

# The perfect withdrawal amount over the historical record

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## Abstract

What has been the perfect withdrawal amount (PWA) from retirement savings accounts in long-term historical data? The PWA is that which, if taken out in the first year of retirement and used again every year adjusted by inflation, leaves exactly the desired final balance on the account. We present the formula for obtaining this measure and evaluate the values it has taken in the past under varying combinations of the relevant parameters. We find that safety-minded investors should enter retirement with a higher stock allocation than what is currently used in most investment funds designed to provide income during retirement. © 2020 Academy of Financial Services. All rights reserved.

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## 1. Introduction

A considerable body of research has explored the question of what is the safe level of withdrawals from a savings account that must provide income during retirement. A frequent objective of these studies is to examine the historical record to determine the withdrawal figure that has “rarely” depleted the funds before the end of the retirement period. However, if the withdrawal amount chosen does not exhaust the funds in the account, it means that it left a positive balance at the end. If the retiree had not planned to leave a bequest or posthumous gift—or if the final balance is larger than what she intended to leave behind—this is money

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that she would rather have consumed herself. In these cases, it can be said that the amount withdrawn was not perfect; it should have been higher.

Here we use results presented in Suarez, Suarez, and Walz (SSW; 2015) that show that for any given series of return rates on a portfolio, there is one and only one constant withdrawal amount that will take the starting balance to the desired ending balance in a given span of time. If this time span is the number of years for which the retiree wishes to plan, then this is the perfect withdrawal amount (PWA) for her: it's constant (no ups-and-downs in the income stream) and it attains the final goal exactly and just in time (no portfolio "failure," or inheritance shortfall, or overaccumulation at the end). If she withdraws more than her PWA, she will run out of funds—or at least leave behind less money than she intended. If she withdraws less, she will leave money on the table.

It is important to stress that the PWA is a measure inherent in any sequence of return rates. A sequence of returns has a PWA just as it has a mean, a standard deviation, or any other conventional statistical measure, and by calculating these values we are not endorsing any particular viewpoint on retirement withdrawals. Rather, we intend for this paper to be of the same nature as Ibbotson and Sinquefeld's seminal 1976 article (Ibbotson, R. G., & Sinquefeld, R. A., 1976), where long-run historical rates of return were first presented. We argue that the retirement withdrawal question can be understood as the search for what the PWA will turn out to be (eventually) for each retiree's portfolio and objectives. And a first step in this direction should be a revision of the values that this measure has taken, historically, for different values of the relevant parameters. In this sense, the results presented here can significantly assist the decision-making process in retirement.

We also refer to another result presented in SSW, namely that if the retiree is interested in using up her retirement savings in stages—for example, withdrawing a certain amount for a number of periods and then a different, smaller amount as social security benefits start to come in—the "perfect" set of figures is a function of the "regular" PWA that is reported here, which is a constant amount throughout the retirement period (Bridges & Choudury, 2007). The numbers presented here can thus provide guidelines for many different circumstances and cases.

We present results for the PWA over the period 1926–2014. Using data from Ibbotson SBBI Classic Yearbook (Morningstar, Inc., 2015), we start by computing PWAs for a base-case scenario of all-stock portfolios and 30-year long retirement periods, where the investor's goal is to exhaust the account's funds completely. The retirement periods are rolled forward one month at a time, with the first one beginning in January 1926 and ending in December 1955, and the last one running from January 1985 through December 2014. This one-month-shift approach produces a series of 708 figures that, although not all independent, provide a bird's-eye view of how this particular PWA has evolved over time.

We also use a separation result obtained in SSW to look at how the PWA is affected by the sequential order in the portfolio's returns. This is known as sequencing risk, and we examine it by computing the values for the sequencing factor—a measure developed in that paper for this type of risk—over the same period.

We then compute the values corresponding to other asset allocations for the portfolio. By increasing the bond allocation in 10 percentage-point increments—and decreasing the corresponding allocation to stocks—the relation between asset allocation and historical PWAs is explored.

Next, we look at how our results change when the retiree's goal is not to exhaust her savings entirely, but to leave a portion of her balance as bequest or final gift. This is perhaps a more relevant case, especially since the protected balance can also be considered an emergency reserve for unforeseen expenditures and, in this sense, it might be a desirable feature in most retirement plans. This interpretation of a nonzero final balance goal as a safety device suggests an important question that, to our knowledge, has not been discussed before: if the retiree wants to "play it safe," should she withdraw the amount that *rarely exhausts* the balance in her account, or the amount that *frequently produces* a final balance that can cover some unexpected outlays? Are these approaches equivalent? The results in this section provide insight for this discussion.

We close with the PWA results for other horizon lengths. By changing the duration of the retirement period from 30 years to 25, 35, and 40 years we obtain a ballpark estimate of the level of risk incurred by using a specific endpoint in retirement plans, when in reality this closing date is of course unknown.

Taken together, the numbers presented in this paper draw the outline of a surface that can be used for decision-making. The true perfect withdrawal amount awaiting in each retiree's future cannot be known in advance, but by perusing the cross-sections of this object we can make well-educated guesses of what it may end up being in each case. The aim of this paper is to recruit the assistance of financial history to make this guesswork more robust.

## 2. Previous research

The research program on optimal retirement withdrawals began in 1994, with the release of two seminal papers. The first one, Bierwirth (1994), actually computes what we are now calling PWAs, although he obtains them by trial-and-error. Bierwirth does not seek the withdrawal amount that exhausts the starting balance, but rather the figure that leaves the account balance unchanged in nominal terms after 25 years (even though the withdrawal amount is kept constant in real terms). However, the stated goal of Bierwirth's work—and its lasting contribution—is to use historical data instead of simplistic assumptions to evaluate any withdrawal strategy. This key insight is taken up by Bengen (1994) to develop the concept of "portfolio longevity," which he uses to arrive at the conclusion that "a first-year withdrawal of 4%, followed by inflation-adjusted withdrawals in subsequent years, should be safe."

This "4% rule" was later reinforced by a series of papers by Cooley et al. (1998, 1999, 2003, 2011). These studies used a more exhaustive scope, and varying conditions and assumptions, to confirm that this strategy provides a high probability (85%–95%) that the retiree will not run out of funds ahead of time.

The unifying feature of these "first-generation" approaches is that the retiree is asked to select a specific withdrawal amount at the beginning of a very long period, and then she is supposed to take out that same amount, in either nominal or real terms, each and every year. This motivated attempts to develop "adaptive" rules, where the withdrawal amount is modified in some particular way depending on the circumstances faced in every period. In this direction, the field has veritably exploded in recent years (Mitchell & Blanchett, 2011).

Currently there are, for example, rules that modify the withdrawal amount when the rate of return is negative (Guyton & Klinger, 2006), or when it deviates too much from an

average value (Frank et al., 2011), or when the withdrawal amount/rate becomes higher than a predefined number (Pye, 2000; Zolt, 2013). There are rules that change the amount every five years to meet withdrawal rate targets (Spitzer, 2008), and rules that change it every year depending on the length of the remaining horizon (Blanchett & Frank, 2009). Some approaches incorporate mortality tables to the analysis, including the investor's age in the calculation of the withdrawal amount (Stout & Mitchell, 2006), or computing the expected present value of all future withdrawals (Mitchell, 2011; Stout, 2008).

Furthermore, there are even some approaches that forgo the stability criterion altogether, letting the withdrawal amount bear the brunt of the variability in the rates of return. Waring and Siegel (2015), for example, propose a rule in which “the investor will receive 30 years of payments of varying size, each appropriate to the then-current value of the portfolio and the interest rate . . . the only risk is to the size of each payment, which will vary . . . with investment results.” Although these procedures may have some merits, we think that most retirees prefer a stable flow of income year after year.

In a sense, most of these methods are just different ways of “hunting” for one single, elusive number: the constant withdrawal amount that can be used year after year, over a certain period in the future, without causing neither early depletion nor overaccumulation. This paper provides the historical series for that number.

### 3. The perfect withdrawal amount

The perfect withdrawal amount formula can be thought of as a generalization of the payment formula for a fixed-rate loan. In this case, however, the rate is not fixed but changes in every period, so the simplification allowed by geometric series does not occur. Also, here we are talking about payments *from* an investment account, instead of payments *into* a loan that must be amortized. Finally, the generalization includes a final balance term that must be non-negative, but not necessarily zero.

As shown in SSW, this constant value is given by:

$$w = [K_S \prod_{i=1}^n (1 + r_i) - K_E] / \sum_{i=1}^n \prod_{j=1}^i (1 + r_j) \quad (1)$$

where  $w$  is the yearly withdrawal amount,  $K_S$  is the starting balance,  $r_i$  is the rate of return in year  $i$ ,  $n$  is the planning horizon in years, and  $K_E$  is the ending balance sought at the end of these  $n$  years.

This equation shows that a period of time with a large cumulative return is not necessarily associated with a high PWA, because the sequential order by which the total return comes about is also relevant. This effect is seen more clearly by rewriting Eq. (1) as:

$$w = (R_n K_S - K_E) S_n \quad (2)$$

where  $R_n = \prod_{i=1}^n (1 + r_i)$  is the return generated by the assets in the portfolio over the entire period, and  $[S_n = \sum_{i=1}^n \prod_{j=i}^n (1 + r_j)]^{-1}$  can be interpreted as an “adjustment” applied to this total return to take into account the specific sequence of period-by-period return rates that produced it. The expression  $S_n$  is what we call *sequencing factor*.

Ordering matters, of course, because for an investment account that is constantly drawn down—as is the case in the retirement stage—getting high rates at the beginning and low rates later on is better than the other way around, because if the high rates come early they will impact a larger balance. A closer inspection of the sequencing factor formula shows how this feature is captured by our measure:

$$S_n = [\sum_{i=1}^n \prod_{j=1}^i (1 + r_j)]^{-1} = [(1 + r_1)(1 + r_2)(1 + r_3) \dots (1 + r_n) + (1 + r_2)(1 + r_3) \dots (1 + r_n) + (1 + r_3)(1 + r_4) \dots (1 + r_n) + \dots + (1 + r_{n-1})(1 + r_n) + (1 + r_n)]^{-1} \quad (3)$$

We note that the rates that come later in the sequence (higher index number) appear more times in the summation than the rates that come sooner. Because  $S_n$  is the reciprocal of the summation, this means that if we take a set of return rates (unordered) and arrange it into a sequence (ordered), we can increase the value of  $S_n$  by swapping any pair of rates so that the larger of the two figures comes before the lower one<sup>1</sup>. Therefore, as it should, the value of  $S_n$  is higher when the sequence is “better.”

The next step in the PWA approach is just to realize that Eq. (1) defines a stochastic variable. That is, since the rate of return is a random variable, the PWA measure is a random variable as well. In that sense, the results presented here can be considered as snapshots of a stochastic process that could have given us other values as observations. The purpose of this paper is to examine those values—the ones that actually came up and became observable—to derive insights for the management of retirement accounts.

#### 4. The data

For all the calculations in this paper we use data from Ibbotson SBBI Classic Yearbook, 2015 edition. Specifically, we use the series “Large-Capitalization Stocks: Total Returns” as the monthly return on stocks, and “Intermediate-Term Government Bonds: Total Returns” as the monthly return on bonds. All the rates of return are deflated using CPI-U to obtain real-term yields. Because we use real-term returns throughout, the withdrawal amounts that we obtain are constant in terms of the purchasing power they provide (the initial withdrawal amount is adjusted for inflation in every period) and the desired ending balance is attained in real terms as well.

To provide perspective for the results of this study, it is instructive to review these return figures briefly. The geometric average of the real monthly rate of return for stocks over the entire period spanning from January 1926 through December 2014 (1,068 months) was 0.56% (6.98% annualized), with a minimum of –29.26% (in September 1931) and a maximum of 42.56% (in April 1933). The arithmetic mean was 0.71%, and the standard deviation was 5.47 percentage points.

In the case of bonds, the geometric average of the real monthly return over the 89-year period was 0.18% (2.24% annualized). The minimum value was –7.71% (February 1980), and the maximum was 10.74% (April 1980). The mean for this series was 0.19%, and the standard deviation was 1.39 percentage points.

