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Your mileage may vary

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Abstract

The traditional model of retirement planning centers around the accumulation of a portfolio during the earning years followed by a drawdown from this portfolio after retirement. This drawdown is intended to support a desired retirement lifestyle, and central to a successful retirement is the sustainability of the retirement portfolio over the expected planning horizon. We define portfolio success to mean the ability of the retirement portfolio to sustain a desired retirement lifestyle over the desired planning horizon, and use simulations and logistic regressions to evaluate the impact of asset allocation, the profile of portfolio returns, the withdrawal rate, and the length of the planning horizon on portfolio success. Our analysis shows that the likelihood of success is inversely related to withdrawal rate, retirement horizon, and portfolio risk increase, and directly related to portfolio return, allocation aggressiveness, and early experience. The analysis also indicates that portfolio success is highly sensitive to withdrawal rates, with conservative allocations exhibiting greater variation in portfolio outcomes and aggressive allocations providing more dependable portfolio outcomes for retirees who desire higher withdrawal rates.

JEL classifications: G51; G53

Keywords: Retirement planning; Portfolio success; Retirement income; Withdrawal rate; Sequence risk

1. Introduction

In the process of building wealth and trying to ensure a financially secure retirement, individuals typically go through two distinct phases: an accumulation phase over the course of their career, and a postretirement drawdown or withdrawal phase. These withdrawals from

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the accumulated retirement portfolio are intended to sustain the desired lifestyle over the retired lifetime. Traditionally, retirees typically relied on Social Security, pension benefits, and personal savings to sustain their retirement lifestyle. Social security benefits in the United States are designed to only partially replace earnings that workers lose when they retire, and average only about 40% of total retirement income (Biggs & Springstead, 2008). Further, employer-sponsored defined-benefit pension plans have been declining over the years, and are being replaced by defined-contribution pension plans. The prospective retiree has to rely on social security retirement income and their retirement portfolio comprised of the defined-contribution plan balance and personal savings.

The cash flow characteristics of each source of retirement income is different. Social security retirement income is similar to a defined-benefit plan with fixed inflation-indexed payments over the life of the retiree with some survivor benefits and no potential for a bequest. The traditional defined-contribution plan requires the retiree to take minimum distributions (RMD) that fluctuate every year based on the actual plan balance and the relevant life expectancy factor for that year. A retiree who lives longer than expected may face a substantially depleted plan balance and reduced cash flow towards the later retirement years, while a retiree who dies earlier than expected may leave a large bequest. Elective withdrawals can be made from Roth defined-contribution plans and from personal savings, and any balance remaining in either is available for a bequest. These elective withdrawals serve to supplement retirement income available from other sources. An additional consideration affecting retirement cash flows is the different tax implications of each of these sources.

An important financial issue that most prospective retirees contend with, and has been extensively examined by financial planners and in the financial planning literature is: "How much can I spend each year from the retirement portfolio without completely depleting the retirement portfolio?" The most common response to this question is the "4% rule" which in its most general form, suggests that a retiree with a diversified portfolio could make inflation-adjusted annual withdrawals equal to 4% of the initial portfolio balance with a low chance of depleting the retirement portfolio over a 30-year retirement horizon. While it is popularly termed a rule, both financial planners and the academic literature understand that it is a guideline, a starting point for discussion, and that the safe withdrawal rate (SWR) should be modified based on individual circumstances.

This discussion suggests that the ability of the retirement portfolio to successfully sustain the desired lifestyle would depend on various factors such as asset allocation, portfolio return characteristics, the desired retirement horizon, and of course the desired withdrawal rate. While the literature suggests that retirees remain flexible to individual circumstances, there is very little guidance about the manner in which these factors impact portfolio success. All retirees do not make the same retirement portfolio choices and consequently may face very different retirement outcomes. We create plausible retirement scenarios and simulate variations in these factors to determine portfolio success or failure. We then use logistic regression to measure the impact that these factors have had in determining portfolio success.

Unlike previous studies, we define portfolio outcome as a binary variable and use logistic analysis to highlight the likelihood of portfolio success as a function of five retirement planning horizons, three asset allocation strategies, and five fixed real withdrawal rates to arrive at 75 plausible unique retirement scenarios. For each scenario, we use four distinct monthly return generating distributions and run 100 simulations to arrive at 30,000 portfolio outcomes and use these in a logistic regression to examine the determinants of portfolio success or failure. We show that the probability of portfolio success is inversely related to withdrawal rate, retirement horizon, and portfolio risk, and directly related to portfolio return, allocation aggressiveness, and the returns experience in the five years immediately after retirement.

This paper is important because it applies an interesting methodology to a large number of retirement scenarios and portfolio outcomes to enable a retiree better understand how their unique set of circumstances and choices impact retirement success. The rest of this article is organized as follows: We begin by reviewing some of the relevant literature to provide the necessary context for the current research. We then present the empirical design of our research, data definitions, sample statistics, and the results of our regression analysis. Finally, we present our conclusions, practical implications, and suggestions for future research.

2. Background and literature review

The traditional view of retirement is that in the early years people earn, save and invest (the "accumulation" stage) to subsequently retire and withdraw from their retirement portfolio to finance consumption over their retirement horizon (the "decumulation" or "asset distribution" stage). In the decumulation stage, individuals balance the competing goals of maintaining consumption in retirement without prematurely depleting their retirement portfolio.

One of the early solutions to the problem of creating retirement cash inflows was the state retirement pension program created in 1889 by Chancellor Bismark of Germany that provided a pension starting at age 70 when life expectancy was merely an additional two years. However, with improved working conditions and increased life expectancy, the amount of time spent in retirement today is now both longer and also a larger proportion of total life expectancy. Retirement income is now needed for a few decades rather than a few years.

