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Portfolio insurance using leveraged ETFs

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Abstract

This study examines the use of Leveraged Exchange Traded Funds (LETFs) within a constant proportional portfolio insurance (CPPI) strategy. The advantage of using LETFs in such a strategy is that it allows a greater percentage of the portfolio to be invested in the risk-free rate relative to a traditional CPPI. Where a standard CPPI strategy may require 50% of the portfolio to be invested in equities, using a 2x LETF only requires 25%, and a 3x LETF only requires 16.7% to attain the same effective exposure to equities. Results show when the risk-free asset is yielding at least 3% or the 1 year minus 90-day Treasury exceeds 1%, the use of LETFs within a CPPI framework results in annual returns approximately 1–2% higher with better Sharpe, Sortino, Omega, and Cumulative Prospect Values while reducing Value at Risk (VaR) and Excess Shortfall (ES) below VaR.

JEL classification: G11; G17

Keywords: Portfolio insurance; Leveraged Exchange Traded Funds

1. Introduction

With two major market crashes in the last 17 years, portfolio insurance, or the protection of downside risk, has become increasingly important as “once in a century” events are occurring multiple times instead. The two main types of portfolio insurance are option based (Leland and Rubinstein, 1976) and constant proportionate portfolio insurance (CPPI) strategies set forth by Black and Jones (1987). Most option based ideas are premised on purchasing or creating synthetic puts on an index, effectively a protective put. The cost of

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this protection is usually quite high and although it reduces downside exposure, gains tend to be moderated significantly. CPPI strategies are based on a portfolio floor value where a percentage of the portfolio is invested in the risky asset and the remainder in a risk-free asset. If the risky asset value declines, the percentage in the risky asset is reduced. This exposure can decrease to zero if a decline in the risky asset causes the portfolio value to reach the floor, where the floor is defined as the minimum value that the portfolio can fall to over an investment period.

Research on portfolio insurance strategies is extensive with most research coming down on the side of CPPI. Cesari and Cremonini (2003) test different dynamic strategies including CPPI and option based portfolio insurance among buy-and-hold and constant mix strategies. They find CPPI strategies are dominant in bear and no-trend markets and considered more beneficial overall. Zieling, Mahayni, and Balder (2014) review an extensive research list and show that using a time varying multiple to dictate the amount of market exposure improves CPPI results. Pezier and Scheller (2014) show CPPI strategies are superior to option based strategies when implemented in discrete time. Annaert, Osselaer, and Verstraete (2009) compare strategies under a stochastic dominance approach and generally find one strategy does not outperform another, including buy-and-hold when considering first, second, and third order stochastic dominance. However, Maalej and Prigent (2016) finds CPPI outperforms option based portfolio insurance based on third order stochastic dominance. Bertrand and Prigent (2011) analyze option based portfolio insurance and CPPI strategies using downside risk measures and performance measures that consider the nonnormality of returns, otherwise known as Kappa performance measures. They find the CPPI method outperforms option based portfolio insurance using the Omega measure.

Kahneman and Tversky's (1979) Prospect Theory, which assumes investors weigh losses more than gains, is particularly relevant in terms of portfolio insurance for mitigating downside risk. Tversky and Kahneman (1992) expand this idea with the introduction of cumulative prospect theory. Dichtl and Drobetz (2011) use this idea to show portfolio insurance strategies are superior to buy-and-hold based on higher cumulative prospect values (CPVs).

This study attempts to combine aspects of both CPPI and option based portfolio insurance. Specifically, by incorporating Leveraged Exchange Traded Funds (LETFs) within a CPPI strategy, this study shows CPPI results can be improved. In a typical CPPI strategy where 50% of the portfolio is invested in equities, the use of a 2x LETF only requires 25% to be invested in equities while attaining the same effective equity exposure. Thus, rather than having only 50% earning the risk-free rate, 75% of the portfolio is earning the risk-free rate. Using a 3x LETF only requires 16.67% in the risky asset to attain the same 50% exposure.

The drawbacks of using LETFs are their higher expense ratios, inherent financing costs through their use of derivatives, and return decay over time relative to what the daily leverage ratio might imply. For instance, a 2x LETF usually falls short of multiplying the index return by two over longer holding periods. However, with more of the portfolio earning the risk-free rate relative to a standard CPPI using the underlying index, the gains on average exceed the costs. This study finds that if the risk-free rate is at least 3% or the one year minus the 90-day Treasury exceeds 1%, the simple substitution of LETFs for the underlying index within a CPPI strategy results in higher returns with better Sharpe, Sortino, Omega, and CPV values while reducing Value at Risk (VaR) and Excess Shortfall (ES) below VaR. Treasury rates less than 3% or flat yield curves generally eliminate the value of using LETFs in a CPPI framework.

However, even under low interest rate conditions, using LETFs within a CPPI framework does not result in extreme underperformance (less than 0.4% annually) relative to a standard CPPI. The drawback is with low T-bill rates, the advantage of having a LETF effectively "borrow" short so the investor can earn a higher return with excess funds over the year is eliminated. This drawback can be overcome to the extent an investor is willing to take on more risk by investing excess funds in higher performing but riskier assets such as a diversified bond fund.

2. Review of LETFs

Although leveraged mutual funds have been around since at least 1993, they did not gain traction until ProShares introduced the first 2x LETF in 2006. Since that time, they have expanded dramatically and as of 2017, there are more than 170 LETFs with \$40+ billion in assets on a variety of assets and indexes including, gold, oil, foreign currencies, Treasury-bond futures, and a myriad of equity indexes.

In general, most LETFs magnify the daily return of an underlying index up to ±3.0x, although there are few funds that magnify the monthly return. Recently, several funds have been proposed to deliver ±4.0x, (Hunnicutt and McCrank, 2017). Strictly speaking, the proposed 4x funds will magnify index futures, but the effect will generally be the same because the cost of carry is mitigated because of the underlying dividends paid.

LETFs attain their leverage by using derivative assets such as futures and swaps. It should be noted that although the swaps are based on the underlying daily index returns and Libor, there is counterparty risk. Thus, a large gain to the LETF could theoretically not be paid by the counterparty. Although an unlikely scenario, not an impossible one.

