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Bond laddering and bond indexing: An empirical comparison

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Abstract

Bond laddering and bond indexing have been widely accepted approaches to bond investing among retail investors. However, bond laddering has virtually been ignored in both the academic literature and most of the popular investment textbooks. One thing both approaches have in common is that they are passive strategies with no attempt whatsoever to beat the market. There are many unresolved issues about the two seemingly similar approaches. First, which approach should an investor favor? Is there any room for both to be used at the same time? Second, if an investor decides to use a ladder, what is the appropriate term to maturity for the ladder? There is hardly any theoretical or empirical guidance as to which is a better approach to use and the right term of a ladder. The relative attractiveness of the above two approaches are empirically examined in this study. We identify conditions that favor one over the other. Conditions under which both instruments should be held within an optimal portfolio are also identified. We also identify conditions in which a longer term ladder is more appropriate than a shorter term ladder.

Jel classification: G10; G11

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1. Introduction

Both laddering and indexing are popular approaches to bond investing. Holding a bond portfolio that replicates an index can be entirely consistent with asset pricing theories in which an investor in equilibrium holds a market portfolio of stocks and bonds. Bond

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laddering is somewhat of an enigma to the academic profession. Bond laddering consists of investing a roughly equal dollar amount in bonds with different maturities and holding the bonds until maturity. When the shortest term bond matures, the proceeds will be reinvested in the longest term bond to maintain the same maturity structure of the bond portfolio over time. The idea is to avoid locking into a low return bond for a long period of time. The ladder approach allows investors to stay fully invested and take advantage of rising yields when the opportunity arises. The approach is widely accepted by practitioners but virtually ignored in investment textbooks.[1] Except the few studies mentioned below, the academic literature contains virtually no serious discussions of the laddering approach to bond investing. For example, specialized fixed-income books by Garbade (1996), Kaufman, Bierwag, and Toevs (1983), and Sundaresan (1997) and make no mention of the topic.[2] The ladder approach is not considered to be important enough to be mentioned in some of the most popular investment textbooks, such as Alexander, Sharpe, and Bailey (2001), Bodie, Kane, and Marcus (2007), and Jones (1996).[3]

One thing both the approaches of bond laddering and indexing have in common is that they are passive strategies with no attempt whatsoever to add value by betting on interest rate movements or the selectivity of individual bonds. While laddering may be a lower risk approach as maturities are shorter, its return at the same time may be lower. How an investor should choose between these two passive strategies is far from obvious. This study provides an empirical guide.

Judd, Kubler, and Schmedders (2011) build an intertemporal model that justifies the use of a ladder bond portfolio in an investor's investment holding. Their model shows the familiar separation theorem in which an investor will hold a risk-free asset and a common risky portfolio. The ladder bond portfolio acts like a risk-free asset delivering a steady stream of income in an intertemporal world. Their model assumes no human capital, labor income, and inflation. It is entirely possible any labor income from the human capital will serve the same purpose, thus negating the need for a ladder bond portfolio. Further, as pointed out by Campbell and Viceira (2001), inflation makes nominal bonds used in the ladder portfolio risky in real term. More important, Canner, Mankin, and Weil (1997) have documented empirically that separation theorem Judd, Kubler, and Schmedders rely on to justify the existence of a bond ladder portfolio has no empirical support in the real world.

Cheung, Kwan, and Sarkar (2010) justify the existence of bond ladders based on bound rationality. A rationale for the existence of ladders, however, does not help investors to choose between seemingly similar alternatives.

The present study on bond indexing complements the growing literature on stock indexing. The performance of stock indexing has been investigated in many studies. Prather, Chu, Mazumder, and Topuz (2009) compare alternative passive ways to invest in the Standard and Poor's (S&P) 500 index. While the Standard and Poor's Depository Receipts (SPDRs) have lower advertised annual expenses, investors in SPDRs face bid-ask spreads and commissions. The S&P 500 index mutual funds involve no additional trading costs. The question for them is whether the undisclosed trading costs alter the choice between SPDRs and the S&P 500 index mutual funds. Chang and Krueger (2010) examine whether enhanced index funds (EIFs) live up to their name and enhance portfolio performance. They find that EIFs have mostly lower returns, much higher risks, and lower risk-adjusted returns. Lu, Wang, and

Zhang (2012) examine leveraged and inverse exchange trade funds (ETFs) and note how, over time, these products do not accurately track the leveraged or inverse return of the benchmark they are designed to mimic. DiLellio and Stanley (2011) look at actively and passively managed investment strategies utilizing only ETFs. They test whether ETF-only strategies such as sector rotation strategy can typically outperform the S&P 500 index, and more appropriately a representative benchmark on an absolute and/or risk-adjusted basis and find that these strategies may allow investors to capture inefficiencies in equity markets.

2. Optimal portfolio of a representative United States investor

To examine the contribution of a bond index or a bond ladder in an investment portfolio, we consider a representative U.S. investor who is interested in holding an optimal portfolio consisting of U.S. equities, non-North American equities, and U.S. bonds. For this investor, we examine the effects of using either a bond index or a bond ladder on the risk-return tradeoff of the portfolio. Specifically, we estimate the Sharpe ratio of the optimal portfolio under a mean-variance optimization framework when either the bond index or the bond ladder is held with U.S. equities and non-North American equities. Therefore, we compare: (1) the maximum Sharpe ratio of the portfolio with the two equities and the bond index, and (2) the maximum Sharpe ratio of the portfolio with the two equities and the ladder strategy. The bond strategy that offers the higher Sharpe ratio is the superior approach to adopt.

To ensure that any difference in the attractiveness between the bond ladder and the bond index is not because of sampling errors, we test if the extra diversification benefit offered by either approach is indeed statistically significant. If short selling is allowed, the maximum Sharpe ratio can be derived by first finding out the weights w of the tangency portfolio.[4]

$$w = \frac{\Sigma^{-1}\,(\bar{z} - R_f \cdot 1)}{B - A \cdot R_f}, \tag{1}$$

where $\bar{z}$ is the vector of the mean returns of assets

Σ is the variance-covariance matrix of asset returns

1 is the unit vector

R_f is the constant risk-free rate

$$A = 1' \cdot \Sigma^{-1} \cdot 1$$

$$B = 1' \cdot \Sigma^{-1} \cdot \bar{z}$$

The Sharpe ratio of the tangency portfolio is then equal to:

$$\Theta_p = \frac{\bar{z}_p - R_f}{\sigma_p} = \frac{\bar{z}' \cdot w - R_f}{(w' \cdot \Sigma \cdot w)^{0.5}} \tag{2}$$

where $\bar{z}_p$ and σ_p are, respectively, the mean and standard deviation of the return of the tangency portfolio. Because most investors do not short sell an entire asset class even they may short a stock occasionally, we consider the portfolio allocations where short selling is disallowed throughout this article. When short selling is disallowed, the weights of the tangency portfolio can be solved with the extra inequality constraint that the vector of weights is nonnegative.

3. Data

The test period covered in this study is from 1980 to 2015 inclusively. Unlike other asset classes such as stock and bond indices where returns can be readily obtained, returns on ladder strategies must be constructed from individual bonds. Let us take the five-year ladder as an example. The return on the ladder at time t can be calculated as the equally weighted average return of the one-year, two-year, three-year, four-year, and five-year bonds at time t. This ladder portfolio will be updated at time $t + 1$ by dropping the return of the one-year bond that is maturing while adding the five-year bond return observed at time $t + 1$. This process will be repeated over time. By adding five-year bonds repeatedly as we roll over the ladder strategy, the portfolio eventually consists of four past five-year bonds and a current five-year bond. Thus, the return on the ladder portfolio is simply an equally weighted average of past and current five-year bond returns.[5] Likewise, the return on a seven-year ladder is the equally weighted average of past and current seven-year bond returns. To the extent that longer term bonds tend to offer a higher coupon, longer term ladders will offer a higher coupon returns. The bond yields used in calculating the time-series of ladder returns of various maturities are obtained from the Federal Reserve website. Specifically, historical annual yield data on three-year bonds, five-year bonds, and seven-year bonds are obtained to construct a three-year ladder, a five-year ladder, and a seven-year ladder, respectively.[6] Annual return data are used as the maturing bonds in ladders tend to be reinvested annually. By using an equally weighted average of current and past five-year bonds as the return on the five-year ladder, we implicitly ignore any mark-to-the-market price changes before the maturity dates of the bonds. Consider a five-year bond issued now at par with the yield-to-maturity equals to its coupon rate. This yield-to-maturity will be used in computing the return on the ladder for the current and also the next four periods. This approach ignores the price gain or loss of the bond in the next four periods as a result of any changes in the market interest rates.[7] This approach is deemed to be valid if the investor holds the bond until maturity as in the case of a ladder and is adopted in our main analysis. We will consider the implications of mark-to-market price changes on the performance of the ladder strategy later in our analysis.