The reduction in the number of defined-benefit plans and an increase in defined-contribution plans, transfers risk from the employer to the retirement saver. Employees are now responsible for the saving decision, the asset allocation decision, and the asset withdrawal decision, while simultaneously accepting the real danger of premature retirement portfolio depletion ("portfolio failure").

The determination of a sustainable withdrawal rate that would reduce the probability of portfolio failure has been addressed in a number of prior studies generally using the overlapping periods methodology or simulation methodology. A historical analysis of the overlapping retirement experiences of individuals retiring between 1926 and 1980 led to the early consensus that a retiree with a diversified portfolio could make inflation-adjusted annual withdrawals equal to 4% of the initial portfolio balance with a low chance of depleting the retirement portfolio over a 30-year retirement horizon. This 4% rule refers to the popular withdrawal rate that originated from studies like Bierwirth (1994), Bengen (1994), and Cooley, Hubbard, and Walz (1998) that were meant to dispel the notion that higher withdrawal rates that matched the historic average real returns on a diversified portfolio (between 5% to 6%) were sustainable over the retirement horizon.

Saving for retirement is challenging, and most employees have little training upon which to draw in making the relevant decisions (Benartzi & Thaler, 2007, p.102). Similarly, most retirees lack the skills required to manage their retirement portfolio successfully, highlighting the need for practical and easily understood solutions to generating sustainable retirement income (or, alternately, the need for an experienced and reputable financial advisor). Merton (2014, p.1408) states that requiring people to save for retirement is reasonable, but expecting them to acquire the expertise necessary to make investment and withdrawal decisions is not reasonable. Further, cognitive functions may decline in retirement (Bonsang, Adam, & Perelman, 2012) and while the portfolio decisions of older investors may reflect greater knowledge about investing, their investment skill does deteriorate with age due to the adverse effects of cognitive aging (Korniotis & Kumar, 2011).

This is where retirees may find the 4% rule to be useful, and it is certainly a reasonable and intuitive starting point in the retirement planning process. Indeed, Cooley, Hubbard, and Walz (1998, p.16) argue that individual experiences may vary due to personal behavioral traits, circumstances, and goals, and that no single rate appears appropriate for every investor. Moreover, the finding of a 4% SWR itself has been subject to various challenges. Pfau (2010) contends that the early consensus may be an artifact of the data used in the analysis for a couple of reasons. First, the use of rolling 30-year periods emphasizes data from the middle of the period and hence introduces temporal bias in these analyses. Second, from an international perspective, a 4% real withdrawal rate would have been "safe" in only four of 17 developed countries. These results are consistent with Dimson, Marsh and Staunton (2004) who explained that the United States has had higher real returns and lower market volatility during the 1900 to 2002 period when compared with many other countries. These results are also consistent with Estrada (2018, p. 62) who examined the retirement experience across 21 countries and 115 years using 11 asset allocations. Using equally weighted returns to a balanced portfolio across all countries in the sample, they find that a retiree with a 30-year retirement horizon would face a 50% probability of failure with a 4.8% withdrawal rate and could only withdraw 2.6% if a 5% probability of failure was desired. Further, the maximum withdrawal rate varied substantially, leading them to conclude that individuals who retired in some countries or at certain points in time had vastly different standards of living than those who retired in other countries or at other points in time.

The success of any retirement portfolio certainly depends on the expected return assumptions used in the analysis. Pye (2000) showed that 4% withdrawals from an equity portfolio with 8% real return and 18% standard deviation could be sustained for 35 years with an 81% chance of success, but a higher 4.5% withdrawal rate could be achieved by allocating 60% of the portfolio to Treasury Inflation Protected Securities (TIPS) assuming a certain 3.7% real return. Finke, Pfau, and Blanchett (2013) used simulations to review the safe withdrawal rate in the current low-yield environment to conclude that a 30-year retirement portfolio would have a failure rate of 18% if yields revert to their historic mean in 5 years. Similarly, Blanchett, Finke, and Pfau (2014) test the sustainable withdrawal rate in a low bond-yield environment and use a drift model of bond yields to show that a 4% initial withdrawal rate has just a 50% probability of success over a 30-year retirement horizon.

Thus, there is some evidence that the demonstrated success of the 4% rule is partly an anomaly of historic U.S. market returns and assumptions of expected returns. The implication from these studies is that the historical asset returns used in the overlapping periods model are not suitable for forward-looking forecasts on which retirement withdrawal strategies should be based. Further, asset returns experienced in the last decade appear to have disrupted the conventional thinking about the safe withdrawal rate, and Athavale and Goebel (2011) reinforced the notion that the 4% rule constitutes a probabilistic model and past success does not guarantee future success.

In addition to returns, there is some evidence that the standard deviation of returns and the sequence of returns may impact the success of a retirement portfolio. Blanchett and Blanchett (2008) investigate the relative importance of portfolio return and standard deviation on portfolio success using the standard 4% withdrawal rate over a 30-year period. They find that a 1% reduction in returns is likely to result in an increase in the probability of failure that is approximately four times greater than a 1% increase in portfolio standard deviation, leading them to conclude that portfolio returns have greater impact on the probability of portfolio success compared with standard deviation. Clare, Seaton, Smith, and Thomas (2017, 2021) suggest that the sequence of returns matters in both the accumulation and decumulation stages, and show that a portfolio-timing strategy using the cyclically adjusted price-to earnings (CAPE) ratio can help investors mitigate sequence risk and achieve higher withdrawal rates.

