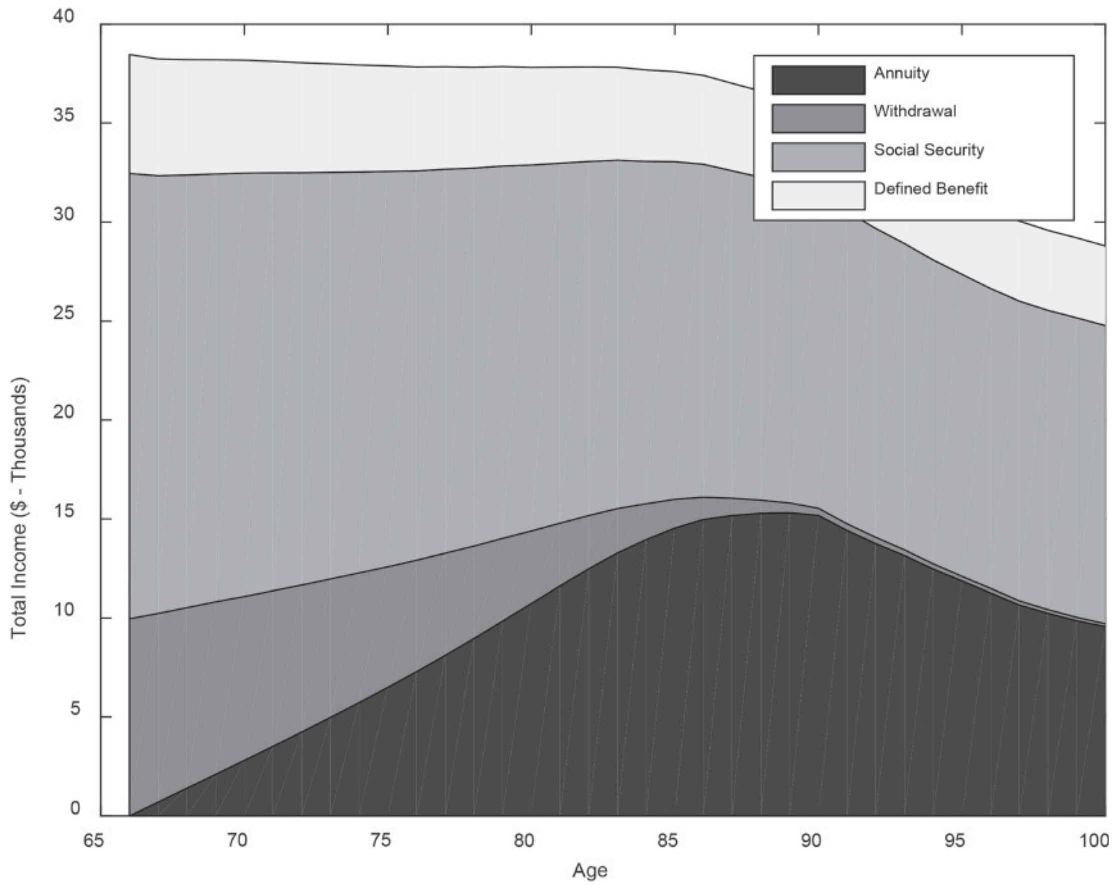

Fig. 1. Sources of total annual income, mean, by age ($ real). *Source:* Author's simulations, for household C in Table 11.

accounts. The second household is composed of an older sick person with significant asset holdings. Because of his large bequest motive and his poor health, no annuitization is optimal while the moderate withdrawal rate, combined with Social Security and pensions, gives plenty of income and preserves most of the tax-deferred and taxable assets to grow and to leave as an inheritance. The third household has even more modest retirement means than household A, but is more risk-averse, more desirous of future than current spending, and much less interested in leaving a bequest. Therefore, the optimal strategy for household C places a much higher allocation to annuitization, laddered over an extended period, which in turn produces a less volatile income flow over the household's retirement lifetime.

Figs. 1 through 4 show some of the outcomes graphically for household C, using the optimal strategy indicated above.

