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Personal Financial Planning and the Allocation of Disposable Wealth

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In the process of personal financial planning individuals are confronted with a time dependent wealth allocation problem. Oftentimes the solution involves selecting financial products based on objective criteria, for example, product cost and expected return. While objective criteria are important to the selection process, an individual's subjective valuation *of all criteria, objective and subjective, relevant to the decision plays the crucial role. A goal programming model parameterized by the analytical hierarchy process is presented to determine the allocation of an individual's disposable wealth to present and future consumption bundles and investable assets, conditional on the preference ordering of the individual.*

I. INTRODUCTION

The personal financial planning process has in recent years increased in complexity, requiring individuals to seek outside assistance in developing a plan for their financial needs (Cooper and Ulivi, 1983). Yet, to date there has been relatively little rigorous academic research addressing the personal financial planning function.[1] One important dimension of personal financial planning involves the allocation of an individual's disposable wealth to investable assets, and present and future consumption bundles. The process of personal financial planning, most often undertaken with the counsel of a financial planner, involves an individual making subjective assessments with regard to the risk and expected return of alternative assets, and consumption preferences. For example, a commonly used tool by financial planners is a risk profile evaluation which crudely gauges an individual's level of risk tolerance. In the personal financial planning literature, however, there is no decision model that fully integrates an individual's subjective valuation of all

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objectives and constraints relevant to this time dependent wealth allocation problem.

This paper posits such a decision paradigm. We present a multiple-objective model that generates a portfolio consistent with an individual's preference toward current and future consumption and desired portfolio characteristics. Our model is based on multi-objective goal programming (GP) with the parameters of the model derived through the analytical hierarchy process (AHP). Thus, we offer a determinate model that unifies the characteristics of the alternative portfolios with an individual's preference set to ascertain an individual's optimal allocation of disposable wealth.

In Section II we provide a brief description of the two decision-making techniques employed in our model, goal programming and the analytical hierarchy process. In Section III we outline our financial planning model and in Section IV we discuss the model implementation and illustrate its use.

II. Decision-Making Methodologies

A. Goal Programming

The concept of goal programming (GP) was introduced by Charnes and Cooper (1961) and applied to the decision-making environment by Lee (1972). GP is best viewed in the context of Simon's (1955) seminal work on the "satisficing" nature of managers. In an environment where conflicting goals cannot be achieved simultaneously the solution that achieves a set of goals to the manager's satisfaction is implemented. In a GP model goals are formulated and the underachievement of these goals is minimized based on the relative priority or weighting of the goal. In other words, GP allows for a "satisficing" solution when an optimal solution with all goals attained is not feasible.

In financial planning the decision-maker is faced with a number of conflicting goals, for example the maximization of return and liquidity while minimizing risk. Therefore, the decision maker must derive a priority weighting scheme to obtain a satisfactory solution where goals are achieved in order of importance.

In the context of financial planning and decision-making, GP has been applied to both personal and corporate financial planning problems. Recent examples include Batson (1989), Puelz and Puelz (1989) and Kvanli and Buckley (1986). In these GP models the deviations from portfolio and/or consumption goals are minimized at weights established by the decision-maker. These goal weights can be preemptive or relative in nature. The difficulty in utilizing GP to solve the wealth allocation problem for an individual is in the establishment of relative objective and subjective goal weights. We employ the analytical hierarchy process (AHP) to generate the weights for the portfolio and consumption goals in the asset allocation problem. These weights are used to parameterize the GP model which generates the portfolio that minimizes an individual's goal unattainment. The actual GP model formulation is presented in detail in Section III.

B. Analytical Hierarchy Process

As suggested in the preceding section, the personal financial planning problem has associated with it many subjective and objective criteria important to an individual. For example, the value an individual places on the liquidity characteristics of the portfolio is subjective while the expected return is objective. Comparisons among subjective and objective criteria by an individual are, however, inherently judgmental reflecting a preference weighting after all comparisons have been held. In our model it is this subjective valuation of the relevant criteria in the personal financial planning process which is captured. Multiple criteria weights are derived through the AHP by incorporating the individual's judgment into an objective ratio scale through pair-wise comparisons of preference orderings.[2] In the context of the time dependent wealth allocation problem, the objective is investor satisfaction which is dependent on short and long-term consumption goals, and portfolio goals which include the characteristics we model: risk, liquidity and asset preference. Future consumption takes the form of M period short-term bundles and an additional bundle classified simply as "Long-Term Consumption." This hierarchy of goals for the personal financial planning problem is presented in Figure 1.

