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Risk and reward of fractionally leveraged ETFs in a stock/bond portfolio

James DiLellio[a,*]

[a]*Department of Decision Sciences, Information Systems and Strategy, Pepperdine Graziadio Business School, 24255 Pacific Coast Highway, Malibu, CA 90263, USA*

Abstract

This article investigates using 1.25X leveraged stock and bond exchange-traded funds (ETFs) as an asset allocation strategy. Performance is analyzed by replicating funds from 1989 to 2017, including all relevant costs. Conditions for excess returns are derived analytically and confirmed empirically. Simulations are conducted to assess performance under a variety of market conditions, and demonstrate opportunities for excess returns over unlevered funds in a 60/40 stock/bond allocation. These results are accomplished with a small reduction in the Sharpe ratio and no need to access margin. We conclude that this asset allocation strategy could be well suited for investors more interested in total returns during upward trending markets.

JEL classification: G11; C53

Keywords: Leveraged ETFs; Simulation; Geometric Brownian motion; Bootstrapping

1. Introduction

Individual use of financial leverage is fairly common by individual investors. Many first-time home buyers use a mortgage where the buyer provides 20% of the closing price of a home, and finance the remaining 80%. The use of leverage in the stock market is also common, where most brokers offer margin accounts to support individual investors purchasing securities above their cash balance, shorting a stock, or investing in financial derivatives.

* Corresponding author. Tel.: +1-714-403-0085; fax: +1-714-832-3654.
E-mail address: james.dilellio@pepperdine.edu

These margin accounts can amplify gains and losses. However, retirement accounts do not permit this use of leverage, because of the need to provide collateral and their tax exempt status.[1] Nevertheless, individual investors and financial planners seeking to boost returns in retirement accounts can invest in leveraged funds.

Because of the effect of daily rebalancing, most leveraged exchange-traded funds (ETFs) have largely remained in the hands of short-term traders. Many academic articles have discussed the technicalities behind why leveraged ETFs are not intended for passive buy-and-hold investors. Cheng and Madhavan (2009) and Guedj et al. (2010) show that, because of the path dependency of leveraged ETFs, longer holding periods reduce the value of the leverage. Lu et al. (2012) show that, for holding periods up to one month, a 2X leveraged ETF produces approximately twice the return of its underlying. However, for longer holding periods, the ability of the 2X fund providing twice the return of its underlying diminishes. Somewhat surprisingly, some authors indicate that there are potential opportunities.

In Trainor and Carroll (2013), the term "decay" is defined when the difference between a leveraged ETF's return and the leverage factor multiplied by underlying index is negative. This article showed that low volatility coupled with a significant upward price trend can substantially offset decay. Avellaneda and Zhang (2010) show that with a dynamic hedging strategy, it is possible for active traders to manage leveraged ETFs over longer time horizons. However, the complexity of employing such an active strategy is likely beyond the skill of many individual investors. DiLellio et al. (2014) find potential portfolio diversification exists with inverse and leveraged ETFs used as an alternative asset class in a long-term passive investment strategy. Diversification benefits were shown to exist, dependent on the behavior of equity and debt markets, and were shown to be generalizable to a variety of core stock and bond funds often utilized by individual investors and financial planners.

Trainor and Baryla (2008) point out that, for individual investors interested in leverage, the cost of obtaining it via a leveraged ETF can often be less than a typical margin account. Barnhorst and Copcozza (2010) discuss investors taking "volatility risk," and show how a 2X leveraged fund can under (over) perform its underlying benchmark because of higher (lower) price volatility. Giese (2010) summarizes the positive benefits of holding leveraged funds in bullish markets, but recognizes that these benefits can be offset by increased volatility.

Given the general downward trend in volatility in the equity markets since the 2008 financial crises, there may be future opportunities for individual investors to take this so-called volatility risk. As noted above, the speed at which the leveraged ETF can diverge from its underlying index is proportional to the amount of leverage. Therefore, individual investors may have interest in using ETFs with smaller amounts of leverage than the 2X and 3X ETFs that have dominated the leveraged ETF marketplace. To this end, this article investigates whether less leverage, at 1.25X, provides a viable investment opportunity for a passively managed portfolio of stocks and bonds without taking on excessive risks. This article also contributes to work by Ott and Zimmer (2016), where theoretical and empirical results support the use of fractional leverage on return maximizing a portfolio of risky and risk free assets. While their work determined optimal leverage at 1.77–1.85, we chose to focus on a value of 1.25, which was the level available in the financial markets at the time this paper was written. A summary of current stock and bond ETFs offering 1.25X leverage appear in Table 1 below, along with their unleveraged counterparts.

Table 1 1.25X leveraged stock and bond exchange-traded funds (ETFs) and their unleveraged counterparts*

ETF symbol	Underlying index	Assets under management	Expense ratio	Median daily share volume	Average spread	Premium/discount median (range)	Issue date
PPLC	S&P500	$81.11M	0.34%	25,223	0.04%	−0.03% (−2.28% to 2.25%)	January 7, 2015
IVV†		$144.83B	0.04%	3,870,076	0.01%	0.00% (−0.11% to 0.13%)	May 15, 2000
PPTB	Barclay's US Aggregate Bond	$25.05M	0.34%	15,100	0.04%	0.01% (−0.39% to 0.17%)	February 15,2018
AGG		$53.62B	0.05%	3,627,118	0.01%	0.07% (−0.16% to 0.26%)	September 22, 2003

Note: *Obtained from www.etf.com in January 2018.

†IVV was selected as one of three exchange-traded funds (ETFs) that track the S&P 500 Index, and is issued by Blackrock. The other two, VOO and SPY, issued by Vanguard and State Street Global Advisors, have similar expense ratios and spreads, and could have also been chosen. The results that follow are generalizable to using any of these S&P 500 Index ETFs.

Table 1 illustrates that the assets under management for the 1.25X leveraged ETF tracking the stock and bond indices are extremely small, compared with the unleveraged ETFs with the same underlying index, and have only existed for a few years. Similar to other leveraged ETFs, the 1.25X leveraged ETFs also have a higher expense ratio. However, despite the 1.25X leveraged ETF's median daily share volume being hundreds of times smaller than the unleveraged version, the average spread is less than ten times larger, thanks to the liquidity of underlying index. This low volume and low spread outcome differs from what occurs for the majority of ETFs, and as shown in Agrrawal and Clark (2009), who show low volume more often leads to exponentially higher spreads. Consequently, transaction costs of rebalancing these leveraged ETFs may not significantly reduce returns, so will be investigated in more detail in this article.

The 1.25X leveraged stock and bond ETFs also exhibit larger premium/discounts. Therefore, it is possible that during times of market stress, these ETFs may be trading in the lower or higher price range, producing a difference in holding period performance. For purposes of the analysis that follows, we assume the net effect of this premium/discount to be negligible.

