

## A portfolio of leveraged exchange traded funds

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### Abstract

This study demonstrates how a portfolio of leveraged exchange traded funds (ETFs) targeting a unit exposure to their underlying indexes outperforms a portfolio using traditional ETFs while simultaneously reducing downside risk. By extension, a 3x ETF portfolio designed to mimic 2x ETFs outperforms the underlying 2x ETF portfolio. The results are primarily a function of ETFs borrowing short while the investor lends the additional wealth generated from this leverage in one- to seven-year Treasury bonds or similar type of assets. For every one percent earned above the implied borrowing rate, a portfolio of 2x and 3x ETFs outperforms a traditional portfolio by 0.41% and 0.63%, respectively, corresponding roughly to the additional return on the 50% and 67% of the wealth invested in bonds. More than 90% of ETFs outperformance is explained by the borrowing lending differential. © 2020 Academy of Financial Services. All rights reserved.

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### 1. Introduction

Leveraged exchange traded funds (ETFs) were first listed in 2006 by Proshares, although leveraged mutual funds have been around since 1993. While Proshares introduced  $\pm 2x$  products, Direxion upped the leverage ante with  $\pm 3x$  funds in late 2008. Because ETFs are designed to return a daily multiple, the constant daily leverage results in uncertain realized

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leverage over longer periods of time. In general, realized leverage tends to fall over time because of the volatility of returns (Avenllaneda and Zhang, 2010; Carver, 2009; Cheng and Madhavan, 2009; Trainor and Baryla, 2008).

Historically, investors have used margin to create leverage in their investments. However, with LETFs expansion into everything from equity indexes to oil, gold, currencies, and treasuries, it is now possible to create a diversified portfolio of LETFs that mimic a typical investor's portfolio. This can be done by creating unit exposure to the underlying asset classes freeing up wealth to enhance returns. Specifically, 2x and 3x LETFs only require 50% and 33% of investor's wealth to create the same exposure as investing 100% of an investor's wealth in the underlying ETFs or mutual funds. The advantages of using LETFs, instead of margin, are leverage can be theoretically increased to 3x; no explicit interest costs, no possibility of margin calls, and unlike margin, LETFs can be used in most retirement accounts.

However, LETF's leverage is not free as there are implicit lending costs. In addition, LETFs have higher expense ratios and general leverage decay. To the extent an investor seeks to maintain a specific asset allocation, frequent rebalancing is required along with the associated trading costs and taxes on realized gains. A LETF portfolio structured for unit exposure will outperform its traditional counterpart if and only if the return to the invested excess wealth exceeds the implicit financing costs and higher costs of using LETFs.

In contrast to previous research, this study examines how a portfolio composed almost entirely of LETFs can be created to maintain unit exposure to the underlying indexes without increasing the overall risk. George and Trainor (2017) use similar methodology, but where they use a single LETF to demonstrate how it can be used within a portfolio insurance setting, this study uses a portfolio of LETFs to maintain an asset allocation comparable with traditional buy-and-hold portfolios created from mutual funds or ETFs.

In addition, for risk-seeking investors looking to invest in a diversified portfolio with up to 2x leverage, this study examines if 3x LETFs may be used to the same effect. Investing 67% in a 3x LETF is equivalent to 100% in a 2x LETF. If the return to the remaining 33% of the investor's wealth exceeds the additional borrowing cost, the 3x LETF will outperform.

A critical question for the 2x or 3x strategy suggested above is where to invest the freed-up wealth. This study investigates leveraging bonds instead of equity as this will maintain the risk and structural characteristics of the underlying unleveraged portfolio. Because LETFs typically borrow short to attain their exposure, the portfolios investigated in this study are in effect borrowing short to lend long. This strategy should moderately increase expected return and reduce downside risk as most of the portfolio is invested in less risky bonds.

Results suggest a portfolio composed of 2x or 3x LETFs outperforms a portfolio using standard ETFs based on the same underlying indexes. Even in the low interest environment from 2010 to 17 and using an aggregate bond index for the remaining wealth, a portfolio of 2x or 3x LETFs outperforms a portfolio of standard ETFs by 0.9% and 1.8%, respectively, on an annual basis. For risk seekers, using 3x LETFs to mimic the exposure of 2x LETFs outperforms by 1.5% annually during this period. The critical input is the borrowing-lending differential that explains 90% or more of the LETF portfolio's greater return. The results are essentially confirmed with simulated LETF returns since 1946.

The caveat to this type of strategy is LETF portfolios must be rebalanced more often as their initial positions deviate from “optimal” asset allocations even faster than standard portfolios. A 10% barrier threshold defined as a relative deviation of 10% from the initial allocation is used to determine when portfolios are rebalanced. This active management keeps the risk exposure within reasonable bounds while keeping 2x and 3x rebalancing requirements to approximately quarterly and monthly, respectively. Because of tax effects, the strategies outlined above are more suited to qualified nontaxable accounts.