The AHP is carried out by the individual through a pair-wise comparison of the goals at each level of the hierarchy. In other words, the individual compares the relative importance of one goal with respect to another which is quantified through a pair-wise comparison scale (see Table 1). For example, a ratio of nine between any two goals means that one goal is absolutely more important than the other. All the ratios are stored in a criteria matrix which is positive reciprocal. That is, all diagonal elements equal one, elements above the diagonal range in integers from one to nine and their reciprocals, and the j,i element below the diagonal is the reciprocal of the i,j element above the diagonal.

At each level of the hierarchy relative importance weights represented by the eigenvector are determined by the solution to the equation:

$$(Q - \lambda^* I)w = 0 \tag{1}$$

where Q is the $n \times n$ criteria matrix of pair-wise comparisons over n goals, I is the $n \times n$ identity matrix, and λ is the eigenvalue which is the solution to the characteristic polynomial of Q.[3]

For the personal financial planning model the first level of the hierarchy is comprised of two criteria (consumption goals and portfolio goals) which will reveal a 2×1 column vector of importance weights among these criteria when the individual undertakes the pair-wise comparison of relative importance. The eigenvectors for the second level of the hierarchy represent important weights for each of the M short-term consumption periods and the long-term consumption period, the N asset alternatives, and the risk and liquidity characteristics, with respect to the two characteristics of the first level. This will yield a $(M + N + 3) \times 2$ matrix of