As a final note about 1.25X leveraged funds, there are currently two mutual funds available to individual investors that offer this fractional leverage, but on a monthly rather than daily basis. The target index for these mutual funds is the NASDAQ-100 total return index, and tickers DXNLX and DXNSX provide positive and inverse leverage. In the analysis that follows, we include the bull 1.25 monthly fund as part of a sensitivity analysis to highlight the effect of a monthly versus daily return matching 125% of the underlying.

2. Data source and replication technique

As shown and discussed in the previous section, there is very limited historical data available on fractionally leveraged ETFs. So, to study long-term investment performance under a variety of market conditions, we replicated returns of two equity index ETFs in Table 1 using the S&P 500 total return index (Bloomberg index code SPXT). The first fund was an unleveraged ETF, and the second was a 1.25X leveraged ETF. Because we are also

Table 2 Parameters to replicate unleveraged and leveraged ETFs

Expense ratio, unleveraged fund (stock, bond)	Leverage factor	Expense ratio, leveraged fund (stock, bond)	Additional financing cost
e	f	$\tilde{e}$	X
0.04%, 0.05%	1.25	0.32%, 0.32%	0.25%

Note: ETF = exchange-traded funds.

interested in a mixed stock/bond portfolio, we similarly replicated returns for a 1.25X leveraged and unleveraged bond ETF, which are based on the Barclay's aggregate total return bond index (Bloomberg index code LBUSTRUU). We will refer to these funds simply as our leveraged or unleveraged "stock ETF" and "bond ETF" for the remainder of the paper. Because we will be replicating the effect of constant daily leverage, we obtained our stock and bond index data on a daily basis. The total return data from Bloomberg provided full years of daily returns from 1989 through 2017.

We began by replicating the unleveraged stock and bond ETF daily returns. Let r_i be the i^{th} day's total return (including dividends) of the underlying stock or bond index. To construct the unleveraged ETF, we impose an expense ratio e, so that the yearly return R can be expressed as

$$R = -1 + \prod_{i=1}^{n}\left(1 + r_i - \frac{e}{n}\right) \tag{1}$$

where $\prod_{i=1}^{n}$ represents the product over each i^{th} day of a year with n total days. Typically, $n = 252$, corresponding to an average of 21 trading days per month.

We similarly constructed leveraged stock and bond ETF daily returns. Let $\tilde{r}_i$ be the leveraged ETF daily return (including dividends) so that, before imposing any expenses or fees,

$$\tilde{r}_i = f * r_i \tag{2}$$

where f is the leverage factor. For a 1.25X leveraged ETF, we set $f = 1.25$. Then, the annual return $\tilde{R}$ of the leveraged ETF is

$$\tilde{R} = -1 + \prod_{i=1}^{n}\left[1 + \tilde{r}_i - \frac{\tilde{e}}{n} - \left(\frac{Libor_i + X}{n}\right)(f - 1)\right], \tag{3}$$

where $\tilde{e}$ is the expense ratio of the leveraged fund, $Libor_i$ is the annual rate for the 1-month LIBOR,[2] and X is an additional financing cost imposed on the ETF provider to borrow the money and leverage the underlying index's assets. Note that, with the exception of the potentially different expense ratios, Eq. (3) reduces to (1) when $f \to 1$. Table 2 lists values that will be used for the analysis that follows. It is also important to note that no attempt to model divergence from the daily benchmark will be made, which can occur when such a fund trades at either a discount or premium. We also assume that these investments are held in a tax deferred or exempt retirement account. Lastly, we assume that slippage from incomplete baskets and managing leverage has a negligible long-term effect on returns. In the case of

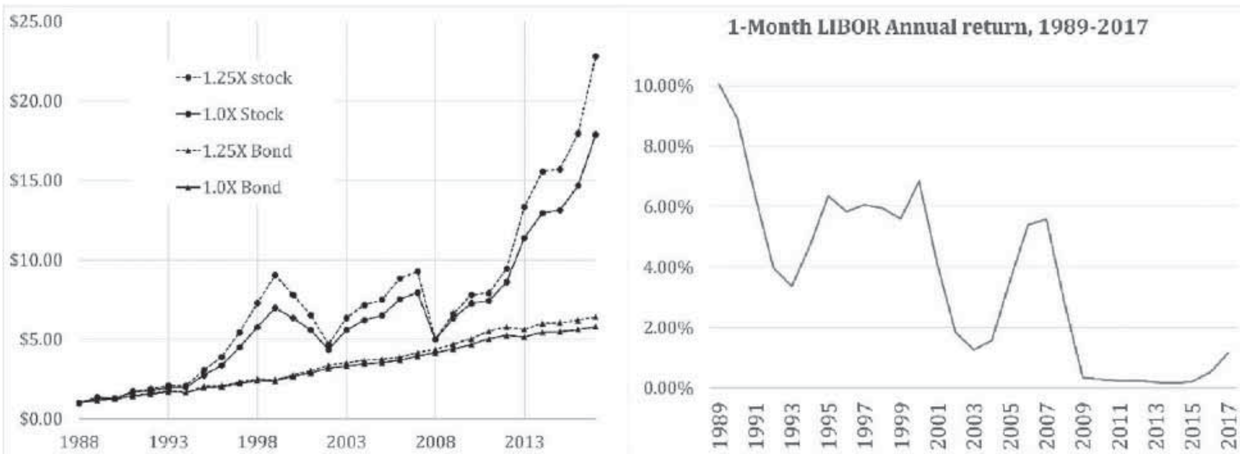

Fig. 1. (left pane) The 29 Years of growth of a $1 investment in replicated 1.25X leveraged and underlying stock and bond ETFs (right pane) 1-month LIBOR Annual returns over same periods.

PPLC, the fund holds IVV (iShares S&P 500 Index ETF) instead of individual securities included in the underlying index. We believe that IVV is well known to track the S&P 500 index very well. Leverage is obtained by the fund investing in total return swaps on the S&P 500. Data provided by Direxion Investments on PPLC show that from the fund's inception at the end of 2016, the tracking error because of managing this leverage was less than 0.001% on an annualized basis.

We can now replicate annual returns and volatility for unleveraged and leveraged stock and bond ETFs. Annual return data and volatility for the stock funds appear in Table A1, while Table A2 contains annual return and volatility of the bond funds appear in Table A2 of the Appendix. The growth of a $1 investment in the 1.25X leveraged and underlying index ETF constructed with these assumptions appear in the left pane of Fig. 1. The right pane of Fig. 1 provides a reference to the effect that borrowing costs have on limiting return growth, where higher LIBOR rates reduce the total return for the leveraged stock and bond funds.

The *excess return*, or the positive difference between the 1.25X stock ETF and unleveraged stock ETF, grows during bull markets, then shrinks during market downturns. The longest run of growing excess returns began in 2009, helped by two conditions. First, there has not been a significant stock market downturn since 2009, and volatility has generally trended down. Additionally, the excess returns have benefited by the near 0% LIBOR rates following the 2008 financial crisis. In the next section, we derive the conditions of annual returns and volatility of the underlying ETF that produce excess returns, and compare it to these replicated returns.