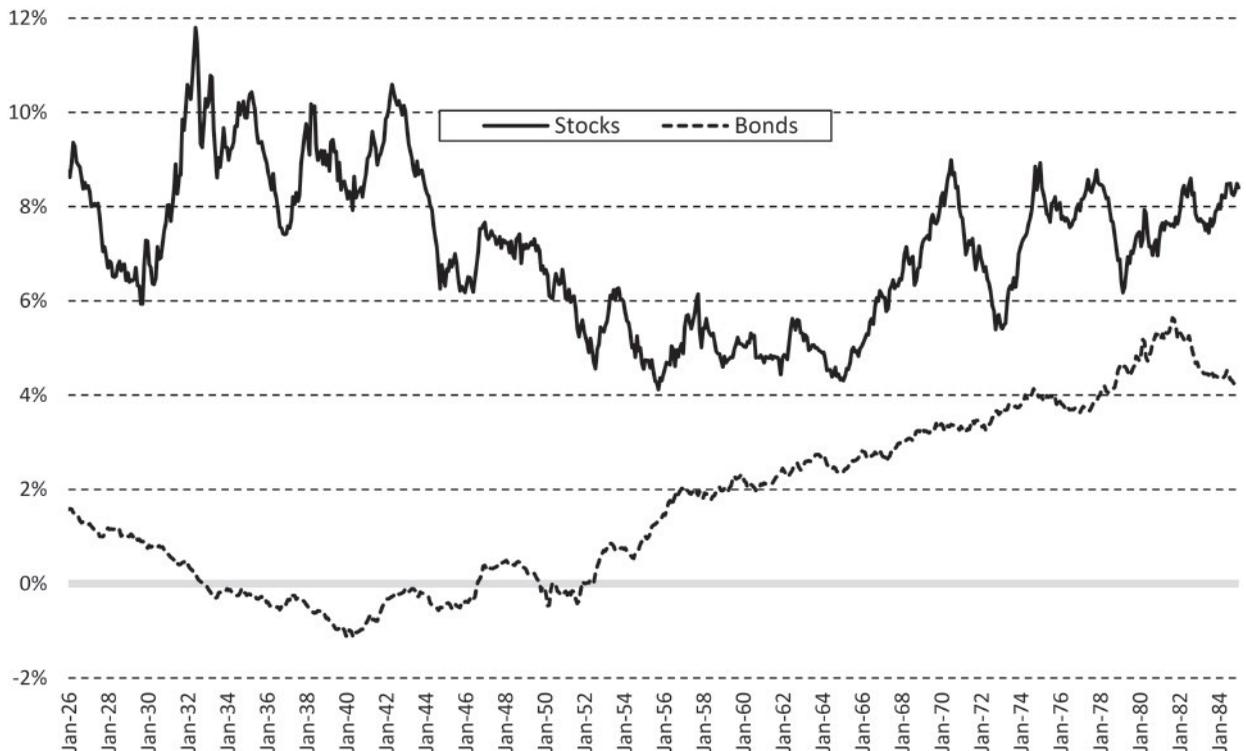


Fig. 1. Real rates of return for stocks and bonds, over 30-year periods beginning on the indicated date. Shown are yearly geometric averages over the 30-year periods.

We now ask what the real rate of return has been, for these two financial instruments, over 30-year periods in the historical record. This 30-year horizon is what we use below as our “standard scenario,” which in turn corresponds to the set-up that is most frequently used in the literature on optimal withdrawals.

Fig. 1 shows these 30-year rates, expressed as annual rates, and we notice a major upward trend in the long-term real returns for bonds. However, although no more recent data can be included because 30-year periods starting after December 1984 have not yet ended (at least not in our dataset), using 20- or 15-year periods we see that this series will probably end up hovering around 3% annual, or even lower, when the next decade of observations becomes available. In any case, these figures are well known; we show them here just as a reference for the analysis of the next sections.

## 5. Results for the standard scenario

The scenario we use as our “base case” is an investor that enters her retirement period with a portfolio fully invested in stocks, and whose aim is to deplete her balance entirely over a period of 30 years. In other words, our investor profile in this section is a person of approximately 65 years of age who wants her money to last until she’s 95 and has no heirs, or otherwise has no desire to leave behind any inheritance or gift.

When there is no bequest sought,  $K_E$  in Eq. (2) is zero and the expression simplifies to:

$$w = R_n S_n K_S \quad (4)$$



Fig. 2. Perfect withdrawal rate for 100%-stocks portfolio over 30-year period with zero final balance. For the indicated retirement start dates, these withdrawal rates (applied to the starting balance and updated by inflation every year) would have exhausted the available funds exactly after 30-years.

Before moving on to the actual results, we make one last modification to express our perfect withdrawal amount as a perfect withdrawal *rate* (PWR), relative to the starting balance in the account:

$$w/K_S = R_n S_n \quad (5)$$

Fig. 2 presents the PWR for the standard scenario as a monthly variable, assigning to each month the PWR value corresponding to the 30-year period beginning on that month. This computation assumes that withdrawals are made only once per year, on the very first day and adjusted by the previous year's inflation, whereas the returns are credited to the account on the last day of the year (or, equivalently, at the end of every month). As mentioned before, under this convention for dating the data the PWR cannot be computed beyond December 1984 because the 30-year periods beginning after that date are not yet complete in our data set.

The first thing one notices in Fig. 2 is the wide range of variation, and how these up-and-downs do not follow closely the changes in the rate of return for stocks (see Fig. 1). To see how this can come about, and to further pin down the PWA concept, Fig. 5 shows the explicit sequence of debits and credits on the account in two retirement periods, beginning

Retirement period beginning in June 1949						Retirement period beginning in February 1969					
12-mo period begins	Rate of return	Inflation	W-drawal amount	W-drawal amt in June '49 dollars	Ending balance	12-mo period begins	Rate of return	Inflation	W-drawal amount	W-drawal amt in Feb '69 dollars	Ending balance
Jun-49	42.4%	-0.4%	\$ 139,929	\$ 139,929	\$ 1,224,744	Feb-69	-14.7%	6.2%	\$ 39,612	\$ 39,612	\$ 819,013
Jun-50	23.8%	9.3%	\$ 139,341	\$ 139,929	\$ 1,343,229	Feb-70	17.0%	5.3%	\$ 42,060	\$ 39,612	\$ 909,415
Jun-51	18.0%	1.9%	\$ 152,276	\$ 139,929	\$ 1,405,168	Feb-71	11.8%	3.3%	\$ 44,285	\$ 39,612	\$ 967,444
Jun-52	8.8%	1.1%	\$ 155,216	\$ 139,929	\$ 1,359,677	Feb-72	14.9%	3.6%	\$ 45,732	\$ 39,612	\$ 1,058,658
Jun-53	25.8%	0.7%	\$ 156,979	\$ 139,929	\$ 1,513,572	Feb-73	-14.0%	9.4%	\$ 47,401	\$ 39,612	\$ 869,496
Jun-54	35.9%	-0.7%	\$ 158,155	\$ 139,929	\$ 1,842,542	Feb-74	-16.5%	11.8%	\$ 51,851	\$ 39,612	\$ 682,559
Jun-55	24.1%	1.1%	\$ 156,979	\$ 139,929	\$ 2,091,350	Feb-75	36.6%	6.7%	\$ 57,971	\$ 39,612	\$ 852,914
Jun-56	9.0%	3.7%	\$ 158,743	\$ 139,929	\$ 2,105,762	Feb-76	5.3%	5.2%	\$ 61,866	\$ 39,612	\$ 832,602
Jun-57	-3.2%	3.2%	\$ 164,623	\$ 139,929	\$ 1,878,417	Feb-77	-8.1%	6.8%	\$ 65,092	\$ 39,612	\$ 705,049
Jun-58	37.7%	0.3%	\$ 169,914	\$ 139,929	\$ 2,352,118	Feb-78	18.1%	9.3%	\$ 69,543	\$ 39,612	\$ 750,287
Jun-59	-1.7%	1.7%	\$ 170,502	\$ 139,929	\$ 2,145,466	Feb-79	20.6%	13.9%	\$ 75,997	\$ 39,612	\$ 813,498
Jun-60	23.2%	1.0%	\$ 173,442	\$ 139,929	\$ 2,428,740	Feb-80	19.5%	11.8%	\$ 86,567	\$ 39,612	\$ 868,820
Jun-61	-7.6%	1.3%	\$ 175,206	\$ 139,929	\$ 2,081,537	Feb-81	-2.1%	8.4%	\$ 96,804	\$ 39,612	\$ 756,071
Jun-62	22.9%	1.0%	\$ 177,557	\$ 139,929	\$ 2,340,677	Feb-82	27.7%	3.7%	\$ 104,927	\$ 39,612	\$ 831,835
Jun-63	17.1%	1.3%	\$ 179,321	\$ 139,929	\$ 2,531,835	Feb-83	17.5%	4.2%	\$ 108,821	\$ 39,612	\$ 849,529
Jun-64	13.4%	1.6%	\$ 181,673	\$ 139,929	\$ 2,665,351	Feb-84	15.2%	3.5%	\$ 113,383	\$ 39,612	\$ 847,950
Jun-65	0.5%	2.9%	\$ 184,613	\$ 139,929	\$ 2,493,839	Feb-85	22.9%	3.9%	\$ 117,389	\$ 39,612	\$ 897,878
Jun-66	7.1%	2.8%	\$ 189,904	\$ 139,929	\$ 2,466,966	Feb-86	33.9%	1.5%	\$ 121,951	\$ 39,612	\$ 1,038,909
Jun-67	14.3%	3.9%	\$ 195,195	\$ 139,929	\$ 2,597,601	Feb-87	-3.3%	4.0%	\$ 123,731	\$ 39,612	\$ 884,624
Jun-68	8.1%	5.5%	\$ 202,839	\$ 139,929	\$ 2,588,781	Feb-88	20.1%	4.7%	\$ 128,738	\$ 39,612	\$ 907,787
Jun-69	-23.6%	6.0%	\$ 214,009	\$ 139,929	\$ 1,815,269	Feb-89	14.5%	5.2%	\$ 134,747	\$ 39,612	\$ 884,849
Jun-70	34.8%	4.4%	\$ 226,944	\$ 139,929	\$ 2,140,791	Feb-90	8.4%	5.7%	\$ 141,757	\$ 39,612	\$ 805,458
Jun-71	13.3%	3.2%	\$ 236,939	\$ 139,929	\$ 2,157,434	Feb-91	22.7%	2.6%	\$ 149,768	\$ 39,612	\$ 804,473
Jun-72	-1.4%	5.5%	\$ 244,582	\$ 139,929	\$ 1,886,647	Feb-92	10.6%	3.3%	\$ 153,663	\$ 39,612	\$ 719,666
Jun-73	-13.9%	10.7%	\$ 258,105	\$ 139,929	\$ 1,402,945	Feb-93	12.9%	2.5%	\$ 158,670	\$ 39,612	\$ 633,236
Jun-74	9.6%	9.5%	\$ 285,738	\$ 139,929	\$ 1,223,914	Feb-94	0.5%	2.8%	\$ 162,675	\$ 39,612	\$ 473,057
Jun-75	14.4%	6.2%	\$ 312,783	\$ 139,929	\$ 1,041,906	Feb-95	38.7%	2.7%	\$ 167,237	\$ 39,612	\$ 424,062
Jun-76	0.1%	6.7%	\$ 332,185	\$ 139,929	\$ 710,340	Feb-96	26.3%	3.0%	\$ 171,800	\$ 39,612	\$ 318,713
Jun-77	6.6%	7.0%	\$ 354,527	\$ 139,929	\$ 379,220	Feb-97	26.9%	1.6%	\$ 177,029	\$ 39,612	\$ 179,811
Jun-78	7.4%	10.9%	\$ 379,220	\$ 139,929	\$ (0)	Feb-98	32.5%	1.7%	\$ 179,811	\$ 39,612	\$ (0)

Fig. 3. All-stocks portfolio with \$1 M starting balance, to be depleted over 30 years, begins retirement at two different dates. An all-stocks portfolio of \$1 million that entered retirement in June 1949 would have been exhausted after 30 annual withdrawals of \$139,929 (plus inflation), but this same portfolio would have provided only \$39,612 (plus inflation) per year over 30 years if it had entered retirement in February 1969—even though the average annual real return over both 30-year periods was practically the same (7.2%).

in June 1949 and February 1969, respectively, chosen to illustrate some of the points we make below.

Another salient feature of Fig. 2 is that the 4% rule would seem overly conservative, as the chart rarely pierces the 4% level. However, in fact, the “90% safe” withdrawal rate is only slightly higher, at 4.27%. That is, out of the 708 observations in this series, 70 were lower than 4.27%. Fig. 4 shows the frequencies by PWR range.

Still, the fact that the PWR has been higher than 4% so frequently—and higher by so much, with more than half of the periods registering over 7%—raises the question of what would have happened, in the past, to the final balance of an investor who followed the 4% rule. Under the PWA framework this is easy to answer, as we can rewrite Eq. (2) to solve for the final balance<sup>2</sup>:

$$K_E = R_n K_S - w/S_n \quad (6)$$

PWR range	# of instances	Relative frequency
Less than 4%	35	4.9%
Between 4% and 5%	130	18.4%
Between 5% and 6%	94	13.3%
Between 6% and 7%	77	10.9%
Between 7% and 8%	87	12.3%
Between 8% and 9%	53	7.5%
Between 9% and 10%	86	12.1%
Between 10% and 11%	71	10.0%
Between 11% and 12%	34	4.8%
Between 12% and 13%	26	3.7%
More than 13%	15	2.1%
Total	708	100%
Less than 4.27%	70	9.9%

Fig. 4. Frequency distribution of PWR in standard scenario. In the standard scenario the portfolio is invested entirely in stocks and is completely depleted (no bequest) after 30 years. The first decile cutoff is at 4.27%, so this is the “90%-safe” withdrawal rate in this case.

If we express the final balance as a fraction of the starting balance and define the withdrawal rate  $\rho = w/K_S$ , we get:

$$K_E/K_S = R_n - \rho/S_n \quad (7)$$

Fig. 5 presents the results of Eq. (7), over 30-year periods, for an all-stocks portfolio using  $\rho = 4\%$ —the 4% rule. This chart brings forward a point that was made in SSW, namely, that choosing a withdrawal amount that provides a high safety level is tantamount to accepting a high probability of ending up with an account balance much larger than what was intended.

We find that in 81% of the periods, following the 4% rule would have left the investor with more money—in real terms—than what she had at the beginning (instead of exhausting the balance). In almost two-thirds of the historical instances the final balance would have been more than twice the starting balance. An in fully one-third of the cases the account would have closed with five times or more the original money!

Since overaccumulation means that the retiree’s standard of living was unnecessarily restricted, the fact that it has happened so frequently under the 4% rule (and that it is so large) is in itself a significant drawback for this common recommendation. Additionally, when we consider that retirement savings accounts are rarely the only source of income for retirees<sup>3</sup>—so that even if withdrawals exhaust the balance, there is usually another source of funds—a likely culprit for this excess balance comes to the fore: using 90% as the chosen safety level is probably an inadequate requirement for an adaptive plan.