The primary drawback to LETFs is realized leverage over time is usually less than what the daily multiple might imply (Trainor and Baryla, 2008). Thus, while the realized return over time often falls short of the daily leverage ratio, the risk does not. To enumerate, assume an underlying index falls 5% on Day 1, and increases 10% on Day 2 for a 2-day return of 4.5%. A 3.0x LETF would lose 15%, then gain 30% for a 2-day return of 10.5% resulting in an effective leverage ratio of 2.3 (10.5%/4.5%) instead of 3.0. This is often referred to as leverage decay and is a function of time, leverage, return trend, and volatility, with volatility usually being the significant determinant.

Realized leverage can mathematically be expressed by Eq. (1):

$$\frac{LR_T}{XR_T} = \beta + \frac{T\frac{(\beta^2 - \beta)}{2} \times [\mu_r^2(T-1) - \sigma_r^2]}{XR_T} \tag{1}$$

where LR_T is the return to the leveraged fund, XR_T is the underlying index return, β is the daily leverage ratio, T is time in days, μ_r is the mean daily return, and σ_r^2 is the standard daily population variance (Avellaneda and Zhang, 2010; Cheng and Madhaven, 2009; Trainor and Caroll, 2013). When $[\mu_r^2(T - 1) - \sigma_r^2]$ is negative, the realized leverage over time will be less than the daily leverage ratio β. This effect is greater with higher leverage since a daily leverage ratio of 2x multiplies this term by one, $[(2^2 - 2)/2]$, but a daily leverage ratio of 3x multiplies this term by three. Lu, Wang, and Zhang (2012) generalized this leverage decay and suggest over holding periods no greater than one month, an investor can assume that a 2x/-2x LETF will maintain its leverage ratio and provide the expected return applied to the underlying index.

Most research generally concludes LETFs should be used for short-term trading strategies only, and the providers market them as such. However, if the trend μ is high enough, Trainor (2011) shows an investor can end up with a great deal more than the daily leverage ratio might indicate. A perfect example of this is ProShare's 3.0x UltraPro (UPRO) fund that magnifies the daily return of the S&P 500. Since the fund was introduced in June of 2009, the S&P 500 increased 183% through December 2016, while the 3.0x fund increased 1,045% for an effective ratio of 5.7x. This occurred over a period with high return trend and lower than average volatility.

Within a portfolio setting, DiLellio, Hesse, and Stanley (2014) suggest there may be a place for long-term holdings of LETFs as their results show LETFs may reduce a portfolio's standard deviation. From an option based portfolio insurance strategy, Trainor and Gregory (2016) show the results of using covered calls and protective puts with LETFs. Both strategies reduce risk, but significant returns are often sacrificed. Scott and Watsun (2013) suggest a floor-leverage rule where an investor places 85% of wealth in a risk-free asset and 15% in a 3x LETF. With annual rebalancing, they find this strategy can be used to manage risk and appears to be optimal for sustainable investment in retirement.

This study considers the use of LETFs in a CPPI context. The logic for using LETFs within a CPPI setting is straight forward. In a standard CPPI portfolio, the investor may start out with 50% in the risk-free rate and 50% in the risky asset. With a 2x LETF, the investor only needs to invest 25% in the LETF leaving 75% to earn the risk-free rate. If a 4.0x LETF becomes available, only 12.5% is needed. If the benefits of the additional return from having a greater percentage of the portfolio in the risk-free asset exceeds LETF's decay and higher expenses, then a CPPI using LETFs will outperform a standard CPPI strategy using the underlying index. This study determines this is indeed the case when the risk-free asset yields at least 3% or the one year minus 90-day Treasury exceeds 1%.

3. Methodology

This study explores the benefits of LETFs within a CPPI format. The S&P 500 is used as the underlying index and the 1-year treasury is set as the risk-free asset. Four different CPPIs labeled CPPI S&P, CPPI 2x, CPPI 3x, and CPPI 4x are compared using a 90% floor. Although there currently are no 4x funds, both ForceShares and ProShares have proposed them. The 90% floor is used to account for a typical 50/50 stock/bond portfolio where an investor wants to limit a stock loss to approximately 20% over any given year. Assuming no change in the value of a bond fund, a 20% loss in equities would hit a 90% floor value.

Table 1 Initial positions for CPPI strategies

CPPI	Floor (F)	Cushion	Multiplier (m)	% in risky asset	% in risk-free
CPPI S&P	$90,000	$10,000	5	$50,000	$50,000
CPPI 2x	$90,000	$10,000	2.5	$25,000	$75,000
CPPI 3x	$90,000	$10,000	1.67	$16,667	$83,333
CPPI 4x	$90,000	$10,000	1.25	$12,500	$87,500

Constant proportional portfolio insurance (CPPI) positions are initially set based on a 90% floor and are rebalanced with every 2.5% move in the S&P 500. Floor values are reset each year based on the portfolio value at the end of the previous year.

The proportion in the risky asset for the CPPI S&P at time t is calculated as the $\max\{\min[(m(V_P - F),V_P],0\}/V_P$ where m is the multiplier, V_P is the value of the portfolio, and F is the floor. The index multiplier is set at 5 which implies an initial 50% investment in the risky asset. Following Cesari and Cremonini (2003), the portfolio is rebalanced only when the underlying risky-asset (the S&P 500 in this study) increases or decreases by 2.5% since the last rebalance. Rebalancing once a week or once a month is also tested. For the latter, even if the market sheds 20% over the month, the floor would not be breached. This seems reasonable even for very risk averse individuals since a 20% loss in a single month has only occurred once post-WWII in October 1987 (−21%). The drawback of monthly rebalancing for LETFs is the leverage decay that can be experienced over a month. With weekly or 2.5% price limits, this is less likely to be an issue.

The benefit of using LETFs is 50% exposure can be attained with a smaller equity position. In the case of a 2x, the multiplier only needs to be 2.5 to attain the same 50% exposure. For a 3x, the multiplier only needs to be 1.67, and for the 4x, 1.25. Because it is assumed the investment in the risky asset is capped at 100%, the LETFs maximum exposure must be additionally constrained. The proportion in the risky asset for CPPI 2x at time t is calculated as the $\min\{\max(\min[(m(V_P - F),V_P],0)/V_P,0.5\}$ while the proportion in the risky asset for CPPI 3x at time t is $\min\{\max(\min[(m(V_P - F),V_P],0)/V_P,0.33\}$. For a 4x, the maximum exposure is 25%. The four initial positions using a portfolio value (V_P) of $100,000 for exposition are shown in Table 1 below.