2.1. Derivation of return and volatility conditions that produce excess returns

It is well understood that excess returns of a leveraged ETF can be amplified by lower volatility, but returns on the underlying return are also important. Here, we consider the *annual* return expectations for a leveraged ETF, and compare it to the unleveraged return, net of fees and financing costs. Leveraged ETF *annual* expected returns can be expressed as

$$E(\tilde{R}) = f\mu - \frac{f^2}{2}\sigma^2 \tag{4}$$

as shown in Lu et al. (2012), where μ is the rate of return and σ is volatility. Thus, the ability to generate excess returns occurs when $\tilde{R} > R$. In terms of continuously compounded expected returns, we solve when

$$E(\tilde{R}) > E(R) \tag{5}$$

for a given year. Including expenses implies that returns are reduced accordingly. Thus,

$$\mu \rightarrow \mu - \tilde{e} - (Libor_i + X)(f - 1) \tag{6}$$

for the leveraged ETF and

$$\mu \rightarrow \mu - e \tag{7}$$

for the unleveraged ETF. Then, the inequality to produce excess returns becomes

$$f[\mu - \tilde{e} - (Libor_i + X)(f - 1)] - \frac{f^2}{2}\sigma^2 > \mu - e - \frac{1}{2}\sigma^2. \tag{8}$$

Solving for μ yields the quadratic relationship between volatility and returns of the underlying ETF to produce excess returns.

$$\mu > \frac{(f^2 - 1)}{2(f - 1)}\sigma^2 + \frac{f[\tilde{e} + (Libor_i + X)(f - 1)] - e}{f - 1}. \tag{9}$$

The first term on the right hand side of Eq. (9) contains the quadratic coefficient, and the second is a constant. Because there is little correlation between the LIBOR rate and volatility,[3] the average LIBOR rate from 1989 to 2017 of 3.57% was used that, along with the values in Table 2 for the stock ETF, simplify Eq. (9) to $\mu > 0.062 + 1.125\sigma^2$.

Fig. 2 is a scatter plot of the annual returns and volatility from 1989 to 2017 of the unleveraged stock ETF, where the caption for each point shows the excess return. The solid line represents Eq. (9), so as expected, positive call-outs occur above the quadratic line and negative ones are below it. The data used to produce Figs. 2 can be found in the Appendix.

Similar results occur for leveraged bond funds, where lower volatility and higher returns often produce higher excess returns, as shown in Fig. 3. However, there is less sensitivity to higher volatility, shown by a flatter quadratic. Also, for Eq. (9) to apply in Fig. 3, we restricted annual returns to appear from 2009 to 2016, when LIBOR rates were nearly constant, and averaged 0.27%. Note that from Eq. (9) that for every 1% increase in the annual LIBOR rate, the quadratic in Fig. 3 increases by 1.25%.

2.2. Relationship of excess returns and underlying risk adjusted returns

From the previous section, we established that for sufficiently large returns and sufficiently small volatility, excess returns occur. That is, excess returns obtained from using

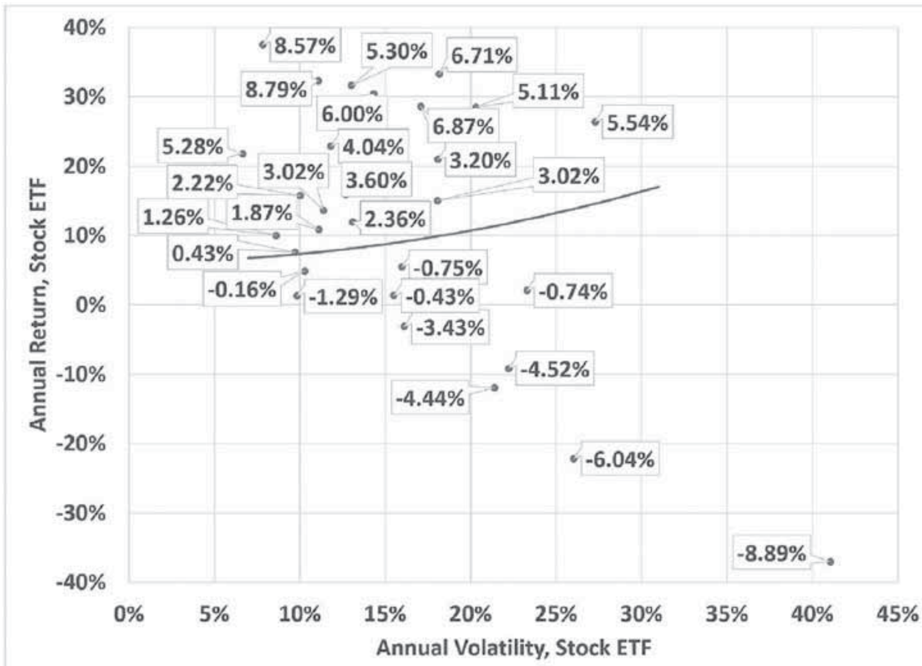

Fig. 2. Excess returns and their relationship to underlying volatility and returns, stock ETF, 1989–2017.

leverage is not only the function of upward trending markets, but the market's associated volatility. In this section, we quantify this strong relationship from our dataset using a risk-adjusted return measure. We consider a simple linear regression of where the dependent variable is excess returns and the independent variable is the Sharpe ratio of the underlying

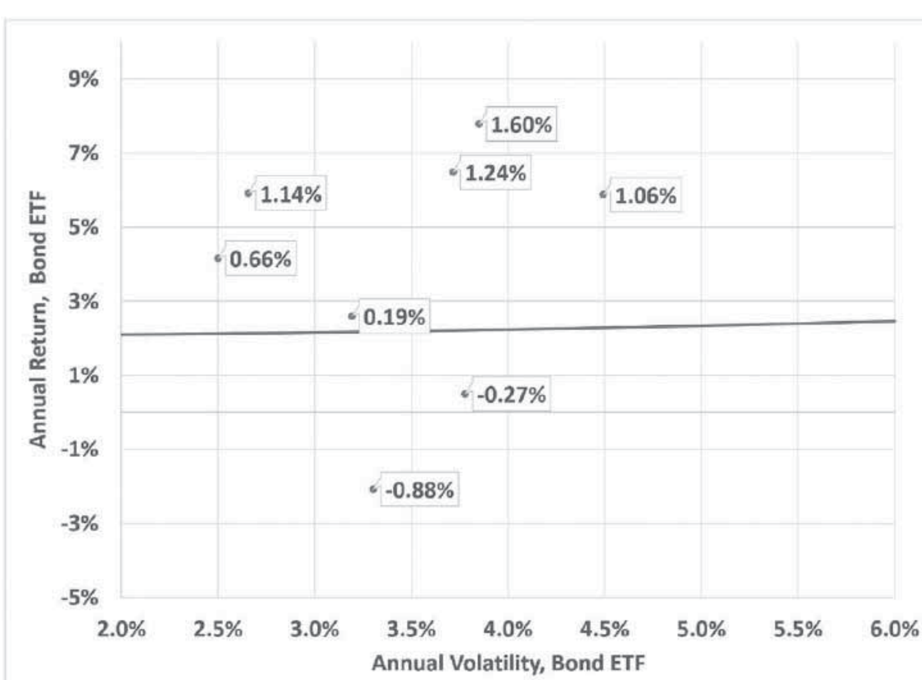

Fig. 3. Excess returns and their relationship to underlying volatility and returns, Bond ETF, 2009–2016.