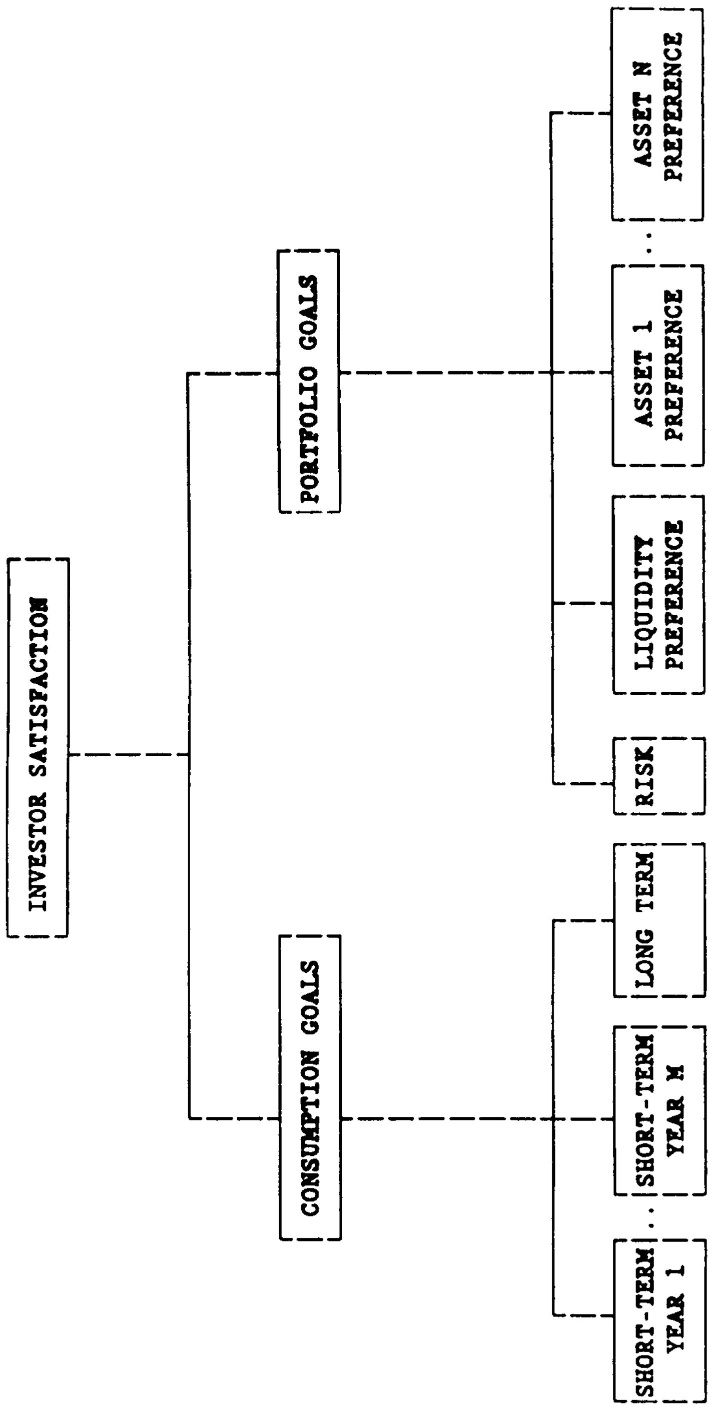

Figure 1. Goal hierarchy for financial planning allocation.

TABLE 1.
Importance Scale

Intensity of Importance	*Definition*
1	Equal importance
3	Weak importance of one over another
5	Strong importance of one over another
7	Demonstrated importance
9	Absolute importance
2,4,6,8	Intermediate values between the two adjacent judgments
Reciprocals	If attribute i has one of the above non-zero numbers assigned when compared with activity j, then j has the reciprocal value when compared to i.

eigenvectors for the second level of the hierarchy. The overall ranking of each of the portfolio alternatives is obtained by pre-multiplying the 2×1 column vector of importance weights from the first hierarchy by the $(M + N + 3) \times 2$ matrix of second level eigenvectors. The result is a $(M + N + 3) \times 1$ vector which weights the $(M + N + 3)$ bottom level goals from highest to lowest preference.

Finally, the reason for the joint AHP/GP process is two-fold. First, if the AHP is used to allocate directly to assets as, for example, in the asset allocation problem addressed by Khaksari, Kamath, and Grieves (1989) or the life insurance selection problem by Puelz (1991) then the individual must directly compare categories based on objective and subjective goals. The typical individual who contracts for personal financial planning services does not have sufficient knowledge to perform such a comparison.[4] Second, the financial planning model proposed in this paper is based on a multiperiod horizon. In order to incorporate AHP into a multiperiod planning model a different model is required for each future period. By solving the multiperiod problem as a set of independent models the integration of a single portfolio decision for multiple periods is lost. By contrast, in our GP model the multiperiod nature of the personal financial planning decision is captured.

III. The Financial Planning Model

We present the model in the following manner. First, we formulate feasibility constraints which impose the restrictions that (a) dollars invested each period do not exceed disposable income for that period plus liquidated investment from prior periods, and (b) that the liquidated amount of any asset does not exceed the principal amount plus earnings on that asset. Second, we present the portfolio and consumption goals and identify the deviation variables that are placed in the objective function of the GP model with a weight established from the AHP. The math program minimizes the weighted deviations from the portfolio and consumption goals subject to the feasibility constraints. The solution entails the quantity of assets which are selected for purchase and sale each period, consistent with the individual's preference ordering. The notation used throughout the GP model formulation is presented in Table 2.

A. Model Constraints

Model constraints are those conditions that must be met in order for a feasible portfolio to be generated. The first set of constraints equates for each of the M periods in the short-term horizon the net dollar investment, $\Sigma(\tau_{bn}X_{nm} - \tau_{sn}Y_{nm})$, to disposable wealth for that period, A_m, less consumption for that period, Z_m[5]

$$\sum_{n=1}^{N} [\tau_{Bn}X_{nm} - \tau_{Sn}Y_{nm}) + Z_m = A_m \qquad \forall \quad m = 1 \text{ to } M \tag{2}$$

The second set of constraints assure that the amount divested from an asset in any period, $Y_{n,j+1}$, is not greater than the principal investment and earnings on that asset,