While Table 1 shows the initial positions, Table 2 demonstrates how the CPPIs are adjusted after each rebalance using the S&P 500 SPY ETF and Proshare's 3x S&P 500

Table 2 Percentage changes in the CPPI strategies

2016	SPY Ret	% in S&P	CPPI S&P V_P	3x UPRO Ret	% in 3x	CPPI 3x V_P
1/4-1/7	−4.82%	50.00%	$0.98	−14.12%	16.67%	$0.98
1/7-1/13	−2.69%	39.00%	$0.97	−8.16%	13.12%	$0.97
1/13-1/29	2.59%	34.26%	$0.98	7.29%	11.52%	$0.98
1/29-2/5	−2.98%	38.75%	$0.96	−9.15%	12.99%	$0.96
2/5-2/11	−2.71%	33.91%	$0.96	−7.71%	11.16%	$0.96

Results show the returns of the underlying risky asset designated as the SPY ETF, the allocation to the risky asset (% in S&P, % in 3x), and the portfolio value with a start value of $1 for two of the constant proportional portfolio insurance CPPI strategies (CPPI S&P Vp, CPPI 3x Vp) from 1/4/16 to 2/11/16. UPRO is Proshare's 3x S&P LETF.

(Ticker UPRO) as an example. With each 2.5% change in the underlying index, the exposure is adjusted. The floor is rebalanced annually to 90% of the value of the portfolio at the end of the year. This implies an investor could be 100% in equities if the returns are positive enough. Because the floor is only reset annually, more than a 10% loss could occur within the year, but not for the entire year assuming there are no historic losses in any given day. The floor could be breached if equities declined by at least 20% in any given day before the portfolio could be rebalanced. Even this loss is likely not enough to breach the floor as the value of the bond portion of the portfolio would increase dramatically in such a scenario. In addition, even if a 100% equity position was held because of the increase in the portfolio value over the year, a 20% daily decline in equities would still be required to breach the 90% floor.

As an example of how the rebalancing is implemented, Table 2 shows the SPY falls −4.8% in the first 3 days of 2016. This breaches the 2.5% limit and the portfolios are rebalanced. This results in a reduction in the risky asset from 50% of the portfolio to 39% for the CPPI S&P. A similar type of reduction is made in the CPPI 3x portfolio as UPRO's 3x return is −14.12%. Alternatively, from 1/13/16 to 1/29/16, the market increases 2.6% leading to a percentage increase in the risky asset. Rebalancing occurs every time the index changes by 2.5% or more since the previous rebalancing. On average, 23 trades a year are required using a 2.5% barrier.

In January of the following year, the percentages are reset to their original values with a 90% floor based on the value of the portfolio at the end of December. It should be noted the results in this study do not explicitly account for brokerage costs or for bid-ask spreads, although the latter are usually a few cents a share at most. Because all the CPPI strategies require equal number of transactions, the relative results between the CPPI strategies are not biased, but depending on the size of the portfolio, may overstate results relative to buy-and-hold. For a $100,000 portfolio, brokerage costs would be approximately 0.2% annually.

Because the two major LETFs on the S&P 500 were not introduced until 2006 and 2009, respectively, theoretical LETF returns are calculated to attain a clearer picture of the risk/return characteristics from the CPPI strategies. The Center for Research in Security Price's (CRSP) S&P 500 value weighted portfolio is used as the risky asset and 2x, 3x, and 4x returns are calculated assuming a 1.2% annual expense ratio which is approximately 0.2 percentage points higher than the expense ratio for S&P 500 LETFs. The reason for the higher expense ratio is explained below.

Following Scott and Watsun (2013), the daily returns for the 2x, 3x, and 4x LETFs are calculated as:

$$R_L = L \times R_{S\&P} - R_{exp} - (L\text{-}1) \times R_f \quad (2)$$

where R_L is the daily return to the LETF with a daily leverage ratio of L, $R_{S\&P}$ is the daily return of the S&P, R_{exp} is the daily expense ratio, and R_f is the borrowing rate using the 90-day T-bill rate as a proxy. Strictly speaking, the one-week/month Libor rate should be used, but Libor data begins in 1986 and to remain consistent with sampled returns before this date, the 90-day T-bill rate is used. The 90-day T-bill has a 98% correlation with Libor and averages 0.2% less than Libor.

The logic behind Eq. (2) is a 2x has increased exposure by borrowing \$1 for every \$1 invested. A 3x borrows \$2 for every \$1 invested. Since LETFs primarily attain their exposure using swaps, there are imbedded financing costs increasing with leverage (Charupat and Miu, 2014). In addition, there are additional transactions costs not reflected in LETFs expense ratio that are generally higher for these funds because of the use of derivative contracts.

To test this pricing equation, theoretical daily, monthly, and annual returns for a 2x and 3x are compared with the actual daily, monthly, and annual returns of ProShare's 2x SSO and 3x UPRO on the S&P 500, and their 2x SQD and 3x TQQQ on the Nasdaq 100. To create near equivalence for daily, monthly, and annual returns between the LETFs and the simulated returns, the annual expense ratio is increased from 1.0 to 1.2% which coincides with the exact amount of the average difference between the 90-day T-bill and Libor. This results in average daily and monthly differences at or close to zero and average annual differences less than 0.1%. As an example of the differences, for 2016, the 2x SSO return is 21.5% while the theoretical return was 21.6%. For the Nasdaq 2x, both the 2x SQD and simulated 2x had returns of exactly 10.2%.

Based on the reliability of the results above, LETF returns are calculated from Jan. 1947 to Dec. 2016 using daily data based on Eq. (2). In addition to these results, empirical data using ProShare's 2x SSO is presented for 2007–2016. Finally, to determine the robustness of the results, block bootstrapping is used to resample 252 daily return windows to create 10,000 unique annual returns. The 252-day blocks are used to keep the continuity of the interest rate environments associated with the stock returns during that 252-day period, although 5, 22, and 63-day blocks are also examined with no substantial change in the average results. Because the floor is reset each year, this covers thousands of historically relevant 252-day periods. The drawback of using shorter blocks is that the interest rates experienced both in terms of financing and the risk-free asset used for investing excess funds tends to average out, and does not show what may occur in sustained extreme interest rate environments. Thus, sampling shorter blocks would bias the results.