The bond index used as an alternative to the ladder strategy is the Barclays U.S. Aggregate Bond Index, a leading benchmark for fixed-income investors.[8,9] We obtain the annual index returns over our sample period of 1980–2015 from Bloomberg.

We use the annual total return series of the Center for Research in Security Prices' (CRSP's) value-weighted index and the MSCI Europe, Australasia and Far East (EAFE) Index to proxy for the United States and non-North American equity returns, respectively.

4. Analytical framework

We perform five types of analyses on bond laddering and bond indexing. First we examine the historical risk, return, and the Sharpe ratio for each of the two alternatives to study their risk and return characteristics in isolation. This univariate analysis allows us to determine the attractiveness of each asset class. While bond laddering and bond indexing are similar in that they are both passive in style with no attempt to beat the market, they are not identical. The bond index contains bond assets with a higher credit risk and a longer duration and, therefore, it is important to quantify the resulting difference in the risk and return characteristics because of credit risk and duration.[10]

Second, we compare the performance of ladder strategies of different term structures. For investors who pursue the ladder approach, what should be the ideal term structure? Should the investor pursue a five-year ladder instead of a seven-year ladder? There is no theoretical guidance in the literature. We intend to provide some empirical guidance on this issue by examining the attractiveness of a three-year ladder, a five-year ladder, and a seven-year ladder versus a passive bond index.

Third, the attractiveness of each bond strategy is examined in a portfolio context. Because most investors own both bonds and stocks, the modern portfolio approach demands that any bond strategy must be judged by its contribution to an overall diversified portfolio.

Fourth, it is quite possible that neither the ladder nor the bond index dominates each other uniformly for all investors with different risk preferences. The difference in their risk and return characteristics could be in such a way that they are both appropriate depending on the risk aversion of the individual investor. To examine this possibility, we investigate the ladder with different maturities versus the bond index in a portfolio context for investors with different risk aversion parameters. Finally, we also perform statistical tests to ensure any difference in the attractiveness of the two passive fixed-income approaches is not because of sampling variations.

5. Results

Table 1 provides the descriptive statistics for the key individual asset classes. Here we focus on the five-year ladder without marking to market.[11] Later in the article we will

Table 1 Summary statistics of annual returns on different assets (36 years from 1980 to 2015)

		CRSP VW	MSCI-EAFE	Five-year ladder	Barclays bond index
Mean		0.1252	0.1106	0.0631	0.0814
Standard deviation		0.1715	0.2219	0.0307	0.0694
Sharpe ratio[a]		0.4645	0.2929	0.5704	0.5163
Correlation	CRSP VW	1.000	0.683	0.155	0.169
	MSCI-EAFE		1.000	0.292	0.048
	Five-year ladder			1.000	0.575
	Barclays bond index				1.000

Note: [a]Sharpe ratio calculated based on a risk-free rate of 4.56%, which is the average yield of the three-month T-bill over the sample period.

consider ladders of other maturities and ladders that are marked to market. Among the four asset classes under consideration here, U.S. equities and international equities offer the best average annual returns at 12.52 and 11.06%, respectively. As expected, these two asset classes also have the highest risk based on the standard deviations of their returns. Between the two fixed-income assets, the bond index has a higher return but also comes with a higher risk. The five-year ladder is the safest asset class with the worst return. The difference in the risk and return between the two fixed income approaches is probably because of the higher credit risk and longer duration associated with the bond index. The Sharpe ratio points to the five-year ladder being the best asset class because of its extremely low risk. As expected, the correlation coefficients between equities and the two fixed-income assets are in general quite low.

The next step is to examine the attractiveness of the five-year ladder versus the bond index in a portfolio context. We gauge their attractiveness by comparing the Sharpe ratios of optimal portfolios making up of U.S. equities, international equities, and each of the two fixed-income assets. The correlation structure between the ladder and the two equity asset classes and the ladder's low standard deviation may make the ladder more attractive despite its low return.

The most common way to construct the optimal portfolio is the *ex post* approach. Specifically, the tangency portfolio on the efficient frontier is obtained by using the historical average return of T-bill as the risk-free interest rate. We then examine the relative attractiveness of the two fixed-income assets by comparing the Sharpe ratios of the tangency portfolios based on a ladder with United States and international equities and the bond index with United States and international equities according to Eq. (2). In addition to the *ex post* approach, we also consider the resulting Sharpe ratios at different values of the risk-free rate R_f in Eq. (2). By varying R_f, we in effect trace out the efficient frontier by solving for different tangency portfolios.[12] This approach allows us to solve for different optimal portfolios on the frontier for investors with different risk aversion parameters. Thus, we can ascertain the diversification benefits to investors with different degrees of risk aversion based on the Sharpe ratios and their statistical significance. This should be a useful exercise for dissecting the performance of the ladder strategy since its lower standard deviation of returns (Table 1) suggests it is a somewhat less risky way to invest in bonds relative to simple bond indexing. This may make laddering a more suitable bond strategy for conservative investors holding a diversified portfolio of equity and fixed-income assets. By allowing for different degrees of risk aversion, we will be able to examine the suitability of the two bond strategies for investors of different risk preferences.

Table 2 reports the attractiveness of the ladder approach versus the bond index in a portfolio context. The results are consistent with the univariate results in Table 1, which show that both fixed-income assets have the highest Sharpe ratios among all asset classes. They continue to play a dominant role in the optimal portfolios. Panel A in Table 2 pertains to the case where portfolios are formed with the two equity assets and the five-year ladder. When the risk-free rate is zero, the ladder makes up about 96% of the optimal portfolio. Note that a zero risk-free rate corresponds to a highly risk averse investor who loads up with bonds at the expense of equities. As the risk-free rate increases, U.S. equities start entering into the optimal portfolios. Higher risk-free rates correspond to investors with higher risk tolerance.

Table 2 Optimal portfolio allocations with short sale disallowed at different levels of risk-free interest rates

Panel A: With five-year ladder strategy

Risk-free rate	Portfolio weights			Mean return	Standard deviation of return	Sharpe ratio
	CRSP VW	MSCI-EAFE	Five-year ladder			
0.0000	0.0366	0.0000	0.9634	0.0654	0.0312	2.096
0.0100	0.0426	0.0000	0.9574	0.0658	0.0314	1.776
0.0200	0.0515	0.0000	0.9485	0.0663	0.0317	1.459
0.0300	0.0656	0.0000	0.9344	0.0672	0.0324	1.147
0.0400	0.0917	0.0000	0.9083	0.0688	0.0341	0.845
0.0500	0.1569	0.0000	0.8431	0.0729	0.0401	0.570
0.0600	0.6074	0.0000	0.3926	0.1008	0.1067	0.383
0.0700	1.0000	0.0000	0.0000	0.1252	0.1715	0.322
0.0456[a]	0.1191	0.0000	0.8809	0.0705	0.0363	0.686

Panel B: With Barclays U.S. aggregated bond index

Risk-free rate	Portfolio weights			Mean return	Standard deviation of return	Sharpe ratio
	CRSP VW	MSCI-EAFE	Barclays bond index			
0.0000	0.1397	0.0323	0.8281	0.0885	0.0683	1.295
0.0100	0.1527	0.0292	0.8181	0.0890	0.0687	1.149
0.0200	0.1699	0.0252	0.8050	0.0896	0.0693	1.004
0.0300	0.1932	0.0197	0.7871	0.0905	0.0703	0.861
0.0400	0.2270	0.0117	0.7613	0.0917	0.0719	0.720
0.0500	0.2792	0.0002	0.7207	0.0937	0.0749	0.583
0.0600	0.3523	0.0000	0.6477	0.0969	0.0812	0.454
0.0700	0.5193	0.0000	0.4807	0.1042	0.1002	0.341
0.0456[a]	0.2533	0.0055	0.7412	0.0927	0.0733	0.643

Note: Optimal portfolios are constructed based on the estimated risk-return parameters of the asset classes during the sample period of 1980–2015 as reported in Table 1.