$$\sum_{m=1}^{j} [1 + \rho_n)^{j-m+1} (X_{nm} - Y_{nm})] \geq Y_{n,j+1}$$

$$\forall\, j = 1 \text{ to } M - 1 \text{ and } n = 1 \text{ to } N. \tag{3}$$

TABLE 2.
Model Notation

X_{nm}	=	The amount invested in asset n at the beginning of period m.
Y_{nm}	=	The amount divested in asset n at the beginning of period m.
Z_m	=	Idle funds used for short-term consumption in period m.
τ_{Bn}	=	The dollar amount necessary to buy $1.00 of asset n (includes transaction costs).
τ_{Bn}	=	The dollar amount received from the sale of $1.00 of asset n (includes transaction costs).
ρ_n	=	Expected after tax annual return on asset n.
α_n	=	Liquidity parameter for asset n.
β_n	=	Risk parameter for asset n.
A_m	=	Dollars available to invest at the beginning of period m.
P_m	=	Desired consumption level period m.
P_L	=	Desired long-term consumption level.
L	=	Desired maximum portfolio average liquidity.
R	=	Desired maximum portfolio average risk.
S_n	=	Desired maximum (minimum) percent held of asset n.
d_i^-	=	Negative deviation from a particular goal level in goal constraint i.
d_i^+	=	Positive deviation from a particular goal level in goal constraint i.
W_i	=	The AHP generated weight attached to the achievement of goal i.
N	=	Number of assets.
M	=	Number of periods in the short-term horizon.

B. Model Goals

Goals are classified in the personal financial planning model as consumption and portfolio goals. Short-term consumption goals are anticipated cash expenditures in the short-term horizon, and might include such things as the purchase of a car, or a planned vacation. The dollars required for each period's short-term consumption is estimated by the individual with the aid of the financial planner and serves as desired short-term consumption goal attainment levels. The set of constraints for the short-term consumption goals are formulated as follows:

$$Z_m + d_i^- - d_i^+ = P_m \qquad \forall \quad m = 1 \text{ to } M \tag{4}$$

where P_m is the desired short-term consumption and Z_m is cash available for consumption during period m. Deviations from these goals are penalized at a level established in the AHP framework. In other words, if cash available during any period falls below the desired level, P_m, penalties are assessed due to underachievement of the goal. In the GP model the negative deviation variable, d_i^-, indicates the amount below the desired consumption level and is therefore minimized in the objective function at the AHP established weight.[6]

Long-term consumption is simply the individual's desired savings at the end of the planning period. Goal levels are established and are discounted back to period M. P_L is the discounted long-term consumption level and the right-hand side is the value of the portfolio at period M. The long-term consumption goal is formulated as

$$\sum_{n=1}^{N} \left[\sum_{m=1}^{M} (1 + \rho_n)^{(M-m+1)} (X_{nm} - Y_{nm}) \right] + d_i^- - d_i^+ = P_L \tag{5}$$

As in the short-term consumption goals, the negative deviation, d_i^-, represents underachievement of the goal and is minimized in the objective function at the weight established in the AHP model.[7]

In the development of an individual's portfolio, the financial planner considers not only the individual's desired short and long-term consumption goals but also the individual's attitudes towards various portfolio characteristics such as risk and liquidity.[8] In addition, the individual may have a preference for certain asset categories. These preferences are modeled in the portfolio goals.

Portfolio liquidity is measured by the liquidity of the underlying assets. The liquidity parameter, α_n, is the percentage penalty required to immediately liquidate asset n.[9] This parameter is estimated by the bid-ask spread for the asset. The goal constraints for liquidity are formulated for every period in the short-term horizon.

$$\frac{\sum_{n=1}^{N} \alpha_n \left[\sum_{m=1}^{j} (X_{nm} - Y_{nm}) \right]}{\sum_{n=1}^{N} \left[\sum_{m=1}^{j} (X_{nm} - Y_{nm}) \right]} + d_i^- - d_i^+ = L \qquad \forall \quad j = 1 \text{ to } M \tag{6}$$

The right-hand side of equation (6) represents the average portfolio liquidity in period m. The left-hand side of equation (6) is the maximum desired portfolio liquidity desired, L, as measured by the percentage penalty required to immediately liquidate a dollar of the portfolio. The positive deviation variable, d_i^+, measures the unattainment of the liquidity goal and is minimized in the objective function of the GP model at the established AHP weight.

The risk of a portfolio is measured by the risk of the underlying assets. The asset's beta, β_n, is a measure of the non-diversifiable risk associated with that asset. R is the portfolio risk level above which penalties are assessed. The goal constraints for risk are formulated for every period in the short-term horizon

$$\frac{\sum_{n=1}^{N} \beta_n \left[\sum_{m=1}^{j} (X_{nm} - Y_{nm}) \right]}{\sum_{n=1}^{N} \left[\sum_{m=1}^{j} (X_{nm} - Y_{nm}) \right]} + d_i^- - d_i^+ = R \qquad \forall \; j = 1 \text{ to } M \tag{7}$$

Analogous to the liquidity goal, the right-hand side of equation (7) represents the average portfolio risk and the left-hand side is the maximum desired portfolio risk, R. In the objective function the positive deviation, d_i^-, represents unattainment and is minimized in the objective function at the appropriate AHP weight.