Results are analyzed using a variety of measures. These include the average return, minimum return, maximum return, standard deviation, Sharpe ratio, Sortino ratio, VaR, ES, Omega ratio, and cumulative prospect values. The risk metrics are explained in Appendix 1.

4. Results

4.1. Historical results

Table 3 shows the annual performance and risk measures from 1947 to 2016 for the CPPI S&P, CPPI 2x, CPPI 3x and CPPI 4x. The multipliers of 5, 2.5, 1.67, and 1.25, respectively, determine the exposure to the risky asset that is rebalanced with a 2.5% or greater move in the S&P 500 relative to the last rebalance. The remaining allocation is invested in one-year T-bills. The floor is set to 90% of the portfolio value and is reset annually. Return data are provided for the respective risky assets with a 1.2% annual expense ratio and daily financing costs assumed for the LETFs based on Eq. (2).

Table 3 Annual returns from 1947 to 2016 for the underlying indexes and CPPI portfolios

	S&P	S&P 2x	S&P 3x	S&P 4x	CPPI S&P	CPPI 2x	CPPI 3x	CPPI 4x
Average	12.53%	21.91%	33.97%[b]	47.55%[b]	10.13%	10.60%	11.15%	11.48%
Standard deviation	17.02%	36.11%	58.89%	87.00%	11.79%	12.23%	12.58%	12.78%
Median	13.81%	24.51%	33.07%	42.43%	8.18%	8.38%	8.87%	9.24%
Minimum	−36.65%	−66.82%	−85.24%	−94.55%	−8.65%	−8.64%	−8.63%	−8.64%
Maximum	52.85%	127.45%	239.66%	402.99%	46.32%	46.65%	47.54%	48.15%
Sharpe Ratio	0.49	0.49	0.51	0.50	0.51	0.53	0.56	0.57
Sortino	0.80	0.78	0.87	1.02	2.21	2.39	2.62	2.78
VAR 5%	−15.47%	−34.35%	−49.77%	−62.56%	−6.58%[b]	−6.64%[b]	−6.34%[b]	−6.31%[b]
ES	−5.62%	−12.87%	−19.01%	−24.55%	−0.23%[b]	−0.40%[b]	0.33%[b]	0.38%[b]
Omega	5.96	4.67	4.66	4.79	13.52	14.56	16.06	17.11
CPV	7.47	5.76	5.34	5.33	14.20	14.56	15.01	15.24

Results show average annual returns from January 1947 to December 2016 for the S&P 500 along with three Leveraged Exchange Traded Funds (LETFs) and four constant proportional portfolio insurance (CPPI) strategies. 2x, 3x, and 4x CPPI strategies replace the standard S&P 500 with 2x, 3x, and 4x LETFs.

[a]Significantly better than the CPPI S&P at the 5% level.

[b]Significantly better than the S&P 500.

The first item to note is that all the LETFs suffer decay and their returns are positively skewed. For example, the S&P 3x average annual return of 33.97% is only 2.71 times the average annual return of the S&P. The riskiness of the LETFs is also apparent with extreme minimums, VaRs, and ES values. Based on Sharpe ratios, there is not a huge difference between them although LETF's minimums are daunting ranging from −67% to −95%.

However, there does appear to be benefits for LETFs within a CPPI strategy as all the LETF CPPIs display better average annual returns over the standard S&P CPPI ranging from 0.47% for the CPPI 2x to 1.35% for the CPPI 4x. There is some increase in the standard deviation for these higher returns, but the minimums, VARs, and ES are lower as reflected in their higher Sortino, Omega, and CPV values.

From a risk-return perspective, all four CPPI strategies dominate the S&P 500 based on risk metrics, but they do give up 2.46% to 1.37% annually moving from the CPPI S&P to the CPPI 4x. Because the LETFs are rebalanced after every 2.5% move, the −95% possibility of buying and holding a 4x is eliminated along with the fact the 4x can only be a maximum of 25% in the portfolio. Downside protection for the leveraged CPPIs is confirmed as the 90% floor is never breached for any of the CPPI strategies. For prospect theory type investors, the CPVs of the CPPIs do exceed the S&P 500 with increasing levels of CPV from CPPI S&P to the CPPI 4x.

Graphically, Fig. 1 demonstrates the value of CPPI strategies relative to a 100% investment in the S&P, and the value of using LETFs in a CPPI strategy as opposed to a standard CPPI portfolio. All of the CPPI strategies avoided the major drawdowns of wealth in the early 1970s, as well as in 2001 and 2008. There is a cost for CPPI strategies as their cumulative values all fall short of the S&P 500. It is also the case all the LETF CPPIs outperform the standard S&P CPPI.

Results thus far suggest LETF CPPI strategies appear to outperform a standard S&P CPPI strategy. However, part of the outperformance of LETF CPPI strategies has been the greater percentage of wealth that earns the risk-free rate. In effect, the LETFs are borrowing to attain