[a]The optimal portfolio weights in the last row are obtained by assuming a risk-free rate equals to the average three-month T-bill yield of 4.56% over the sample period of 1980–2015.

Based on the annual *ex post* risk-free rate of 4.56%, the optimal portfolio consists of about 12% in U.S. equities and 88% in the five-year ladder. International equities do not play any role in the optimal portfolios. An interesting finding is that the home bias, which is deemed to be undesirable in the finance literature, virtually causes no harm here because of the unattractive risk-return characteristics of the international equities during our sample period.

Panel B in Table 2 contains the results for the bond index. Similar to the above portfolio results for the five-year ladder, the bond index is a dominant asset class especially at lower levels of risk-free rates. This comes as no surprise as low levels of risk-free rates correspond to investors with high risk aversion.

Comparing the ladder portfolio results in Panel A with those for the bond index in Panel B, note that the efficient frontier with the ladder intersects the efficient frontier with the bond index at about the *ex post* risk-free rate. Below the *ex post* level, the frontier with the ladder dominates (higher Sharpe ratios); whereas the frontier with the bond index dominates when the risk-free rate is above the *ex post* level (higher Sharpe ratios). The reason for this result is because of the ability of the ladder in risk reduction. As reported in Table 1, the risk level of the ladder in isolation is about one half of the bond index (standard deviation of 3.07% vs. 6.94%) and thus becomes very attractive to own for a risk averse investor. Hence the frontier with the ladder dominates at lower level of risk-free rates. For example, Table 2 indicates that, at the zero risk-free rate, the *portfolio* risk (i.e., standard deviation of return) with ladder is 3.12% versus that of 6.83% for the bond index portfolio. This meaningful risk reduction leads to a much better Sharpe ratio for the ladder portfolio than the bond index portfolio (2.10 vs. 1.30). For an investor with higher risk tolerance, the lower risk associated with the ladder becomes less of an attraction. The ladder has the lowest return among all asset classes under consideration and its significance in the optimal portfolios drops at risk-free rates above the *ex post* level of 4.56%. In fact, U.S. equities become the dominant assets for investors with somewhat high risk tolerance. Because U.S. equites have a lower standalone Sharpe ratio than the ladder (see Table 1), the introduction of this asset class results in lower portfolio Sharpe ratios under the higher risk tolerance cases. It is this dominance of U.S. equities that causes the Sharpe ratios of the ladder portfolios to be below those of the bond index portfolios in high risk tolerance cases.

The evidence so far points to the superiority of the ladder over the bond index for risk averse investors as our five-year ladder offers more meaningful risk reduction and hence better Sharpe ratios at lower levels of risk-free rates. The next question is whether the difference in the two efficient frontiers (or Sharpe ratios) is indeed statistically significant. The statistical technique used here builds on the spanning tests used in the finance literature.[13] The spanning tests address the question that when an additional asset class is added to a benchmark portfolio of existing asset classes, does the resulting efficient frontier of the combined asset classes indeed display a statistically significant improvement over that of the original benchmark asset classes? If the frontier of the combined asset classes displays a statistically significant shift to the left, the additional asset class should be added to the investor's benchmark portfolio. The usual spanning tests are not used here as they require the testing of the entire frontier. As mentioned earlier, we are interested in the suitability of the ladder versus bond indexing for investors with different risk aversion. This requires the test of difference in the Sharpe ratios given by Eq. (2) at different levels of risk-free rates. Let Θ_1 defined by Eq. (2) be the Sharpe ratio of the tangency portfolio based on the benchmark asset classes and Θ_2 be that after the addition of another asset class corresponding to the same risk-free rate. The null hypothesis to be tested is H_0: $\Theta_1 = \Theta_2$. The benchmark portfolio consists of U.S. equities, international equities (i.e., non-North American equities as proxied by EAFE), and the Barclays bond index. We estimate the maximum Sharpe ratio corresponding to a given level of risk-free rate for this benchmark portfolio. The combined portfolio then contains the benchmark asset classes plus the five-year ladder. If the Sharpe ratio for the combined portfolio is statistically no different from that of the benchmark at the same level of risk-free rate, the ladder will be considered as a redundant asset. This test is repeated at different levels of risk-free rates. If the ladder is shown to be redundant at all levels of risk-free rates, portfolio investors should be content with only holding bond index,

Table 3 Diversification benefits of five-year ladder: Portfolio weights and Sharpe ratios of tangency portfolios with and without ladder are reported at different levels of annual risk-free interest rates

Risk-free rate	Portfolios with five-year ladder					Portfolios without five-year ladder				GJ p-value
	Portfolio weights				Sharpe ratio	Portfolio weights			Sharpe ratio	
	CRSP VW	MSCI-EAFE	Barclays bond index	Five-year ladder		CRSP VW	MSCI-EAFE	Barclays bond index		
0.0000	0.0366	0.0000	0.0000	0.9634	2.096	0.1397	0.0323	0.8281	1.295	0.000
0.0100	0.0426	0.0000	0.0007	0.9567	1.776	0.1527	0.0292	0.8181	1.149	0.000
0.0200	0.0511	0.0000	0.0214	0.9275	1.460	0.1699	0.0252	0.8050	1.004	0.000
0.0300	0.0648	0.0000	0.0555	0.8797	1.152	0.1932	0.0197	0.7871	0.861	0.001
0.0400	0.0911	0.0000	0.1208	0.7881	0.861	0.2270	0.0117	0.7613	0.720	0.016
0.0500	0.1613	0.0000	0.2952	0.5435	0.610	0.2792	0.0002	0.7207	0.583	0.135
0.0600	0.3523	0.0000	0.6477	0.0000	0.454	0.3523	0.0000	0.6477	0.454	—
0.0700	0.5193	0.0000	0.4807	0.0000	0.341	0.5193	0.0000	0.4807	0.341	—
0.0456[a]	0.1198	0.0000	0.1920	0.6883	0.712	0.2533	0.0055	0.7412	0.643	0.057

Note: Optimal portfolios are obtained with short-sale disallowed and based on risk-return parameters estimated over the sample period of 1980–2015. The p-values of Glen and Jorion (GJ) tests on equal Sharpe ratios are reported in the last column of the table.

[a]The portfolio results presented in the last row are obtained by assuming a risk-free rate equals to the average three-month T-bill yield of 4.56% over the sample period of 1980–2015.

without bothering with bond laddering. In designing the test this way rather than directly testing the statistical significance of the difference in the two Sharpe ratios in each row across Panels A and B of Table 2, we can also entertain the possibility that both the ladder and the bond index may coexist in an investor's optimal portfolio to enhance performance in a way that either one alone cannot do. The details of the test procedure as developed by Glen and Jorion (1993) are outlined in the Appendix A.

Table 3 reports the test results. Columns 7–9 are the portfolio allocations for the benchmark asset classes at various levels of risk-free rates and Columns 2–5 are allocations for the combined asset classes. The results for the combined asset classes in Columns 4–5 are most interesting. At very low levels of risk-free rates, the bond index is completely displaced by the ladder and the opposite is true at high levels of risk-free rates. At moderate risk-free rates, both fixed-income assets coexist. The last column reports the p-value of the null hypothesis that the Sharpe ratio for the benchmark case is equal to that for the combined case. A p-value of 0.05 means that the equality can be rejected at five-percent level. At both zero and one percentage risk-free rates, the Sharpe ratio for the combined portfolio that happens to contain no bond index is indeed statistically better than that of the benchmark portfolio. At six percent and seven percent risk-free rates, the combined portfolio contains no ladder. The combined and benchmark portfolios are in effect the same with the same asset allocations and the same Sharpe ratios. The test of equal Sharpe ratios is therefore irrelevant at these two levels of risk-free rates. Between two percent and four percent risk-free rates, the difference between the Sharpe ratios are statistically significant. The two fixed-income assets together in fact do a better job than either one alone. At five percent risk-free rate, the difference between the two Sharpe ratios is no longer statistically significant. Finally, at the *ex post* risk-free rate of 4.56%, the difference is marginally significant (p-value equals to 0.057).