## 2. Mathematics of LETFs

LETFs magnify the daily return of an underlying index. Because of this constant daily leverage, the realized leverage over any multiday period can be virtually anything and is a function of the daily leverage ratio, time, return, and variance with the latter generally having the largest effect. Realized leverage can mathematically be expressed by:

$$\frac{LR_T}{XR_T} = \frac{(1 + XR_T)^L \exp\left(\frac{(L - L^2) \sigma^2 T}{2}\right) - 1}{XR_T} \quad (1)$$

where  $LR_T$  is the return to the leveraged fund,  $XR_T$  is the underlying index return,  $L$  is the daily leverage ratio,  $T$  is time in days, and  $\sigma^2$  is the standard daily population variance, (Avellaneda and Zhang, 2010; Cheng and Madhavan, 2009).

On average, the variance dominates and realized leverage over time tends to decline. This effect is greater with higher leverage since a daily leverage ratio of 2x multiplies the term  $(1 + XR_T)^L$  by  $\exp(-\sigma^2 T)$  but a daily leverage ratio of 3x multiplies this term by  $\exp(-3\sigma^2 T)$ .<sup>1</sup> Equation (1) is related to volatility drag, which is the difference between geometric and arithmetic average returns and is a major hindrance to LETF returns over extended periods of time. The relationship between the geometric and arithmetic return of any asset is written as:

$$X_{pT} = X_{RT} - 0.5 \sigma_t^2 \quad (2)$$

where  $X_{pT}$  is the geometric return,  $X_{RT}$  is the arithmetic return, and  $\sigma_t^2$  is the variance of returns.

As an example, assume a daily return and standard deviation of 0.045% and 1.0%, respectively, which roughly corresponds to the S&P 500 market averages from 1946 to 2017. A 3x LETF multiplies these numbers by 3. Thus, the geometric return over a year for the underlying index assuming 252 trading days is  $252 * 0.045\% - 0.5 * 2.52\% = 10.08\%$ , and for a 3x this return is  $252 * 0.135\% - 0.5 * 22.68\% = 22.68\%$ .<sup>2</sup> For long-term holdings of LETFs, this volatility drag is a major drawback and clearly shows why leveraged funds usually do not return the daily multiple of the underlying index over time.

Expanding on the work of Scott and Watson (2013), Ott and Zimmer (2016) show the return of an investment with leverage  $L$  is:

$$L_{RT} = X_{RT} + (X_{RT} - R_B)(L - 1) - \frac{1}{2}(\sigma^2 L^2) \quad (3)$$

where  $L_{RT}$  is the leverage return,  $X_{RT}$  is the underlying index return,  $R_B$  is the borrowing rate,  $L$  is the leverage ratio, and  $\sigma^2$  is the variance of the underlying index.

To reduce volatility drag relative to the underlying index, this study suggests using only a portion of the investor's portfolio to invest in the ETF. This percentage is set so the effective exposure to the index is the same as if the investor invested only in the underlying index. In effect, the volatility of the position in a ETF is no greater than investing directly in the underlying index. This results in additional wealth available to offset the implied borrowing costs and higher expense ratio of the ETFs. In this way, Equation (3) can be modified to account for the return of a portfolio of ETFs or a portfolio of ETFs.

To simplify, assume a portfolio is composed of just one underlying index. Accounting for the ETF's higher expense ratio along with the additional wealth created from using ETFs, Equation (3) becomes the following:

$$L_{PT} = \frac{1}{L}[X_{RT} + (X_{RT} - R_B)(L - 1) - R_{exp}] - \frac{1}{2}\sigma^2 + \left(1 - \frac{1}{L}\right)R_f \quad (4)$$

where  $L_{PT}$  is the return to the portfolio using the ETF,  $R_{exp}$  is the ETF expense ratio, and  $R_f$  is the return to the risk-free asset.<sup>3</sup> If this asset is not risk-free, there is volatility drag to this asset's return which could easily be adjusted for in Equation (4).

The returns are multiplied by  $1/L$  because only a portion of the portfolio relative to the underlying index is invested in the ETF. When  $L$  is 1, signifying no leverage, Equation (4) basically reduces to Equation (2). With leverage, the ETF return is multiplied by the inverse of the leverage. For example, with  $L$  equal to 3, the investor attains exactly  $X_{RT}$  assuming no borrowing costs or expenses. This result attains since one third of an investor's wealth in a 3x is basically the same as if they had invested 100% in the underlying index. With borrowing costs and higher expenses, they are not equivalent. However, the difference is offset by the return in a risk-free or some other alternative asset represented by  $R_f$ .

Volatility drag for wealth in the ETF or in the underlying index is the same as both have equal variance. The volatility drag will only become an issue if the percentage in the underlying ETF varies to the point where the effective exposure of the ETF portfolio diverges from a portfolio using the underlying index. With constant rebalancing, this discrepancy can be eliminated or at least effectively managed. Constant or daily rebalancing is infeasible for most investors because of transaction costs but can be mitigated by periodic rebalancing to keep the  $1/L$  ratio relatively constant over time. Lu, Wang, and Zhang (2012) find an investor can assume a 2x/-2x ETF will maintain its leverage ratio for holding periods up to one month. Thus, daily rebalancing is likely not needed.