In addition to the overlapping periods model and the simulations model, researchers have explored the use of other sophisticated models to design withdrawal strategies. For example, Milevsky and Robinson (2005) use investment risk and return, mortality estimates, and spending rates in a stochastic present value framework to investigate the relationship between withdrawal rates and the probability of portfolio failure. They find that 4% withdrawals by a 65-year old retiree invested in a balanced portfolio has a 9% chance of portfolio failure, and a 3.24% withdrawal rate has a 5% chance of portfolio failure, leading them to conclude that payout ratios should be lower than generally recommended. Scott, Sharpe, and Watson (2009) suggest a strategy that includes buying and selling 30-year European call options on the market portfolio over the planning horizon to replicate the traditional 4% withdrawals, but acknowledge that many practical issues remain to be addressed before this utility maximizing methodology can be incorporated in retirement planning.

Recent studies have focused on dynamic rule-based multi-asset allocation and liquidation strategies, and switching from fixed withdrawals to variable withdrawals to improve the probability of portfolio success. A case study of the retirement experience of a 1973 retiree invested in a balanced multi-asset portfolio led Guyton (2004) to conclude that systematic decision rules and some restriction on subsequent inflation adjustments could allow for a 5.8% initial withdrawal rate.

Other examples of dynamic withdrawal strategies include adjusting the withdrawal rate based on portfolio performance and remaining life expectancy to improve retirement portfolio success and average lifetime withdrawal rates (Stout & Mitchell, 2006); using a multi-asset portfolio with periodic adjustments to the asset mix and a "bonds first" withdrawal strategy that mitigates the higher volatility in equity returns (Liu, Chang, De Jong, &

Robinson, 2009); and calculating the probability of portfolio failure each year and changing the withdrawal rate based on decision rules (Blanchett & Frank, 2009).

The importance of analyzing portfolio success rates in determining withdrawal rates has previously been emphasized by Cooley, Hubbard, and Walz (2011). They assert that changes should be made to withdrawal rates in response to unexpected changes in financial market conditions, and use the overlapping periods methodology to present portfolio success rate tables for various combinations of withdrawal rates, portfolio compositions, and payout periods.

Dynamic adjustment strategies have been shown to be relevant in both the accumulation stage (Estrada, 2019) and in the drawdown stage (Estrada, 2020) of the retirement portfolio. Specifically, Estrada (2020) shows that dynamic strategies outperform a static strategy of sticking to the plan, and periodic adjustments to the withdrawal rate is superior to adjusting portfolio asset allocations. Similarly, Robinson and Tahani (2010) treat portfolio return, longevity, and consumption as stochastic variables and use an analytical model to conclude that changing consumption to match changes in wealth could reduce the risk of portfolio failure.

These dynamic withdrawal strategies do have intuitive appeal. It is logical to calibrate asset allocations and withdrawals to changing circumstances and economic realities. An unresolved question is whether retirees would have the discipline to follow decision rules and would have the flexibility to reduce consumption. These strategies are still in the early stages of their development and we need a better understanding of the manner in which relevant variables impact portfolio success (DeJong & Robinson, 2017). Retirees may become better equipped to make these changes if they understand the implications of economic circumstances and their actions on the probability of portfolio success. Our current research, therefore, is an effort to better understand the determinants of retirement portfolio success.

3. Hypothesis development and empirical design

Each individual entering retirement has to decide about the asset allocation for their retirement portfolio, the expected retirement horizon, and the desired withdrawal rate from their retirement portfolio. In making these decisions, the retiree faces the tradeoff between maximizing consumption during retirement while minimizing the probability of prematurely exhausting the retirement portfolio.

In this context it is important to understand the composition of the retirement portfolio. As previously described, the retirement portfolio may comprise some proportion of the traditional defined-contribution plan balance, the Roth defined-contribution plan balance, and personal savings. Each of these is taxed differently; consequently, a million dollars in a traditional retirement account is not equivalent to a million dollars in personal savings which, in turn, is not equivalent to a million dollars in a Roth retirement account. Generally, withdrawals from the traditional plan balance are taxable as ordinary income; the capital gains arising from assets liquidated from personal savings prior to withdrawal are taxable at a reduced capital-gains rate; and withdrawals from Roth plan balances are not taxable. Therefore, all references to a retirement portfolio should be on a tax-equivalent basis, and for the purpose of this research, to avoid the differential effect of taxes on retirement withdrawals, we implicitly assume that the retirement portfolio referenced here has been aggregated on an after-tax basis.

It is also necessary to differentiate between the terms withdraw and consume. While the purpose of withdrawals is to finance a desired level of consumption, RMD rules may result in a withdrawal different from that necessary to finance that level of consumption. In the event RMD rules require a withdrawal greater than that needed for a desired lifestyle, we implicitly assume that the prudent retiree would reinvest the excess so as to reduce the risk of premature portfolio depletion.

Prior research has documented that the probability of portfolio success is impacted by withdrawal rates, asset allocation, retirement horizon, and measures of actual portfolio performance, including return, standard deviation of returns, and the sequence of returns. The retiree makes decisions about the withdrawal rate, the planning horizon, and the asset allocation for the retirement portfolio. However, the retirement portfolio will be affected by economic circumstances and chance, factors which are outside the retiree's control, but which will nevertheless affect the actual return, standard deviation, and sequence of returns that the retirement portfolio may experience.

We define a retirement portfolio to be a success if the portfolio can sustain a specified level of withdrawal over the entire retirement horizon. Conversely, a portfolio that is fully consumed within the retirement horizon is a "failure." Retirees need to be cognizant of the factors that can lead to portfolio failure, and while some of these factors cannot be controlled, other factors (most commonly, the withdrawal rate) can be managed to mitigate the risk of portfolio failure.