Table 4 Summary statistics of annual returns on different assets (36 years from 1980 to 2015)

		CRSP VW	MSCI-EAFE	Three-year ladder	Five-year ladder	Seven-year ladder	Barclays bond index
Mean		0.1252	0.1106	0.0583	0.0631	0.0672	0.0814
Standard deviation		0.1715	0.2219	0.0347	0.0307	0.0278	0.0694
Sharpe ratio[a]		0.4645	0.2929	0.3660	0.5704	0.7770	0.5163
Correlation	CRSP VW	1.000	0.683	0.118	0.155	0.158	0.169
	MSCI-EAFE		1.000	0.177	0.292	0.306	0.048
	Three-year ladder			1.000	0.968	0.928	0.590
	Five-year ladder				1.000	0.983	0.575
	Seven-year ladder					1.000	0.532
	Barclays bond index						1.000

Note: [a]Sharpe ratio calculated based on a risk-free rate of 4.56%, which is the average yield of the three-month T-bill over the sample period.

In summary, the above empirical evidence based on the five-year ladder conclusively supports the argument for adding the ladder to a diversified equity portfolio for risk averse and moderately risk averse investors.

5.1. Optimal term structure of a ladder portfolio

The choice of the five-year maturity for our ladder is somewhat arbitrary. The practitioners' literature is silent on the optimal term. As mentioned earlier, the return on the five-year ladder portfolio is simply an equally weighted average of past and current five-year bond returns. Likewise, the return on a seven-year ladder is the equally weighted average of past and current seven-year bond returns. To the extent that longer term bond tends to offer a higher coupon, the longer term ladder will offer a higher coupon return. One would therefore expect a longer term ladder such as a seven-year ladder to deliver a higher return than a three-year ladder. At the same time, the conventional wisdom is that longer term fixed-income assets are deemed to be more volatile than shorter term ones. The question is whether the extra return associated with the longer term ladder is high enough to justify its higher risk. This is essentially an empirical question.

Table 4 provides the summary statistics for the three-year and seven-year ladders together with other asset classes being considered earlier. Again, we assume each bond in the ladder portfolio is held till maturity and make no attempt to mark the value of each bond to its market value as interest rates change. The mark-to-the-market issue will be addressed in the next section. The surprise here is the higher return at lower risk for the longer term ladder. Although we expect longer term ladders to earn higher returns, we do not expect at the same time they exhibit lower return volatility. How can we explain this seemingly counterintuitive finding? First, we need to realize that, since all bonds in each ladder are held till maturity without marking to the market, the current interest rate environment has no impact on the returns of the bonds already in the ladder. Thus, the only reason why the return on a five-year ladder changes from one period to the next is because of the reinvestment of the proceeds obtained when the oldest five-year bond matures. This reinvestment risk for most part drives the risk of the ladder. The key to understand the absence of the expected result is the amount of reinvestment associated with ladders of various maturities. A three-year ladder in-

Table 5 Diversification benefits of three-year ladder: Portfolio weights and Sharpe ratios of tangency portfolios with and without Ladder are reported at different levels of annual risk-free interest rates

Risk-free rate	Portfolios with three-year ladder					Portfolios without three-year ladder				GJ p-value
	Portfolio weights				Sharpe ratio	Portfolio weights			Sharpe ratio	
	CRSP VW	MSCI-EAFE	Barclays bond index	Three-year ladder		CRSP VW	MSCI-EAFE	Barclays bond index		
0.0000	0.0616	0.0000	0.0584	0.8800	1.771	0.1397	0.0323	0.8281	1.295	0.000
0.0100	0.0703	0.0000	0.0862	0.8435	1.496	0.1527	0.0292	0.8181	1.149	0.001
0.0200	0.0835	0.0000	0.1287	0.7878	1.229	0.1699	0.0252	0.8050	1.004	0.003
0.0300	0.1063	0.0000	0.2016	0.6921	0.975	0.1932	0.0197	0.7871	0.861	0.028
0.0400	0.1545	0.0000	0.3560	0.4895	0.749	0.2270	0.0117	0.7613	0.720	0.135
0.0500	0.2792	0.0001	0.7207	0.0000	0.583	0.2792	0.0002	0.7207	0.583	—
0.0600	0.3523	0.0000	0.6477	0.0000	0.454	0.3523	0.0000	0.6477	0.454	—
0.0700	0.5193	0.0000	0.4807	0.0000	0.341	0.5193	0.0000	0.4807	0.341	—
0.0456[a]	0.2154	0.0000	0.5514	0.2332	0.646	0.2533	0.0055	0.7412	0.643	0.328

Note: Optimal portfolios are obtained with short-sale disallowed and based on risk-return parameters estimated over the sample period of 1980–2015. The *p*-values of Glen and Jorion (GJ) tests on equal Sharpe ratios are reported in the last column of the table.

[a]The portfolio results presented in the last row are obtained by assuming a risk-free rate equals to the average three-month T-bill yield of 4.56% over the sample period of 1980–2015.

volves one third of the portfolio to be reinvested each year; whereas a seven-year ladder churns over only one seventh of its portfolio every year. Hence the three-year ladder exposes one third of its portfolio to reinvestment risk, comparing to one seventh for a seven-year ladder. The longer the term of a ladder, the smaller amount of reinvestment is required each year and thus the smaller its exposure to interest rate risk. This explains why, in Table 4, we observe a monotonic decreasing standard deviation of return as the term of the ladder increases.

Analogous to Table 3 for the five-year ladder, Tables 5 and 6 provide the portfolio results for the three-year and seven-year ladders, respectively. The seven-year ladder in Table 6 completely displaces the bond index in the combined portfolio at risk-free rates up to three percent. The Sharpe ratio for the combined portfolio without bond index at the zero risk-free rate is almost twice that of the benchmark one with bond index (2.44 vs. 1.30). The dominance of the ladder at these levels of risk-free rates is statistically significant according to the *p*-values of Glen-Jorion test (last column in Table 6). At the very high risk-free rate of seven percent, the bond index becomes more important again as this aggressive investor pursues return at the expense of risk and the safe seven-year ladder no longer fits her high risk preference profile. At the *ex post* risk-free rate of 4.56%, both fixed-income assets coexist to provide a statistically better Sharpe ratio than the benchmark with the bond index alone. Note that, in this case, the presence of the bond index in the combined portfolio is somewhat low at merely 5.4% of the overall combined portfolio.

The three-year ladder in Table 5 does contribute significantly to the Sharpe ratio of the combined portfolio at risk-free rates up to three percent, demonstrating its attractiveness to risk averse portfolio investors. However, this superiority is achieved with the presence of the bond index in the combined portfolio, whereas the seven-year ladder delivers the superiority without any help from the bond index (see Table 6).