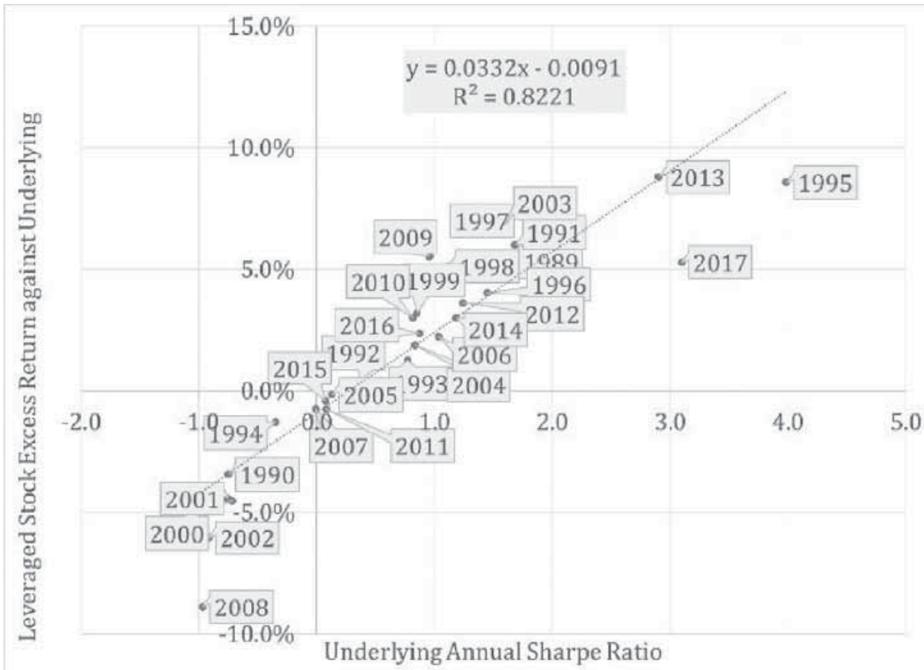

Fig. 4. Excess returns from 1.25X leveraged stock ETF versus its underlying ETF, net of fees and expenses, 1989–2017.

ETF. We hypothesize that annual excess returns, $\tilde{R} - R$, *increase* with higher underlying Sharpe ratio. Formally, we express this relationship as a linear function of Sharpe ratio θ, as described in Sharpe (1994), as

$$\tilde{R} - R = \beta_0 + \beta_1\theta + \varepsilon \tag{10}$$

where β_0 is the intercept, β_1 is the slope, and ε is a normally distributed random variable. We define the Sharpe ratio in Eq. (10) as

$$\theta = \frac{R - r_f}{\sigma}, \tag{11}$$

where R is the annual return of the underlying ETF, r_f is the annualized risk free rate obtained from the 30-day LIBOR rate, and σ is annualized volatility. Fig. 4 shows this relationship for stocks, while Fig. 5 shows a similar result for bonds based on their replicated annual values from 1989 to 2017, where the call outs in these figures show the year.

The values of R^2 at 82% for stocks and 97% for bonds produced in Figs. 4 and 5 supports our hypothesis that the underlying Sharpe ratio has a direct and strong correlation to excess returns before leverage. The coefficients and their statistical significance appear in Table 3. All coefficients are statistically significant at either the 0.05 or 0.001 level.

From these results, we can confirm that periods of upward trending prices with lower volatility will produce the highest excess returns. Conversely, periods of flat prices and high volatility will produce lower or negative excess returns.

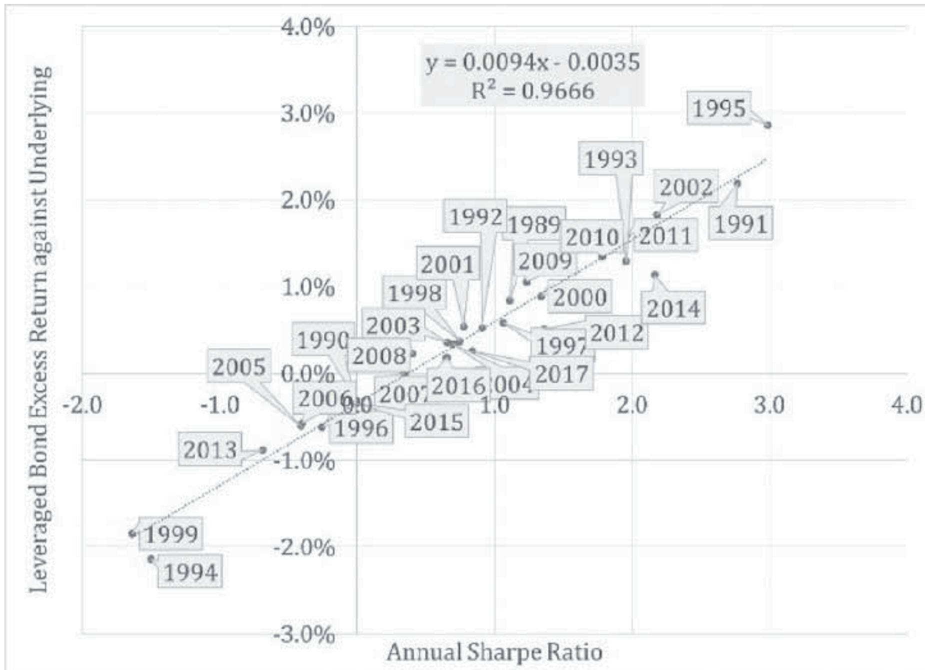

Fig. 5. Excess returns from 1.25X leveraged bond ETF versus its underlying ETF, net of fees and expenses, 1989–2017.

3. Research methods

We next investigate how a 1.25X stock and bond ETFs perform in a periodically rebalanced portfolio. Simulation methods are used to determine a distribution of 10 years of future price paths of the underlying assets. From these paths, we determine a distribution of annualized returns and Sharpe ratios for each portfolio. We selected 10,000 trials, because they produced the half-width of a 95% confidence interval for returns which is less than 0.001.[4] All simulation models were completed in Excel using VBA macros, and are available upon request.

Both of our sampling methods assume that price changes are independent of one another, so follow a Markov or memoryless process, as suggested by Fama (1965a, 1965b). Our first approach is to simulate prices from a geometric Brownian motion process. Our second simulation approach is often termed "bootstrapping," because it samples from a historical distribution. Two excellent references for this methodology are Davison and Hinkley (1993) and Efron and Tibshirani (1993).