While financial planners and prior academic research encourage retirees to remain flexible with their retirement expenditures, retirees may not be aware of the impact that these variables may have on the probability of a successful retirement and may therefore be ill-equipped to make these decisions or respond to circumstances. We seek to identify those factors that contribute to portfolio success, and measure the impact that each of these factors will have on the probability of portfolio success.

Portfolio success lies at the intersection of the planning horizon, asset allocation, and withdrawal strategy (Collins, Lam, & Stampfli, 2015, p.194), and the probability of portfolio failure (also called "ruin") is a useful risk metric that can help retirees understand the link between their withdrawal strategy, planning horizon, and the asset allocation of their retirement portfolios (Milevsky & Robinson, 2005, p. 99). We use portfolio success as the dependent variable in our analysis.

We model the retirement experience as a sequence of annual adjustments, with the initial retirement portfolio growing or shrinking according to the portfolio returns in the first year followed by a withdrawal at the end of the year to finance retirement expenditures. The remaining portfolio balance then grows or shrinks according to the portfolio returns in the second year followed by an inflation-indexed withdrawal, and this progression continues over the duration of the retirement.

The decisions that the retiree makes about asset allocation, the retirement horizon, and the desired withdrawal rate are used as inputs in our analysis. We assume that the retiree prefers fixed real withdrawal rates as they are easy to understand and provide the retiree with constant purchasing power. Annual portfolio returns and standard deviation are a function of the selected asset allocation but reflect the uncertain external environment, and are determined by simulation. The simulated annual portfolio returns also determine if the retirement portfolio encounters an unfortunate sequence of negative returns during the early retirement years. The early sequence of negative returns may decimate the portfolio and significantly impair the portfolio's ability to grow and generate income, decreasing the probability of a successful retirement (Blanchett et al., 2014, p. 55).

Evaluation of retirement strategies involves the consideration of a large number of simulated or historical retirement periods and the subsequent estimation of their failure rate. The use of simulations in retirement planning has both proponents and opponents, and Sandidge (2020) explains that that simulations are ineffective because most people lack the numeric skills needed to assess probability. Collins, Lam, and Stampfli (2015) reviewed retirement income modeling strategies and state that the simulation methodology overcomes the limitation of relying on past returns as the basis of potential outcomes, thus allowing for a much greater range of potential outcomes. However, the inputs that drive these models need to be realistic. Cooley, Hubbard, and Walz (2003, p. 128) find that success rates differ when using Monte Carlo simulation methodology compared with the overlapping periods methodology, and recommend the use of simulation methodology for the longer payout periods which are important in retirement planning. We intend to simulate a large number of scenarios representing the wide spectrum of possible retirement experiences. We then follow the previously described sequence of annual adjustments over the retirement horizon to determine the outcome of each scenario, that is, whether each of the scenarios ends in portfolio success or portfolio failure. And finally, we will model the portfolio outcome (success or failure) using a logistic regression that generally takes the form:

$$Ln\left(\frac{p}{1-p}\right) = \alpha + \sum \beta_i X_i + \varepsilon$$

where p is the probability of portfolio success, and the left-hand term is the log-odds. The explanatory variables include the retiree choice variables (withdrawal rate, asset allocation, and retirement horizon) and the chance variables (return, risk, and the early returns experience). These variables are defined in Table 1.

The binary logistic regression is typically preferred when modeling a dichotomous outcome variable. We intend to use a logit model because the dependent variable is a binary categorical variable, equaling one when the portfolio successfully sustains the desired withdrawal rate over the entire retirement horizon, and zero when the outcome is portfolio failure. The logistic regression allows us to identify and analyze the impact of factors that influence portfolio success in a multivariate setting.

4. Data and method

At the start of the retirement period, individuals can make choices about the retirement planning horizon, asset allocation, and the withdrawal rate. Individuals retiring today are living longer than prior generations and spending longer periods of time in retirement. For the

Table 1 Variable names and descriptions

Name	Description
Withdrawal rate	A retiree determined withdrawal rate expressed as a percentage of the initial portfolio balance. Withdrawals from the portfolio at this rate, adjusted for inflation, are intended to finance a fixed level of real consumption over the retirement horizon. We have used five withdrawal rates in the analysis (2.50%, 3.25%, 4.00%, 4.75%, and 5.50%).
Allocation	A retiree determined portfolio allocation. We have used three allocations in the analysis (Conservative, Balanced, and Aggressive) that differ based on expected return and risk. This is treated as a categorical variable.
Horizon	Expected longevity of the retiree's portfolio. This is expected to match the retirement horizon and is determined by the retiree at retirement based on age at retirement, life expectancy, health, and lifestyle choices. We have used five horizons in the analysis (23, 26, 29, 32, and 35 years).
Return	The simple average annual real rate of return that would have been earned by the portfolio over the retirement horizon. Return is a chance variable and is a function of not only portfolio allocation but also selection, fees, and so forth. This variable introduces differences in outcomes between retirees and deviation from expected return in small samples. This information becomes known at the end of the planning horizon and cannot be used for midterm course correction. This variable is used in the analysis as an environmental control variable.
Risk	The standard deviation of the annual real rate of return, and like Return, is a chance variable. This information becomes known at the end of the planning horizon and cannot be used for midterm course correction. This variable is used in the analysis as an environmental control variable.
Early experience	The ratio of the actual portfolio balance at the end of the fifth year to the expected balance assuming certainty in returns. This is a chance variable that captures a sequence of unfavorable returns early during the retirement experience. While this variable is not determined by the retiree, it can nevertheless be useful in considering midterm corrections.