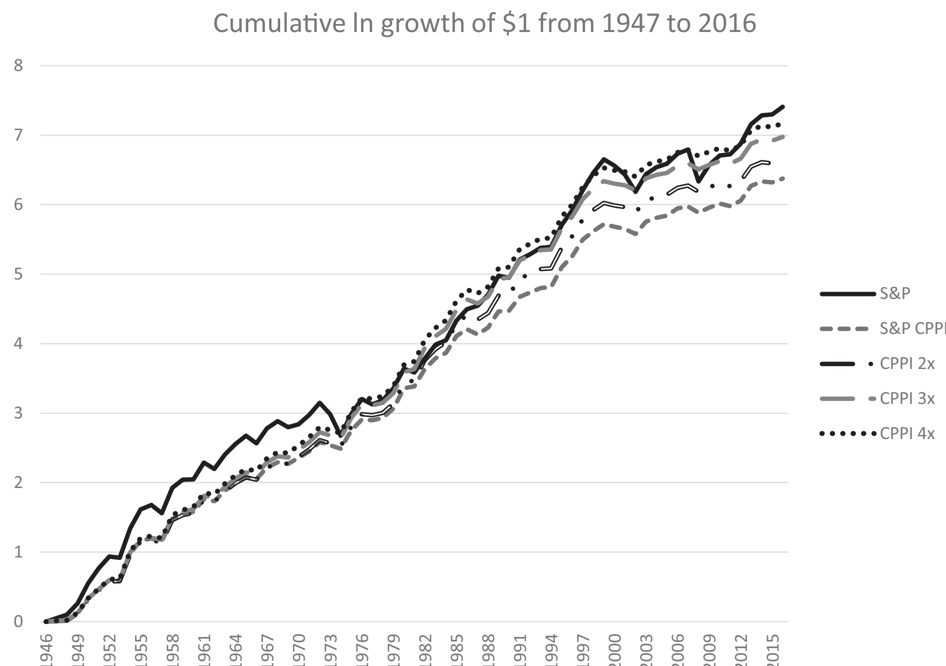

Fig. 1. Cumulative ln growth of $1 for the four CPPI strategies and the S&P 500 from 1947 to 2016.

increased exposure to the index. An investor using a LETF CPPI strategy needs to attain additional return with the excess funds to overcome the financing costs from the leverage and the associated higher expense ratio. To ascertain how well this works in different interest rate environments, Table 4 shows historical subperiods from 1947 to 1959, 1960–1978, 1979–1991, 1992–2008, and 2009–2016. These periods correspond to average rates of 2.0%, 5.59%, 10.87%, 4.71%, and 0.39% respectively.

As might be expected, when rates are low with a relatively flat yield curve as seen in 1947–1959 and especially during 2009–2016, there is little advantage to using a LETF CPPI with average returns less or not much greater than a traditional CPPI. During more "normal" interest rate periods such as 1960–1978 and 1992–2008, LETF CPPIs outperform a traditional CPPI by 0.3% to as much as 1.5%. An interesting find was that the CPPI portfolios have higher average returns than the S&P 500 during the 1960–1978 period as they avoided

Table 4 Average annual returns for different interest rate environments during 1947–2016

Time period	1-year T-bill	S&P	CPPI S&P	CPPI 2x	CPPI 3x	CPPI 4x
1947–2016	5.10%	12.53%	10.13%	10.60%	11.15%	11.48%
1947–1959	2.00%	18.30%	13.57%	13.38%	13.78%	14.06%
1960–1978	5.59%	7.51%	7.94%	8.50%	9.11%	9.51%
1979–1991	10.87%	17.56%	14.96%	16.72%	17.80%	18.40%
1992–2008	4.71%	8.81%	7.93%	8.20%	8.60%	8.82%
2009–2016	0.39%	14.81%	6.59%	6.20%	6.36%	6.41%

Results show average annual returns from January 1947 to December 2016 for the S&P 500 along with four constant proportional portfolio insurance (CPPI) strategies for various sub-periods corresponding to changing interest rate environments.

Table 5 CPPI annual returns based on SPY and ProShares 2x SSO LETF from 2007–2016

Year	1-year T-bill	SPY	SSO 2x	CPPI S&P	CPPI 2x	CPPI S&P w/AGG	CPPI 2x w/AGG
2007	5.94%	5.14%	1.04%	2.45%	2.26%	2.92%	3.23%
2008	4.60%	−36.81%	−67.94%	−8.62%	−8.60%	−2.94%	−2.51%
2009	0.56%	26.37%	47.26%	8.25%	8.13%	9.70%	10.58%
2010	0.55%	15.06%	26.84%	5.81%	5.66%	11.37%	13.70%
2011	0.37%	1.89%	−2.92%	−3.40%	−3.58%	−1.06%	0.40%
2012	0.16%	15.99%	31.04%	8.04%	7.64%	9.33%	10.68%
2013	0.34%	32.31%	70.47%	23.66%	22.92%	23.10%	20.34%
2014	0.24%	13.46%	25.53%	6.25%	5.81%	9.77%	12.08%
2015	0.16%	1.25%	−1.19%	−1.49%	−1.79%	−1.16%	−1.36%
2016	0.76%	12.00%	21.55%	6.22%	6.03%	9.41%	9.89%
Geo. Ann. Ret.	1.35%	6.87%	7.06%	4.40%	4.15%	6.79%	7.47%

Results show annual returns from 2007 to 2016 for the S&P 500 SPY, the S&P 500 2x (SSO), along with constant proportional portfolio insurance (CPPI) strategies using the S&P and S&P 2x. The last two columns replace the one-year T-bill with IShares AGG aggregate bond portfolio within the CPPI portfolios.

large losses and gave up little relative performance by investing in one-year T-bills. LETF CPPI's performed best during the 1979–91 period when the one-year T-bill return averaged 10.87%. Although this was much less than the S&P's 17.56% average, the 3x and 4x CPPI still outperformed the S&P itself. Relative to the standard S&P CPPI, all the LETF CPPIs outperformed by 1.76% to 3.44%.

Thus, during low interest rate or flat yield curve environments, there appears to be little advantage of creating LETF CPPI portfolios. In average or high interest rate environments, they outperform a standard CPPI. With LETF ETFs not being introduced until mid-2006, the results using actual LETFs are unlikely to be favorable relative to a traditional CPPI since the Federal Reserve has followed a near zero interest rate policy since the 2008 financial crisis. However, it is informative to examine how a LETF CPPI performs in practice.

Table 5 shows the returns from 2007 through 2016 using the SPY for the S&P 500 and ProShares 2x SSO. Although not shown, ProShares 3x UPRO has similar results. Both CPPI strategies perform as advertised as downside risk is mitigated and the −10% floor is not breached even in 2008 when the S&P fell 37%. Both CPPI strategies track each other, but the S&P CPPI strategy outperforms the CPPI 2x every year with the exception of near equal results in 2008. These results are mainly because of the fact the 2x still must deal with decay along with higher expenses and cannot make up the difference with additional earnings on the risk-free rate. These results reinforce the conclusions from Table 4 when rates are low.