Fig. 1 shows the mean of stochastic outcomes under the optimal strategy in terms of pretax income, by source and by age. Social Security and, in this case, pensions are inflation-indexed, so the income flow from them is steady, although they do decline, on average, with age, because of the possibility that one member of the couple dies. Income from the series of nominal life annuities being purchased over time grows through age 90, reaching about $12,000 annual income, in real terms, but then declines because purchases of the

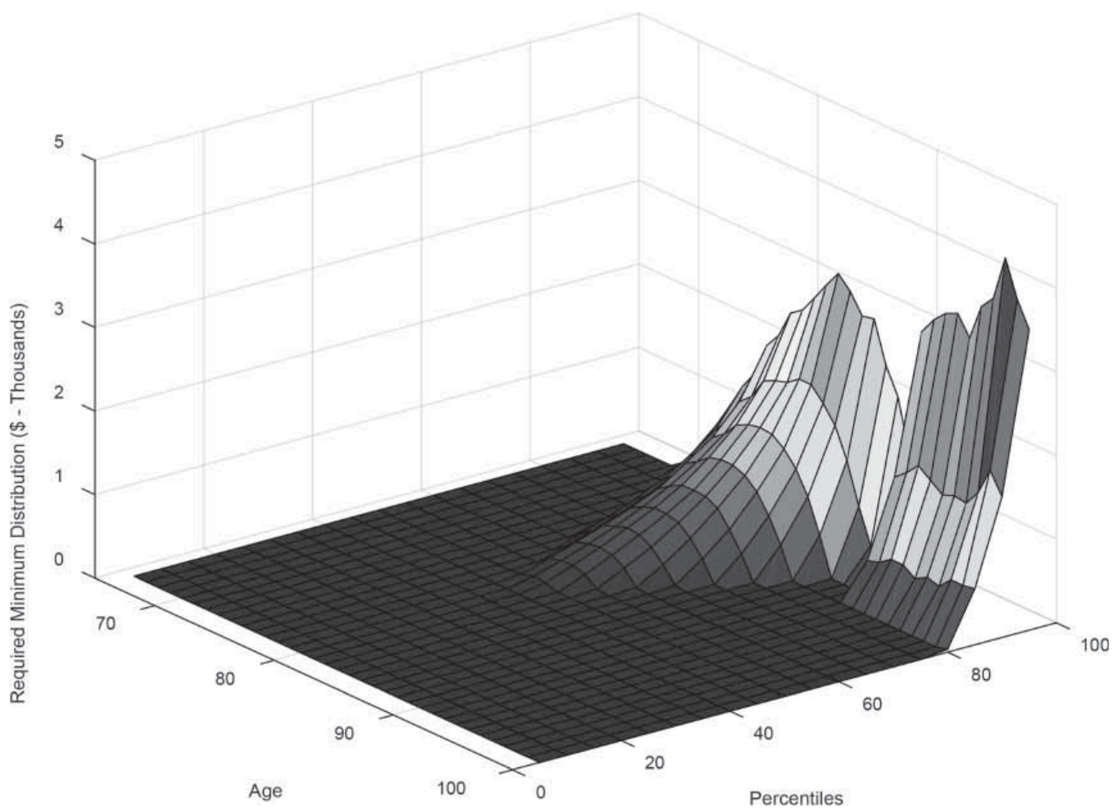

Fig. 2. Stochastic range of required minimum distributions, by age ($ real). *Source:* Author's simulations, for household C in Table 11.

immediate life annuities stop and owing to the force of inflation. Even though this household prefers future spending over current spending, which keeps optimal income high and steady over time, eventually income will decline with age because the large force of mortality makes current spending more salient, and, for a couple, one member is likely to have passed away and spending for him/her inevitably drops. Withdrawals from the retirement investment portfolio, although fixed in percentage terms, decline and eventually disappear as the retirement investments are withdrawn and also transferred, because of the minimum distribution requirements, to a taxable portfolio, and life annuities are purchased.

Minimum distributions (shown in Fig. 2) will be made and transferred to a taxable portfolio (see Fig. 3) only if the withdrawal rate and annuity purchases are less than the minimum legally required (which initially is less than 4% at age 71 but increases with age), and, obviously, if there is value left in the retirement investment portfolio, which in turn depends on return performance. Here minimum distributions are made only in the upper percentiles and at upper ages.