Finally, asset preference goals are in the form of the desired minimum or maximum proportion of the portfolio to be allocated to asset n. For example an individual may desire that at least twenty-percent of the portfolio be placed in growth stocks.

$$\frac{\sum_{m=1}^{j} (X_{nm} - Y_{nm})}{\sum_{k=1}^{N} \left[\sum_{m=1}^{j} (X_{km} - Y_{km}) \right]} + d_i^- - d_i^+ = S_n \qquad \forall \; n = 1 \text{ to } N \text{ and } j = 1 \text{ to } M \tag{8}$$

The right-hand side of equation (8) is the proportion of thc portfolio allocated to asset n and the left-hand side is the maximum or minimum proportion desired for asset n. These goals are formulated for every asset where a maximum or minimum percent is desired. If S_n is the minimum (maximum) proportion to be held of asset n, then the negative (positive) deviation variable, d_i^- (d_i^+), is minimized in the objective function at the AHP established weight.

The GP objective function is of the form

$$MIN\, Z = \sum_{i=1}^{K} W_i d_i^{+/-} \tag{9}$$

where W_i is the weight attached to the attainment of goal i. In words the objective function minimizes the sum of the weighted deviations from goal attainment levels. The deviational variables in equation (9) are those selected from each set of constraints that represent goal unattainment. The W_i values are generated through the AHP process as discussed in Section IIB.

In summary, the GP model generates the portfolio plan for the M-period short-term horizon. The plan consists of the periodic amounts to buy and sell of each asset. The model constraints assure that dollars invested each period do not exceed disposable income for that period plus liquidated investments from prior periods, and that the liquidated amount of any asset does not exceed the principal investment and earnings on that asset. The goal constraints consider short and long-term consumption and the portfolio characteristics of risk, liquidity and asset preference. In each goal constraint, the deviation variable representing underachievement of the goal is minimized in the objective function at the AHP established weights.

IV. Model Implementation

Two types of data are required for our model: investment alternatives and the individual's AHP established goal weights. The data on available investment vehicles is maintained by the financial planner and contains estimates on expected returns, liquidity parameters, risk parameters, and all relevant transaction costs, for each investment option. Investment options are grouped into broad homogeneous categories (i.e., growth stocks or insured municipal bonds). This is preferred over individual security investment options for two reasons. First, the amount of data to be maintained and the size of the goal programming model are greatly reduced. Second, the model will generate the portfolio in terms of broad investment categories, giving the planner and individual flexibility in selecting individual assets or mutual funds within the established broad categories.

To illustrate the use of the AHP/GP framework, an example portfolio is structured. We consider ten asset categories that represent the choice set of investment alternatives (see Table 4). In this example, these categories were determined by the authors to be important, however other categories may be important to another, and the choice set would be altered to reflect the addition or subtraction of such categories. We assume a short-term annual planning horizon of three years with projected disposable wealth (including current savings) for years one, two and three at $100,000, $16,000, and $17,000 respectively.

The individual, with the aid of the financial planner, sets consumption and portfolio goal levels. The goal levels used in this example are presented in Table 3. Through the AHP, the individual performs a pair-wise comparison of goals at each level of the hierarchy represented in Figure 1. For example, at the bottom level of the hierarchy, the individual compares the relative importance of the short-term consumption for years one and two, years one and three, and years two and three using the importance scale in Table 1. The weights established in the AHP for this example are presented in Table 3.[10] For example, the individual places primary

TABLE 3.
Example Problem-Goal Levels and AHP Established Weights

Goal Categories	*AHP established relative weights**
Consumption	
Short-term	
Year 1 $P_1 = \$10{,}000$	.161
Year 2 $P_2 = \$\ 9{,}000$	.064
Year 3 $P_3 = \$20{,}000$	.032
Long-term $P_L = \$130{,}000$	.201
Portfolio	
Liquidity $L = .02$	.030
Risk $R = .65$	.167
Asset Preference	
Proportion in long-term growth > 5%	.069
Proportion in tax-exempt < 10%	.069
Proportion in government and U.S. agencies < 7.5%	.069
Proportion in low-risk corporate bonds < 15%	.069
Proportion in precious metals < 10%	.069