Fig. 5. Final balance for 100%-stocks portfolio under 4% rule after 30 years, relative to starting balance. The 4% rule would have led to large (possibly unwanted) final balances in the historical record.

Returning to Fig. 2, we see basically three “times” when the PWR dipped below the 4% level. First, there are the periods starting in December 1928 through November 1929, or in March 1930 through June 1930. Anybody who started retirement in these specific months, with stocks as their savings vehicle, would have met the brunt of the Great Depression at a very early stage of their retirement period, which is when the retiree’s financial situation is most vulnerable. In the first period mentioned, for example, they suffered the huge crashes of October and November 1929 (–19.7% and –12.5%, respectively, in real terms) during their very first year of retirement. Everybody in this group bore the losses of June and September 1930 (–15.8%, –13.3%) and also took, only one year later, the largest one-month drop in the history of the stock market (–29.3%, September 1931). Furthermore, a few months after that, stocks fell an additional 43.2% in the three-month period of March through May 1932.

All this would seem obvious if not for the fact that the PWR turned out to be much higher (in the 6%–8% range) for people who retired only a few months earlier. Indeed, for people who retired in July 1927 (only 17 months before the first of our ill-fated groups) the PWR was 6.2%. If the retirement period had started in August 1931—taking the “mother of all crashes” head on, and then going through the ordeal of the spring of ‘32—the PWR would still have been 5.9%. And if the retirement period had started only a little after this turmoil, in July 1932, the retiree would have obtained the highest PWR ever recorded—an eye-popping 15.3%—even though the awful returns of 1937 would have hit her while still in the first stage of the retirement period.

The examination of this first segment in the PWR series throws into sharp relief the complex relationship between rates of return and optimal withdrawal rates. Although this volatile behavior could be attributed to the general market turbulence of the time, the sensitivity of the results to what might seem like minor disturbances is still remarkable. In particular, we see how market rebounds sometimes manage to “save” the portfolio, but they have to happen at a specific time to produce a meaningful impact—sequencing is crucial.

The sequencing effect is crucial because it *determines* the perfect withdrawal amount. We stress the point that a set of return rates that might be considered “good”—in the sense that upon compounding it produces a high total return for the entire period—will nonetheless lead to a low PWA if the particular ordering of the set is unfavorable (from low returns to high returns, in general). In particular, a “bad” ordering will be associated with a relatively low PWA, and this cannot be changed by adjusting the withdrawal amount from year to year as it would only zig-zag around, hover about or glide towards an inevitably low value.<sup>4</sup>

Next in our analysis of very low PWR periods is the six-month stretch beginning in October 1965. This instance of “bad times to start retirement” is puzzling because not much happened market-wise in that period. Rather, it would seem that it was just a few instances of negative monthly returns (March, May, August 1966) that, though none terribly bad, combined with a generally lackluster market in the first few years to produce a very deleterious effect.

The most recent period with a very low PWR is the one spanning from August 1967 to June 1969. In this 23-month period, 13 months had a PWR of less than 4%, with the remaining 10 months barely managing to surpass this benchmark. As in the previous case, this instance is hard to understand except as a case of peculiar timing. It would seem that a series of early punches to the investment account (October 1967, Jan-Feb 1968) combined with the bear market of 1969-1970 to create the kind of “perfect storm” that hurts new retirees very much.

It would seem appropriate to also discuss periods when the PWR was particularly high, but by now it is clear that the relationship between rates of return and PWRs is too complex to allow any definite conclusions. For example, the lowest 30-year real rate of return on record for the all-stocks portfolio corresponds to the period starting in October 1955 (4.1% annualized), but the PWR for the cohort that began retirement on that month is 6.2%—not particularly low. The so-called Kennedy Slide of 1962 produced very low returns at the beginning of these retirement periods, yet the PWR ended up in the 4.5%–5.5% range—higher than at other, less remarkable times. Although the results for 30-year periods starting just before the crash of 1987 are not yet available, Fig. 2 shows that retiring at the end of 1984—only three years before that big hit—nonetheless provided an extremely high PWR (10.7%).

The complicated relationship between rates of return and PWRs is illustrated in Fig. 6. Panel (a) superimposes the rate of return for stocks shown in Fig. 1 with the PWR presented in Fig. 2. Panel (b) shows the corresponding scatter plot, where a linear regression between these two variables attains a coefficient of determination of just 14%.

As can be seen, it is hard to establish a clear-cut connection between these two variables over the historical record. We notice, for example, a long stretch of time spanning from late 1946 until mid-1954 when the PWR was much higher than what the return rates at the time would seem to warrant. And the opposite holds true from mid-1967 to mid-1972: high return

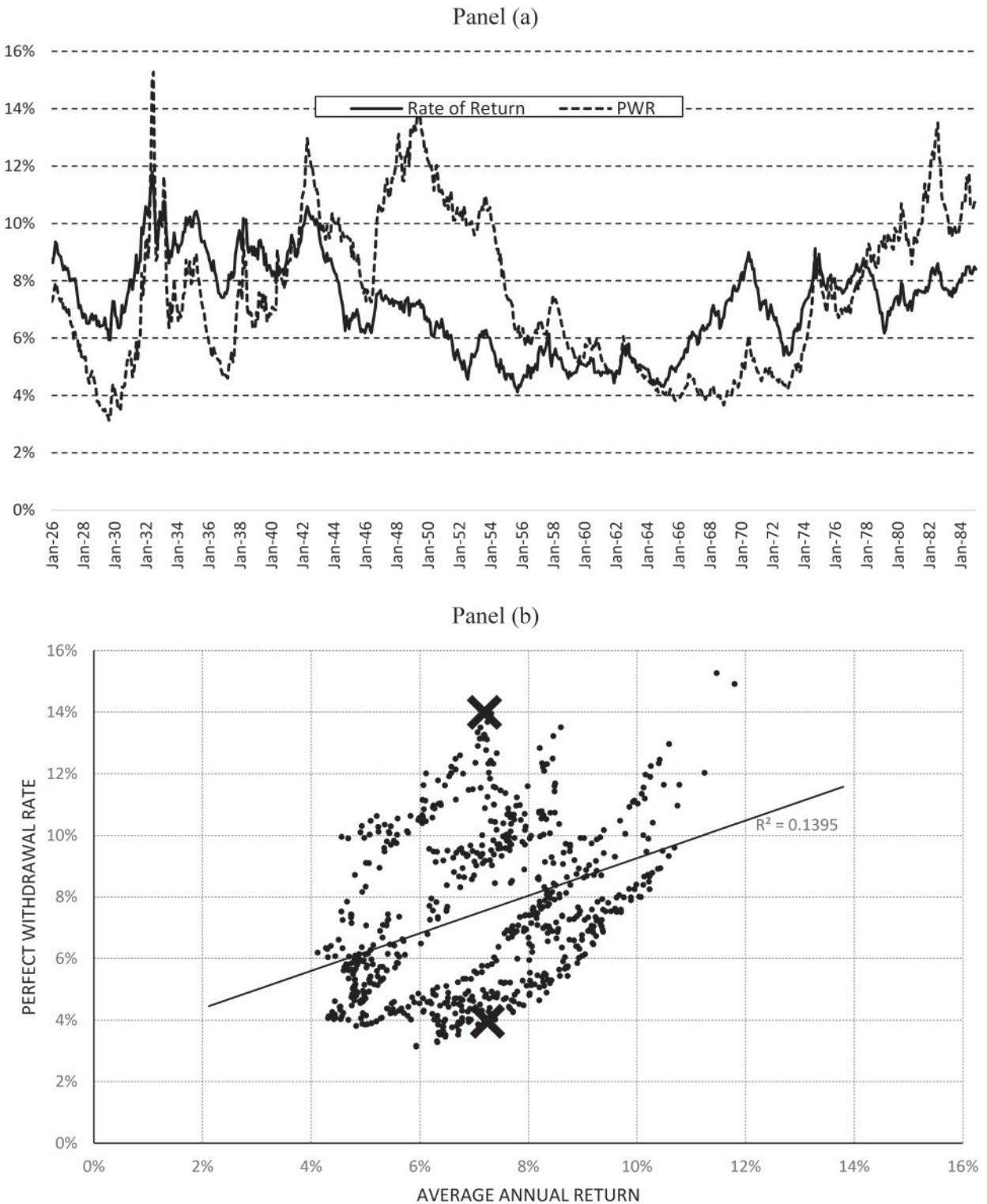


Fig. 6. Real rate of return (annualized) and standard-scenario PWR. Panel (a) compares the all-stocks average rate of return over 30-year periods with the corresponding PWR over the historical record. Panel (b) plots the same-date values of these two variables against each other. The X's in panel (b) mark Jun '49 and Feb '69, the two dates used to construct Fig. 3.

rates, but low PWRs. As shown in Fig. 3, the annualized real rate of return for the 30-year period beginning in June 1949 was 7.2% and the PWR was 14.0%. For the period beginning in February 1969 the rate of return was also 7.2%, but the PWR was 4.0%—10 percentage points lower!<sup>5</sup> The period starting in August 1952 obtained an average rate of return of just 4.6%, yet managed to attain a PWR of 10.0%. The period starting in January 1933 got a much higher rate of return, at 10.1%, but ended up with the same PWR of 10.0%!

It is now that the explanatory power of Eq. (5) becomes truly useful, as all these puzzling cases become just specific instances of one single relationship. This expression tells us concisely and unambiguously that the gist lies in the sequencing part of the equation, and it also tells us how to explore this further. Therefore, even if a specific sequence of return rates will, in all likelihood, never be repeated, the problem of forecasting the PWR in any particular case is no longer the problem of forecasting the whole series of rates. Rather, as the derivations presented here show, what the investor needs to do is to obtain an estimate of the average annual return for the portfolio she holds (for which there is abundant literature that can help her), and then address the problem of the sequencing factor value that she must use to adjust it. The next section focuses on how can the historical evidence shed light on this latter problem.

## 6. The sequencing factor over the historical record

The sequencing factor can be interpreted as an “adjustment” that has to be applied to the rate of return to express the effect of the specific sequence of results that produced that total return. It can be “good” or “bad”; in general, a series of returns that goes from high to low is better than one that goes from low to high. The periods discussed at the end of the previous section show that the magnitude of the sequencing impact can be downright stunning.

Eq. (3) shows the explicit formula for the sequencing factor which, as discussed in SSW, is not a proxy measure. It comes directly from an analytic dissection of the problem and it is a measure of *orientation*, in the same sense that variance is a measure of dispersion—and the orientation of return rates (going up, going down, up a little then down a lot, etc.) is the crucial element that the adjustment factor should capture. Fig. 7 displays the values that this formula has taken over the historical record in the standard scenario, with the corresponding frequency distribution shown in Fig. 8.

The first salient feature in Fig. 7 is that the sequencing factor is low (unfavorable ordering) and relatively stable for a very long time at the beginning of the chart. For the first 173 months (almost 14 and a half years!) the sequencing factor stays inside the lowest two brackets of the frequency distribution in Fig. 8, not surpassing the 0.0075 mark until June 1940. The reason for this is, of course, the general instability that affected equity markets during most of the 1930s—the Great Depression.

This “shaky ground” situation meant that if an investment portfolio entered the withdrawal stage at any time during this period, it didn’t take long for it to reach one or more months when the stock market crashed violently. For the 30-year periods beginning during 1926, 1927, or 1928, the crash of October 1929 came along very quickly, hurting the sequencing factor irreparably even though the average rate of return ended up in the 6%–8%



Fig. 7. Sequencing factor for 100%-stocks portfolio over 30-year periods. The sequencing factor is a measure of the orientation of an ordered set of numbers. If it is large it means that the set goes from high to low, instead of the other way around—which is good, if the numbers in the set are rates of return.

range. The periods beginning shortly after Black Tuesday did not fare much better, sequence-wise, since “just around the corner” they would meet terrible results in June 1930, September 1931, and the March-April-May stretch of 1932. Later periods in this era would be similarly crippled by major downturns in September 1937, March 1938, and May 1940.

Sequencing factor for 100%-stocks portfolio	# of instances	Relative frequency
Less than 0.0050	79	11.2%
Between 0.0050 and 0.0075	238	33.6%
Between 0.0075 and 0.0100	70	9.9%
Between 0.0100 and 0.0125	141	19.9%
Between 0.0125 and 0.0150	79	11.2%
Between 0.0150 and 0.0175	33	4.7%
Between 0.0175 and 0.0200	46	6.5%
Between 0.0200 and 0.0225	14	2.0%
Between 0.0225 and 0.0250	6	0.8%
Between 0.0250 and 0.0275	2	0.3%
Total	708	100%

Fig. 8. Frequency distribution for sequencing factor in standard scenario. The sequencing factor is a measure of orientation in return rates (“going up” vs. “going down”). In general, a low value of the sequencing factor means that the sequence of returns went mostly downwards, which in turn is correspondingly worse for the retiree.

Therefore, we could say that in these years the sequencing factor was low because the return rates could not sustain a normal level for long, instead crashing constantly. The cumulative rate of return over 30 years was very acceptable, but the initial years were bad and the damage was done.

Next in Fig. 7 we come across a stretch of rising values culminating in August 1952, when the sequencing factor reached its maximum value of 0.0261. Indeed, the sequencing factor was over 0.02 for 17 months—between September 1951 and January 1953—which produced PWRs around 10% even though the average return rate was quite low. And none of these 17 months produced spectacularly high returns (the maximum was 5.7% in November 1952); rather, it was a period when returns followed more or less an increasing pattern: from slightly negative in its first two months to moderately strong at the end of 1952. The return sequences then managed to stay clear of major crashes for a number of years, but otherwise these moderately favorable starts were all it took for the sequencing factor to attain very high values. It would seem that a favorable return sequence is not so much one that starts with a bang, but one that doesn't get its legs cut off too quickly.