To calculate the return from using a ETF relative to investing in the underlying index, Equation (2) is subtracted from Equation (4) to attain the following:

$$L_{PT} - X_{PT} = \left(1 - \frac{1}{L}\right)R_f - 1/L[(R_B)(L - 1) + R_{exp}] \quad (5)$$

Table 1 Portfolio composition using standard ETFs and LETFs

Asset class	ETF	2x LETF	3x LETF
S&P 500	SPY, 30%	SSO, 15%	UPRO, 10%
Mid cap	IJH, 10%	MVV, 5%	MIDU, 3.33%
Small cap	IWM, 10%	UWM, 5%	TNA, 3.33%
International developed	EFA, 10%	EFO, 5%	EURL, 3.33%
Emerging markets	EEM, 10%	EET, 5%	EDC, 3.33%
Real estate	IYR, 5%	URE, 2.5%	DRN, 1.67%
20-year T-bonds	TLT, 5%	UBT, 2.5%	TMF, 1.67%
7 to 10-year T-bonds	IEF, 5%	UST, 2.5%	TYD, 1.67%
Aggregate bonds	BND, 15%	BND, 57.5%	BND, 71.67%

*Note:* The base percentage in each asset class is set by the initial percentages in the ETFs. Ticker symbols used to calculate results are shown with each percentage.

Equation (5) can be extended to account for multiple underlying asset classes that may be used by an investor. Similarly, one can compare a 3x LETF with a 2x LETF. However, the basic implication of Equation (5) remains the same. If the excess wealth from using LETFs to create a portfolio (the first term in Equation [5]) is greater than the borrowing costs and higher expense ratio of the LETFs, then the return from using LETFs will be greater than the return from using the underlying indexes. For the comparison of two LETFs, the expense ratios will be approximately the same which reduces Equation (5) to whether the additional wealth gained from the higher leveraged LETF earns a return that exceeds its additional financing costs. This study determines whether this is the case.

### 3. Data and methodology

Because most LETFs were only recently introduced to the market, empirical research is limited. However, there are now 2x and 3x LETFs that cover the main asset categories found in a typical diversified portfolio including small, mid, and large cap equity funds, international funds, short and long-term bond funds, REITs, and a variety of commodity funds including gold, oil, and currencies for those using less traditional portfolios.

To compare portfolio results using tradeable ETFs, Table 1 shows a portfolio of ETFs and their 2x and 3x counterparts along with the effective percentages in each asset. The percentages are based on an investor who has 50% in domestic equity, 20% in international equities, five percent in a REIT, and 25% in bonds. The domestic equity is split between 30% in the S&P 500 with 10% each in mid and small caps. The international equity is split with 10% in developed and 10% in emerging markets. The 25% in bonds is further delineated by five percent in 20+ year T-bonds, five percent in seven to 10-year T-bonds, and the remaining 15% in an aggregate bond portfolio. A similar mix is used by Considine (2006) comparing portfolios of ETFs to mutual funds. The exact percentages are not critical to the results but are created to represent what a typical investor might have.

The portfolio described above can be also be created using LETFs. For an investor using 2x LETFs, only half as much wealth is needed in each LETF to obtain the same amount of

exposure using the underlying ETFs. For 3x LETFs, only a third of the wealth is needed. Thus, to attain 30% exposure to the S&P 500, an investor needs 15% in a 2x S&P 500 LETF or 10% in a 3x S&P 500 LETF.

All the asset classes, except for aggregate bonds, have a corresponding LETF. The aggregate bond asset class is used as the alternative for the excess wealth available when using LETFs to build a portfolio. This results in the 2x and 3x LETF portfolios investing 57.5% and 71.67% of the portfolio, respectively, in a relatively safe bond portfolio. In absolute terms, 80% of the ETF portfolio is at moderate to high risk based on volatility of the underlying assets whereas only 40% and 27% of the 2x and 3x portfolios, respectively, are exposed to equity markets. For comparison of the 3x LETF portfolio to the 2x LETF portfolio, it is assumed the percentage in the underlying 2x LETFs are the same as the underlying index portfolio, that is, 30% is invested in the 2x S&P, and so forth. For the 3x to attain the same exposure as the 2x, 20% is invested in the 3x S&P and so forth.