Table 6 Diversification benefits of seven-year ladder: Portfolio weights and Sharpe ratios of tangency portfolios with and without Ladder are reported at different levels of annual risk-free interest rates

Risk-free rate	Portfolios with seven-year ladder					Portfolios without seven-year ladder				GJ p-value
	Portfolio weights				Sharpe ratio	Portfolio weights			Sharpe ratio	
	CRSP VW	MSCI-EAFE	Barclays bond index	Seven-year ladder		CRSP VW	MSCI-EAFE	Barclays bond index		
0.0000	0.0240	0.0000	0.0000	0.9760	2.442	0.1397	0.0323	0.8281	1.295	0.000
0.0100	0.0280	0.0000	0.0000	0.9719	2.086	0.1527	0.0292	0.8181	1.149	0.000
0.0200	0.0338	0.0000	0.0000	0.9662	1.733	0.1699	0.0252	0.8050	1.004	0.000
0.0300	0.0427	0.0000	0.0002	0.9571	1.382	0.1932	0.0197	0.7871	0.861	0.000
0.0400	0.0578	0.0000	0.0274	0.9149	1.039	0.2270	0.0117	0.7613	0.720	0.003
0.0500	0.0911	0.0000	0.0885	0.8204	0.717	0.2792	0.0002	0.7207	0.583	0.015
0.0600	0.2298	0.0000	0.3423	0.4279	0.462	0.3523	0.0000	0.6477	0.454	0.267
0.0700	0.5193	0.0000	0.4807	0.0000	0.341	0.5193	0.0000	0.4807	0.341	—
0.0456[a]	0.0724	0.0000	0.0543	0.8733	0.855	0.2533	0.0055	0.7412	0.643	0.002

Note: Optimal portfolios are obtained with short-sale disallowed and based on risk-return parameters estimated over the sample period of 1980–2015. The p-values of Glen and Jorion (GJ) tests on equal Sharpe ratios are reported in the last column of the table.

[a]The portfolio results presented in the last row are obtained by assuming a risk-free rate equals to the average three-month T-bill yield of 4.56% over the sample period of 1980–2015.

The evidence so far points to the dominance of longer term ladders for risk averse investors. For the shorter term ladder, an investor will also need to invest in the bond index for its higher return and yet to retain the stability of the fixed-income asset class.

5.2. Mark to the market effects

One potential criticism of the above analysis is that we have ignored marked-to-market (MTM) price changes. A falling interest rate will not only lower the reinvestment return for a ladder but also produce price gain if the bond portfolio is to be liquidated. Given the longer time to maturity of its constituent bonds, such price gain should be larger for a longer term ladder. In the above analysis, we ignore the pricing impact while focusing only on the reinvestment return. This approach can be justified by the assumption that all bonds in a ladder portfolio are held to maturity, which automatically eliminates any price risk from the ladder portfolio.[14] What will be the implications if we also take the pricing effect into account when we measure the risk-return characteristics of our ladders? It is natural to expect return to be more volatile as the price risk also comes into play. More important, the significance of the MTM effect is expected to be different for ladders of different terms. Given the longer time to maturity of its constituent bonds, the MTM effect should be stronger for longer term ladders. Will the consideration of the MTM effect invalidate our previous conclusions drawn regarding the attractiveness of a ladder to a portfolio investor? To address this question, we replicate the MTM annual returns of our three-year, five-year, and seven-year ladders over our sample period of 1980–2015 using the yields of one-year, two-year, three-year, five-year, and seven-year bonds obtained from the Federal Reserve website.[15] Let us take a three-year ladder as an example. A three-year ladder makes up of three bonds that mature in one, two, and three years, respectively. At the beginning of each year, we find out the initial value of the ladder by calculating the prices of the three bonds with the prevailing one-year, two-year, and three-year yields, respectively. Without any information regarding the specific coupon structure of the constituent bonds, here we assume they are all zero-coupon bonds and the observed yields are all zero-yields. To find out the year-end value of the ladder, we revalue the originally two-year (three-year) bond with the one-year (two-year) yield observed at the end of that year. Note that the one-year bond held at the beginning of the year matures at year end yielding the par value and hence requiring no MTM. With the values of the ladder at the beginning and the end of the year, we can then calculate its MTM annual return for that year. In doing so, we recognize the fact that, at the end of each year, the shortest term bond will mature and the proceeds received will be reinvested in a new three-year bond. The above calculations are repeated for each of the years so as to replicate the historical annual MTM returns of the three-year ladder over our sample period.[16] The historical MTM returns for the five-year and seven-year ladders are calculated in a similar fashion.

Analogous to Tables 1 and 4, Table 7 presents the summary statistics of our three MTM ladders together with all other asset classes under consideration. The MTM consideration seems to have very little effect on the return. The five-year MTM ladder has roughly the same return as the one without MTM at about 6.5% per annum. Likewise, the three-year ladder with or without MTM offers an annual return of about 5.9%. The major difference the MTM makes is on risk. Not surprisingly, the risk difference gets more pronounced as the term of the ladder increases, as the price risk being amplified by the longer time to maturity of the constituent bonds. For example, the standard deviation of the seven-year ladder with MTM at 5.96% is about twice that of the one without MTM at 2.78%. In the case of a three-year ladder, the risk for the three-year ladder without MTM is 3.47%, which is not much lower than the standard deviation of 4.39% for the three-year ladder with MTM.

Note that all ladders with or without MTM still have lower risk but also have lower return than the Barclay bond index. The Sharpe ratio of ladders with MTM is a monotonic increasing function as the term increases, albeit at a slower rate than ladders without MTM. In fact, the Sharpe ratio for the seven-year MTM ladder at 0.436 is much higher than that of the three-year MTM ladder at 0.317. This suggests that the longer term ladder is more attractive than the shorter term one if they are held in isolation of a portfolio. To assess the value-adding ability of MTM ladders in a portfolio setting, we repeat the above portfolio analysis on our three MTM ladders. The results are reported in Tables 8 to 10.

Based on the *p*-value of the Glen-Jorion test reported in the last column of Table 8, the three-year MTM ladder contributes significantly to the optimal portfolio performance at risk-free rates up to about two percent, whereas the same ladder but without MTM consideration (see Table 5) offers diversification benefit at risk-free rates up to three percent. Note that the three-year ladder with MTM is completely displaced from the combined portfolio at risk-free rates four percent or higher. With MTM, the three-year ladder is attractive to the more risk averse investors comparing to the case without MTM. While it is obvious that fixed income assets should play a prominent role in the portfolio of a risk averse or moderately risk averse investor, what should be the composition of the fixed income assets and the effect of MTM on the composition is less obvious. Tables 5 and 8 address the proper composition of fixed income assets and the MTM effect on the composition. First, both ladders (with or

Table 7 Summary statistics of annual returns on different assets (36 years from 1980 to 2015)

		CRSP VW	MSCI-EAFE	Three-year ladder	Three-year MTM ladder	Five-year ladder	Five-year MTM ladder	Seven-year ladder	Seven-year MTM ladder	Barclays bond index
Mean		0.1252	0.1106	0.0583	0.0595	0.0631	0.0658	0.0672	0.0716	0.0814
Standard deviation		0.1715	0.2219	0.0347	0.0439	0.0307	0.0507	0.0278	0.0596	0.0694
Sharpe ratio[a]		0.4645	0.2929	0.3660	0.3166	0.5704	0.3984	0.7770	0.4362	0.5163
Correlation	CRSP VW	1.000	0.683	0.118	0.082	0.155	0.029	0.158	0.029	0.169
	MSCI-EAFE		1.000	0.177	0.026	0.292	-0.021	0.306	-0.018	0.048
	Three-year ladder			1.000	0.888	0.968	0.781	0.928	0.685	0.590
	Three-year MTM ladder				1.000	0.818	0.961	0.772	0.900	0.785
	Five-year ladder					1.000	0.722	0.983	0.648	0.575
	Five-year MTM ladder						1.000	0.675	0.982	0.892
	Seven-year ladder							1.000	0.606	0.532
	Seven-year MTM ladder								1.000	0.937
	Barclays bond index									1.000

Note: [a]Sharpe ratio calculated based on a risk-free rate of 4.56%, which is the average yield of the three-month T-bill over the sample period.