Finally, Fig. 4 illustrates the formula for asset allocation given in Footnote 5 above, as applied to the optimal strategy for this particular household. Here, because annuitization is quite full, so is the allocation to equity in the remaining investment portfolios (both tax-deferred and taxable).

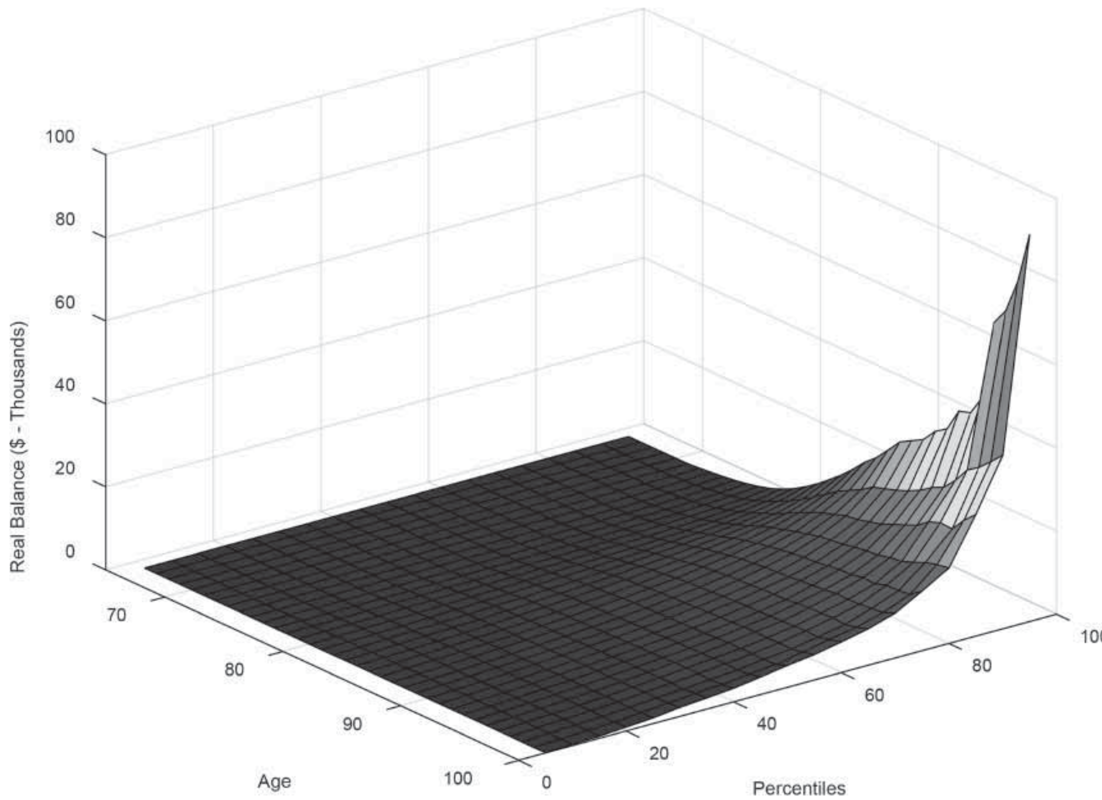

Fig. 3. Stochastic range of after-tax investment portfolio values, by age ($ real). *Source:* Author's simulations, for household C in Table 11.

4.8. Different views on investment returns and interest and inflation rates

Thus far, I have used a model (from Campbell and Viceira, 2005) of investment returns and interest and inflation rates estimated on quarterly data from 1962 through 2011. This is a reasonable approach for, essentially, forecasting the range of possible experience in the future. Nevertheless, some might want to put a greater emphasis on current conditions (in 2013), particularly as interest and inflation rates have declined so dramatically recently, as presumably have overall expected investment returns. It is possible to have the simulations adjusted to reflect this presumed different environment, without, however, completely discarding the longer view, based on more distant past experience. Going a step further, it is even possible to fix interest and inflation rates for the current year, allowing a smaller dispersion in the second year, slowly spreading thereafter; equity returns would continue to be as random as modeled earlier. This latter approach may indeed be the best way to express uncertainty for a household currently retired and planning to implement the produced strategy immediately. For households that are not yet retired, however, the former approach (whether emphasizing current conditions or completely historical), reflecting a wider range of uncertainty, is more appropriate.