Table 3 Regression coefficients and p-values for coefficients appearing in Figures 3 and 4, indicating statistical significance between the underlying Sharpe ratio and excess returns

N = 29 (1989–2017)	Coefficient	p-value
β_1 (stocks)	0.0332	<0.001
β_0 (stocks)	−0.0091	<0.05
β_1 (bonds)	0.0094	<0.001
β_0 (bonds)	−0.0035	<0.001

Table 4 Continuous return and volatility parameters in geometric Brownian motion (GBM) simulation

Stock		Bond		Risk-free asset	
μ	σ	μ	σ	μ	σ
10.6%	17.8%	5.9%	3.9%	3.2%	0.2%

3.1. Simulation of leveraged and unleveraged stock and bond prices with a GBM process

The first model is a Monte Carlo simulation that assumes prices follow a geometric Brownian motion (GBM) process. Here, a future asset price at time (S_{t+1}) is found as

$$s_{t+1} = s_t e^{\left(\mu - \frac{\sigma_2}{2}\right)\Delta t + \sigma \Delta t \varepsilon} \tag{12}$$

where s_t is the current price, μ is the return rate, σ is the volatility or standard deviation of returns, Δt is the time increment between the current and future price, and ε is a random number obtained from the standard normal distribution.

The use of a GBM process has a long history of modeling prices of financial assets. The GBM process generates future prices that are lognormally distributed with variance growing over time, assumes returns are normally distributed, and allows the modeler to either calibrate the GBM process parameters (μ and σ) from market data or set them to some future expectation. The downside of using GBM process to model security prices is that, because it assumes returns are normally distributed, it assumes a very small likelihood of extreme (greater than a few standard deviations) returns. In practice, it is well known that "fat tails" of returns exist, meaning the probability of more extreme returns is not modeled well by the GBM process. Please see the seminal work by Fama (1965a, 1965b) for a discussion of this behavior in commons stock. Nevertheless, modeling stock and bond ETF prices using a GBM process is fairly straightforward, and can be used to illustrate portfolio performance under a variety of parameters selected.

Using a GBM process, 10-year investment returns were evaluated for a variety of rebalancing strategies. Drift and volatility were estimated from the 29 year history from 1989 to 2017, and appear in Table 4 for stocks, bonds, and the risk-free asset. The simulation also included correlation between stock and bond returns set to −9.5%, as observed over this time period.[5] Correlations between bonds and risk-free rate and stocks and the risk-free rate were negligible over this time period, so this correlation was not included in the simulated results.

Additional parameters for transaction costs were also included. The bid-ask spread for stock and bonds was set to 0.01%, and for leveraged stock and bonds to 0.04%. Trading commissions were set to $4.95 per trade for an account with $200,000 at the start of each 10 year simulation.

Results for the baseline GBM simulation appear in Table 5. As expected, the mean annualized return for the unleveraged stock holdings are 9.5%, which is less than the continuous return parameter of 10.6% assumed in Table 5. This effect is aptly described in Winston (2008), because over time, volatility creates a drag on the growth rate of a stock modeled by a GBM process. However, a little surprisingly, average annualized unleveraged bond returns are slightly higher. We attribute this to the much smaller volatility of the bond

Table 5 Averaged annualized returns and Sharpe ratios for 10,000 trial GMB simulation

10,000 trial GBM	Mean annualized return		Mean Sharpe ratio	
	Unleveraged	Leveraged	Unleveraged	Leveraged
100% stock ETF	9.5%	10.3%	0.118	0.114
100% bond ETF	6.0%	6.3%	0.202	0.183
60/40 stock bond portfolios				
Annual	8.5%	9.3%	0.145	0.138
Quarterly	8.5%	9.3%	0.148	0.140
Monthly	8.4%	9.2%	0.145	0.138

Note: ETF = exchange-traded funds; GBM = geometric Brownian motion.

markets effectively eliminating volatility drag, as well as the effect of continuously versus annually compounded rates causing a small perturbation on the numerical results.[6] Table 5 also shows the average annualized returns for the leveraged stock and bond ETFs, where excess stock annual returns are 0.8% and excess bond annual returns are 0.3%. Reviewing the mean Sharpe ratios of the stock and bond ETFs, we see that Sharpe ratios are modestly reduced by the 1.25X stock and bond ETFs. We attribute this to financing costs exceeding the risk-free rate, higher expense ratio of leveraged funds, and the path dependent effect of daily leverage. The effect of leverage on Sharpe ratios is also consistent with Mulvey et al. (2007).

Table 5 also displays the performance of the 60/40 leveraged stock/bond portfolio that is periodically rebalanced. We observe that excess returns are approximately 0.8% annually, and does not appear to depend on the rebalancing interval, which is beneficial to individual investors who may prefer to rebalance once a year. Unfortunately, risk-adjusted return measured by Sharpe ratio reduced by about 5% when rebalanced annually or more. Table 5 also shows that rebalancing on a quarterly basis provided the optimal Sharpe ratio, indicating that the effect of spreads and trading commissions associated with portfolio turnover for a 1.25X leveraged stock and bond ETF do not significantly cause a drag on risk-adjusted performance.

With our baseline results established in Table 5, we next investigated the sensitivity of these results to the volatility of the stock ETF. To this end, we kept all other parameters constant, and considered high and low volatility values. As expected, decreasing volatility increases excess returns. Table 6 shows the results when stock volatility is decreased to 10% and increased to 25% with an annual rebalancing policy to 60/40 stock/bonds. Decreasing the volatility to 10% increases excess return to 1.0%, while increasing the volatility to 25% reduces excess return to 0.4%. Sharpe ratios tell a similar story, where lower volatility

Table 6 Averaged annualized returns and Sharpe ratios for above and below average long-term volatility for an annually rebalanced portfolio of 60/40 stock and bonds

Annual rebalancing policy sensitivity to volatility	Mean annualized return		Mean Sharpe ratio	
	Unleveraged	Leveraged	Unleveraged	Leveraged
σ (baseline from Table 5)	8.5%	9.3%	0.145	0.138
σ (low volatility)	8.9%	9.9%	0.260	0.247
σ (high volatility)	7.9%	8.3%	0.101	0.096

Table 7 Averaged annualized returns and Sharpe Ratios for 10,000 trial GBM simulation (monthly vs. daily leverage)

10,000 trial GBM	Mean annualized return		Mean Sharpe Ratio	
	Leveraged (monthly)	Leveraged (daily)	Leveraged (monthly)	Leveraged (daily)
100% stock ETF	10.2%	10.3%	0.113	0.114
100% bond ETF	6.3%	6.3%	0.179	0.183
60/40 stock bond portfolios				
Annual	9.2%	9.3%	0.138	0.138
Quarterly	9.3%	9.3%	0.140	0.140
Monthly	9.2%	9.2%	0.138	0.138

Note: ETF = exchange-traded funds; GBM = geometric Brownian motion.

increases the Sharpe ratio of both the leveraged and unleveraged portfolios. However, the 5% reduction in Sharpe ratio because of leveraged shows little, if any, significant change because of changing volatility. We can conclude that the reduction in Sharpe ratio because of using the 1.25X stock and bond ETFs is insensitive to the underlying stock ETF volatility, when returns are kept constant.