With limited historical LETF data, additional theoretical LETF returns are calculated for the period before their inception. Because LETFs attain their exposure using a variety of derivative assets including swaps, there are embedded financing costs increasing with leverage (Charupat and Miu, 2014). Based on the methodology of Scott and Watson (2013), LETF returns are calculated based on data going back to 1946. This shows how LETF portfolios are likely to perform in a variety of market environments including the very high interest rate period during the early 1980s. The equation to calculate daily returns using the S&P 500 LETF as an example is:

$$R_L = L * R_{S\&P} - R_{exp} - (L - 1) * R_B \quad (6)$$

where  $R_L$  is the daily return to the LETF with a daily leverage ratio of  $L$ ,  $R_{S\&P}$  is the daily return of the S&P,  $R_{exp}$  is the daily expense ratio, and  $R_B$  is the borrowing rate using the 90-day T-bill rate as a proxy. Strictly speaking, the one-week/month Libor rate should be used, but Libor data begins in 1986 and to remain consistent with sampled returns before this date, the 90-day T-bill rate is used. The 90-day T-bill has a 98% correlation with Libor and averages 0.2% less than Libor. Thus, the borrowing rate is set at the 90-day T-bill yield +0.2%.

The logic behind Equation (6) is a 2x LETF increases exposure by borrowing \$1 for every \$1 invested. A 3x LETF borrows \$2 for every \$1 invested. To determine the validity of Equation (6), theoretical daily, monthly, and annual returns are compared with the actual daily, monthly, and annual returns for the LETFs listed in Table 1. All return data are from The Center for Research in Security Prices (CRSP). Monthly return differences as measured by simulated returns minus the LETF returns average 0.01% assuming an additional 1.2% annual expense ratio. Although LETF's average expense ratio is approximately one percent, there are embedded costs associated with derivatives not accounted for. Using a 1.2% expense ratio reduces the average differences for daily, monthly, and annual returns to near zero for the eight asset classes. Monthly differences ranged from  $-0.07\%$  for the 2x UBT (20+ year Treasury) to  $0.17\%$  for 3x emerging markets EET. Thus, Equation (6) appears to approximate returns accurately enough to simulate LETF returns from index data predating LETF's introduction.

Table 2 Portfolio composition using 1946–2017 historical data

Asset class	Portfolio	2x LETF	3x LETF
S&P 500	50%	25%	16.67%
Mid cap	10%	5%	3.33%
Small cap	10%	5%	3.33%
20-year T-bonds	15%	7.5%	5.00%
7 to 10-year T-bonds	15%	7.5%	5.00%
Bond ladder, 1 to 7 years	0%	50%	66.67%

For historical index data, asset classes are defined as CRSP's S&P 500 index, the 2–4 value weighted deciles proxy for a small-cap fund, and 5–7 value weighted deciles proxy for a midcap fund. The 20-year T-bond and an average of seven to 10-year T-bonds proxy for two additional bond funds. The remainder of a LETF's portfolio is invested equally in one, two, five, and seven-year treasury bonds. No reliable daily international data are available before 1991 so the portfolio comparisons are limited to domestic data. Table 2 shows the portfolio weights for the theoretical historical portfolios. As above, when comparing the 3x to the 2x LETF, it is assumed 100% of the portfolio is in 2x LETFs. This requires two-thirds of the 3x LETF portfolio to be invested in the 3x LETFs.

The final question is rebalancing. If it is assumed the weights in Tables 1 and 2 are optimal, the investor must determine to what extent they can deviate from those percentages. A variety of equity variance thresholds are tested to determine when the portfolios need to be rebalanced. For example, a 10% threshold implies an initial 70% equity exposure is rebalanced when the combined equity position breaches 63% or 77%. The effective exposure for the 2x and 3x portfolios is under the same constraint. For the 2x, this means the amount of wealth in equities can only deviate from 35% by  $\pm 3.5\%$  without initiating a rebalance. Daily, monthly, and quarterly rebalancing is also investigated.

## 4. Results

### 4.1. Rebalancing

To determine how often rebalancing is required to maintain a consistent risk-profile, monthly, quarterly, and allocation thresholds are tested on simulated historical data from 1946 to 2017. Table 3 gives summary statistics using monthly or a 10% variance threshold for rebalancing relative to initial asset allocations described in Table 2. Results for the 2x and 3x are based on their effective exposure.

With monthly rebalancing, a standard portfolio with 70% exposure to equities deviates from 62.65% to 73.44%. However, the 2x LETF portfolio deviates from 43% to 84%, while a 3x LETF portfolio deviates from 27% to 97%.<sup>4</sup> Quarterly rebalancing saw greater extremes as would be expected suggesting a variance threshold must be set for LETF portfolios to keep the risk profile comparable with using standard ETFs.

Using a 10% variance threshold demonstrates improved results in terms of absolute deviations in exposure relative to being invested in the underlying index. Using a 10%

Table 3 Equity portfolio exposure for ETF and LETFs

	Standard ETFs		2x LETFs		3x LETFs	
	Monthly	10% Rebal	Monthly	10% Rebal	Monthly	10% Rebal
Average	70.08%	71.59%	70.44%	71.05%	70.87%	70.94%
Standard deviation	0.98%	2.80%	3.91%	3.10%	6.86%	3.41%
Min	62.65%	62.59%	43.28%	51.22%	26.72%	36.91%
Max	73.44%	77.28%	83.98%	81.03%	96.92%	85.34%

*Note:* Summary of portfolio exposure for ETF and LETFs from 1946 to 2017 based on monthly and 10% threshold rebalancing. Initial and rebalanced weights are 70% equities, 30% bonds. Weights for the LETFs are effective exposure.

variance threshold gives results approximately equal to daily rebalancing. For portfolios using either 2x or 3x LETFs, maximum exposure to equities is reduced. A 3x still reaches a maximum equity exposure of 85%, but this is the same exposure reached using daily rebalancing. Thus, a tighter variance threshold is not warranted.