In Table 12 below, I give the optimal strategies and range of outcomes for cases 10A and 10C. The first simulation, "Current conditions," gives lower expected interest and inflation rates and overall investment returns than the completely historical approach; indeed the real

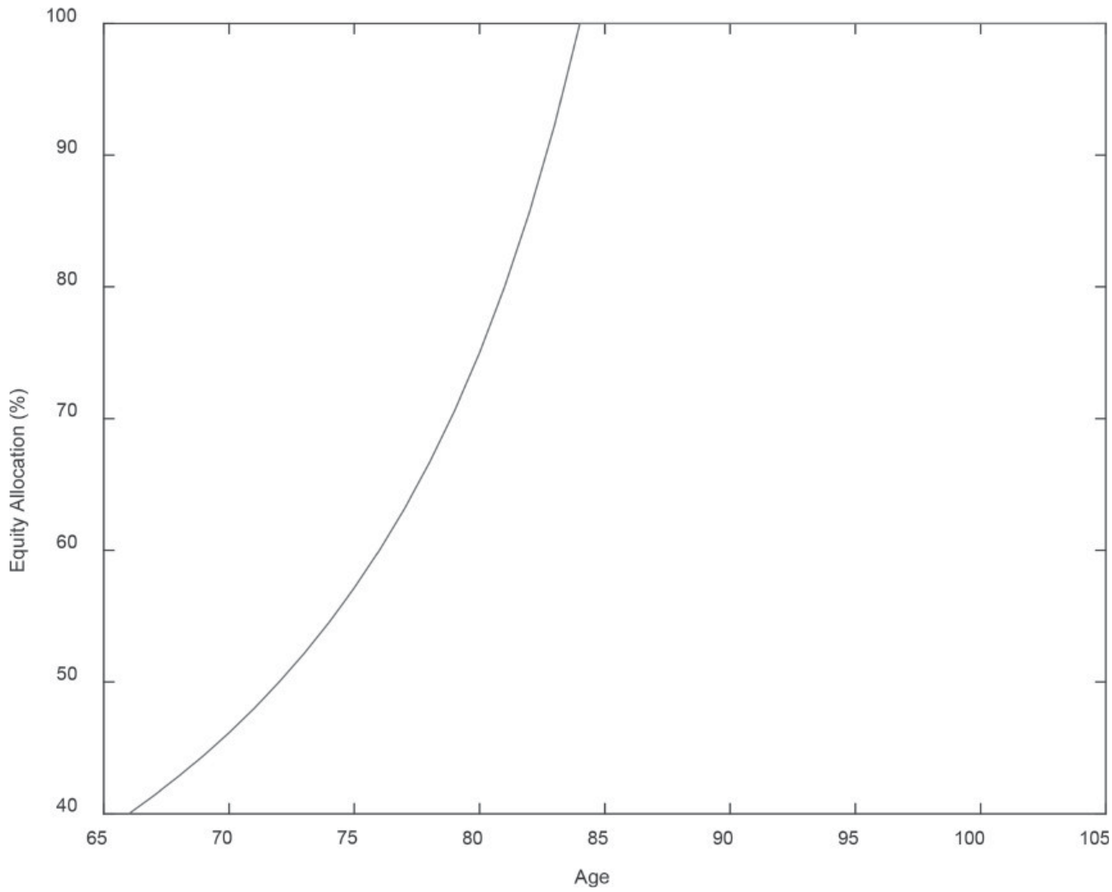

Fig. 4. Optimal equity allocations of investment portfolios, by age, percent. *Source:* Author's simulations, for household C in Table 11.

interest rate is reduced more than 100 basis points here. This may reduce the attractiveness of annuitization somewhat, as it certainly will reduce the ACE. The second simulation, "Current conditions with fixed initial interest rates," uses, somewhat arbitrarily, the average interest rate from the simulation as the fixed initial rate. It should be noted that a fixed initial interest rate reduces considerably the uncertainty around purchasing a life annuity in the first couple of years of the plan horizon; this should have the effect of increasing the attractiveness of initial annuitization, compared with either historical approach.