The last row in Table 6 provides for an interesting extension to the monthly 1.25X mutual fund DXNLX mentioned previously. Using data obtained from Nasdaq Global Indices,[7] 237 monthly returns were available from May 31, 1999 through December 31, 2018. The mean annualized return over this period was 9.3%, and mean annualized volatility was 24.3%. Thus, the sensitivity to higher volatility shown in Table 6 can be representative of a daily leveraged stock fund that had a higher volatility than the S&P 500, like the Nasdaq-100.

Next, we investigated how the simulation results would change in Table 5 change if, instead of 1.25 daily leverage, we used 1.25 monthly leverage. Table 7 shows this effect, with all other model parameters set to their baseline values from Tables 2 and 4, to isolate the effect of monthly versus daily leverage on absolute and risk-adjusted returns.

The results in Table 7 suggest that choosing a fund that has monthly versus daily 1.25 leverage changes stock and bond fund returns by a negligible amount. There is also a small, yet nearly insignificant change in Sharpe ratio in the third decimal place. This very small affect yields no change to the 60/40 leveraged portfolios, regardless of rebalancing period. We can conclude from these results that an investor can be indifferent regarding choosing a broad-based stock or bond fund with monthly versus daily leverage, assuming expenses are kept constant.

3.2. GBM limitations

The GBM process generally models daily returns of stocks and bonds well. However, exceptions occur. The histograms in Fig. 6 shows daily returns from 1989 to 2017 for our replicated stock and bond ETFs. In the top pane of Fig. 6, the normal distribution, labeled "Normal, σ" appears to miss the peak of the observed stock market returns. Reducing σ in the normal distribution to 75% of its original value appears to improve the fit for observations near the center of the distribution, but aggravates the already poor fit of return extremes, such as the daily returns from October of 2008. The normal distribution of bond returns exhibits

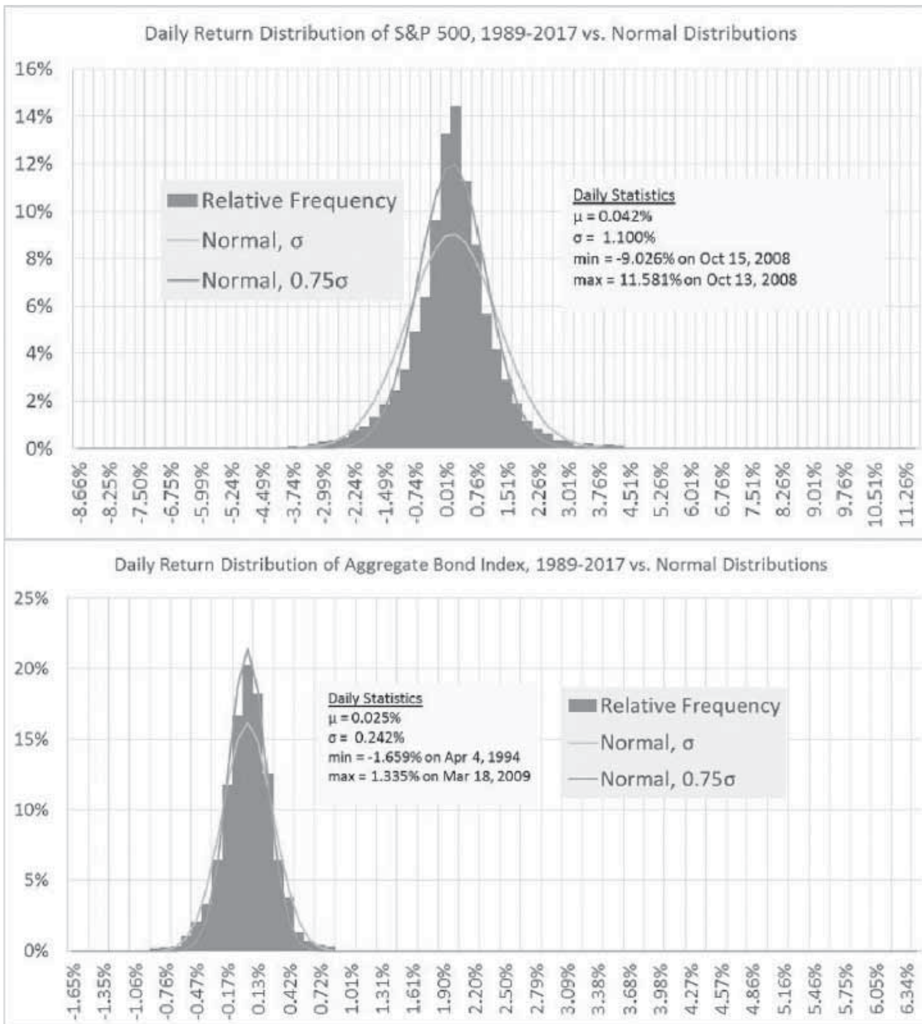

Fig. 6. Daily return distribution of stock (top pane) and bond market (bottom pane) indices. Two normal distributions are also shown, with volatility estimates using historical returns from 1989 to 2017. Reducing the volatility appears to provide a slightly improved fit near the center of the distribution, but worsens the fit in the distribution tails.

similar, but less pronounced differences between observed daily returns and the theoretical expectations from a normal distribution. Thus, the volatility assumption made in a GBM simulation may not fit the return distribution well. Because we know that excess returns are sensitive to volatility, an alternate simulation approach termed "bootstrapping" is used to evaluate the performance of leveraged ETFs under actual market conditions. The bootstrapping approach also benefits from imposing the risk-free rates observed during particular market periods, as opposed to modeling the evolution of the risk-free asset as a GBM process.

3.3. Bootstrapping using 10-year historical periods

Because the return distribution shown above suffers from some modeling issues, we also investigate leveraged ETF performance using a bootstrapping model. Applying the method-

Step	Simulation process
1	Assign a numerical index of 1 to n for each of the daily returns observed in a historical 10-year period. Assuming 252 trading days in a year yields $n \approx 2{,}520$.
2	For each day, determine the cumulative return up to and including the previous 21 days, so that the accumulated returns from these days are representative of the distribution of monthly returns.
3	Generate 120 random numbers ranging from 1 to n.
4	Select returns from the distribution of representative monthly returns determined in Step 2 using the random numbers found in Step 3, and generate 10 years of monthly returns. Use the same set of random numbers for each trial to select monthly returns from each of investment category (stock, bonds, and risk-free asset), ensuring that the historical correlation among assets is preserved.
5	Repeat for 10,000 trials, collecting annualized return and Sharpe ratio for each trial.
6	Determine mean annualized return and Sharpe ratios found from the 10,000 trials generated in Step 5.

ology proposed in DiLellio et al. (2014), we generate returns by randomly sampling empirical return histories observed from market data. By using this alternative simulation approach, the "fat tails" that occurred over the empirical return histories can be properly included in a portfolio performance evaluation, as well as the higher peaks near the center of the distribution of stock and bond returns. The following steps are adapted from the methodology found in the Appendix of DiLellio et al. (2014).