Finally, we comment on a period of nearly two years—from January 1969 to November 1970—when the sequencing factor again dropped below 0.005. This case would seem to be different from the Great Depression period discussed above, because now the low values of the sequencing factor were not caused by “crashes” in the early stages of the 30-year periods. Rather, this was a somewhat peculiar time that registered negative (and positive, but small) returns in the first months, and then rose to strong returns in the second half of 1970. This unfavorable ordering caused the PWR to hover in the 4%–6% range, even though the average return was between 7% and 9% for these periods.

## 7. Results for other portfolio asset mixes

We now examine the PWR results for portfolio mixes that include bonds as well as stocks. We retain the parameters from the standard scenario regarding horizon length (30 years) and bequest goal (zero), but now allow bonds to take up a constant, positive fraction of the asset mix. We identify each asset mix by its percentage of stocks/bonds—as in 90/10 for 90% stocks and 10% bonds—and consider different mixes by varying the proportions in 10-point steps. Thus, we examine the 90/10 portfolio, the 80/20, the 70/30, and so on, all the way to the 0/100 (100% bonds) portfolio.

To maintain the same asset mix in the portfolio at all times, we assume that a rebalancing of asset classes takes place *every month*. That is, after the account obtains the return for each asset class at the end of every month, the investor buys and sells each instrument as needed to restore the corresponding proportions to the asset mix. We clarify this assumption because our set-up would also be consistent with an *annual* rebalancing scheme (withdrawals are made only once a year), and the two rebalancing methods are not equivalent. We chose monthly rebalancing because our retirement horizons are being rolled over month-by-month, and this procedure facilitates comparisons across periods that overlap. It seems to us that both methods produce very similar results, but in any case, the choice is arbitrary.



Fig. 9. PWR for all-stocks portfolio versus 90/10 portfolio. When the asset mix is changed from all-stocks to 90% stocks and 10% bonds, the PWR changes accordingly.

We begin by adding the minimal amount of bonds to the mix (10%), and present the PWR results for this 90/10 portfolio in Fig. 9. The results for the all-stocks portfolio are also shown for comparison. Fig. 10 presents the differential between the two series, as well as the variation in the actual withdrawal amount resulting from the change of mix. Variation charts similar to Fig. 10 could be produced for all the alternate model specifications presented in this paper; we present only this one instance, to convey the idea of the type of analysis that can be conducted.

These charts showcase the enhanced clarity provided by the PWR approach. An examination of Figs. 9 and 10 may lead us to conclude that the all-stocks portfolio is the best alternative, as the instances when the 90/10 mix provides better results are less frequent and less significant. By changing the all-stocks mix to 90/10, the PWR improves in 173 of the periods considered (24% of the 708 total), but in 104 of these instances the increase is less than one-tenth of a point—presumably negligible. The 69 periods where the PWR increased by more than 0.1 points represent just 10% of the total, and are all in the Great Depression era. Also, these increases never surpass 0.42 points, whereas the PWR decreases resulting from the change reach as high as 1.43 points.

However, the periods that were benefited the most by the change in the mix also had a low PWR in the standard scenario, so the impact on the perfect withdrawal *amount* is much more significant. The PWA variation is a more appropriate measure because it presents the situation as the retiree perceives it: her retirement income becoming  $x\%$  higher or lower because of the asset mix change. This is the dashed line in Fig. 10 which, for example, at its highest point informs us that if a person entering retirement in September 1929 had switched



Fig. 10. Change in PWR and variation in PWA when switching from all-stocks to 90/10 mix. PWR differential (how many points is the withdrawal rate affected) is in percentage points, solid line, left axis. PWA variation (how large is the modification in the withdrawal amount) is in percentage, dashed line, right axis.

10% of her retirement funds to bonds instead of stocks, her annual income over the next 30 years would have been 12.5% higher. This is a significant impact but, overall, the chart shows unfavorable PWA variations, so the case against including bonds seems to stand—at least in the marginal proportion examined so far.

What happens if we allot a larger proportion to bonds? For the sake of clarity in the charts, the next results are presented in sets of three portfolio mixes: 100/0 versus 70/30 versus 50/50, and 100/0 versus 30/70 versus 0/100. The summary results for the entire set of 11 mixes (from 100/0 to 0/100, in 10-point increments) are given later in this section.

Now our previous conclusion that bonds should not be included in the portfolio becomes less clear-cut, but the arguments in favor and against are easily grasped from our charts. If we want to make the withdrawal profile more stable, the 30/70 mix seems appealing (see Fig. 12). If what the investor wants is to minimize the probability of a catastrophic outcome, 50/50 looks like the way to go (Fig. 11). The grid shown in Fig. 13 is particularly useful for this kind of decisions.

From the Fig. we see that the 50/50 mix is the one that has breached the 4% PWR level the least number of times, so this could be considered the ideal portfolio for a safety-minded investor. Indeed, regarding the 4% rule it is interesting to note that in *none* of the 708 historical periods covered here has it been the case that the PWR was over 4% for the 100/0 portfolio, but less than this benchmark for the 50/50 portfolio. However, the opposite has indeed happened, as in 32 historical periods the 100/0 PWR was less than 4% but the 50/50 PWR was more than 4%. Therefore, an investor who is satisfied with a 4% withdrawal rate should

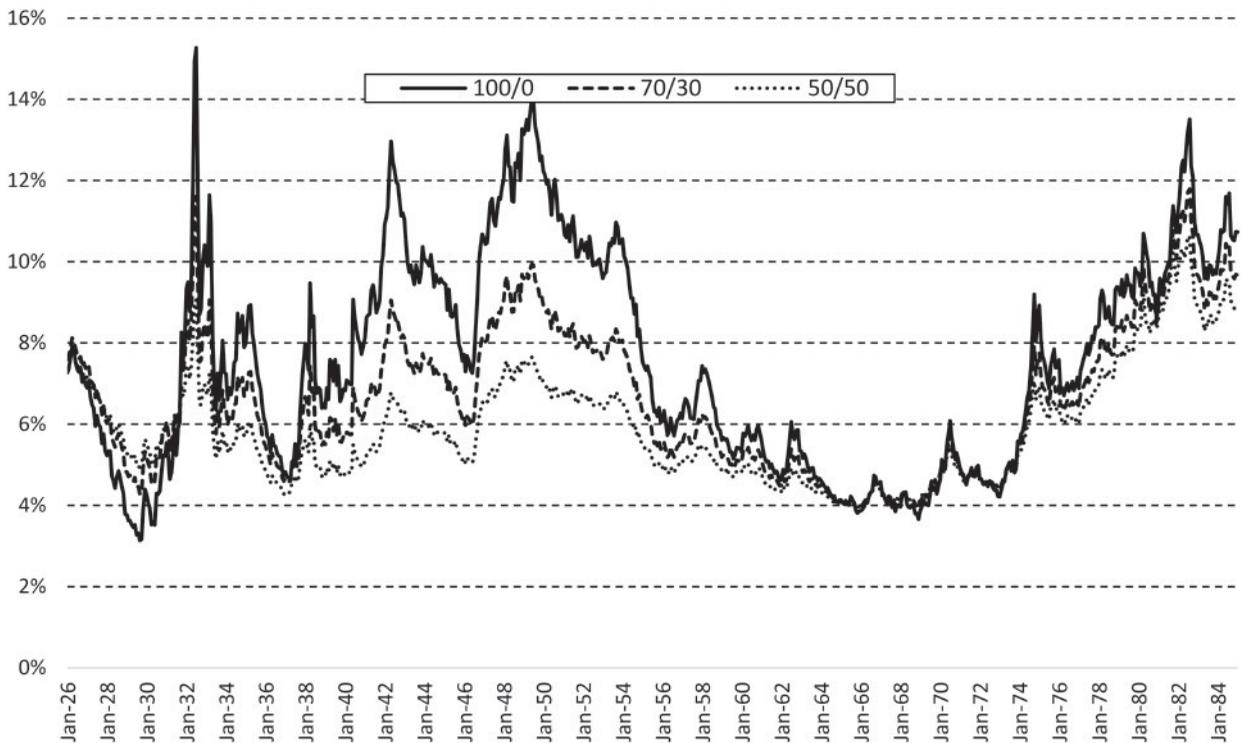


Fig. 11. PWR for 100/0 (all-stocks) portfolio versus 70/30 and 50/50 portfolios.

never hold the all-stocks portfolio: 50/50 is better. By the same token, the idea that an investor who cares a lot about safety should hold more bonds, seems unwise. In particular, if the funds' owner is overly concerned that her withdrawal amount might crash below some subsistence level, she should avoid asset mixes such as 20/80, 10/90, or 0/100.<sup>6</sup>

However, another interpretation of “safe” is that the retiree should pick a withdrawal amount that has a high probability of being sustainable. Under this version of safety, the task of selecting a portfolio mix then consists of finding the proportions that maximize, say, the 90%–safe rate. We have included these values at the end of Fig. 13, where we find that the 70/30 mix is now ideal because the upper bound of the first decile in this PWR distribution is almost 4.5%. Indeed, we can say that by selecting an appropriate asset mix, the 4% rule's withdrawal levels can be improved by over 10% without incurring additional risk.

Using the upper bound of the first decile as the watermark for separating safe from risky is, of course, an arbitrary criterion. Especially if, as discussed in SSW, the procedure for selecting the withdrawal amount calls for periodic reassessments—or if, as argued above, the retiree has other, reliable sources of income—the retiree may be willing to take a less conservative approach. After all, under a periodic revision scheme the retiree's main concern is not so much running out of funds, but having to decrease the withdrawal amount significantly in the future. Be it for those reasons, or simply because the retiree has a higher tolerance for risk, it is useful to review the figures corresponding to other “safety levels” in the historical record. Fig. 14 presents these milestone values by asset mix.

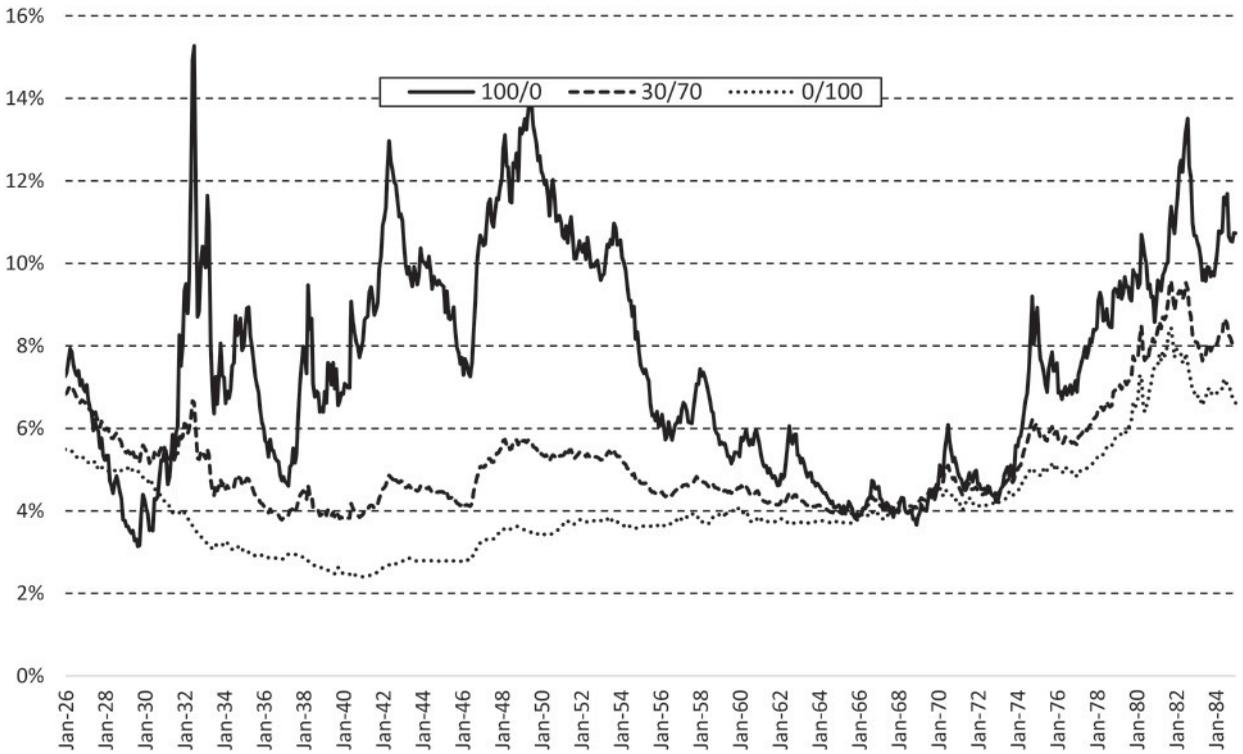


Fig. 12. PWR for 100/0 (all-stocks) portfolio versus 30/70 and 0/100 (all-bonds) portfolios.