Note. This table describes the explanatory variables that are used in this analysis. Some of these variables are retiree choice variables (withdrawal rate, portfolio allocation, and planned retirement horizon) while others are chance variables (realized return, realized risk, and sequence risk proxied by the novel early experience variable).

average American, life expectancy for males, females, and married couples is 82, 85, and 89, respectively, and the probability that one member of a married couple will live to age 95 is 18% (Browning, 2016, p.51). The problem with determining the correct retirement horizon is that retirees do not know precisely when they will die. Further, while in theory a retirement portfolio is meant for consumption, in practice most retirees will feel comfortable under-consuming the retirement portfolio and planning for a long retirement rather than risking portfolio failure (longevity risk). Our analysis assumes that retirees will select one of five retirement planning horizons (23, 26, 29, 32, or 35 years) based on their health and lifestyle. Our analysis recognizes that each retirement situation is different and this uniqueness is captured through the intentional use of a wide range of values for the independent variables, allowing the analysis to apply to many retirement scenarios. For example, assuming life expectancy of 90 years, our choice of planning horizon is wide enough to cover both the traditional age 67 retiree and with a 23-year retirement horizon and the age 55 early retiree with a 35-year retirement horizon.

Our analysis also assumes that a retiree will select one of three asset allocation strategies (a conservative strategy with an emphasis on fixed-income investments, with expected real return of approximately 3.1% and standard deviation of 8%, a balanced strategy with expected real return of 5.1% and standard deviation of 12%, and an aggressive strategy with an emphasis on equity investments, with expected real return of 7.1% and standard deviation of 16%) based on their personal risk tolerance. These numbers reflect the approximate averages of the mean real return and standard deviation reported in prior research referenced in this paper. Finally, our analysis also assumes that a retiree will select one of five fixed real withdrawal rates (2.5%, 3.25%, 4%, 4.75%, and 5.5%) based on their consumption needs. The use of a range of annual withdrawal rates, retirement planning horizons, and stock allocations, is consistent with Cooley et al. (1998). These three decisions about Retirement Horizon, Asset Allocation, and Withdrawal Rate result in $5 \times 3 \times 5 = 75$ unique retirement scenarios.

The next step in the analysis is to consider likely outcomes for each of these scenarios. For example, does a 2.5% withdrawal rate from a conservative portfolio sustain the retirement portfolio over a 23-year retirement horizon? While we know that our conservative portfolio will average annual returns of approximately 3.1% and have a standard deviation of 8% over long periods of time, returns in any particular year can fluctuate away from the average with a wide range of uncertain values. The standard methodology in such cases draws random returns for each year of the retirement horizon from a theoretical distribution. We should test our hypothesis by drawing random annual returns for the first year, making 2.5% withdrawals, noting the portfolio balance at the end of the year, and if the portfolio has not been depleted, continuing this exercise for a total of 23 years.

Drawing random annual returns to the retirement portfolio requires us to impose a priori assumptions about the functional form of the distribution of expected returns, and a standard assumption is that returns are characterized by the normal distribution. This assumption, though convenient, was empirically challenged by Fama (1965) who found that the distribution of monthly stock returns belonged to a non-normal member of the stable class of distributions. Subsequently, Officer (1972) confirmed that the distribution of stock returns has fat-tails, and Gray and French (1990) confirmed that the distribution of stock index returns also deviates from the normal distribution. The preponderance of empirical evidence finds that return distributions are not normally distributed (Kring, Rachev, Höchstötter, Fabozzi, & Bianchi, 2009, p. 272), and have rejected the normal distribution in favor of either a skewed distribution or a fat-tailed distribution (Levy & Duchin, 2004 p. 48). We relax the assumption that returns follow any single distribution and use four different continuous probability distributions (Beta, Kumaraswamy, Pert, and Triangular) from which to draw random annual returns. In the absence of theoretical arguments or empirical evidence to guide our selection of the appropriate distribution, we selected four continuous distributions which allowed negative returns, allowed us to specify bounds, and displayed skewness and fat tails. The parameters of the distributions were set to ensure consistency with the desired mean and standard deviation, and reasonable boundaries were established. Thus, each of the 75 previously mentioned scenarios were tested using four different returns distributions.

Continuing our prior example, we define a retirement portfolio to be a success if 2.5% withdrawals from a conservative portfolio could be sustained over a 23-year retirement horizon, when realized returns followed the Beta distribution. In such cases we assign Outcome = 1, and in cases where the retirement portfolio is prematurely depleted before the end of the retirement horizon, we assign Outcome = 0. This gives us a total of $75 \times 4 = 300$ retirement experiences, reflecting both the choices the retiree has made and the returns uncertainty that impacts portfolio outcomes.

Another returns uncertainty that we considered in the analysis is that portfolio outcomes may also be impacted by the sequence of returns obtained. A sequence of large negative returns early in the retirement period may hasten portfolio depletion. Sequence risk (or serial returns risk) refers to the risk of premature portfolio depletion caused by a combination of withdrawals and significant negative returns early in retirement. We proxy for sequence risk by constructing an Early Experience variable, defined as:

$$Early\ Experience = \frac{Observed\ Balance\ (5,\ 2.5\%,\ 3.1\%,\ 12\%)}{Expected\ Balance\ (5,\ 2.5\%,\ 3.1\%,\ 0\%)}$$

where, the numerator is the observed portfolio balance at the end of the fifth year after 2.5% withdrawals each year from a conservative portfolio earning 3.1% average real returns that follow the Beta distribution with a standard deviation of 12%, while the denominator is what the balance would be assuming certainty in portfolio returns. A better Early Experience implies a greater probability of portfolio success, and we would expect a positive relation between the Early Experience variable and the Outcome variable. The observed portfolio balance at the end of the fifth year and the value of the Early Experience variable at the end of the fifth year are presented in Table 2.