The expected changes occurred. Of further note, given the poorer investment environment, the model favors income over assets, everything else being equal.

5. Policy discussion and conclusions

This analysis is based on a model of rational decision-making by retired individuals and couples, using currently available investment and insurance products. Yet people are not likely to use such strategies without a more formalized structure from which to obtain them. At the least, plan sponsors, plan record keepers, or financial providers or advisors must make them available to participants.

Table 12 Search for optimal strategies among fixed percentage systematic withdrawals and laddered purchases of nominal immediate life annuities—two households, full model, alternative investment views

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Household: 10A, current conditions					
Optimal strategy: Withdrawal 4.5%, annuity initial 0% of wealth, ending 35% by 21 years, ACE 26.0					
After-tax income	47.8	39.9	25.1	38.9	6.8
Tax-deferred balance	428.2	193.3	1.6	205.4	143.1
Taxable balance					
At age 95	90.0	23.1	3.7	32.1	30.4
b. Household: 10A, current conditions with fixed initial interest rate					
Optimal strategy: Withdrawal 3%, annuity initial 21.5% of wealth, ending 52.5% by 21 years, ACE 26.0					
After-tax income	45.6	37.2	24.5	35.9	7.2
Tax-deferred balance	380.0	151.7	0	168.0	130.0
Taxable balance					
At age 95	205.7	54.3	12.3	74.2	66.8
c. Household: 10C, current conditions					
Optimal strategy: Withdrawal 4%, annuity initial 0% of wealth, ending 67.5% by 24 years, ACE 25.2					
After-tax income	37.7	32.5	22.6	31.0	5.0
Tax-deferred balance	250.0	84.1	0.0	100.6	90.4
Taxable balance					
At age 95	20.7	3.3	0.2	5.7	8.2
d. Household: 10C, current conditions with fixed initial interest rate					
Optimal strategy: Withdrawal 3.5%, annuity initial 31.5% of wealth, ending 80% by 24 years, ACE 25.4					
After-tax income	38.0	33.9	22.9	31.6	5.4
Tax-deferred balance	175.9	64.4	0.0	73.5	64.2
Taxable balance					
At age 95	34.3	6.7	1.4	10.9	13.1

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

To encourage the use of life annuities, as strongly recommended by theoretical and simulation models, efforts must go beyond designing the best strategy. As Brown et al. (2008) show, it is critical to frame the issue so that plan participants understand that their account balances—although seemingly a large amount of money in an investment frame—may seem less adequate in a consumption frame as a lifetime annual income flow. Indeed, based on a survey of people ages 50 and older, Brown et al. find that when the alternatives are presented in a consumption rather than investment frame, the vast majority prefer an annuity. Indeed James, Martinez and Iglesias (2006) make a related convincing argument that the high annuitization rate (more than two-thirds) in the Chilean individual account retirement system is because of its regulatory structure. They ascribe the high annuitization rate to a limited range of payout options (effectively only life annuities or systematic withdrawals), and to the absence of a public defined benefit plan except for a minimum pension guarantee.

Given these results, proposed regulations by the Obama Administration, also reflected in proposed bipartisan legislation, could have significantly changed retiree behavior. The Department of Labor proposed guidelines in 2013 to require defined contribution plan sponsors to give participants annual income illustrations. These are attempts to achieve more realistic framing and to provide some incentives for partial annuitization. Given strongly ingrained behavior and market conditions in the U.S. leaning in the opposite direction, the government might have to take an active role to even the playing field with pure asset strategies. Certainly the private sector, especially including life insurers, financial companies, and plan sponsors, need to be more aggressive and creative in their product design and marketing activities in the retirement field, to emphasize the need for lifetime retirement income.

Based on surveys of hypothetical choices, Beshears et al. (2014) find that allowing individuals to annuitize a fraction of their wealth increases annuitization relative to an "all or nothing" decision. The empirical simulation analysis here indicates that some life annuities should indeed be part of the portfolios for many retirees. Life annuities work to establish minimum necessary consumption and a certain level of hedging against longevity risk. While this insurance is being lost with the decline of defined benefit pension plans, it can be restored.