In addition, the standard deviation of the exposure to equities is significantly reduced, especially for 3x LETF portfolios. Over 72 years, using a 10% threshold results in 24 rebalances for the standard portfolio, 243 for the 2x, and 651 for the 3x. This results in rebalancing for a standard portfolio, 2x, and 3x on average every three years, quarterly, and 45 days, respectively.

In terms of transaction costs, assuming six trades at \$5 a trade is needed at each rebalance, a standard portfolio valued at \$100,000 would have 0.01% additional annual expenses, a 2x would have 0.1%, and a 3x would have 0.27%. For taxable accounts, the additional trading required using LETFs will have a greater tax burden that could nullify any excess returns. These tax effect differentials are discussed in the empirical results.

The reported results in this research are based on using a 10% equity variance threshold for rebalancing. It should be noted 24 rebalances over 72 years using standard ETFs might seem small but consider a 20% increase in equities. With an initial \$100 portfolio, the value of equities would increase from \$70 to \$84, but the percentage in equities only increases to \$84/\$114 or 73.7%. This still does not breach the 10% barrier even with no increase in the bond position. Thus, a relatively large move is required to breach the barrier. A variety of variance thresholds are tested but return differences are not significantly different across portfolios based on rebalancing rules. A 10% threshold effectively controls asset exposure without requiring excessive trading. Thus, only the 10% variance threshold is reported.

#### 4.2. 2010 to 2017 portfolio returns

Table 4 shows the portfolio results for using the ETFs and LETFs shown in Table 1 from 2010 to 2017. The average annual portfolio return using standard ETFs is 10.26%. Using 2x LETFs with unit exposure to the underlying indexes increases this average return to 11.19%, while using 3x LETFs increases this return to 12.06%. In any individual year, the portfolio returns are relatively similar demonstrating the use of LETFs does not result in any discernable increases in risk.



Table 4 Portfolio returns from 2010 to 2017

	ETFs	50% in 2x	33% in 3x	100% in 2x	67% in 3x	BND return
2010	17.11%	19.98%	20.17%	30.57%	34.52%	4.94%
2011	0.42%	2.78%	5.34%	−3.77%	−1.82%	7.92%
2012	13.62%	15.11%	16.62%	25.29%	28.44%	3.89%
2013	17.19%	14.85%	14.03%	34.88%	33.28%	−2.10%
2014	8.43%	10.13%	12.55%	14.29%	16.46%	5.82%
2015	−1.77%	−1.99%	−2.06%	−6.17%	−6.88%	0.56%
2016	9.83%	10.25%	11.54%	16.21%	18.02%	2.53%
2017	17.21%	18.44%	18.31%	34.90%	36.62%	3.57%
Average	10.26%	11.19%	12.06%	18.27%	19.83%	3.39%
Standard deviation	7.56%	7.61%	7.31%	16.28%	16.69%	3.12%

*Note:* Annual returns for a portfolio of ETFs, 2x LETFs, and 3x LETFs along with the BND ETF returns to show what return the excess wealth from LETF portfolios attained.

As expected, the LETF portfolio returns relative to using standard ETFs rely heavily on the return to the aggregate bond portfolio (BND). In 2013, the BND ETF has a return of −2.10% which leads to LETF portfolios underperforming. In 2015, the BND return is 0.56% also resulting in slight underperformance by LETF portfolios. However, in 2010, 2011, 2012, 2014, 2016, and 2017 when the aggregate bond fund did relatively well, the LETF portfolios outperform by up to five percent over a traditional portfolio (see the 3x LETF portfolio in 2011). On average, even under a near zero interest rate environment over the last eight years, a portfolio of LETFs outperforms a traditional portfolio.

The 100% in 2x and 67% in 3x columns show the returns for doubling the exposure to the underlying indexes. Returns for most years are approximately doubled, but so are the standard deviations. The compounding issue is easily seen in 2011 as the ETF portfolio is slightly positive, but the fully leveraged 2x portfolio is negative. Finally, the portfolio using 3x LETFs to create exposure equivalent to the 2x LETFs outperforms the 2x LETF portfolio by approximately 1.5% annually.