The process described above was repeated 100 times. Thus, the 300 retirement experiences simulated 100 times each gives us $300 \times 100 = 30{,}000$ portfolio outcomes. Another way of thinking about this is that the simulations are a way of testing the retirement scenarios to see the potential outcome of many possible trajectories and to gauge how vulnerable the scenarios are to portfolio failure. Our model is consistent with Pfau (2012) in that portfolio success is dependent on the interaction of withdrawal rates, capital market conditions, retirement durations, and asset allocation, and the Spitzer, Strieter, and Singh (2007)

Table 2 The early (returns) experience

Portfolio allocation	Withdrawal rate	Average portfolio balance	Early experience		
			Mean	Minimum	Maximum
Conservative	2.50%	1,033,998	100.20	47.78	170.27
	3.25%	994,054	100.20	46.39	172.39
	4.00%	954,110	100.21	44.89	174.71
	4.75%	914,166	100.21	43.26	177.23
	5.50%	874,222	100.22	41.47	179.97
Balanced	2.50%	1,145,076	100.10	38.63	206.12
	3.25%	1,103,597	100.10	37.27	208.77
	4.00%	1,062,118	100.11	35.80	211.64
	4.75%	1,020,638	100.12	34.21	214.73
	5.50%	979,159	100.13	32.48	218.10
Aggressive	2.50%	1,284,861	101.56	22.52	274.61
	3.25%	1,241,476	101.61	21.20	278.86
	4.00%	1,198,091	101.65	19.77	283.43
	4.75%	1,154,706	101.70	18.25	288.35
	5.50%	1,111,322	101.75	16.59	293.65

Note. This table presents information about the observed portfolio balance at the end of the fifth year and the value of the early experience variable. The early experience variable is a proxy for sequence risk which refers to the risk of premature portfolio depletion caused by a combination of fixed withdrawals and significant negative returns in the early years of retirement. There are 2,000 observations in each of the 15 combinations of portfolio allocation and withdrawal rate for a total of 30,000 observations.

assertion that a blanket four percentage withdrawal rule may be an oversimplification of a complex set of circumstances.

5. Descriptive statistics and empirical results

The frequency of success (Outcome = 1) or failure (Outcome = 0) among the 30,000 portfolio outcomes described in the previous section is presented in Table 3.

Portfolio success occurred 83.3% (24,989 of 30,000) of the time among the observed portfolio outcomes. The aggressive portfolio had an average success rate of 87.83% (8,783 of 10,000) while the conservative portfolio has a success rate of 76.34%. As expected, the portfolio with a 2.50% withdrawal rate had an average success rate of 99.13% (5,948 of 6,000) while the portfolio with a 5.50% withdrawal rate had a success rate of 59.08%. And finally, as expected, the portfolio with a 23-year planning horizon had an average success rate of 91.3% (5,478 of 6,000) while the portfolio with a 35-year horizon had a success rate of 75.73%.

We had previously indicated that the retiree decides about portfolio allocation (based on risk tolerance), withdrawal rate (based on consumption needs), and retirement horizon (based on expected life expectancy and lifestyle choice), and we can disaggregate the observed portfolio success rates using these variables. These disaggregated observed portfolio success rates are presented in Table 4.

Table 3 Frequency table of observed portfolio outcomes

		Failure	Success
By portfolio allocation	Conservative	2,366	7,634
	Balanced	1,428	8,572
	Aggressive	1,217	8,783
		5,011	24,989
By withdrawal rate	2.50%	52	5,948
	3.25%	253	5,747
	4.00%	731	5,269
	4.75%	1,520	4,480
	5.50%	2,455	3,545
		5,011	24,989
By retirement horizon	23 Years	522	5,478
	26 Years	750	5,250
	29 Years	1,004	4,996
	32 Years	1,279	4,721
	35 Years	1,456	4,544
		5,011	24,989

Note. $N = 30{,}000$ portfolio outcomes. This table presents the number of successes (or failures) among the 30,000 portfolio outcomes in our sample, aggregated by portfolio allocation, or withdrawal rate, or retirement horizon. These numbers suggest that conservative portfolios, higher withdrawal rates, and longer retirement planning horizons increase the chance of portfolio failure.

A few observations are notable. A withdrawal rate of 2.5% can largely be sustained irrespective of portfolio allocation and retirement horizon. However, higher withdrawal rates (5.5%) over longer retirement horizons (35 years) have a success rate of 65.5% with an

Table 4 Observed portfolio success rates

Portfolio allocation	Withdrawal rate	Retirement horizon				
		23 Years	26 Years	29 Years	32 Years	35 Years
Conservative	2.50%	100.00%	100.00%	99.50%	99.75%	99.00%
	3.25%	99.75%	98.75%	96.00%	95.75%	91.25%
	4.00%	96.00%	93.00%	83.25%	79.50%	71.00%
	4.75%	88.25%	76.75%	59.75%	50.25%	37.25%
	5.50%	66.50%	47.75%	38.75%	23.50%	17.25%
Balanced	2.50%	99.75%	99.50%	99.75%	98.50%	99.00%
	3.25%	99.25%	97.50%	96.00%	94.50%	93.25%
	4.00%	95.50%	92.75%	90.50%	84.50%	84.00%
	4.75%	86.50%	83.50%	83.00%	73.50%	68.25%
	5.50%	77.25%	70.75%	65.50%	56.75%	53.75%
Aggressive	2.50%	98.75%	99.00%	98.00%	98.50%	98.00%
	3.25%	96.50%	96.00%	94.75%	94.00%	93.50%
	4.00%	94.50%	92.50%	88.50%	84.50%	87.25%
	4.75%	89.25%	86.00%	82.25%	77.75%	77.75%
	5.50%	81.75%	78.75%	73.50%	69.00%	65.50%

Note. This table allows us to evaluate the simultaneous impact of portfolio allocation, withdrawal rate, and retirement horizon on portfolio success rates. Portfolio success occurs in 24,989 of the 30,000 portfolio outcomes.

aggressive portfolio allocation, but only 53.75% with a balanced portfolio allocation and 17.25% with a conservative portfolio allocation. A withdrawal rate of 4% over a 29-year horizon has a success rate of 90.50% with a balanced portfolio allocation, 88.50% with an aggressive portfolio allocation and 83.25% with a conservative portfolio allocation. And finally, a conservative portfolio exhibits wide variations in outcomes ranging from 100% to 17.25%, while an aggressive portfolio allocation exhibits variations ranging from 99.00% to 65.50%. These results confirm that aggressive portfolios improve the probability of portfolio success for higher withdrawal rates.