In addition, when the risk-free rate was still relatively high in 2007 and 2008, the SSO 2x still underperforms because of the high volatility during that period causing serious return decay. In fact, for 2007, the 2x SSO only returns 1.04% compared with SPY's 5.14%, although half of the 4% underperformance is because of the large tracking error this fund had when first introduced. This tracking error has not been observed since. Furthermore, 2008 had the next largest tracking error when SSO returned −67.94% relative to the predicted −67.06% from Eq. (2). This tracking error has continually fallen since then and over the last 3 years has been less than 0.1% on an annual basis.

Table 6 Bootstrapped annual returns for the underlying indexes and CPPI portfolios

	S&P	S&P 2x	S&P 3x	S&P 4x	CPPI S&P	CPPI 2x	CPPI 3x	CPPI 4x
Average	12.31%	21.60%[b]	33.30%[b]	46.41%[b]	10.07%	10.59%	11.09%[a]	11.39%[a]
Standard deviation	16.72%	35.68%	57.99%	84.45%	11.88%	12.40%	12.74%	12.93%
Median	12.97%	21.22%	29.35%	35.74%	8.11%	8.36%	8.81%	9.06%
Minimum	−46.99%	−77.09%	−91.77%	−97.60%	−9.79%	−9.79%	−9.79%	−9.79%
Maximum	71.24%	175.88%	327.36%	530.57%	60.62%	66.33%	68.88%	70.19%
Sharpe Ratio	0.49	0.49	0.50	0.50	0.50	0.52[ab]	0.54[ab]	0.56[ab]
Sortino	0.95	1.08	1.29	1.50	1.37	1.51	1.66	1.75
VAR 5%	−15.63%	−34.94%	−53.34%	−70.40%	−5.51%[b]	−5.38%[b]	−5.26%[ab]	−5.17%[ab]
ES	−25.53%	−50.09%	−68.56%	−82.70%	−7.40%[b]	−7.32%[b]	−7.25%[b]	−7.20%[b]
Omega	6.18	4.75	4.73	4.86	14.10	15.18	16.40	17.17
CPV	8.70	9.43	8.71	7.17	14.83	15.48	15.98	16.28

Results show annual statistics based on 10,000 bootstrapped simulations for the S&P 500 along with three Leveraged Exchange Traded Funds (LETFs) and four constant proportional portfolio insurance (CPPI) strategies.

[a]Significantly better than the CPPI S&P at the 5% level.

[b]Significantly better than the S&P 500.

It is also interesting to note that both CPPI strategies lost money in 2011 and 2015 even though the SPY was slightly positive, 1.89% and 1.15%, respectively. This occurs because CPPI strategies are reactionary. When the market falls, the percentage of the portfolio in equities is reduced. When it bounces back, there is less wealth in the portfolio to regain the losses. Similarly, if the market increases (more is allocated to equities) then decreases. Thus, CPPI strategies tend to do poorly in volatile flat markets. The same holds true for LETFs because of their daily rebalancing. In fact, the SSO 2x lost money both in 2011 and 2015 despite the market increasing. This is a perfect demonstration of LETF's return decay.

For the less risk averse, Table 5 shows one alternative to overcoming extremely low interest rates. The last two columns show CPPI returns by combining a composite bond ETF within a CPPI strategy. Instead of using the one-year treasury from 2007 to 2016, IShares AGG bond ETF is used as a proxy for a relatively safe asset with higher expected returns. Both CPPI strategies show improvement and the LETF CPPI outperforms the standard CPPI except for 2013 and is only slightly worse in 2015. In 2013, both CPPI strategies using the IShares bond ETF underperform CPPI strategies using the one-year treasury. This is because of the −1.98% return the IShares bond ETF experienced that year. It was the only negative year for the aggregate bond fund, but highlights the fact that increasing expected return, even with a relatively safe bond fund, does have risks.

4.2. Bootstrapped results

To check on the robustness of the data, the historical data are blocked bootstrapped by sampling 252-day trading periods to create 10,000 annual returns to remove any bias from the January to January annual returns that may have favored one method or another. In addition, this will hopefully encompass most future possibilities while maintaining the relationship between interest rates and returns along with systemic low or high interest rate environments that may be experienced. Table 6 shows the results.

The returns and risk statistics confirm the earlier results with the LETF CPPI portfolios showing better returns and risk metrics relative to the standard CPPI based on Sortino, Value at Risk (VaR), Expected Shortfall (ES), Omega, and CPV. Although there are no statistical tests for significance for the Sortino and Omega values, the differences are monotonically increasing. The Sortino measure also suggests the 4x by itself has better return relative to downside risk even over the CPPI S&P, but this is mainly because of the very positively skewed returns for this asset resulting in a high average return. The median return shows the more likely result and is significantly less than the average for the 4x fund. Reconfirming the historical results, all four CPPI portfolios have better CPV values relative to the S&P 500, with the LETF CPVs greater than the CPPI S&P as well.

Both the CPPI 3x and CPPI 4x have significantly greater returns than the CPPI S&P. The CPPI 4x gives the best results despite using the riskiest asset. The caveat to the CPPI 4x results is there are no 4x LETFs currently in existence, and when and if they do make it to market, the expense ratio and cost of running these funds may be more expensive than what is assumed in this study. With no way to test the pricing model on an actual 4x fund, the 4x results should be interpreted with caution.

From an absolute return standpoint, CPPI portfolios have annual average returns 1% to 2% less and median returns 4% to 5% less than the S&P 500. Thus, CPPI portfolios do not provide "free" downside protection. However, the amount of average return sacrificed to avoid large losses would seemingly be appealing to risk-averse investors. The reduction in the minimums, VaR, and ES values bear this out along with much higher CPV values that measure the value to prospect theory type investors. For the risk-speculators, the average returns to the LETFs by themselves are enticing, up to an average 46% return with the 4x. However, these returns are coupled with up to −98% losses in any given year.

The question going forward is which CPPI strategy is likely to do best? Noting what has happened since 2007 when the T-bill rate is close to zero, a LETF CPPI loses its advantage if the additional funds are simply invested in the risk-free rate if rates are very low. To ascertain what risk-free rate of return one would need to overcome decay and the higher expenses from using a LETF, 40,000 bootstrapped annual returns are sorted based on the average T-bill return attained each year. The average T-bill return was 5.13% with a standard deviation of 3.84%. The top section of Table 7 shows the return data for when the T-bill rate is below 1% to greater than 7%.