Some have proposed advanced life delayed annuities as providing the essence of insurance at lower premiums. Maintaining a sustainable income flow before ALDA payments begin, however, is no easy task, and the ALDA has higher loads. Welfare measures using ALDA strategies are lower. A cheaper, less risky and more transparent strategy is the combination of systematic withdrawals (fixed percentage) with laddered purchases of nominal immediate life annuities. This strategy is supported by the analytical work shown here and in the literature, and is robust across the spectrum of household preferences and situations.

Notes

1. Ameriks, Veres, and Warshawsky (2001) also use a shortfall framework, but with historical, not stochastic, simulated data.
2. These are unpublished results, which have been kindly provided by DMM, based on United States financial and mortality data over the 1967–2002 period for a 65-year-old single man.
3. The Obama Administration enacted a special dispensation, effective in 2014, from the retirement account minimum distribution requirements for longevity insurance with deferred payment up to age 85; the dispensation removes the premium for longevity insurance from the calculation base used in determining required distributions subject to income tax; the delayed payments, however, are taxed when paid. Reportedly, some individuals in high tax brackets are using this dispensation to reduce their tax bill rather than to manage risks.
4. To be consistent with our functional form, we have rearranged the specifications of De Nardi et al. (2010) and Lockwood (2012) and recalculated the parameter values. Ameriks et al. (2011) find their benchmark estimates of $\kappa = 7.28$ and $\eta = 47.6$. These parameters would lead to little consumption and substantial bequest for households in

our model because the consumption threshold $7,280 is easily exceeded ($13,800 from Social Security alone for a single retiree, see assumptions below) and the bequest motive is particularly strong. Nevertheless, we do consider the effect of larger bequest motive parameters than DeNardi's but maintain the consumption threshold, so that lower-income retired households will likely depend more on life annuities, ceteris paribus.

5 Let e denote the equity share in the remaining non-annuitized wealth and a denote the degree of annuitization (i.e., a = aw/w, annuitized wealth divided by total wealth). For the desired 50–50 risk exposure, e*(w-aw)/w = 0.5. Rearranging the equations gives e = 0.5/(1-a).

6 Using some six months of pricing data from August 2011 through April 2012 provided to me by a large, highly rated, United States insurance company and our own computations of fair value using government bond yields and general population mortality, we calculated that the load differential between a nominal ALDA commencing payment at age 85 and a single-premium immediate straight fixed-payout life annuity, both issued at age 65, indeed averaged about five percentage points.

7 There is some rigidity in these withdrawal options. The fixed dollar strategy does not respond to realizations of asset returns and the fixed percentage option does not speed up distribution toward the end of life, as might be desired to avoid leaving too large a bequest. Rather, they are easy to implement in old age, easy to explain, and require minimal governance from product issuers. Households could re-optimize their portfolios any time, perhaps with the help of financial advisors, but to avoid a lot of extra costs and governance requirements, the rerunning of the algorithm should be done sparingly, perhaps only at major life events such as the death of a spouse. Sun and Webb (2012) show that households could spend according to the IRS table for required minimum distributions (RMD), plus interest and dividends, and get better utility than from alternatives in the class of systematic withdrawals. Collins and Lam (2011) use a case study approach to discuss how financial advisors can utilize a credible simulation model to help investors make informed retirement planning decisions.

8 The degree of annuitization at any age is expressed as a percentage of the accumulated value of initial wealth, which reflects the increased with interest (at the average rate). One could have alternatively expressed it as a percentage of the account balance, but this would have been misleading. An extreme example: 100% annuitization of the remaining $1 balance appears as a strong preference for the life annuity but may be trivial if $99 has been withdrawn over prior years.

9 While the strategy of fixed dollar withdrawals and a real annuity gives completely steady real income—a goal perhaps desired by the highly risk averse, the combination of fixed percentage and a real annuity is not examined here because the nominal annuity has a lower load than the real annuity and the increase in income to cover inflation can be accommodated through the laddering purchase of smaller amounts of immediate annuities over time.