### 8. Results for bequest goals other than zero

In this section we restore the asset mix from the standard scenario (100% stocks) to look at the behavior of the PWA when the “perfection” of the measure does not mean that it will

PWR range	Relative frequency for each portfolio mix										
	100/0	90/10	80/20	70/30	60/40	50/50	40/60	30/70	20/80	10/90	0/100
Less than 4%	4.9%	2.8%	1.6%	0.8%	0.6%	0.4%	1.3%	7.9%	22.9%	40.7%	61.4%
4% to 5%	18.4%	20.1%	21.0%	22.7%	25.6%	31.9%	42.1%	47.9%	48.6%	34.0%	19.4%
5% to 6%	13.3%	15.1%	17.7%	19.6%	24.3%	26.7%	26.8%	26.7%	14.1%	14.4%	10.2%
6% to 7%	10.9%	12.7%	14.5%	17.4%	17.4%	22.0%	17.4%	7.5%	5.6%	3.0%	4.8%
7% to 8%	12.3%	11.6%	13.1%	14.3%	17.9%	9.7%	4.1%	3.8%	5.2%	5.5%	3.7%
8% to 9%	7.5%	10.0%	13.7%	14.0%	8.3%	5.1%	5.8%	4.5%	3.2%	2.4%	0.6%
9% to 10%	12.1%	13.7%	10.9%	8.5%	3.7%	3.0%	2.4%	1.7%	0.3%	0.0%	0.0%
10% to 11%	10.0%	7.9%	5.5%	1.7%	2.0%	1.1%	0.1%	0.0%	0.0%	0.0%	0.0%
11% to 12%	4.8%	4.4%	1.4%	1.0%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
12% to 13%	3.7%	1.4%	0.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
More than 13%	2.1%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
90%-safe rate	4.28%	4.32%	4.39%	4.49%	4.46%	4.35%	4.27%	4.05%	3.73%	3.26%	2.79%
Standard deviation	0.026	0.023	0.020	0.018	0.016	0.014	0.013	0.013	0.012	0.012	0.012

Fig. 13. Frequency distribution for historical PWR, by portfolio mix, for 30-year periods and zero bequest goal. Over the historical record, the PWR was lower than 4% in the least number of periods using the 50/50 mix. But the 90%-safe rate was highest when the 70/30 mix was used.

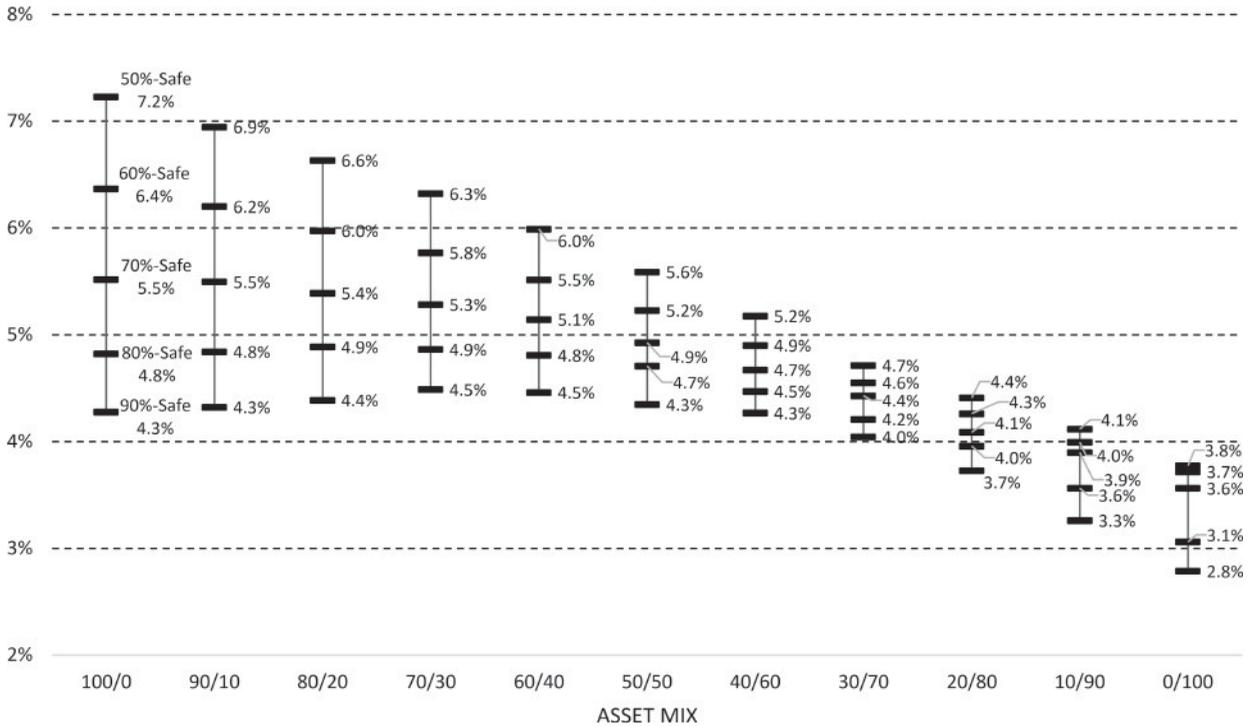


Fig. 14. Cutoff points of the first five deciles in historical PWR distributions, by asset mix. If the retiree wants to be 90%-safe she should use the 70/30 mix, but for lower safety levels (higher risk tolerance) she should add more stocks.

exhaust the account balance in a given number of periods but, rather, that it will make that balance reach a certain figure by the end of the retirement period. As in the previous sections, we assume a horizon length of 30 years and all figures are in real terms, so the bequest goal is attained in the sense that this is the purchasing power that the ending balance provides.

This investigation is important not only because many prospective retirees are actually interested in bequeathing or giving away some of their funds, but also because of the possibility of introducing positive-valued bequest goals as safety devices in retirement plans. That is, if the investor is concerned about getting low returns, or that perhaps a large unforeseen expenditure may arise, she could protect against these contingencies by planning *as if* she intended to leave an inheritance. This maneuver provides the funds’ owner with three adjustment levers to react to new developments: the withdrawal amount, the safety level, and the bequest goal. Thus, for example, in a scenario of poor portfolio performance the retiree can either reduce her withdrawals, accept a lower safety level, or reduce her bequest target/protection balance—or combine these adjustments to spread the impact.

Before presenting our results, we discuss a change in the role that the sequencing factor plays when the bequest goal is nonzero. The zero final balance specification used in the



Fig. 15. PWR for standard scenario (no bequest) versus 10% bequest goal and 30% bequest goal. The adjustment in the withdrawal rate needed to accommodate significant bequest intentions is surprisingly small (see also Fig. 16).

standard scenario produces the very succinct expression shown in Eq. (5), but the corresponding equation for the general case is:

$$w/K_S = R_n S_n - S_n (K_E/K_S) \quad (8)$$

This complete version reveals a peculiar feature of the relationship between the PWR and the sequencing factor. Although a favorable ordering in the sequence of return rates will provide a higher PWR, it will also make the result more sensitive to any change in the bequest goal ( $K_E/K_S$ ).

As we did with the asset mix, we change the bequest goal parameter in 10-point increments from zero to 100%, going from the case where the account's funds are exhausted completely (no bequest) to the case where the balance on the account is unchanged in real terms at the end of the planning horizon (100% bequest). The intermediate points correspond to bequeathing 10% of the starting balance, 20%, 30%, and so forth, always in constant purchasing power terms. We start by comparing, in Fig. 15, the PWR for the standard scenario against the results for 10% and 30% bequest goals.

We will argue that the main thing to see in Fig. 15 is, ironically, that there is not much to see in it. That is, the three variables shown are so close to each other that they “fudge” the chart, making it difficult to distinguish one line from the others. What this means is that the reduction in the PWR necessary to protect or salvage a non-negligible fraction of the

investment balance is surprisingly small. One would think, for example, that if a person who enters retirement with \$1 million in her account wants to leave a \$300,000 estate, she would have to reduce her withdrawal amount very significantly with respect to what she could take out if she didn't want to leave any funds at all. However, the evidence in the historical record indicates that the PWR differential in this case (all-stocks portfolio over 30-year periods) has never been larger than 0.8% points, it has been as low as 0.1 points, and its median value was 0.3 points—that's all she would have to shave off.

Again, it may be more relevant to discuss the required variation in the PWA, that is, how much of her annual income would the retiree have to sacrifice to reach a given bequest goal. Our results for the 30% bequest goal say that the most she would have had to cut back was 8.9%, and that in some periods the necessary decrease in her withdrawal amount was just 0.1%. The median PWA variation was  $-3.7\%$ .

However, it would be wrong to conclude that by committing to reductions as small as these the retiree would immediately have 30% of the balance available for emergencies. The figures presented here leave the desired remainder in the account, yes, but *at the end of the planning horizon*, and they depend crucially on the length of this timespan. Emergencies and unforeseen expenditures can arise at any time, and if they come up in the first few years of retirement we would be "cashing our bequest early" to deal with them. This would render the results in this section inapplicable by, in effect, shortening the retirement horizon. Still, by following these guidelines an investor will come progressively closer to having a free-disposition fund available—if she manages to navigate a number of years in retirement without major mishaps.

The next chart presents the results for bequest goals of 50% and 100% of the starting balance. The results for all the bequest-goal levels considered are presented at the end of this section.

With these two bequest-goal levels we may be moving into unrealistic territory, as perhaps very few people will want to leave behind 50% of their starting balance (let alone 100%) when they could have used it for consumption during retirement. But 50% is psychologically appealing (the "half for me, half for the kids" approach), whereas there is something very realistic about the 100% bequest goal: endowment funds. An institutional endowment fund is, essentially, a retirement investment account that must last forever. The "infinitely-lived retiree" (the institution) is interested in withdrawal strategies that provide a relatively stable flow of funds, but leave the principal value unchanged in the long run, so that the withdrawing process can go on indefinitely. This is similar to leaving a bequest of 100% after a number of years, so we comment on the results for this specification at some length below.

As with the smaller bequest goal levels shown in Fig. 15, in Fig. 16 we find it remarkable that the lines in the chart are not further apart, even though we are now dealing with very ambitious bequest intentions. To leave behind a full 50% of the starting balance as departure gift, there have been periods when the retiree only had to decrease her withdrawal rate by 0.2 percentage points. For the period beginning in June 1935, for example, the PWR for zero bequest was 8.0% and the PWR for 50% bequest was 7.8%. Of course, in some other periods the half-and-half approach has been a much more costly endeavor. For the period beginning in August 1952 (a peculiar period that we have already singled out) the PWR with no

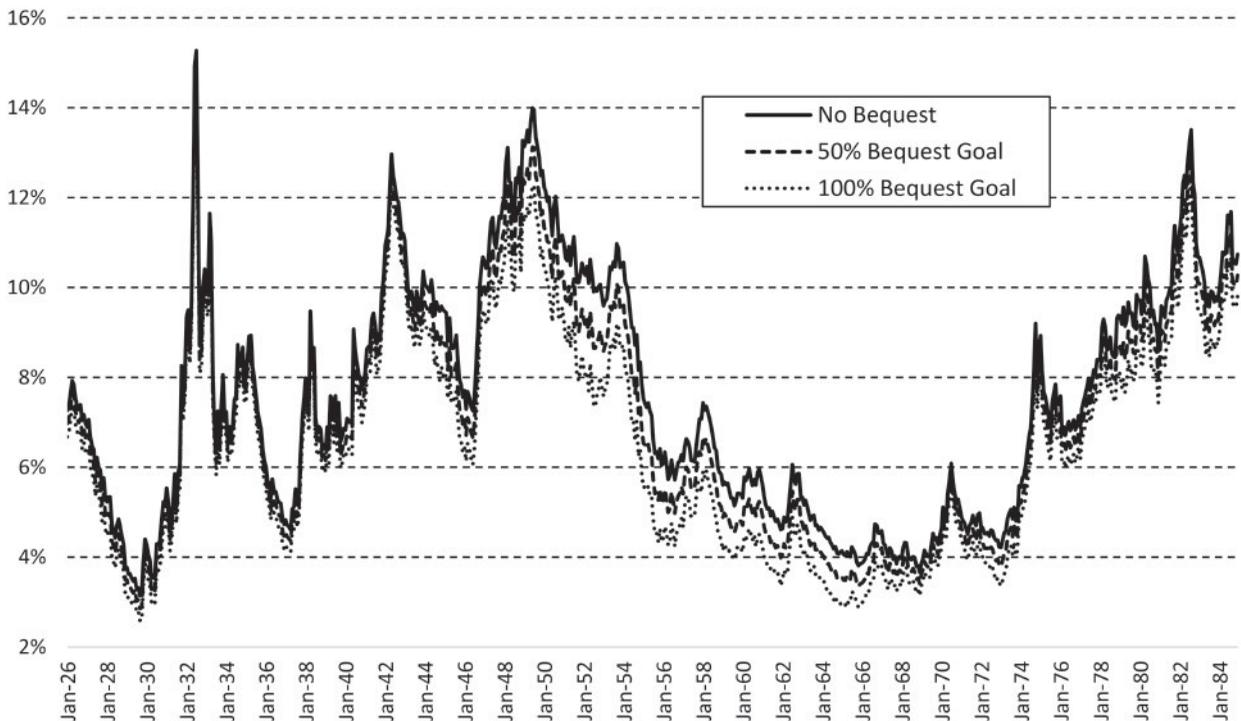


Fig. 16. PWR for standard scenario (no bequest) versus 50% bequest and 100% bequest.

bequest was 10.0%, while for the 50% bequest goal the PWR was 8.7%—a 1.3-point difference. The median reduction in the PWR, when going from zero bequest to 50% bequest, was just 0.4 points.

The variation in the PWA, when moving from a zero-bequest goal to a 50% bequest goal, was also rather small. The maximum decrease in the withdrawal amount was 14.9%, and the minimum was 1.8%. The median variation was  $-6.2\%$ , which means that, by reducing the amount withdrawn by only that much, the retiree would have ended up with a final balance equal to 50% or more of the starting balance in one half of the historical periods covered here—instead of zero dollars at the end.