A key input to any bootstrapping simulation is the time period selected. To select two meaningful periods representing extremes in stock market returns and volatility, we obtained 51 years of daily returns for the S&P 500 total return index from the Center for Research in Security Prices (CRSP). Next, we determined 10-year simple moving average annual returns and volatilities, and divided them by their corresponding 51 year averages of 11.1% for annual returns and 15.1% for annual volatility, respectively. Thus, values above one represent above average returns/volatilities, and values less than one represent below average returns/volatilities. The results appear in Fig. 7, where the call outs designate the final year in each 10-year moving average.

To select two extremes, we selected the 10 years ending in 2000 and 2010. The first period is from 1991 to 2000, when above average returns and below average volatility of stocks occurred. This first period corresponds to the equity bull market often associated with the tech bubble. The second period is from 2001 to 2010, when below average stock returns occurred with higher than average volatility. This second period includes the tech bubble correction as well as the financial crisis of 2008. These periods also benefit from being non-overlapping, so using them eliminates the possibility of temporal correlation.

It should be noted that there are some other interesting 10-year periods that may be worth investigating. For example, the 10 years ending in 1990 and 1979. Unfortunately, our Bloomberg data set for daily stock and bond indices does not extend this far back, as noted previously. Also, the 1-month LIBOR rates were not available on a daily basis before the mid-1980s.

Tables 8 and 9 below highlight how bootstrapping from these unique 10-year periods affect excess returns. In Table 8, the period of 1991–2000 produced excess returns from the leveraged stock(bond) ETF of 2.5% (0.3%). These returns then translate to excess returns for a 60/40 stock bond portfolio, with a value of approximately 1.7% annually when rebalanced annually, quarterly, or monthly. Thus, given the small spreads and low turnover of the 1.25X

Fig. 7. The 10-year moving average of stock market returns and volatilities divided by 51 year average. Values above one represent above average returns/volatilities, and values less than one represent below average returns/volatilities.

portfolio, rebalancing frequency does not affect excess returns. Again, these excess returns are because of this period containing historically lower volatilities and higher returns, as predicted by Trainor and Carrol (2013). Unfortunately, the favorable excess returns do not translate to an increase in the mean Sharpe ratio, which is similar to results from the GBM simulation approach. From Table 8, we see that the stock ETF Sharpe ratio is reduced by 3%

Table 8 Bootstrapping simulated results from 1991 to 2000, where 60/40 portfolio excess returns were approximately 1.7% annually; Sharpe ratio reduced by 4%

10,000 trial bootstrapping 1991–2000	Mean annualized return		Mean Sharpe ratio	
	Unleveraged	Leveraged	Unleveraged	Leveraged
100% stock ETF	17.8%	20.3%	0.269	0.260
100% bond ETF	8.0%	8.3%	0.194	0.175
60/40 stock bond portfolios				
Annual	14.0%	15.7%	0.283	0.272
Quarterly	13.9%	15.6%	0.284	0.273
Monthly	13.9%	15.6%	0.285	0.274

Note: ETF = exchange-traded funds.

Table 9 Bootstrapping simulated results from 2001 to 2010, where 60/40 portfolio excess returns were approximately −0.4% annually; Sharpe ratio reduced by 23%

10,000 trial bootstrapping 2000–2010	Mean annualized return		Mean Sharpe ratio	
	Unleveraged	Leveraged	Unleveraged	Leveraged
100% stock ETF	1.4%	0.1%	0.010	0.003
100% bond ETF	5.8%	6.3%	0.238	0.218
60/40 stock bond portfolios				
Annual	3.6%	3.2%	0.043	0.033
Quarterly	3.5%	3.1%	0.043	0.033
Monthly	3.5%	3.1%	0.043	0.034

Note: ETF = exchange-traded funds.

and bond ETF Sharpe ratio is reduced by 10% with 1.25X leverage. The 60/40 stock/bond portfolios see the Sharpe ratio reductions of 4%, regardless of the rebalancing frequency.

Table 9 shows a very different story, and summarizes the performance by sampling from the 2000–2010 return history with its high volatility and low returns of the stock ETF. Here, excess returns for the leveraged stock fund are no longer positive, with a value of −1.3%. Fortunately, the leveraged bond ETF fared better, with a 0.5% annual excess return. However, this positive excess return from the leveraged bond ETF could not be entirely offset by the leveraged stock loss, so the 1.25X leveraged 60/40 stock bond investment had negative excess returns of 0.4% annually under a variety of rebalancing policies ranging from annually to quarterly.

Results for the mean Sharpe ratio are more adversely affected when the volatility is high and returns are low. Leverage in the stock ETF reduces the averaged Sharpe ratio by 70%, while the bond ETF reduction is a more modest 8% reduction. The stock ETF Sharpe ratio reduction is extreme, because the average daily return of the leveraged stock ETF narrowly exceeded the risk-free rate in 2001–2010. Consequently, the leveraged 60/40 portfolio reduces the Sharpe ratio by 23%, regardless of rebalancing frequency.

4. Conclusions

This article investigated the use of 1.25X leveraged stock and bond exchange-traded funds (ETFs) as a stock and bond asset allocation strategy. Analytical conditions of return and volatility leading to excess returns over unleveraged funds is derived and agree favorably with empirical findings from 1989 to 2017, including all relevant costs. We also show the relationship of excess returns and the underlying's Sharpe ratio, demonstrating high levels of statistical significance.

We use a GBM process to simulate asset prices to determine a first approximation of future portfolio returns. We find that at 60/40 leveraged stock/bond fund generates excess returns of 0.8% annually (net of fees and expenses) but reduces the Sharpe ratio by 5% because of the financing exceeding the risk-free rate, higher volatility, and higher expense ratios of leveraged funds. We also show that changing leverage from daily to monthly has a negligible effect on the simulation results.

By using a bootstrapping simulation approach, non-overlapping periods are also evaluated using specific market conditions. We show the effect the 1.25X leverage can produce excess returns for a 60/40 portfolio of stocks and bonds of 1.7% (−0.4%) annually in higher return/lower volatility (lower return/higher volatility) equity markets, relative to their non-leveraged counterparts, and net of fees and expenses. The resulting Sharpe ratios are reduced between 4 and 23%, depending on the behavior of the stock and bond market.

We can conclude from these findings that some leverage, such as the 1.25X factor assumed here, can have long term net return benefits to a 60/40 stock/bond portfolio under upward trending and lower volatility conditions of the underlying asset. Additionally, while some individual investors may be unwilling to reduce their Sharpe ratio, the modest reduction found here may be acceptable to individual investors seeking to increase their returns and not able or willing to access margin. We conclude that this investment strategy could be well suited for investors more interested in total returns during upward trending markets.