Two conclusions are apparent from examining the top section of Table 7. For a LETF CPPI to outperform, a rate of 2.5% or more is required to make up for the additional costs from using LETFs. Two, when the rate is less than 2%, for those constructing a CPPI, a standard CPPI will be slightly better. Alternatively, an investor could redefine the "risk-free" asset to accommodate a relatively "risk-free" bond fund providing higher yield. Interest rates above 6% are very favorable to LETF CPPIs with returns up to 3% greater over the standard CPPI. In addition, rates over 7% seem to favor any CPPI, even over the S&P itself. They are particularly favorable to LETF CPPIs with average returns 1.5% to 3% greater than both the S&P and the CPPI S&P.

By using LETFs in a CPPI, the investor is in effect borrowing short via the LETF and investing in one-year Treasuries. Thus, the slope of the yield curve is a critical issue. Generally, lower rates are associated with a flatter yield curve and vice versa. Thus, returns

Table 7 Bootstrapped annual returns sorted by 1-year T-bill return and slope of the yield curve

	S&P	S&P 2x	S&P 3x	S&P 4x	CPPI S&P	CPPI 2x	CPPI 3x	CPPI 4x
T-bill return								
0–1%	17.41%	34.46%[b]	53.96%[b]	74.70%[b]	10.21%	9.81%	9.99%	10.09%
1–2%	17.92%	35.14%[b]	55.24%[b]	77.22%[b]	11.31%	11.05%	11.27%	11.39%
2–3%	6.62%	11.54%[b]	19.84%[b]	31.19%[b]	6.06%	6.03%	6.28%	6.44%[a]
3–4%	6.87%	11.64%[b]	19.02%[b]	28.32%[b]	6.36%	6.60%	6.97%[a]	7.22%[a]
4–5%	8.39%	13.59%[b]	20.55%[b]	28.08%[b]	7.05%	7.38%	7.80%[a]	8.06%[a]
5–6%	8.62%	12.45%[b]	16.97%[b]	20.99%[b]	6.92%	7.37%	7.84%[a]	8.15%[a]
6–7%	13.49%	23.08%[b]	34.69%[b]	47.07%[b]	10.90%	11.65%	12.26%[a]	12.62%[a]
>7%	13.44%	21.32%[b]	31.42%[b]	42.69%[b]	13.79%	15.43%[ab]	16.35%[ab]	16.87%[ab]
1 year to 90-day								
<0%	23.47%	47.97%[b]	79.43%[b]	117.76%[b]	17.03%	16.32%	16.51%	16.65%
0–1%	12.55%	23.08%[b]	35.23%[b]	47.77%[b]	7.61%	7.42%	7.64%	7.76%
1–2%	10.39%	17.51%[b]	26.64%[b]	36.83%[b]	8.18%	8.43%	8.83%[a]	9.08%[a]
2–3%	10.48%	17.26%[b]	26.23%[b]	36.41%[b]	9.41%	10.01%	10.54%[a]	10.86%[a]
>3%	12.02%	19.40%[b]	29.07%[b]	40.00%[b]	12.51%	14.12%[ab]	15.02%[ab]	15.54%[ab]

Results show average annual returns based on 40,000 bootstrapped simulations sorted by 1-year T-bill returns and the slope of the yield curve measured by the 1-year minus 90-day T-bill return.

[a]Significantly different from constant proportional portfolio insurance CPPI S&P at the 5% level.

[b]Significantly better than the S&P 500.

are also sorted by the yield curve slope as measured by the one year minus 90-day Treasury. The bottom of Table 7 shows the results. When the 90-day Treasury exceeds the return from the one-year Treasury, the standard CPPI outperforms a CPPI 2x by 0.7% to 0.4% moving from a CPPI 2x to a CPPI 4x. As soon as the slope exceeds 1%, all the LETF CPPIs outperform a tradition CPPI and this outperformance increases the greater the difference in treasury rates. These results reaffirm what may be obvious; the advantage of using a LETF CPPI strategy is what return an investor can attain with the excess funds relative to the intrinsic financing costs of the LETFs leverage.

4.3. Rebalancing rules

There is nothing magic about using a 2.5% market move to rebalance. In fact, using a 2% or 3% market move gives virtually the same results. Taken to the extreme, daily rebalancing could be implemented, but trading costs would increase. Although this study did not account for brokerage costs explicitly in the return calculations, with two trades a day at $5 a trade, even a $1,000,000 account faces an additional 0.25% in trading cost if using daily rebalancing. Table 8 shows the average returns from 10,000 bootstrapped simulations for the CPPI strategies based on the 2%, 2.5%, and 3.0% rule along with daily, weekly, and monthly rebalancing.

Weekly rebalancing is not significantly different from the 2.5% rule, nor is monthly worse suggesting the decay drag, even over a month, is not significant confirming Lu, Wang, and Zhang (2012) findings that investors can generally expect to earn the leverage ratio for up to a month. Daily rebalancing does improve results, but not by enough to overcome transaction costs. With the exception of daily rebalancing, the average returns for the CPPI S&P varied from 10.01% to 10.05%. The same type of range held for the LETF CPPIs.

Table 8 Average annual returns for different rebalancing rules

Rebalance	Avg. no. of trades	S&P	CPPI S&P	CPPI 2x	CPPI 3x	CPPI 4x
2% rule	31	12.33%	10.04%	10.53%	11.01%[a]	11.28%[a]
2.5% rule	23	12.31%	10.01%	10.53%	11.03%[a]	11.32%[a]
3.0% rule	17	12.35%	10.05%	10.52%	10.98%[a]	11.24%[a]
Daily	252	12.30%	10.27%	10.74%	11.17%[a]	11.39%[a]
Weekly	50	12.28%	10.03%	10.53%	11.00%[a]	11.28%[a]
Monthly	12	12.34%	10.01%	10.50%	10.95%[a]	11.22%[a]

Results show average annual returns based on 10,000 bootstrapped simulations for the S&P 500 along with four constant proportional portfolio insurance (CPPI) strategies based on various rebalancing rules. Percentage rules based on absolute return of the S&P 500 relative to previous rebalance.