As for the 100% bequest level, the necessary decrease in the PWR to achieve this goal ranged from 0.4 to 2.6 points with respect to the standard scenario, with a median of  $-0.9$  points. The PWA variation, in turn, was as large as  $-29.8\%$  in some periods, and as small as  $-3.5\%$  in others. Not surprisingly, these extreme values correspond to retirement periods for which the average rate of return was also close to the endpoints of its own range. The smallest required decrease in the withdrawal amount—to leave the real balance unchanged after 30 years instead of exhausting it—occurs in the period beginning in June 1932 (11.8% average real return); the largest, in October 1955 (4.1% average real return, lowest ever, see Fig. 1). The median PWA variation was  $-12.4\%$ .

It could even be argued that the inclusion of a high bequest goal should be the hallmark of a safe retirement plan. The 4% rule, for example, is considered a safe strategy because

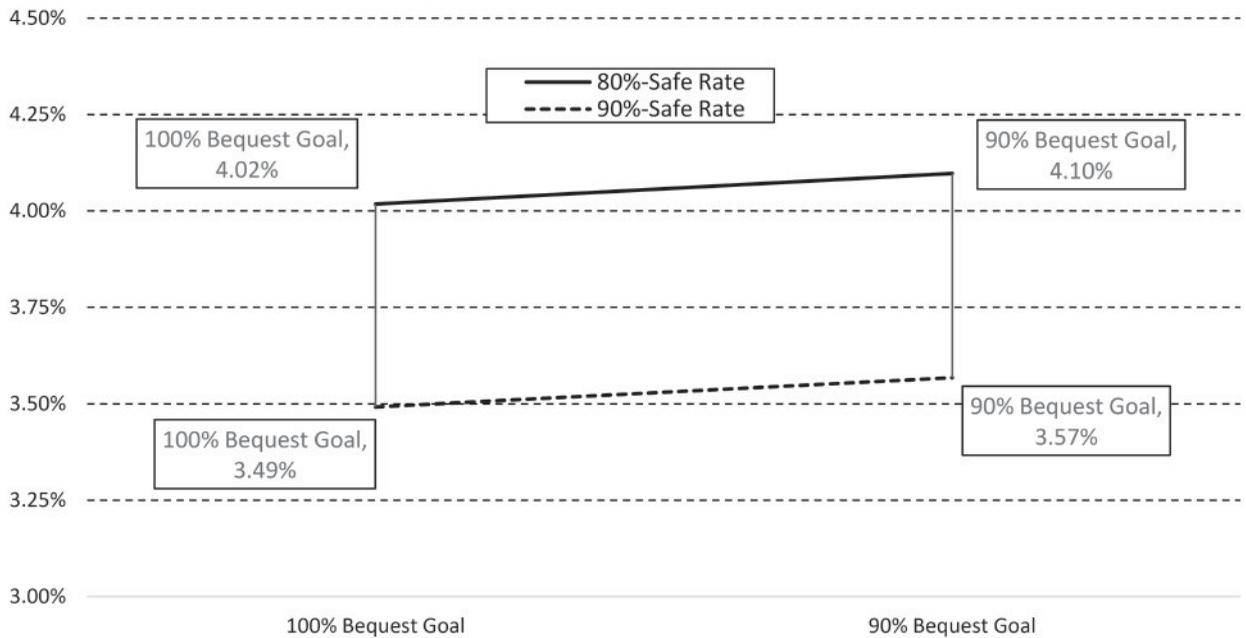


Fig. 17. Milestone safety levels in historical PWR distributions with 100% bequest goal and 90% bequest goal. Possible range of withdrawal rates for balance protection. Portfolios are all-stocks and horizon length is 30 years.

it lowers the probability of “failing,” but it defines failure as running out of funds altogether! Even in the presence of other sources of income, would it not be more prudent to plan so that the worst-case scenario—or at least the most unlikely ones—still provides means for sustenance, or some amount of supplementary funds? A promising idea would be to develop retirement plans that, at the outset, use a very high bequest goal—perhaps 100%. This feature would provide the plan with an “embedded safety” that would support the selection of an otherwise risky withdrawal rate—a 50%–safe level could even be used. Then, if future developments indicate that our case is in the unfavorable range, the bequest goal can be adjusted gradually until a safe situation is reestablished. It should be noted that if we select the median value from the correct PWR distribution, in one half of the cases the withdrawal amount would be revised up, not down.<sup>7</sup>

With respect to endowment funds, the manager’s problem is to determine the maximum amount that can be withdrawn consistently each year without jeopardizing the long-term viability of the fund. This is equivalent, to some extent, to the problem of sustaining a long retirement period as comfortably as possible and yet bequeath a large portion (close to 100%) of the starting balance in the account.<sup>8</sup> For example, an all-stocks fund manager that would like to protect the entirety of the fund’s assets, but who considers acceptable to end up with a 10% decrease in the balance after a 30-year period, may use the values shown in Fig. 17 as a reasonable range for the withdrawal rate.

From Fig. 17 we see that a withdrawal rate between 3.5% and 4.1% would seem to be appropriate in this case. If our manager takes out more than 4.1%, the “probability” that the fund’s balance after 30 years will be lower than 90% of the starting balance would be in excess of 20%—too much risk, perhaps. On the other hand, if she were to withdraw



Fig. 18. Comparison of average real return versus perfect withdrawal rate for all-stocks portfolio with 100% bequest and 30-year horizon. Although it might be argued that the applicable withdrawal rate for endowment funds is simply the average real return on the investment account, in the historical record the correct rate has turned out to be a very different figure.

less than 3.5%, the likelihood that the balance in the distant future will be larger than the current balance would be more than 90%, which seems overly conservative. Because the actual PWR is likely to be higher than these values—at least under our interpretation of past frequency as probability—perhaps the ideal procedure would be to “hunt down” the correct withdrawal rate. Instead of taking out the same amount (in real terms) every year, the fund administrator would use this range of rates to see if they translate into withdrawal amounts with the same purchasing power as what was taken out in the previous year. In case of significant discrepancy, any adjustment would be pondered against the benefits of a stable income flow.

It is interesting to note that a more simplified approach to the endowment fund problem would identify the average real return as the 100%–bequest PWR. However, these two concepts are not the same and, in general, do not coincide. They would be strictly equal only if the rate of return was the same in every period, but otherwise the sequencing effect makes them differ. Fig. 18 superimposes these two variables, and we see that the difference has been very substantial. Fig. 19 displays the relative frequency of the perfect withdrawal rates according to the range of bequest goals.

Finally, our discussion about how it is not so difficult to bequeath a large portion of the retirement funds is just the flip side of what we mentioned in a previous section: if the retiree undershoots her PWR (takes out less than the perfect amount) even by just a fraction of a point, she can end up with huge unwanted balances at the end. These realizations can lead us to conclude that the whole business of determining an optimal withdrawal amount is too

PWR range	Relative frequency for each bequest goal level										
	No Bequest	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Less than 4%	4.9%	7.1%	8.2%	9.5%	10.3%	11.4%	13.1%	15.0%	16.8%	18.6%	19.1%
4% to 5%	18.4%	16.8%	16.7%	16.8%	17.5%	18.4%	18.1%	18.5%	18.5%	17.1%	18.2%
5% to 6%	13.3%	14.1%	14.4%	13.4%	12.7%	11.3%	10.0%	8.9%	8.1%	8.2%	7.5%
6% to 7%	10.9%	11.0%	10.7%	11.9%	11.0%	11.7%	13.0%	13.0%	12.6%	12.4%	12.0%
7% to 8%	12.3%	11.2%	11.0%	10.0%	10.2%	9.5%	9.7%	8.9%	9.6%	10.5%	12.0%
8% to 9%	7.5%	8.2%	8.2%	8.9%	11.6%	11.9%	12.1%	13.3%	13.6%	14.3%	13.3%
9% to 10%	12.1%	13.0%	13.3%	13.4%	11.9%	11.7%	11.3%	10.7%	10.6%	9.0%	8.9%
10% to 11%	10.0%	9.0%	8.5%	7.2%	6.9%	6.9%	6.2%	5.9%	4.7%	4.9%	4.5%
11% to 12%	4.8%	4.8%	4.5%	5.2%	4.7%	4.4%	3.8%	3.8%	4.4%	3.8%	3.4%
12% to 13%	3.7%	3.1%	3.4%	2.8%	2.4%	2.3%	2.3%	1.7%	1.0%	0.8%	0.8%
More than 13%	2.1%	1.7%	1.1%	0.8%	0.8%	0.6%	0.3%	0.3%	0.3%	0.3%	0.3%
90%-safe rate	4.28%	4.22%	4.15%	4.07%	3.97%	3.90%	3.82%	3.73%	3.65%	3.57%	3.49%
Std deviation	0.026	0.026	0.026	0.026	0.026	0.025	0.025	0.025	0.025	0.025	0.025

Fig. 19. Frequency distribution for historical PWR, by bequest goal level, for all-stocks portfolio and 30-year horizon. For any given period in the historical record, the reduction in the withdrawal rate required to accommodate a positive final balance is equal to the sequencing factor for that period times the bequest goal sought. This means that periods that registered a high sequencing factor (good for the retiree) are also more sensitive to the introduction of a bequest goal.

fragile, subject to some kind of “butterfly effect.” However, for us they only underscore the need to develop algorithms, instead of rules of thumb, to navigate this stage of life with some degree of stability. We think that should be the research program for this field and, although we do not present any proposals here, SSW can provide sketches of what a PWA-based algorithm would look like. We close this section with the PWR frequency distribution under different bequest goals, computed using a 100%–stocks portfolio and with a planning horizon of 30 years.

## 9. Results for other planning horizon lengths

The last item in our research list is to see what happens to the PWR when the length of the planning horizon changes. As in the previous two sections, we reset the values of the other two parameters in Eq. (1) to what they are in the standard scenario. Thus, in this section the rates of return  $r_i$  correspond to an all-stocks portfolio and the ending balance  $K_E$  is set at zero, but we use different values for  $n$ .

Exploring the impact of horizon length changes is important for at least two reasons. First, the horizon length used when putting together a retirement plan will be either an educated guess of the person’s remaining life span (based on age, life expectancy data, health outlook, family history, etc.) or an intentional “overshoot” figure to accommodate the possibility of an unusually long life. If she is using the realistic figure, she wants to know what happens if she misses; in particular, she would like to know how large would the



Fig. 20. PWR for standard scenario (30 years) versus 35-year long and 40-year long horizons. All cases computed using 100%-stocks and zero bequest (total exhaustion). The adjustments required to accommodate longer lifespans are smaller than might be expected.

adjustments be if she ends up living longer than she thought. If she is “padding up” the horizon length, she wants to know the price she is paying for doing this, that is, how much is the withdrawal amount lowered by this arbitrary extension of the planning horizon.

The second reason is that the retiree will probably want to change this figure along the way. Perhaps a new medical breakthrough will benefit her, increasing her life expectancy. Or if she adopts healthier habits—exercise, better nutrition, quitting smoking—what happens then? How bad would it be if she suddenly needed to make her funds last a longer number of years? And we must also be ready for the opposite case, when health developments are such that a reduction in the planning horizon seems to be in order.

Indeed, the number of years remaining in the horizon is a different sort of uncertainty that needs to be addressed, perhaps stochastically. In this paper we are treating it as a purely subjective choice, but even a change in the *perception* of the numbers of years still ahead for the retiree (or a change in the current age of the retiree under consideration) is a drastic modification of the estimation problem. Even though the return rates going forward remain of course the same, a change in the planning horizon is a change in our choice of cutoff point for that future sequence. We start by extending the planning horizon to 35 and 40 years. Fig. 20 presents the values that the PWR has taken, over the historical record, for these alternate specifications.

Once again, as in the case of bequest goals, we find that the decrease in the PWR produced by the extension of the planning horizon is remarkably small. For the 40-year specification, the difference in the PWR with respect to the standard scenario is *at most* 1.5

percentage points, and sometimes it is as low as 0.1 points. The median differential is  $-0.4$  points.

If we use the PWA variation as measure, the results remain striking. In the historical record, the cutback to the withdrawal amount required to make the funds last 40 years instead of 30 has never been larger than 11.4%. As for the minimum, an investor who entered retirement in April 1932 with \$1 million could have taken out \$94,143 for 30 years, or \$92,634 for 40 years—a variation of just 3.5%!

The median value for the PWA variation is  $-6.3\%$ , which means that if a retiree decreases her withdrawal from the outset by just 6.3%, she will have a 50% chance of being able to cover 10 more years.<sup>9</sup> We believe that a majority of investors and retirees would think that an increase of one-third in the number of periods that must be covered with the funds available would have a much larger impact on the amount of money they can withdraw.

It is certainly possible that some 35- or 40-year periods had very high returns in the last five or 10 years, so the horizon extension managed to carry its own weight, so to speak, and did not affect the withdrawal amount very much. But this would be a rare occurrence, and Fig. 20 shows that a small decrease in the PWR is the norm, not the exception. Rather, this feature seems to be embedded in the intrinsic dynamics of the drawdown process. This is an important result, as it seems that prospective retirees perhaps do not have to worry so much about underestimating their life expectancy: there will be plenty of time to react and, in all likelihood, the reaction will not have to be severe.