Table 8 Diversification benefits of three-year MTM Ladder: Portfolio weights and Sharpe ratios of tangency portfolios with and without Ladder are reported at different levels of annual risk-free interest rates

Risk-free rate	Portfolios with 3-year MTM ladder					Portfolios without 3-year MTM ladder				GJ *p*-value
	Portfolio weights				Sharpe ratio	Portfolio weights			Sharpe ratio	
	CRSP VW	MSCI-EAFE	Barclays bond index	Three-year MTM ladder		CRSP VW	MSCI-EAFE	Barclays bond index		
0.0000	0.0978	0.0114	0.0560	0.8347	1.494	0.1397	0.0323	0.8281	1.295	0.009
0.0100	0.1098	0.0115	0.1223	0.7564	1.277	0.1527	0.0292	0.8181	1.149	0.022
0.0200	0.1282	0.0116	0.2243	0.6359	1.070	0.1699	0.0252	0.8050	1.004	0.067
0.0300	0.1602	0.0118	0.4015	0.4266	0.880	0.1932	0.0197	0.7871	0.861	0.203
0.0400	0.2270	0.0117	0.7613	0.0000	0.720	0.2270	0.0117	0.7613	0.720	—
0.0500	0.2792	0.0002	0.7207	0.0000	0.583	0.2792	0.0002	0.7207	0.583	—
0.0600	0.3523	0.0000	0.6477	0.0000	0.454	0.3523	0.0000	0.6477	0.454	—
0.0700	0.5193	0.0000	0.4807	0.0000	0.341	0.5193	0.0000	0.4807	0.341	—
0.0456[a]	0.2533	0.0055	0.7412	0.0000	0.643	0.2533	0.0055	0.7412	0.643	—

Note: Optimal portfolios are obtained with short-sale disallowed and based on risk-return parameters estimated over the sample period of 1980–2015. The *p*-values of Glen and Jorion (GJ) tests on equal Sharpe ratios are reported in the last column of the table.

[a]The portfolio results presented in the last row are obtained by assuming a risk-free rate equals to the average three-month T-bill yield of 4.56% over the sample period of 1980–2015.

without MTM) and the bond index coexist in the optimal portfolio of a risk averse or moderately risk averse investor. Secondly, comparing Tables 5 and 8, the allocation to the bond index increases at the expense of the ladder when the MTM is introduced in Table 8. For example, at the risk-free rate of two percent, the optimal allocation to the bond index increases from 12.9% to 22.4% when we incorporate the MTM effect in the returns of the three-year ladder. The reasons are the increase in the correlation between the three-year ladder and the bond index caused by the MTM from 0.590 to 0.785 and also the increase in the standard deviation of the three-year ladder from 3.47% to 4.39% (see Table 7). These increases in correlation and risk dampen the conservative investor's enthusiasm for the three-year ladder in favor of the bond index. Note that MTM also reduces the overall demand for fixed income assets. The total fixed income holding at the zero, one-percent, and two-percent risk-free rates are 94%, 93%, and 92% of the overall portfolio, respectively, without MTM (Table 5). Table 8 shows the total fixed income holding now drops to 89%, 88%, and 86% at the same respective risk-free rates because of the less attractive risk profile of the MTM ladder.

In the case of the five-year ladder results reported in Table 9, the optimal portfolio based on the *ex post* risk-free rate contains no five-year MTM ladder, whereas the five-year ladder without MTM makes up about 69% of the portfolio allocation at the same risk-free rate (see Table 3). Note that the absence of the five-year MTM ladder in the portfolio at the *ex post* risk-free rate is replaced by about 74% of the portfolio in the bond index (see last row in Table 9). The five-year MTM ladder is clearly an inferior fixed-income product comparing to the bond index. The reason for this outcome is not difficult to understand. The five-year MTM ladder and the bond index has a high correlation at 0.892 (see last column of Table 7). In effect, the five-year MTM ladder and the bond index are close substitutes of each other. Yet the bond index has a higher Sharpe ratio (0.516 vs. 0.398). Hence it is no surprise that the five-year MTM ladder fails to make the cut when comparing to the bond index. Nevertheless,

Table 9 Diversification benefits of five-year MTM Ladder: Portfolio weights and Sharpe ratios of tangency portfolios with and without Ladder are reported at different levels of annual risk-free interest rates

Risk-free rate	Portfolios with five-year MTM ladder					Portfolios without five-year MTM ladder				GJ p-value
	Portfolio weights				Sharpe ratio	Portfolio weights			Sharpe ratio	
	CRSP VW	MSCI-EAFE	Barclays bond index	Five-year MTM ladder		CRSP VW	MSCI-EAFE	Barclays bond index		
0.0000	0.1244	0.0149	0.0000	0.8607	1.473	0.1397	0.0323	0.8281	1.295	0.009
0.0100	0.1369	0.0130	0.0000	0.8501	1.274	0.1527	0.0292	0.8181	1.149	0.021
0.0200	0.1545	0.0102	0.0000	0.8353	1.078	0.1699	0.0252	0.8050	1.004	0.061
0.0300	0.1765	0.0095	0.1739	0.6402	0.890	0.1932	0.0197	0.7871	0.861	0.151
0.0400	0.2177	0.0086	0.5333	0.2404	0.722	0.2270	0.0117	0.7613	0.720	0.375
0.0500	0.2792	0.0002	0.7207	0.0000	0.583	0.2792	0.0002	0.7207	0.583	—
0.0600	0.3523	0.0000	0.6477	0.0000	0.454	0.3523	0.0000	0.6477	0.454	—
0.0700	0.5193	0.0000	0.4807	0.0000	0.341	0.5193	0.0000	0.4807	0.341	—
0.0456[a]	0.2533	0.0055	0.7412	0.0000	0.643	0.2533	0.0055	0.7412	0.643	—

Note: Optimal portfolios are obtained with short-sale disallowed and based on risk-return parameters estimated over the sample period of 1980–2015. The p-values of Glen and Jorion (GJ) tests on equal Sharpe ratios are reported in the last column of the table.

[a]The portfolio results presented in the last row are obtained by assuming a risk-free rate equals to the average three-month T-bill yield of 4.56% over the sample period of 1980–2015.

the five-year MTM ladder completely replaces the bond index at risk-free rates up to two percent. Thus, the five-year MTM ladder is still very attractive to investors with high risk aversion. As in the case of the three-year ladder, the MTM reduces the percentage allocation to the total fixed income holding and also tilts the composition of the fixed income asset away from the less attractive ladder toward bond index for the moderately risk averse investors. In the only two cases in Table 9 where the two fixed income assets coexist at the risk-free rates of three percent and four percent, the p-values indicate that the coexistence of the two fixed income components adds no statistically significant diversification benefit. The results in Table 9 favor the five-year MTM ladder for the risk averse investors and the bond index otherwise.[17]

The results in Table 10 provide the case of the seven-year MTM ladder in which the effect of the MTM is strongly felt. At first brush, the seven-year MTM ladder appears to be very dominant as it completely or almost completely displaces the bond index at risk-free rates up to three percent. Also recall from the summary statistics reported in Table 7 that the seven-year MTM ladder has the best Sharpe ratio among the three MTM ladder strategies. It therefore manages to make a presence, albeit small, in the optimal portfolio at the *ex post* risk-free rate in Table 10, as opposed to the cases for the three-year and five-year MTM ladders that are completely absence at the same *ex post* rate. Unfortunately, the p-values indicate the presence of the seven-year MTM ladder adds no statistically significant benefit except at the zero risk-free rate. Its dominance at one-percent risk-free rate is weakly statistically significant at the 10% level. For most part, the ladder is not a potent instrument except to the most risk averse investors. As in the cases of the three-year and five-year ladder, the MTM reduces the percentage allocation to the total fixed income holding and also tilts the composition of the fixed income asset away from the less attractive ladder toward bond index for the moderately risk averse investors. Among the cases of risk-free rates being considered, the two fixed income assets coexist only at risk-free rates of three percent and

Table 10 Diversification benefits of seven-year MTM Ladder: Portfolio weights and Sharpe ratios of tangency portfolios with and without Ladder are reported at different levels of annual risk-free interest rate

Risk-free rate	Portfolios with seven-year MTM ladder					Portfolios without seven-year MTM ladder				GJ p-value
	Portfolio weights				Sharpe ratio	Portfolio weights			Sharpe ratio	
	CRSP VW	MSCI-EAFE	Barclays bond index	Seven-year MTM ladder		CRSP VW	MSCI-EAFE	Barclays bond index		
0.0000	0.1555	0.0156	0.0000	0.8289	1.389	0.1397	0.0323	0.8281	1.295	0.039
0.0100	0.1680	0.0133	0.0000	0.8187	1.217	0.1527	0.0292	0.8181	1.149	0.064
0.0200	0.1850	0.0101	0.0000	0.8049	1.047	0.1699	0.0252	0.8050	1.004	0.097
0.0300	0.2039	0.0079	0.1187	0.6696	0.880	0.1932	0.0197	0.7871	0.861	0.192
0.0400	0.2299	0.0063	0.4250	0.3388	0.723	0.2270	0.0117	0.7613	0.720	0.351
0.0500	0.2792	0.0002	0.7207	0.0000	0.583	0.2792	0.0002	0.7207	0.583	—
0.0600	0.3523	0.0000	0.6477	0.0000	0.454	0.3523	0.0000	0.6477	0.454	—
0.0700	0.5193	0.0000	0.4807	0.0000	0.341	0.5193	0.0000	0.4807	0.341	—
0.0456[a]	0.2534	0.0050	0.7026	0.0390	0.643	0.2533	0.0055	0.7412	0.643	0.473

Note: Optimal portfolios are obtained with short-sale disallowed and based on risk-return parameters estimated over the sample period of 1980–2015. The *p*-values of Glen and Jorion (GJ) tests on equal Sharpe ratios are reported in the last column of the table.