To further break down the performance of LETFs relative to comparable ETFs, Table 5 shows the returns as if 100% is invested in each asset class relative to 50% in the 2x LETF

Table 5 100% effective exposure for ETF and corresponding LETFs

ETF Ticker	Asset Class	ETF Average return	ETF Standard deviation	2x Relative Performance	3x Relative Performance
SPY	Large cap	14.50%	10.15%	−0.99%	−0.67%
IJH	Mid cap	15.11%	12.88%	−0.68%	−1.08%
IWM	Small cap	14.51%	15.31%	−0.88%	−1.23%
EFA	Developed	7.28%	13.78%	−1.49%	−0.84%
EEM	Emerging	5.84%	19.76%	−1.14%	−1.58%
IYR	Real estate	12.78%	11.81%	−0.70%	−0.18%
TLT	20-year	8.20%	15.48%	−0.47%	−0.95%
IEF	7–10 year	4.29%	6.42%	−0.41%	−0.44%
Average		10.31%	13.20%	−0.85%	−0.87%

*Note:* Comparison of annual results for 100% in the underlying ETF, 50% in a 2x, and 33.3% in a 3x for each ETF or LETF from 2010 to 2017. Remaining wealth in LETFs assumed to earn zero. No statistical return differences between the ETFs or LETFs.

and 33.3% in the corresponding 3x LETF. The remaining wealth for the LETF portfolios is assumed to earn zero. The 10% variance rebalancing rule is applied to each asset class to keep the exposure close to 100% while still being able to estimate beta decay and the higher costs of LETFs.

On average, 50% of a 2x or 33% of a 3x underperforms its 100% ETF counterpart by 0.85% for 2x LETFs and 0.87% for 3x LETFs. For 2x LETFs, the underperformance ranges from  $-0.41\%$  for 7 to 10-year Treasuries to  $-1.49\%$  for developed equity. The 3x LETFs underperformance ranges from  $-0.018\%$  for Real Estate to  $-1.58\%$  for Emerging Markets. The discrepancy in ranges is due in part to differences in the volatility/return differences across assets and tracking error. It is not surprising the worst underperformance for the LETFs occurs in the volatile international markets that have substandard returns given their volatility. The standard deviations for the ETFs and LETFs are virtually identical and thus, only the ETF standard deviations are shown. In summary, these results show the inherent costs of the higher expense ratio and financing costs of LETFs. From a portfolio standpoint, the remaining funds available from using LETFs need to overcome this performance lag.

To determine how well each LETF tracks their ETF counterpart, LETF returns are also regressed on their corresponding ETF returns from 2010 to 2017. Beta coefficients range from 0.97 to 1.04 with 12 of 16 at 0.99 or 1.0. The 2x emerging market LETF is the only exception with a beta of 0.90 and an  $r^2$  of 90%;  $r^2$  for all other LETF regressions are 98% or better. Thus, of the 16 LETFs, only one did not attain almost exact unit exposure. These results contrast to Tang and Xu (2013) who show even realized daily leverage was less than advertised from 2006 to 2010.

However, that period was more volatile and the LIBOR rate was higher. In addition, small tracking errors are not as magnified in this study as only the inverse of the leverage is held in each fund. The market was also generally upward trending during this study's time period resulting in a positive compounding effect, and finally, the funds themselves are possibly doing a better job of maintaining their daily leverage ratios. Thus, the results suggest using LETFs along with the 10% variance threshold achieves the goal of 100% exposure to the underlying indexes, albeit with an annual financing and expense ratio drag of approximately  $-0.85\%$ .

### 4.3. 1946 to 2017 simulated historical returns

To get a better idea of portfolio returns going forward, simulated returns from 1946 to 2017 are created using Equation (6). Table 6 shows the portfolio results using the weighting shown in Table 2. Subperiod returns are also shown roughly corresponding to different interest rate environments. For the entire 1946–2017 period, LETF portfolios on average outperform a traditional portfolio from 0.63% to 1.41% a year.

However, if this strategy is implemented in nonqualified accounts, the differences do not overcome trading costs and taxes. Assuming a long-term capital gains tax of 15% and a 24% marginal income tax bracket, the approximate returns after taxes and trading costs for the 1946 to 2017 time period for the ETF, 2x, and 3x portfolios are 10.54%, 9.75%, and 10.45%, respectively. This assumes all gains to the ETF are taxed at 15% every three years and all

Table 6 Portfolio average annual returns using 1946 to 2017 historical data

Time period	Portfolio	50% in 2x	33% in 3x	100% in 2x	67% in 3x	Treasuries
1946–2017	11.12%	11.75%	12.53%	18.96%	20.58%	5.46%
1946–1959	11.45%	10.78%	11.22%	21.91%	22.23%	1.63%
1960–1978	8.03%	8.45%	9.35%	13.26%	14.82%	5.39%
1979–1991	16.56%	18.81%	20.05%	25.40%	28.75%	11.42%
1992–2009	10.13%	11.00%	11.65%	15.40%	17.22%	5.78%
2010–2017	11.29%	11.47%	12.16%	23.80%	24.72%	1.96%
1946–2017 risk statistics						
Standard deviation	12.67%	13.23%	13.51%	26.52%	26.93%	5.09%
Min	−23.38%	−19.45%	−18.64%	−46.33%	−44.81%	−1.66%
Max	38.84%	39.53%	44.18%	89.92%	90.62%	25.77%
VaR	−5.31%	−4.24%	−3.19%	−14.57%	−12.52%	0.51%
Sharpe	0.55	0.57	0.62	0.56	0.61	N/A
Sortino	2.56	2.93	3.34	1.97	2.26	N/A

*Note:* Table shows average annual returns from 1946 to 2017 along with several sub-periods corresponding to different interest rate environments. Because of limited data and large standard deviations, there are no statistical differences between the mean returns between the unit exposure portfolios or between the 2x exposure.

gains to the LETFs are taxed at 24% each year. Thus, after taxes and trading costs, using LETFs do not outperform a standard ETF portfolio.