It is important to stress, however, that for this result to hold the horizon extension needs to be acknowledged and applied in the early stages of the retirement period. Thus, for example, if at the very beginning of retirement, the investor is unsure whether to use a 30-year long or a 35-year long planning length, it is useful to know that the respective PWA distributions are not very different. However, this is not the same as traversing, say, one half of a 30-year retirement horizon and then decide *at that point* that we would like to cover five more years. In that case we would need to craft a 20-year plan—what we need to cover from that point on—to substitute for a 15-year plan—the remaining horizon according to the original plan. This is a different situation, although it can be assessed in a similar fashion. To provide an idea of what these “midterm” adjustments might entail, we present summary results for 20-, 15-, 10-, and 5-year long horizons later in this section.

Although the literature in this field has settled on 30 years as the typical horizon length for planning purposes, there are reasons to consider it excessively long. The full retirement age to receive social security benefits is currently 66 years and, according to data compiled by the Social Security Administration (SSA), an American man reaching age 65 today can expect to live, on average, until age 84.3. An American woman turning 65 today can expect to live, on average, until age 86.6. Only about one out of every four 65-year-olds today will live past age 90, and one out of 10 will live past age 95.<sup>10</sup> This means that the standard scenario has a large additional safety margin, as this horizon length will actually be required by just 10% of all retirees.

Our position here is that it is advisable to use a withdrawal amount with a good chance of being sustainable, but this must be taken from a PWA distribution with a realistic horizon length. Otherwise, the sustainability of the chosen amount would be incorrectly estimated, and the true safety level will likely be much higher than what was intended. Under this

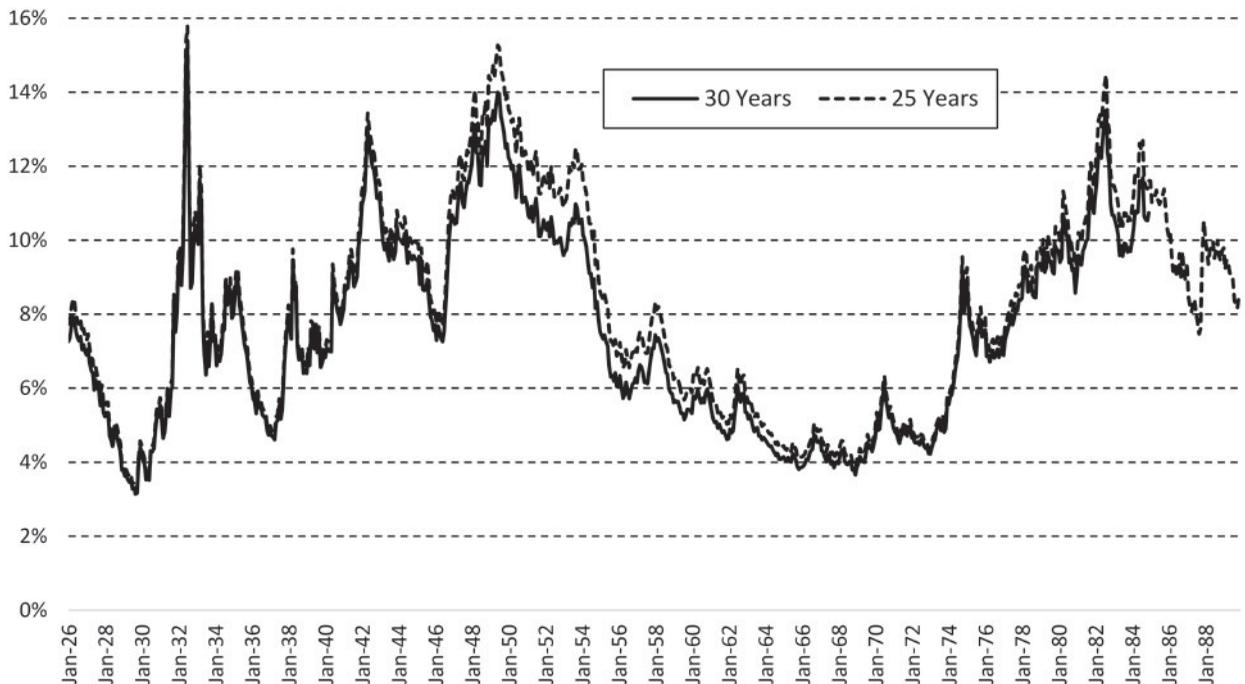


Fig. 21. PWR for standard scenario (30 years) versus 25-year planning horizon. The 25-year planning horizon is more realistic for people retiring at the age of 66.

view, the SSA's life expectancy data indicate that 25 years would be more appropriate than 30 years for first-approximation calculations. Fig. 21 presents the PWR results under this horizon length,<sup>11</sup> keeping in mind that about 25% of all retirees—particularly women—will need to apply corrections along the way to cover additional years.

We see that the PWA for 30-year periods has been between 2.9% and 13.1% lower than for the 25-year periods beginning on the same month, with a median of  $-5.3\%$ . This means that an intervention in the early years of a 25-year plan, aiming to turn it into a 30-year plan instead, should call for reductions of the withdrawal amount in the 3% to 10% range to realistically expect to cover the additional longevity.

The relatively small size of these adjustments supports our notion that it is a good idea to build retirement plans using 25-year horizons, and then operate on these plans using prudent withdrawal strategies. The 90%-safe rate in 25-year horizons with no bequest is maximal in the 70/30 mix, reaching 4.80%. The 50%-safe rate in 25-year horizons with 100% bequest—the version of conservative approach that we advocate in this paper—is highest in the all-stocks allocation, at 6.6%. Per life expectancy data, this timespan should be appropriate for most retirees and, if this is not the case, then the conservative withdrawal program will probably provide sufficient leverage to cover the extra years. Furthermore, even if the perfect storm materializes in the form of a longer-than-usual life span *and* a terrible sequence of returns, the option of curtailing the withdrawal amount does not look catastrophic because the required reduction is probably not so large.

As for the timing of the actual adjustments to be made, we advocate in favor of the procedure proposed in SSW. As time moves forward—and either the planning horizon shortens or the retiree decides to choose an endpoint farther into the future—the withdrawal amount

Panel (a)					Panel (b)				
PWR range	Relative frequency for each planning horizon length				PWR range	Upper bounds of first five deciles in PWR distributions (interpreted as safety levels)			
	25 years	30 years	35 years	40 years		25 years	30 years	35 years	40 years
Less than 4%	2.1%	4.9%	8.5%	11.1%	50%-safe rate	7.9%	7.2%	6.7%	6.1%
4% to 5%	15.1%	18.4%	19.1%	21.9%	60%-safe rate	7.1%	6.4%	5.8%	5.3%
5% to 6%	11.7%	13.3%	15.4%	16.5%	70%-safe rate	6.1%	5.5%	5.1%	4.8%
6% to 7%	9.1%	10.9%	13.0%	12.6%	80%-safe rate	5.2%	4.8%	4.6%	4.4%
7% to 8%	13.2%	12.3%	11.3%	8.2%	90%-safe rate	4.5%	4.3%	4.1%	3.9%
8% to 9%	10.2%	7.5%	10.3%	10.7%					
9% to 10%	11.2%	12.1%	11.1%	9.2%	Std deviation	0.028	0.026	0.024	0.024
10% to 11%	8.1%	10.0%	5.1%	4.8%					
11% to 12%	9.8%	4.8%	4.0%	3.9%					
12% to 13%	4.9%	3.7%	1.9%	0.9%					
More than 13%	4.7%	2.1%	0.3%	0.3%					

Fig. 22. Frequency distribution for historical PWR, by horizon length, for 100%-stocks portfolio and zero bequest goal. Panel (a) shows bins in integer percentage points. Panel (b) shows cutoff points for first five deciles.

under consideration must be assessed in terms of the percentage it represents of the account balance at that point (i.e., that is, the implied withdrawal rate). The withdrawal amount is to be modified when the location of this withdrawal rate in the corresponding PWR distribution falls outside the confidence range (probability interval) with which the retiree feels more comfortable: if the confidence level becomes too high, it is advisable to increase the

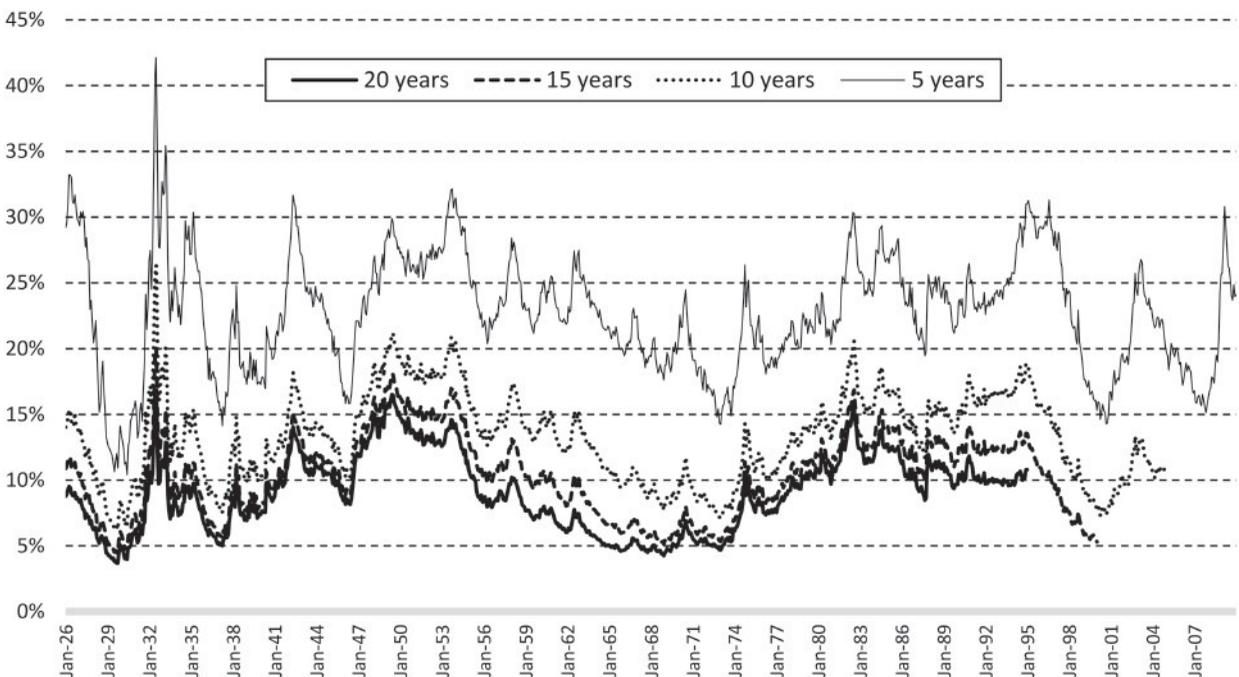


Fig. 23. PWR for standard scenario (30 years) versus 20-, 15-, 10-, and 5-year planning horizon. These shorter timeframes may be used for midcourse adjustments, once inside the retirement stage.

PWR range	Relative frequency for each planning horizon length				PWR range	Upper bounds of first five deciles in PWR distributions (interpreted as safety levels)			
	5 years	10 years	15 years	20 years		5 years	10 years	15 years	20 years
Less than 4%	0.0%	0.0%	0.0%	0.6%	50%-safe rate	22.7%	13.4%	10.5%	9.1%
4% to 5%	0.0%	0.0%	1.1%	8.0%	60%-safe rate	21.7%	12.3%	9.6%	8.2%
5% to 6%	0.0%	0.0%	8.1%	12.1%	70%-safe rate	20.3%	10.9%	8.5%	7.3%
6% to 7%	0.0%	1.1%	10.9%	6.6%	80%-safe rate	18.8%	10.1%	7.0%	5.9%
7% to 8%	0.0%	3.9%	6.0%	10.6%	90%-safe rate	16.8%	8.7%	6.1%	5.1%
8% to 9%	0.0%	8.1%	8.2%	11.7%					
9% to 10%	0.0%	6.5%	9.4%	11.8%	Std deviation	0.045	0.034	0.031	0.029
10% to 11%	0.3%	11.1%	12.0%	14.2%					
11% to 12%	0.5%	7.4%	11.6%	6.3%					
12% to 13%	0.9%	8.9%	12.5%	7.4%					
13% to 14%	0.5%	11.8%	6.6%	5.5%					
14% to 15%	1.3%	9.1%	5.2%	3.1%					
15% to 16%	3.8%	9.3%	4.8%	1.4%					
16% to 17%	4.3%	8.7%	2.5%	0.6%					
17% to 18%	4.8%	5.3%	0.6%	0.0%					
18% to 19%	4.9%	5.0%	0.2%	0.0%					
19% to 20%	6.7%	2.0%	0.1%	0.0%					
More than 20%	72.2%	2.0%	0.1%	0.0%					

Fig. 24. Same as Fig. 22, but with shorter planning horizons.

withdrawal amount; if it gets too low, the amount withdrawn should be reduced. The magnitude of the adjustments, in turn, may conceivably be based on the results presented in this paper. Thus, for example, if the investment account is all-stocks, the retiree is trying to use all her funds for consumption, and the appropriate planning horizon length for the current year is deemed to be, say, 15 years, then the third column on the right side of Fig. 24 is applicable. There we find that she should take out at least 6.1% of her balance, lest she takes on too much risk of running up an excess balance, and perhaps no more than 10.5% of the balance—unless she is not troubled with being more than 50% likely of having to reduce her withdrawal amount in the future. This is what the evidence in the historical record tells us.