[a]The portfolio results presented in the last row are obtained by assuming a risk-free rate equals to the average three-month T-bill yield of 4.56% over the sample period of 1980–2015.

four percent. However, the *p*-values indicate that the coexistence of the two fixed income components adds no statistically significant diversification benefit. The results in Table 10 also favor the seven-year MTM ladder for the risk averse investors and the bond index otherwise.

What can we say about the implication of the MTM effect across ladders of different terms? First, note that none of the shorter term MTM ladders enters the optimal portfolio at the *ex post* risk-free rate. Moreover, the weights of MTM ladders drop quite dramatically at higher risk-free rates including the *ex post* risk-free rate comparing to the ladders without MTM. The weights at the *ex post* risk-free rate for the three-year, five-year, and seven-year ladders without MTM are 23.32%, 68.83%, and 87.33%, respectively. The MTM effect reduces the weights of first two ladders to zero and the seven-year ladder to a mere 3.9%. There seems to be a relationship between the magnitude of the impact caused by the MTM and the term of the ladder with a greater reduction in the weight of the longer term ladder. To understand the above effect of MTM on the allocation of money to ladders, one has to see the allocation behavior with MTM and then the allocation behavior without MTM. First, in the case of allocations with MTM, notice that the correlation between the MTM ladders and the bond index rises as the term increases. The correlation coefficient increases monotonically with the term ranging from 0.785 for the three-year MTM ladder to 0.937 for the seven-year MTM ladder according to Table 7, whereas the correlations between the ladders without MTM and the bond index are no higher than 0.6. The seven-year MTM ladder is an almost perfect substitute for the bond index. The high correlations reduce any potential diversification benefit of holding both fixed income assets simultaneously and invalidate the rationale for both fixed income assets to coexist. The decision to own predominantly only one fixed income asset depends on the risk preference of the individual investor. At higher risk-free rates, the bond index offers the investor with higher risk tolerance a better return and thus dominates the longer-term ladders with MTM. This explains in part the drop in the weight of the seven-year ladder from 87.33% to a mere 3.9% when MTM is considered. This also explains the earlier result that the seven-year MTM ladder with a lower risk and lower return profile is preferred at lower risk-free rates. The net outcome is to hold either the bond index or the seven-year ladder depending on the levels of the risk-free rates. The second reason for the larger drop in the allocations for the longer term ladders in the presence of MTM can be explained by examining the allocation behavior without the MTM. The longer term ladders without MTM not only have lower risk than the shorter term ladders without MTM because of the lower reinvestment risk mentioned earlier, they also have slightly lower correlations with the bond index. These attractive risk and correlation characteristics translate into a greater diversification benefit as the term increases. In fact, one can argue that it is the lack of MTM consideration that makes ladders unusually attractive. This is especially true in the case of the seven-year ladder without the MTM. The seven-year ladder without MTM has the lowest standard deviation among all fixed income assets at 2.78% and at the same time the lowest correlation with the bond index. The above attractive risk-return characteristics brought about by the absence of the MTM consideration make the seven-year ladder the most dominant asset at 87.33% in the optimal portfolio at the *ex post* risk-free rate. This unusual advantage disappears with MTM and hence the dramatic reduction in portfolio weight.

The above results of higher correlations between longer term MTM ladders and the bond index also lead to another interesting result. Longer term MTM ladders do not statistically add significant diversification benefit to the benchmark portfolio with the bond index. None of the *p*-values supports the coexistence of a seven-year ladder or a five-year ladder with the bond index whenever MTM is introduced. In the absence of MTM, the *p*-values support far more incidences of statistically significant improvement in the Shape ratios when the ladders and the bond index coexist for investors with moderate risk aversion.

The above insights can only be gained through our use of tests of statistical significance. It is our statistical tests that pinpoint the effect of MTM of longer term ladders and its investment implications.

The previous conclusion that ladder portfolios are more attractive to more risk averse investors still holds when the MTM is introduced. The MTM also increases the correlations and the standard deviations of longer term ladders and reduces its attractiveness in a portfolio context. These two factors lead to binary outcomes for longer term ladders in which an investor will hold either the bond index or the longer term ladders depending on the investor's risk aversion but not both fixed income assets at the same time.

6. Conclusion

This article attempts to bridge a major gap in personal finance by addressing a popular approach to bond investing that is rarely mentioned in the academic literature. Investors are confronted with a choice between two similar fixed-income investment instruments that are passive in nature with no way to choose either one or both together. Further, there is absolutely no objective guidance as to the proper term of the ladder strategy in the existing literature.

This article offers insights into the above missing pieces. First, by allowing the risk aversion behavior to change, we show that the two instruments are suitable for investors with different risk preference. Because of their risk reduction ability, ladders are particularly suitable for conservative investors with somewhat higher degrees of risk aversion. Second, longer term ladders appear to offer investors a better risk-return tradeoff than shorter term ladders. This is especially true for investors who believe that marking to market is not necessary as all bonds are held till maturity. Third, marking to the market in general raises the correlations between ladders and the bond index, and increases the risk of ladders as the term increases. This dampens the usefulness of longer term ladders except for the most conservative investors. The higher correlations also make the coexistence of longer term ladders and the bond index less likely. Fourth, ladders and bond index may coexist within an optimal portfolio to improve the risk-adjusted return especially for investors with moderate risk aversion in the world without MTM. In the case of MTM, the coexistence of both fixed income assets is likely only for shorter term ladders.

Marking to the market has a profound effect on the composition of the fixed-income assets in an optimal portfolio. First, it makes fixed-income assets as a group less attractive as MTM heightens the correlation structure of fixed-income assets. Second, it affects the decision to go with a long term ladder or a short term ladder. If an investor believes MTM is unimportant because the bonds are held till maturity, the longest term ladder offers the best outcome. Finally, it reduces the likelihood that the two fixed-income assets will coexist in a portfolio as longer term ladders act like the bond index.

Notes

1 See, for example, the coverage of the strategy in Hirt, Block, and Basu (2008, Chapter 12), Bernstein (2003), Bohlin and Strickland (2004), and Scatizzi (2009).

2 Fabozzi (2007) mentions the ladder strategy in the context of active yield curve strategies for investment professionals. His focus there is on comparing the return performance of a barbell strategy versus a bullet strategy when the yield curve shifts, whereas retail investors/advisors use laddered portfolios as a passive strategy. Laddered portfolios as examined in the academic literature are usually in conjunction with barbells and bullets. Again, as in the case of Fabozzi, the emphasis is on the risk-return characteristics of various active bond strategies as interest rates changes.

3 Fabozzi (1995) is the refreshing exception in which laddering is mentioned. Again, its mention is in the context of interest rate risk and the comparison is against active barbell and bullet strategies.