Appendix

Table A-1 Search for optimal strategies among systematic withdrawals—singles

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Options: Fixed dollars, inflation adjusted Optimal strategy: Withdrawal $15, ACE 25.1					
Income	28.8	28.8	13.8	27.0	4.8
Balance	315.5	174.2	0.0	163.0	106.3
b. Options: Fixed percentage of balance, nominal Optimal strategy: Withdrawal 9%, ACE 25.7					
Income	36.7	26.7	17.6	27.2	6.4
Balance	254.1	140.3	41.0	146.3	71.4

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

Table A-2 Search for optimal strategies among systematic withdrawals— couples

Real $000	95th percentile	50th percentile	5th percentile	Mean	Standard deviation
a. Options: Fixed dollars, inflation adjusted Optimal strategy: Withdrawal $15, ACE 23.2					
Income	35.7	35.7	13.8	30.4	7.2
Balance	314.0	158.3	0.0	151.7	109.4
b. Options: Fixed percentage of balance, nominal Optimal strategy: Withdrawal 9%, ACE 23.9					
Income	43.1	30.0	17.5	30.2	8.5
Balance	250.3	126.8	37.0	136.8	71.6

Source: Author's simulations. Higher ACEs (average certainty equivalent consumption) indicate better outcomes for households, for a given set of preference parameters.

Appendix A: Simulations of rates and returns

Asset returns are simulated as a vector autoregressive process (VAR). The VAR coefficients and variance matrix are first empirically estimated and then embedded in the simulations with stochastic shocks. Technically, let V be a vector containing the variables. The vector evolves in an autoregressive pattern:

$$V_t = \beta_0 + \sum_{k=1}^{K} \beta_k V_{t-k} + \mu_t$$

where $\mu \sim (0, \Sigma)$ denotes a vector of serially uncorrelated normal errors with $E\mu_t\mu_s = 0, t \neq s$. The contemporaneous correlations of shocks are incorporated via the variance-covariance matrix Σ and serial correlations of the variables via the coefficients β. The econometric regression on historical data yields estimates of the coefficients, $\hat{\beta}'s$ and the variance-covariance matrix $\hat{\Sigma}$. The simulations follow several steps: First, a Cholesky factorization decomposes the variance-covariance matrix to a triangle matrix. That is, the factorization finds a triangle matrix W so that $W'W = \hat{\Sigma}$. Second, a vector of random values are generated according to *IID N*(*0, 1*). Multiplying this vector by the Cholesky factor matrix generates correlated shocks to rates and returns. Third, multiplying the VAR coefficients by previous period returns, plus the shocks, gives current period returns. The procedure is repeated forward until the end-of-time horizon under consideration.

Following the specification in Campbell and Viceira (2005), asset classes include money market (90-day T-bills), stocks (proxied by the S&P 500 Total Return index), and bonds (proxied by the 10-year U.S. Government Bond Total Return index). These rates and returns in the VAR estimation are expressed in logarithm real terms (after adjusting for inflation measured by the change in the CPI-U index), using quarterly data. Additionally, three forecasting variables (state variables), which help form expectations of future rates and returns, include short-term nominal interest rate (nominal T-bills), equity dividend yield, and the slope of the yield curve (yield spread as the difference between U.S. 10-year T-note zero-coupon yield and the yield on 90-day T-bills). The entire system is estimated on 1962–2011 quarterly data.

Bankruptcy of insurance companies may occur. It is assumed that an insurance provider fails with a probability of 0.15% per annum (uniform distribution), based on Moody's global analysis of default probability for corporate bonds rated A for 1970–2005. The size of the loss of insurance contract value is simulated, within the empirical range of economic contractions estimated by Barro (2006).

Appendix B: Results for additional scenarios

Table A-1 shows the optimal results for single retirees considering systematic withdrawals alone. For the optimal fixed-dollar inflation-indexed withdrawals, the median level of real consumption is $28,800, including $13,800 from Social Security. The median real balance is $174,200 among survivors.

Table A-2 shows the optimal results for married retirees who are considering systematic withdrawals alone, either fixed real dollars or a fixed percentage of balances.

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