Fig. 22 summarizes the main features of the historical PWR distributions for the four different horizon lengths that we discussed in detail in this section. Fig. 23 presents the corresponding values for shorter horizons, and Fig. 24 shows the historical evolution of the PWR for these shorter timeframes.<sup>12</sup>

As for how does the sequencing factor change when the length of the planning horizon is modified, an algebraic analysis indicates that it should vary inversely with the horizon length parameter. First, in general, a longer horizon should lead to lower withdrawal rates simply because the starting balance would need to cover a larger number of periods (a longer life span). Second, the total compounded return generated by the investment portfolio over the entire period should become larger as the planning horizon lengthens—again, in general—because there will now be more periods over which the return rates will compound to provide the total return. Finally, from the decomposition of the PWR

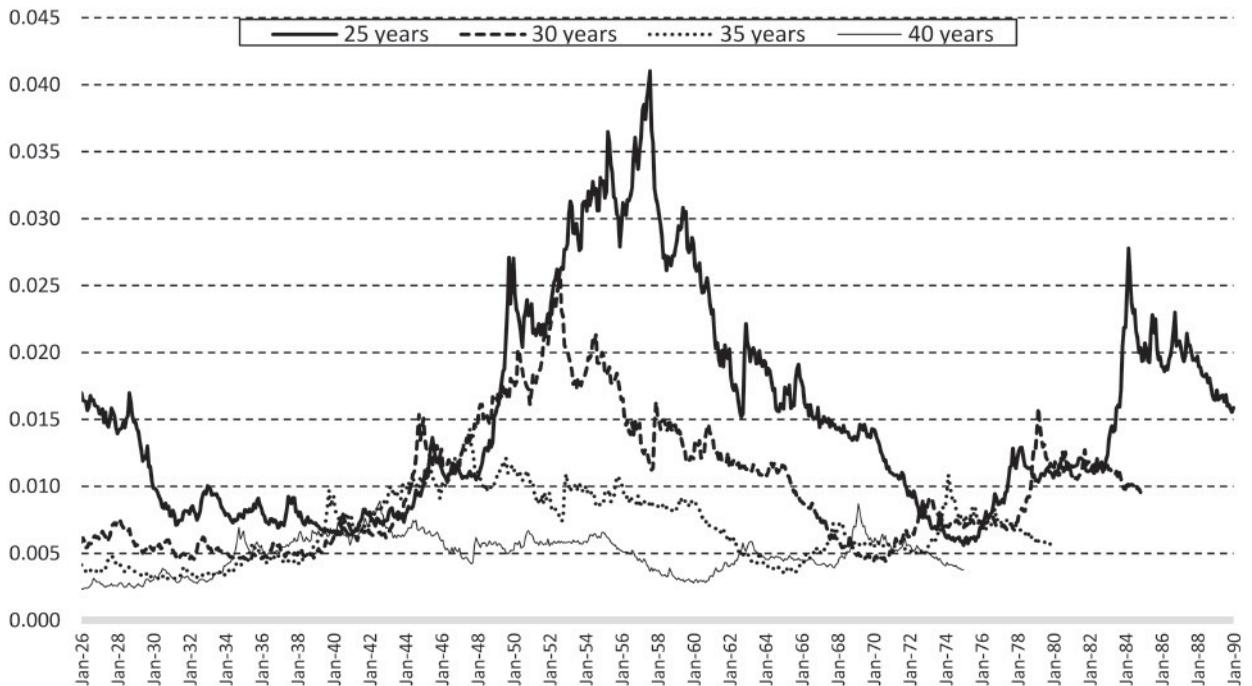


Fig. 25. Sequencing factor for 100%-stocks portfolio over different horizon lengths. In general, longer time-frames should be associated with lower values for the sequencing factor but, although true in general in the historical record, we see that this has not always been the case.

provided by Eq. (5) ( $w/K_S = R_n S_n$ ) we see that a lower withdrawal rate (left-hand side of the equation) may be reconciled with a larger total return ( $R_n$ , in the right-hand side of the equation), only if the corresponding sequencing factor decreases, because  $S_n$  is the sole remaining term in the right-hand side of the equation.

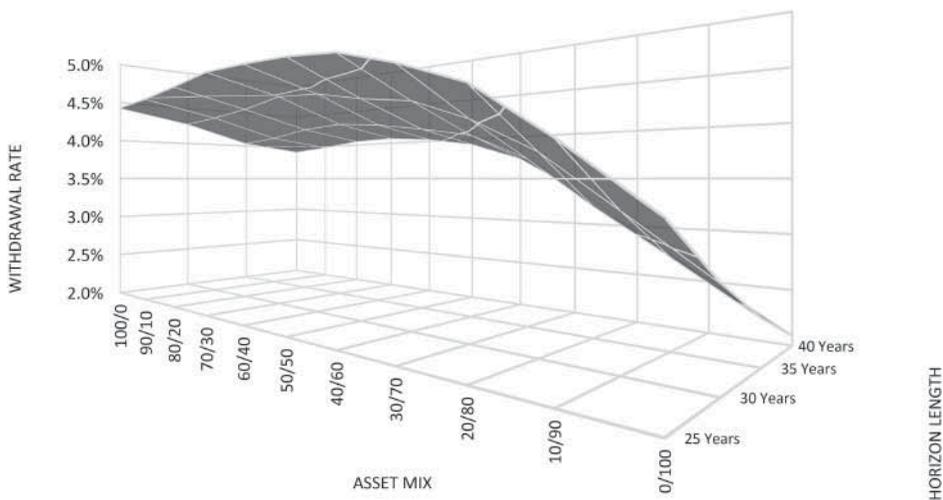


Fig. 26. Historical 90%-safe PWR for zero bequest goal, as asset mix and horizon length vary. This is a particular 3-day section of the historical PWR hypercube; similar sections can be produced for other safety levels or bequest goals. Also, any of these two control parameters may be swapped for one of the three variables in the axes of this chart, so as to explore different situations (see, for comparison, the work of Frank et al. (2011)).

Fig. 25 shows the actual values of the sequencing factor, over the historical record, for the four horizon lengths that we examined more closely in this section. It is interesting to note that, the arguments of the previous paragraph notwithstanding, there are several periods in the historical record where the sequencing factor for shorter horizons was lower than the factors for longer horizons beginning on the same date. In general, however, the arguments presented above are confirmed by our results.

It would also be useful to understand the PWR interplay between horizon lengths and asset mixes in the historical record, as this would provide guidelines for the construction of glide path strategies and for the management of funds that are to provide income during retirement. Of course, all the algorithms and rules currently used by serious practices have a strong statistical foundation, but it would still be instructive to know what these programs look like when placed under the lens of historical experience. We offer the results shown in Fig. 26 as a point of entry for this discussion.

This chart shows that for the 40, 35, and 30-year horizon lengths, the highest 90%-safe rate is attained with the 70/30 mix, whereas for the 25-year horizon this rate is slightly higher with the 60/40 mix. From the point of view of mutual funds designed to provide income during retirement, the 25-year and 30-year timespans can be considered reasonable or even short, and even so the “very safe” withdrawal rate is highest at equity allocations that are larger than what is normally used in these funds.<sup>13</sup> Also, a strategy that switches the portfolio mix away from stocks when a certain accumulation goal is attained (Basu et al., 2011) may also be called into question, because presumably the retiree’s goal is not to reach the payout phase with a large balance in her account, but to make a series of large withdrawals from it. Keeping a high stocks allocation can help her achieve this, even after the phase transition point is passed.

## 10. Conclusions

The examination of historical data using the PWA approach yields useful results for the management of retirement investment accounts. The historical record would recommend the use of relatively stock-heavy portfolios, with about 70% of total funds allocated to equity. Even with horizons of just 25 years—that would be well inside the retirement stage according to the assumptions of most models—the maximization of the 90%-safe withdrawal rate calls for stock allocations of at least 60%.

For retirement plans that intend to exhaust the available funds after 30 years—the most frequent setup in optimal withdrawal studies—portfolios containing 70% stocks and 30% bonds produced a PWR higher than 4.5% in 90% of the historical periods covered here, and in none of the 708 periods examined did it drop below 3.9%. In our opinion, this calls for a reexamination of the 4% rule because “safe” withdrawal amounts can be increased by more than 10% through a judicious selection of asset allocation. The 4% rule would seem to be more appropriate for managing endowment funds, instead.

We argue, however, that safe withdrawal strategies should include a bequest goal—even if the retiree has no such intentions—for at least three reasons. The first is that

retirement plans should strive to avoid undesirable outcomes, but running entirely out of funds is too drastic as worst-case scenario. The second reason is that the retiree needs to have a recourse in case that unexpected large outlays arise for any reason. And the third is that the presence of a bequest goal in the plan creates an additional adjustment lever for midcourse corrections, should these become necessary in case of poor portfolio performance.

Under this interpretation, safety would be provided by the bequest goal embedded in the plan, not by the location of the withdrawal amount in a histogram. For an all-stocks portfolio with a 100% bequest goal and a 30-year planning horizon, the PWR historical series produced a median of 6.4%. This could then be used as a safe withdrawal guideline, with the bequest goal as an adjustment lever that will have to be lowered in about 50% of all cases. If an adjustment via the bequest goal becomes insufficient and the withdrawal amount has to be reduced, our results suggest that the impact on the PWA for the remaining horizon length will not be very large, if the correction is applied in the first five to 10 years.

Indeed, the sensitivity of PWA distributions to changes in parameter values—be it the horizon length or the bequest goal level—is surprisingly small. Upon changing the planning horizon from 30 to 35 years, the median decrease in the zero-bequest PWR for all-stocks portfolios was just 0.3 percentage points. If the horizon is extended to 40 years, the median PWR decrease was only slightly higher at 0.4 points. This would indicate that the adjustments demanded by longer lifespans are not very large, if detected and implemented in the early stages of retirement.

In turn, if the horizon length is kept at 30 years but the bequest goal is increased to 50%—one-half of the starting balance’s purchasing power remaining in the account at the end—the median PWR decrease for all-stocks portfolios was 0.4 points. Compared with the zero-bequest case, a reduction of just 6.2% in the amount withdrawn would be sufficient to achieve (or surpass) the 50% bequest goal in one-half of the historical periods examined.

Researchers are already applying the PWA methodology with some profit, with Clare et al. (2016a) using it to implement a trend-following overlay that improves the 90%-safe rate by 2.2 percentage points in 20-year horizons. In another paper, these authors employ the PWA concept to discuss ways to address the problem of sequence of returns in decumulation portfolios (Clare et al., 2016b). In turn, Estrada (2018) takes the methodology to an international context and computes PWAs for 21 different countries.

The PWA framework was conceived as a toolkit for constructing withdrawal strategies. The results presented here should be considered guidelines, provided by financial history, to assist in the design and calibration of these strategies.

## Notes

- 1 This characteristic of non-commutativeness (changing the order in the sequence *does* change the result) lies at the heart of the sequencing factor formula. It is thus

- fundamentally different from the “volatility drag” formula ( $\frac{1}{2} \sigma^2$ ), and we thank an anonymous reviewer for comments that prompted this clarification.
- 2 This expression is valid only when it produces non-negative values. If it is negative it must have happened that the withdrawal amount was larger than the available balance at some point, which is either not allowed or implies a change in the rate of return (to a rate of interest).
  - 3 Bridges and Choudhury (2007) report that 89% of the population aged 65 and older receive Social Security benefits. In turn, Coile and Milligan (2009) present data from the Health and Retirement Study that show that for households where the older member of the couple is in the 65-69 age bracket, 39.1% of their *financial* assets are not in their IRAs, nor as stocks, bonds, or bills, but in checking, savings, and money market accounts.
  - 4 We thank an anonymous referee for bringing to our attention the need to establish this point clearly.
  - 5 This example, borne out of actual historical data, illustrates the extent to which sequencing can be crucial indeed. The return rates in both periods produced the same total return, yet the corresponding PWAs were very different solely by the effect of sequencing. If the corresponding retirees had made periodic adjustments to the withdrawal amount it would not have made any difference: the one retiring in '49 would have fared well, the one retiring in '69 would have done poorly.
  - 6 This is perhaps a very relevant observation. Studies of actual asset allocation choices have found more than 55% of people of retirement age having no equity at all in their IRA portfolios (Waggle and Englis, 2000).
  - 7 The median for the 100% bequest all-stocks PWR with 30-year horizons over the historical record is 6.4%. With the caveats mentioned regarding mid-course corrections, this could then be called the “6.4% rule”.
  - 8 Strictly speaking, the problem should be posed as the limit convergence of Eq. (1) with  $K_S = K_E$  as  $n$  increases. We use the results for  $n=30$  as an approximation, with the next section discussing their sensitivity to extensions of this horizon. We also simplify by ignoring additional future contributions.
  - 9 The number of PWR figures that can be produced with our dataset is 648 for the 35-year horizon length and 588 for the 40-year length. This is less than the 708 data points we had in the standard scenario, so the validity of these findings for inference purposes is correspondingly weaker.
  - 10 Social Security Administration (2016).
  - 11 For the 25-year horizon length, the number of PWR figures that can be produced with our dataset is 768. The dashed line in Fig. 21 (25-year long horizon) is thus longer than the solid line (30-year long horizon).
  - 12 The number of data points for these horizon lengths is 828 (20 years), 888 (15 years), 948 (10 years), and 1,008 (5 years). So the lines in Fig. 24 become longer as the planning horizon shortens.
  - 13 John Hancock Investments' Retirement Living Portfolios enter the retirement stage with a 50% stock allocation. Fidelity Investments' Income Replacement Funds offers information that implies allocations of 53%, 34%, and 13% to stocks, bonds, and cash

in Year 1 of retirement. Vanguard's Managed Payout Fund invests 57.5% in stocks, 17.9% in bonds, and 24.6% in "other". Charles Schwab's Monthly Income Fund-Enhanced Payout allocates between 10% and 40% to equities, and the Monthly Income Fund-Maximum Payout uses 0% to 25% as its equity allocation range.

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