4 See, for example, Ingersoll (1987, Chapter 4), for the derivation.

5 Some may construct a ladder by using bonds maturing every six months or every three months. In the case of a five-year ladder using bonds maturing every six months, the ladder will consist of the six-month bond, one-year bond, 1 1/2-year bond, . . . , and the five-year bond. The six-month bond maturing six month later will be reinvested in a new five-year bond. The initial one-year bond maturing one year later will be reinvested in a five-year bond to maintain the same maturity structure. A new five-year

bond will be bought every 6 months in this case. The return on this ladder portfolio is again simply an equally weighted average of past and current five-year bond returns. While ladders can be built with different "steps" such as six months or three months, our statement that the portfolio return is simply an equally weighted average of past and current five-year bond returns remains true. We use bonds maturing every year in the construction of ladder portfolios in this article.

6 The annual yield data of bonds of other maturities are also obtained to address the "mark-to-the-market" issue as explained later.

7 It would appear that we have ignored the reinvestment of coupons at returns different form the initial yield. This is not an issue if the investor uses certificates of deposit (CD) with compounding of interest at the initial rate. For investors who use treasury notes with regular coupons, the reinvestment returns on the intermediate coupons may be different from the initial yield. We will consider the scenario that the reinvestment returns happen to be the prevailing T-bills rates at the time the coupons are received later in the article. See Note 17.

8 For a description of the origin of the Barclays U.S. Aggregate Bond Index, see Cui (2013).

9 Both Vanguard and BlackRock offer exchange traded funds (ETFs) replicating the index. The iShares Core US Aggregate Bond ETF offered by BlackRock is the largest bond ETF among its offerings at about $35 billion of assets under management as of March 31, 2016. It has a management expense ratio of 0.08 percent.

10 We do not expect the bond index to be subject to much credit risk. As of December 31, 2015, about 73 percent of the Barclays U.S. Aggregate Bond Index consists of AAA bonds.

11 While the choice of five-year ladder is somewhat arbitrary, it is not highly unusual. US treasury notes, which are ideal for the construction of ladders, are available in terms of 2, 3, 5, 7, and 10 years. Most banks offer CDs with maturities up to 10 years. The five-year ladder happens to be the medium term of available treasury notes or CDs.

12 Elton, Gruber, Brown, and Goetzmann (2006, Chapter 6) suggest the variation of the risk-free rate as a way to trace out the frontier.

13 See Jobson and Korkie (1989) for a discussion of the spanning tests.

14 The bond index requires portfolio re-balancing to maintain its term structure and hence the use of market prices for their bond holding can be justified.

15 Unfortunately, yield information for four-year and six-year bonds are unavailable from the Federal Reserves website. We impute the four-year (six-year) yield by interpolating the yields between the three-year (five-year) and five-year (seven-year) bonds.

16 See Appendix B for a detailed numerical illustration of the procedure.

17 Instead of using zero coupon bonds in constructing the marked-to-the-market portfolio, we repeated the exercise for the five-year ladder using coupon bonds issued at par and the coupons are invested at the prevailing risk-free rates until maturity. The bonds are marked to the market at the end of each year. The return on this five-year ladder over the same sample period is slightly lower than those reported in Tables 7 and 9. The conclusion that the ladder and the bond index do not coexist remains unaffected. To conserve space, the detailed results are not produced here and available on request.

Appendix A: Statistical test for the difference in portfolio Sharpe ratios

Let us define T as the number of monthly observations. Also, let Θ_1 be the Sharpe ratio of the tangency portfolio based on the three benchmark asset classes and Θ_2 be the Sharpe ratio after the addition of an addition asset class. The null hypothesis to be tested is H_0: $\Theta_1 = \Theta_2$. When short selling is allowed, the test statistic presented in Gibbons, Ross, and Shanken (1989) and Jobson and Korkie (1989) is:

$$F = (T - 4) \times \frac{\Theta_2^2 - \Theta_1^2}{1 + \Theta_1^2}, \tag{3}$$

where F follows a F-distribution with 1 and T-4 degrees of freedom.

In this study, we consider the case where short selling is disallowed. Under this condition, the F statistic of Eq. (3) will not be following the F-distribution and needs to be simulated before hypothesis tests can be conducted. We conduct the simulations by following the method proposed by Glen and Jorion (1993). We first estimate the means, variances, and the covariances using historical return data. The expected return of the additional asset class (i.e., a ladder strategy in this study) is then modified so that the original tangency portfolio is still mean-variance efficient after this additional asset class is included. This in effect ensures the null hypothesis is satisfied in the subsequent simulation exercise. With these modified parameters, T random samples of joint returns are drawn from a multivariate normal distribution. Based on these simulated returns, a new set of means and variance-covariance matrix are estimated. Sharpe ratios of the tangency portfolios with and without the additional asset class can then be estimated. Finally, the value of the test statistic of Eq. (3) is computed and recorded. The empirical distribution of the statistic is generated under the null hypothesis by repeating this process 1,000 times.

Appendix B: A numerical example of calculating the annual MTM ladder return

Suppose we start implementing a three-year ladder strategy at the end of 1979. By assuming the observed one-year, two-year, and three-year bond yields of 11.98%, 11.39%, and 10.71% as zero yields, the prevailing prices of the one-year, two-year, and three-year zero-coupon bonds with face value of \$100 are \$89.30, \$80.59, and \$73.70, respectively. Consider a notional starting investment value of \$100 for our three-year ladder portfolio. It is equally split among the three bonds (i.e., we are investing \$33.33 in each of the three bonds). After one year (i.e., at the end of 1980), the first bond matures and its price is \$100. The second bond now has a remaining time to maturity of one year; whereas the third bond will mature in two years. Suppose the one-year and two-year bond yields are now 14.88% and 14.08%. Based on these yields, the prevailing market prices of the second and third bonds become \$87.05 and \$76.84. The annual MTM return of the ladder portfolio in 1980 is, therefore, the weighted average return of the three bonds over that year, i.e.,

$$\frac{\$33.33}{\$100}\times\frac{(\$100.00-\$89.30)}{\$89.30}+\frac{\$33.33}{\$100}\times\frac{(\$87.05-\$80.59)}{\$80.59}+\frac{\$33.33}{\$100}\times\frac{(\$76.84-\$73.70)}{\$73.70}=8.09\%$$

At the end of 1980, the market values of the positions on the three bonds are respectively \$37.33 (= \$33.33 × 100/89.30), \$36.01 (= \$33.33 × 87.05/80.59), and \$34.75 $\left(=\$33.33 \times \frac{76.84}{73.70}\right)$. The overall value of the ladder portfolio is, therefore, \$108.09. As the first bond has matured, the \$37.33 is reinvested in a newly issued three-year bond. With a three-year bond yield of 13.65%, a newly issued three-year zero-coupon bond should be selling at \$68.12. Now, the ladder portfolio consists of a three-year bond (the first bond), a one-year bond (the second bond), and a two-year bond (the third bond).

After another year (i.e., at the end of 1981), the second bond matures and its value is \$100. The third bond now has a remaining time to maturity of one year; whereas the first bond will mature in two years. Suppose the one-year and two-year bond yields are now 12.85% and 13.29%. Based on these yields, the prevailing market prices of the first and third bonds become \$77.91 and \$88.61. The annual MTM return of the ladder portfolio in 1981 is, therefore, the weighted average return of the three bonds over that year, i.e.,

$$\frac{\$37.33}{\$108.09}\times\frac{(\$77.91-\$68.12)}{\$68.12}+\frac{\$36.01}{\$108.09}\times\frac{(\$100.00-\$87.05)}{\$87.05}+\frac{\$34.75}{\$108.09}\times\frac{(\$88.61-\$76.84)}{\$76.84}=14.84\%$$

At the end of 1981, the market values of the positions on the three bonds are respectively \$42.69 (= \$37.33 × 77.91/68.12), \$41.37 (= \$36.01 × 100.00/87.05), and \$40.07 $\left(=\$34.75 \times \frac{88.61}{76.84}\right)$. As the second bond has matured, the \$41.37 is reinvested in a newly issued three-year bond. Now, the ladder portfolio consists of a two-year bond (the first bond), a three-year bond (the second bond), and a 1-year bond (the third bond).

The above calculations are repeated for each of the subsequent years of the sample period to obtain the time-series of annual MTM return of the three-year ladder.

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