[a]Significantly better than the CPPI S&P at the 5% level.

In summary, the additional return from using a LETF CPPI strategy relative to just using the index appears robust to the rebalancing method chosen with a CPPI 2x, CPPI 3x, and a CPPI 4x earning approximately 0.5%, 1.0% and 1.3% more, respectively.

5. Conclusion

LETFs are proclaimed to be risky-short term trading vehicles with plenty of warnings, (Carver, 2009; Justice, 2009; Zweig, 2009, 2017). As an individual asset, there is no denying the extremes that can be experienced by buy-and-hold investors. However, more active traders can moderate this risk by periodic rebalancing which fits in perfectly with a CPPI strategy. By managing the exposure as LETFs change in value, downside losses can be mitigated.

One of the disadvantages of a CPPI is the need for constant rebalancing. However, with only periodic rebalancing based on market movements, this study shows using LETFs instead of the underlying risky asset in a CPPI portfolio leads to greater returns with less risk. This outcome is possible because the same amount of exposure to the risky asset can be attained with a smaller percentage of the portfolio, leaving a larger amount available to earn the risk-free rate. If the return from the additional amount in the risk-free asset exceeds the LETF decay and higher expenses, the LETF CPPI will outperform. Results suggest risk-free yields of 3% or if the one-year exceeds the 90-day Treasury by more than 1% appear to be sufficient for LETF CPPIs to outperform a standard CPPI using the index itself.

Both simulated results from 1947 to 2016 and bootstrapped data show CPPI strategies created with LETFs outperform a CPPI strategy using the underlying index. Average annual returns over all interest rate environments are 0.5% to 1.3% higher with better minimums, Sharpe ratios, Sortino ratios, Omegas, VARs, ES, and CPVs. Using LETFs in a CPPI strategy will underperform slightly when the risk-free rate is extremely low as it has been for the last seven years with the yield below 1%. There is simply no additional return from having a greater percentage of wealth in the risk-free rate to compensate for LETFs higher expenses and leverage decay.

For the less risk averse, more aggressive CPPI portfolios could be created by using a smaller multiplier, lower floors, letting the amount in the 2x, 3x, or 4x LETFs exceed 50%, 33%, or 25%, respectively, and/or using a more risky "risk-free" asset such as longer-term treasuries or some type of composite bond ETF as demonstrated in the last two columns of Table 5 where a bond ETF was substituted for the one-year Treasury. The latter adjustment is likely the safest when yields are extremely low, while allowing the amount in the LETF to increase up to 100% is certainly the riskiest. Two days like October 16 and October 19, 1987 when the market fell 5.1% then 19.5% would see a CPPI 4x lose 84% of its value if exposure to a 4x LETF is allowed to increase to 100%. This would seem to defeat the purpose of a CPPI strategy in the first place.

LETFs are relatively new instruments. The reception from investors has been mostly positive despite their risk as seen by the phenomenal growth both in number and in asset growth. Like options and futures, LETFs can be used for highly speculative gambles, hedging, or risk management. This study demonstrates that LETFs, even a 4x LETF if it becomes available, can be used to enhance return, and reduce risk within the right context.

Appendix 1

The risk metrics used to evaluate the results are described below:

1. The Sharpe ratio is the excess return divided by the standard deviation and is reported for completeness, even though it is not the best measure for assessing the downside risk portfolio insurance portfolios are attempting to mitigate, (Sharpe, 1964). Opdyke's (2007) testing procedure is used to determine whether the LETFs CPPI Sharpe ratios are better than the S&P 500 and the CPPI S&P.
2. The Sortino ratio is a modification of the Sharpe ratio and only considers the downside deviation removing the aspect of "good volatility" (Sortino and Price, 1994). It is more appropriate for analyzing portfolio insurance strategies that are designed to mitigate large losses and whose returns may not be normally distributed. The Sortino ratio is written as:

$$S = \frac{R - T}{\text{TDD}} \text{ where TDD} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\text{Min}(0, X_i - T))^2} \quad (3)$$

where R is the return, T is the target return, N is the total number of returns and X_i is the ith return. The higher the ratio, the greater the return per unit of downside risk. T is set at the one-year treasury return for this study.

3. The Omega ratio, reported by Keating and Shadwick (2002) also measures downside risk. Omega is the sum of the returns above a certain threshold divided by the sum of the returns below that threshold, which is also set at zero in this study. The Omega ratio is written as,

$$\Omega_X[\Gamma] = \frac{\int_{\Gamma}^{\infty}(1 - F_x(x)dx}{\int_{-\infty}^{\Gamma} F_x(x)dx} \tag{4}$$

where F is the cumulative distribution function and Γ is the threshold return. In this study, it is simply the sum of the returns above zero divided by the absolute value of the sum of the returns below zero.

4. Value at Risk measures the expected maximum loss with a given confidence level over a specific time; 5% of the observations are less than the VaR. To test for differences in VaR, an unconditional coverage test is applied as put forth in Annert, Osselaer, and Verstraete (2009) and is written as:

$$\frac{\left(\frac{1}{N}\sum_{n=1}^{N} Hit_n - \alpha\right)}{\sqrt{\frac{\alpha(1-\alpha)}{N}}} \tag{5}$$

where Hit_n equals one if the LETF CPPI return is lower than the S&P CPPI VaR, zero otherwise, N is the number of returns, and α is the 5% VaR. All statistical tests for significant differences are set at the 95% confidence level.

5. As pointed out in Acerbi, Nordio, and Sirtori (2001), Excess Shortfall (ES) shows the average loss beyond the VaR threshold and represents the severity of a dramatic loss. It addresses the "what if" factor and makes up for the discrepancies with the VaR calculation. Acerbi and Tasche (2002) confirm the appropriateness of this definition compared with other shortfall calculations. Annaert et al. (2009) is followed to test for differences in ES values.
6. Because there is an expectation portfolio insurance appeals more to prospect theory investors, cumulative prospect values (CPV) are calculated using the function and parameters set forth in Tversky and Kahnemen (1992). The probability weighting parameter for gains is 0.61 and 0.69 for losses. Dichtl and Drobetz (2011b) use a similar methodology in comparing dollar cost averaging to lump sum investing.

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