We next use regression analysis to analyze the impact of factors which influence portfolio success in a multivariate setting. Our variable of interest is portfolio success that is a binary dependent variable. In such cases, the ordinary least squares technique can be nonconforming and the estimates of the dependent variable can go out of bounds (0, 1). The logistic regression technique is well suited to examining the relation between portfolio success and the predictor variables as it keeps the predicted values of the dependent variable within expected bounds.

At the start of the retirement period, the retiree makes decisions about the withdrawal rate, the planning horizon, and the asset allocation for the retirement portfolio. The retirement portfolio will also be affected by actual returns, standard deviation, and sequence of returns that the retirement portfolio may experience. While these factors are outside the retiree's control and will nevertheless impact portfolio outcome, the retiree may be able to observe and act on any early unfavorable sequence of returns that the portfolio may experience. These then, are the explanatory variables used in the analysis.

The logistic function is used to estimate, as a function of unit changes in the independent variables, the probability that the event of interest will occur. Our logistic model provides a good fit for the data if we can demonstrate an improvement over the intercept-only model, and we check this using the Akaike Information Criterion and the Schwarz Criterion. We also note that the Cox and Snell R^2 is 46%, Nagelkerke's rescaled R^2 is 78%, and McFadden's pseudo R^2 is 69%. In addition, the maximum likelihood coefficient estimates are all individually significant at 1% using the Wald χ^2 test.

Direct interpretation of the logistic regression coefficients is difficult since coefficient estimates are in terms of log-odds. The estimated coefficients do not represent the marginal effects of the independent variables on the probability of portfolio success. Instead of the coefficients being the rate of change in the dependent variable as the independent variable changes, a coefficient derived from a logistic regression is interpreted as the rate of change in the log-odds as the independent variable changes. Exponentiating the coefficient gives us the odds ratio, which can range from 0 to infinity, and which allows for somewhat easier interpretation. The model coefficients and the odds ratio are presented in Table 5.

While the odds ratio is somewhat easier to interpret than the coefficients of the logistic regression, neither is as useful as the traditional marginal effect. Unlike a linear regression, the marginal effect is not constant across the entire range of values of the explanatory variable, and hence the marginal effect is calculated at each observation in the dataset, and then averaged. This average marginal effect indicates expected changes in the predicted probability of portfolio success as a function of a change in an explanatory variable while keeping

Table 5 Determinants of portfolio success

	Coefficient	Wald χ^2	Odds ratio	Marginal effect
Intercept	18.4342 (0.6080)	919.2899		
Withdrawal rate	−3.1056 (0.0552)	3161.9001	0.045	−0.1311
Allocation aggressive	2.7052 (0.7069)	14.6443	14.957	0.1142
Allocation conservative	−2.3530 (0.5821)	16.3402	0.095	−0.0993
Horizon	−0.3638 (0.00870)	1746.6138	0.695	−0.0154
Return × Allocation Conservative	1.5348 (0.0412)	1384.7595	4.641	0.0648
Return × Allocation Balanced	1.3722 (0.0358)	1466.0887	3.944	0.0579
Return × Allocation Aggressive	1.2288 (0.0320)	1477.4365	3.417	0.0519
Risk × Allocation Conservative	−2.648 (0.0424)	38.9240	0.767	−0.0112
Risk × Allocation Balanced	−0.3202 (0.0377)	72.1836	0.726	−0.0135
Risk × Allocation Aggressive	−0.4087 (0.0317)	166.5041	0.664	−0.0173
Early experience	0.0781 (0.00178)	1916.2730	1.081	0.0033
R^2	69%			

Note. $N = 30{,}000$. Standard errors are placed below the coefficient. All coefficients are significant at the 1% level. This table presents the results of the logistic regression analysis used to model the probability of portfolio success, where portfolio success is defined as the ability of a retirement portfolio to sustain a desired lifestyle over a desired retirement horizon.

other covariates constant. The average marginal effect for each explanatory variable is also presented in Table 5.

The marginal effect of the Withdrawal Rate variable indicates that the probability of portfolio success changes by −0.1311 for a 1-level change in the Withdrawal Rate. Similarly, the marginal effect of the Retirement Horizon variable indicates that the probability of portfolio success changes by −0.0154 for a 1-level change in the Retirement Horizon. Early Experience is a continuous variable and hence the marginal effect is defined as the partial derivative of the probability of portfolio success with respect to Early Experience. Similarly, higher levels of Return increase the probability of portfolio success while higher levels of Risk decrease the probability of portfolio success, and this is consistent across all allocations. Finally, an Aggressive Allocation changes the probability of portfolio success by 0.1142 compared with a Balanced Allocation, and a Conservative Allocation changes the probability of portfolio success by −0.0993 compared with a Balanced Allocation.