For qualified nontaxable accounts, LETF's outperform, especially when the interest rate environment is relatively high such as 1979 to 1991. Alternatively, during periods of extremely low rates such as the 1946–1959, a portfolio of LETFs tends to underperform. However, low rates by themselves do not relegate LETF portfolios to underperformance. Both the empirical data in Table 4 and simulated data in Table 6 show LETF portfolios outperforming during 2010–2017. The critical factor is the return to the freed-up wealth from using LETFs relative to their implicit financing costs and higher expense ratios.

The bottom of Table 6 shows the standard deviation, minimum, maximum, 10% Value at Risk (VaR), along with the Sharpe and Sortino ratio, the latter of which measures downside risk (Sortino and Price, 1994). Even though there is a marginal increase in the standard deviation when using a portfolio of LETFs, the minimum and 10% VaR for the LETF portfolios is better than the traditional portfolio while at the same time having higher maximums. Thus, the standard deviation is misleading when measuring risk for the LETF portfolios as downside risk for LETFs is mitigated because of the higher percentage in a relatively safe bond ladder. The large percentage in the bond ladder is also valuable during market panics as investors flee to safer assets. Results from the simulated returns reinforce the empirical data from 2010 to 2017.<sup>5</sup>

For investors seeking 2x leverage, the 100% in 2x and 67% in 3x columns show the results. Average returns are less than double while the standard deviations and minimums are doubled with a tripling of the VaR. Using the 3x LETF to attain 2x exposure shows improved results. Overall and in every subperiod, 67% in a 3x outperforms 100% in a 2x with a 1.6% annual average increase. The minimum and VaR using 3x LETFs are also better than 2x LETFs along with a higher Sharpe and Sortino ratio.

Table 7 Return differences regressed on T-bond ladder or AGG ETF minus borrowing rate

	$r^2$	Intercept ( $t$ -stat)	Coefficient ( $t$ -stat)
2010 to 2017			
2x LETF-ETF portfolio	92.53%	-0.001 (-1.87)	0.41 (13.7) <sup>a</sup>
3x LETF-ETF portfolio	95.36%	0.000 (-1.17)	0.63 (17.36) <sup>a</sup>
3x LETF-2x LETF	64.70%	0.000 (0.35)	0.34 (4.65) <sup>a</sup>
1946 to 2017			
2x LETF-ETF portfolio	93.13%	-0.0071 (-6.14) <sup>a</sup>	0.56 (21.39) <sup>a</sup>
3x LETF-ETF portfolio	95.15%	-0.005 (-3.54) <sup>a</sup>	0.76 (25.88) <sup>a</sup>
3x LETF-2x LETF	79.75%	0.003 (1.15)	0.54 (11.06) <sup>a</sup>

Note: <sup>a</sup>Statistical difference at the one percent level.

The disadvantage to this strategy is the rebalancing frequency required for a 3x. Using a 10% variance threshold to maintain the initial 2x exposure to the underlying indexes requires 79 and 381 rebalances for the 2x and 3x, respectively, over the 72 years. Thus, although using the 3x to mimic the exposure of a 2x increases the returns, trading costs and the more onerous tax effects in nonqualified accounts eliminate the after-tax return differential.

For a LETF portfolio to outperform, the returns to the T-bond ladder need to exceed the financing and expense ratio costs of LETFs. However, this only occurs 45% of the time from 1946 to 2017. Despite this, 2x and 3x LETF portfolios outperform 54% and 61% of the time, respectively, during this time period. This is possible because a highly positive trending market allows LETFs to return more over time than their daily leverage multiple implies. This can make up for small interest rate spreads. To more accurately estimate the effect of the lending minus borrowing differential on returns, the 50% in 2x and 33% in 3x LETF returns minus the standard portfolio returns are regressed on the bond minus borrowing rate differential for both the 2010 to 2017 empirical data and the 1946 to 2017 simulated data. The regression results are shown in Table 7.

Not surprisingly, the bond return minus the borrowing cost explains more than 90% of the return differences of the 2x and 3x LETF portfolios relative to a standard portfolio for both the empirical and simulated historical data. Based on the empirical data, a one percent return over the borrowing rate leads to a 0.41% increase for a 2x and 0.63% increase for a 3x LETF over a standard portfolio. These regression results held when comparing the LETFs to their corresponding ETFs at an individual level and roughly correspond to the 50% and 67% freed up wealth from using 2x or 3x LETF portfolios, respectively.