5. Future work

There are several additional avenues for future work in this area. Perhaps the most obvious question is related to the best amount of fractional leverage. This determination would likely have a behavioral component, since additional leverage will likely magnify the marginal reduction in Sharpe ratio, which at some point an investor may not be willing to consider. One could also expand this work to include other equity asset classes to enhance diversification benefits and reduce portfolio risk. These additional asset classes could include small caps, international, commodities, and real estate investment trusts. Lastly, we could evaluate how fractionally leveraged ETFs may exist on the efficient frontier, and develop optimal portfolio strategies based on these new insights.

Notes

1 https://www.fool.com/knowledge-center/can-you-trade-on-margin-in-an-ira.aspx.
2 https://fred.stlouisfed.org/series/USD1MTD156N.
3 From 1989 to 2017, the correlation between annual volatility of the S&P500 and the annual LIBOR rate was −8.8%, suggesting no significant relationship between these two variables.
4 The primary driver on sample size was the underlying asset's volatility.
5 In the article by Johnson et al. (2013), the correlation between stocks and treasury bonds is shown to change over time because of macroeconomic conditions. However, we did not attempt to include an econometric model like the one developed by these authors, instead using the long-term correlation.
6 In this example, the continuous rate for bonds was 5.9% and volatility 3.9%, so that exp(5.9% - 0.5*3.9%2) = 1.060, implying an annually compounded rate was 6%, as shown in Table 5.
7 https://www.quandl.com/data/NASDAQOMX/XNDX-NASDAQ-100-Total-Return-XNDX.

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Table A1 Annualized returns and volatility of unleveraged and leveraged stock ETF

Year	*n*	S&P 500 Index annual return	S&P 500 Index annual volatility	S&P 500 ETF annual return	1.25X annual volatility	1.25X annual return	Excess return
1989	252	31.68%	13.03%	31.63%	16.28%	36.934%	5.30%
1990	253	−3.10%	16.10%	−3.14%	20.13%	−6.58%	−3.43%
1991	253	30.47%	14.32%	30.41%	17.90%	36.41%	6.00%
1992	254	7.62%	9.73%	7.58%	12.16%	8.01%	0.43%
1993	253	10.08%	8.63%	10.03%	10.79%	11.30%	1.26%
1994	252	1.32%	9.82%	1.28%	12.28%	−0.01%	−1.29%
1995	252	37.58%	7.83%	37.52%	9.78%	46.09%	8.57%
1996	254	22.96%	11.82%	22.91%	14.78%	26.95%	4.04%
1997	253	33.36%	18.16%	33.31%	22.70%	40.02%	6.71%
1998	252	28.58%	20.29%	28.53%	25.37%	33.64%	5.11%
1999	252	21.04%	18.08%	20.99%	22.60%	24.19%	3.20%
2000	252	−9.10%	22.22%	−9.14%	27.77%	−13.66%	−4.52%
2001	248	−11.89%	21.38%	−11.92%	26.72%	−16.36%	−4.44%
2002	252	−22.10%	26.04%	−22.13%	32.55%	−28.18%	−6.04%
2003	252	28.68%	17.07%	28.63%	21.34%	35.51%	6.87%
2004	252	10.88%	11.10%	10.84%	13.87%	12.71%	1.87%
2005	252	4.91%	10.29%	4.87%	12.86%	4.71%	−0.16%
2006	251	15.79%	10.02%	15.75%	12.53%	17.97%	2.22%
2007	251	5.49%	15.96%	5.45%	19.95%	4.70%	−0.75%
2008	253	−37.00%	41.05%	−37.02%	51.31%	−45.91%	−8.89%
2009	252	26.46%	27.28%	26.41%	34.09%	31.95%	5.54%
2010	252	15.06%	18.06%	15.02%	22.58%	18.03%	3.02%
2011	252	2.11%	23.29%	2.07%	29.11%	1.33%	−0.74%
2012	250	16.00%	12.70%	15.96%	15.88%	19.56%	3.60%
2013	252	32.39%	11.07%	32.34%	13.84%	41.12%	8.79%
2014	252	13.69%	11.38%	13.64%	14.23%	16.66%	3.02%
2015	252	1.38%	15.49%	1.34%	19.37%	0.91%	−0.43%
2016	252	11.96%	13.10%	11.92%	16.37%	14.28%	2.36%
2017	251	21.83%	6.67%	21.78%	8.33%	27.07%	5.28%

Note: ETF = exchange-traded funds.

Table A2 Annualized returns and volatility of unleveraged and leveraged bond ETF

Year	n	Aggregate bond index annual return	Aggregate bond index annual volatility	Aggregate bond ETF annual return	1.25X annual volatility	1.25X annual return	Excess return
1989	250	14.53%	3.99%	14.47%	4.98%	15.32%	0.84%
1990	251	8.96%	4.26%	8.91%	5.32%	8.61%	−0.30%
1991	248	16.00%	3.47%	15.95%	4.34%	18.17%	2.22%
1992	251	7.40%	3.73%	7.35%	4.66%	7.88%	0.54%
1993	249	9.75%	3.23%	9.69%	4.04%	11.01%	1.31%
1994	249	−2.92%	5.13%	−2.97%	6.41%	−5.09%	−2.13%
1995	248	18.47%	4.04%	18.42%	5.05%	21.31%	2.89%
1996	249	3.63%	4.74%	3.58%	5.92%	2.73%	−0.85%
1997	250	9.65%	3.34%	9.60%	4.18%	10.20%	0.60%
1998	250	8.69%	3.60%	8.63%	4.50%	9.01%	0.38%
1999	250	−0.82%	3.94%	−0.87%	4.93%	−2.70%	−1.83%
2000	251	11.63%	3.54%	11.57%	4.43%	12.47%	0.90%
2001	247	8.44%	4.46%	8.39%	5.57%	9.16%	0.77%
2002	251	10.25%	3.84%	10.20%	4.79%	12.03%	1.83%
2003	250	4.10%	4.27%	4.05%	5.33%	4.41%	0.36%
2004	251	4.34%	3.89%	4.29%	4.86%	4.63%	0.35%
2005	250	2.43%	2.89%	2.378%	3.61%	1.78%	−0.60%
2006	250	4.33%	2.78%	4.28%	3.47%	3.71%	−0.57%
2007	250	6.97%	3.51%	6.91%	4.38%	6.95%	0.03%
2008	251	5.24%	5.91%	5.19%	7.39%	5.42%	0.24%
2009	250	5.93%	4.49%	5.88%	5.61%	6.94%	1.06%
2010	252	6.54%	3.72%	6.49%	4.64%	7.73%	1.24%
2011	250	7.84%	3.85%	7.79%	4.81%	9.39%	1.60%
2012	250	4.22%	2.50%	4.16%	3.12%	4.82%	0.66%
2013	250	−2.02%	3.30%	−2.07%	4.13%	−2.96%	−0.88%
2014	250	5.97%	2.66%	5.91%	3.32%	7.05%	1.14%
2015	251	0.55%	3.78%	0.50%	4.72%	0.23%	−0.27%
2016	250	2.65%	3.19%	2.60%	3.99%	2.78%	0.19%
2017	250	3.54%	2.80%	3.49%	3.50%	3.75%	0.26%

Note: ETF = exchange-traded funds.

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