Table 6 Determinants of portfolio success (partitioned by portfolio allocation)

	Aggressive		Balanced		Conservative	
	Coefficient	Marginal effect	Coefficient	Marginal effect	Coefficient	Marginal effect
Intercept	14.62		16.12		24.98	
Withdrawal rate	−2.11	−0.0814	−2.84	−0.1137	−4.62	−0.1962
Horizon	−0.26	−0.0101	−0.32	−0.0126	−0.55	−0.0232
Early experience	0.06	0.0023	0.07	0.0030	0.11	0.0045
Return	0.93	0.0360	1.26	0.0506	2.18	0.0926
Risk	−0.31	−0.0120	−0.29	−0.0117	−0.38	−0.0163
R^2	66%		68%		75%	

Note. $N = 10{,}000$ for each of the three partitions. All coefficients are significant at the 1% level. This table presents the results of the logistic regressions (sample partitioned by portfolio allocation) analyzing the impact of the explanatory variables on the probability of portfolio success.

We also analyzed the determinants of portfolio success by partitioning the data based on Allocation. This facilitates validation of the full-sample results and also allows for easier interpretation of the results. The results of this analysis are presented in Table 6.

As before, retiree-determined variables (Withdrawal Rate and Retirement Horizon) are significant across all three values of Allocation, as are the chance variables (Early Experience, Return, and Risk). The negative sign on withdrawal rate indicates a lower probability of portfolio success at higher withdrawal rates. This effect is most pronounced for the conservative portfolio allocation—the probability of portfolio success changes by −0.1962 for a 1-level change in Withdrawal Rate. Retirees who select a conservative portfolio allocation will find that the probability of portfolio success is more sensitive to the determinants of portfolio success, as compared with selecting a conservative or aggressive allocation.

6. Concluding comments

This analysis has examined the impact of various retiree-determined and chance variables on the probability of portfolio success. The results of this analysis provide retirees with specific information about the impact their actions may have on the probability of portfolio success. While the chance variables, by definition cannot be controlled, retirees may be able to mitigate the risk of portfolio failure by using intermediate targets like Early Experience to determine the need for midterm corrections to the retirement plan.

The results of the analysis confirm that portfolio success is impacted by both retiree-determined variables (Allocation, Withdrawal Rate, and Planning Horizon) and chance variables (the profile of portfolio returns includes Return, Risk, and Early Experience). However, the relative impact of each of these variables differs. Withdrawal Rate is the most significant driver of portfolio success, and though relevant, Early Experience is not as significant. Retirees who select a conservative portfolio allocation will find that their portfolio success is much more sensitive to the explanatory variables, when compared with retirees who select other allocations. Conservative portfolio allocations also result in wide variations in portfolio success outcomes, while aggressive portfolio allocations result in relatively narrow variations across different withdrawal rates. These results are consistent with the Ho, Milevsky, and Robinson (1994) assertion that equity should have a bigger role in retirement portfolios than is recommended by most financial planners. The actual return to a particular retiree may differ from that suggested by the portfolio allocation. Both Return and Risk are significant in the analysis suggesting that asset selection within a portfolio is important to mitigating any adverse effect that these variables would have on portfolio success. The Early Experience variable could serve as an early indicator of the need for midterm course corrections to the retirement plan with the withdrawal rate serving as the transmission mechanism for portfolio success.

Planning for success in sustaining a retirement portfolio would be incomplete without also discussing other issues which arise even in the event of portfolio success. In the event of portfolio success, by definition, there is a residual (unconsumed) portfolio balance. This reduced consumption over the retirement horizon is the prudent reality of dealing with an uncertain future in the absence of well-accepted instruments that can capture the value of a potential future surplus. Another issue is the possibility of the retiree living beyond the planned retirement horizon. While any unconsumed portfolio balance may serve to mitigate longevity risk, longevity insurance is also available in the form of a single-premium deferred inflation-indexed fixed life annuity, and this could allow real consumption at the same level as that experienced during the expected retirement years. And finally, many financial planners recommend that retirees maintain a cash bucket outside their invested portfolio as part of their overall strategy. This cash bucket is intended to meet unexpected consumption needs, reduce the need to liquidate portions of the invested portfolio during market downturns, and finance any mismatch in the timing of cash flows.

When it comes to retirement planning, the cost of failure is high. Nevertheless, most retirees lacking the knowledge and tools, engage in wishful thinking rather than structured planning. Retirement planning is complex and has inherently uncertain outcomes. This analysis is a simple and imperfect representation of the complex realities of retirement planning. Although the analyses are simplistic, the results provide guidance to the manner in which various retiree-determined and chance variables impact portfolio success. The research proposed in this study is a topic of active policy debate, and may serve as a baseline for additional research and sophisticated and dynamic models for generating lifetime income for retirees, pension funds, endowments, and managed payout mutual funds.

This research also has various limitations. The simulation methodology requires us to specify the unknown future distribution from which returns might obtain, and the expected parameters of that distribution. We have assumed that the retiree makes withdrawals at the end of every year, and have not considered monthly or quarterly withdrawals. We have also implicitly assumed that the retire may engage in additional retirement planning outside the invested portfolio (e.g., bequests, longevity insurance, and a cash bucket for liquidity) and that retirees desire a constant level of real consumption over the retirement horizon. And finally, our analysis examines portfolio success as a binary (success/failure) variable but does not consider the size of the bequest or the timing of the portfolio failure as measures of the extent of success or failure (Estrada and Kritzman, 2019).

There is no "one size fits all" single right answer when it comes to addressing the various tradeoffs and interactions associated with retirement planning, and outcomes will vary based on retiree choices, economic circumstances, and chance. It is nevertheless important to understand them so that a realistic initial plan and consumption target can be established, and mid-term adjustments can be initiated if necessary. Just be aware that your mileage may vary.

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