Results are similar for the 3x LETF relative to the 2x LETF as a one percent return over the borrowing rate corresponds to a 0.34 percentage point increase using the empirical data and 0.54 percentage point gain for the simulated data. Both results are associated with moderately lower  $r^2$  as there is much more volatility with returns leveraged to 2x. Thus, both the empirical and simulated data confirms the importance of the lending-borrowing spread.

## 5. Conclusion

Despite the early negative press about the dangers of investing in LETFs, they have become popular investment vehicles with 265+ funds growing to more than \$68 billion in

assets over the last 12 years ([www.etf.com](http://www.etf.com)). While LETFs have higher expense ratios and suffer from leverage decay, LETFs can be used effectively to improve average returns and reduce downside risk if properly managed. One key is to periodically rebalance LETFs to minimize both the impact of decay and maintain a set risk exposure.

This study compares a diversified portfolio of typical ETFs to portfolios comprised of 50% in 2x LETFs or 33% in 3x LETFs with the remainder invested in a relatively safe T-bond ladder or a bond fund. In addition, a 3x LETF portfolio is compared with a 2x LETF portfolio where the underlying exposure of the 3x is set equal to the 2x. Using a 10% variance threshold for rebalancing, this study finds a portfolio using LETFs since 2010 or one based on simulated data from 1946 to 2017 outperforms a standard portfolio. By extension, the 3x portfolio created to mimic the 2x portfolio outperforms the 2x portfolio.

Specifically, combining tradeable LETFs since 2010 with an aggregate bond ETF, a 2x LETF portfolio averages 0.9% more per year while a 3x averages 1.8% more relative to a standard ETF portfolio. The 3x portfolio created to mimic the 2x portfolio outperforms by 1.5% annually. Using simulated data from 1946 to 2017, a 2x LETF or 3x LETF portfolio combined with a one, three, five, and seven-year treasury ladder outperforms a standard ETF portfolio by 0.6% and 1.4%, respectively, on an annual basis with no increase in risk and better downside risk metrics as measured by minimums, VaR, and Sortino ratios. The results extend for the 3x over the 2x with an average annual outperformance of 1.6%.

The critical component for LETF portfolio outperformance is how the return to the remainder of the portfolio not invested in LETFs compares to the implicit borrowing costs and higher expense ratios of LETFs. Regression results show the difference in the bond return minus borrowing rate explains more than 90% of the difference in returns between a portfolio created with LETFs versus one created with standard mutual funds. For every one percent return earned over the borrowing rate, the 2x LETF portfolio outperforms by 0.41% and a 3x LETF portfolio outperforms by 0.63%. This corresponds closely to the freed-up wealth invested in bonds.

From a practitioner's point of view, LETFs require more active management and in any given day, these instruments are more volatile. If the rebalance rule is breached on the downside, an investor will need the "stomach" to buy more of a fund that theoretically could lose 60% or more of its value in a day (recall October 19, 1987 when the S&P 500 fell 20%). Thus, a certain degree of behavioral fortitude may be needed before creating a portfolio of 2x or 3x LETFs. In addition, the tax liability from the greater rebalancing neutralizes the excess returns from using LETFs found in this study. Thus, the implementation of the strategy should probably be limited to qualified accounts. Although tax issues are detrimental, the proposed LETF strategy does have the ability to be used in IRA type accounts—accounts that typically do not allow margin trading.

In summary, this study shows LETFs can successfully be held long-term as major components of a portfolio while improving returns and reducing downside risk. Whether piecemeal such as only using an S&P LETF to partially or fully take the place of an S&P ETF, or to completely replicate an investor's entire portfolio of ETFs, LETFs, when properly managed, can enhance returns and reduce risk.

## Notes

- 1 As an example, a 10% return to the index over a 252-day trading period with a one percent daily standard deviation will result in a theoretical realized leverage ratio for a 2x and 3x LETF of 1.80 and 2.34, respectively.
- 2 The variance over time is  $\sigma_t^2 = t\sigma^2$  so for the index,  $\sigma_t^2 = 252*0.01^2 = 2.52\%$ . For the 3x, the daily returns and standard deviations are  $3*0.045$  (.135%) and  $3*1$  (3%), respectively. Thus, for the 3x,  $\sigma_t^2 = 252*0.03^2 = 22.68\%$ .
- 3 Note  $\sigma^2 L^2$  after dividing  $\sigma$  by L becomes  $(\sigma/L)^2 L^2$  that simplifies to  $\sigma^2$ .
- 4 The max and min exposure levels are for the entire period and do not necessarily sum up to 100%. In addition, for the LETF portfolios, the exposure does not include the allocation to the Treasury ladder meaning the combined equity and bond allocation will deviate from 100% between rebalances.
- 5 Block bootstrapping of the 1946–2017 data was performed to create 20,000 unique one-year returns. The results are qualitatively like the original historical results with the only differences being the mean returns and VaRs across the portfolios are statistically significantly different.

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