Note: *Consumption weights are normalized by dividing P_m or P_L.

importance on long-term consumption (relative weight = .201), and relatively more importance on the risk of the portfolio than the portfolio's liquidity. After all pair-wise comparisons are performed the AHP generated weights are incorporated into the objective function of the GP Model in equation (9) and the GP model formulated in equations (2) through (9) is solved to generate the financial plan that maximizes the goal attainment level for the individual.

All relevant transaction and tax cost are incorporated in the model. The optimal allocation of disposable wealth to short and long-term consumption bundles, and investable assets for this example is detailed in Table 4.

The dollar amounts of assets bought and sold in each year are listed in columns titled *Assets Purchased and Assets Sold*. Initial investments in the first year are high because of the assumption that the individual has substantial savings available to invest the first year. Each year, thereafter, the disposable income is assumed to come only from current income. Assets are liquidated in year three to achieve the short-term consumption goal during that year. It should be noted that five assets are liquidated during year three in order to achieve the liquidity and risk goals for that year. If the individual desires that transactions be in larger blocks, then the solution procedure for the GP model would be altered.[11]

The percentage goal attainments for this example, as measured by the actual level of attainment divided by the desired level of attainment for each goal, are presented in Table 5.

TABLE 4.
Example Problem-Model Output

Asset category	*Year*	*Amount purchased*	*Amount sold*
Growth, small companies	1	$26,166	—
	2	0	$ 0
	3	0	2,257
Growth, long-term	1	4,354	—
	2	313	0
	3	0	181
Income	1	11,973	—
	2	826	0
	3	0	537
Tax-exempt	1	8,708	—
	2	561	0
	3	0	437
Tax-exempt, high-risk	1	0	—
	2	0	0
	3	0	0
Government and U.S. agencies	1	6,531	—
	2	638	0
	3	0	72
Corporate bonds, high-risk	1	0	—
	2	0	0
	3	0	0
Corporate bonds, low-risk	1	20,639	—
	2	2,508	0
	3	347	0
Precious metals	1	8,708	—
	2	1,001	0
	3	79	0
Real estate	1	0	—
	2	0	0
	3	0	0

Both long-term consumption and asset preference goals have attainment levels below 100%. This is due to the fact that the GP technique maximizes the weighted attainment of all goals. In other words, the generated portfolio plan maximizes the individual's *overall* satisfaction.

TABLE 5.
Example Problem-Goal Attainment

Goal categories	*Percentage goal attainment*
Consumption	
Short-term	100%
Long-term	91%
Portfolio	
Liquidity	100%
Risk	100%
Asset preference*	89%

Note: *Average attainment for all asset preference goals.

V. Conclusion

We have presented a multi-period model to allocate an individual's disposable wealth to short and long-term consumption, and investable assets through the use of goal programming and the analytical hierarchy process. Our model integrates an individual's subjective valuation of all relevant goals associated with the allocation problem into a math programming model which generates an allocation solution consistent with the individual's consumption and portfolio goals.

We illustrated the model's operation for a particular individual's valuation of consumption goals, and characteristics of ten asset categories. The model, however, is sufficiently flexible to accompany a broader range of goals and assets.

Notes

1. See Cohen (1988) for a detailed literature review in the area.
2. An exposition on AHP can be found in Saaty (1980) or Saaty and Vargas (1982).
3. The characteristic polynomial of Q is the determinant of $Q - \lambda I$.
4. For example, an individual may be asked to assign relative weights to the liquidity characteristics of a municipal bond and a precious metal.
5. Disposable wealth is defined independent of sufficient funds to pay for all essential consumption and to abstain a financial emergency, and adequate insurance cover for common perils such as pre-mature death, disability, and property-liability losses.
6. The AHP established weight is normalized by dividing by P_m.
7. The AHP established weight is normalized by dividing by P_L.
8. The client may have other concerns such as capital appreciation, current income, inflation protection, tax reduction, etc. These preferences may be incorporated in the model in the same fashion as risk and liquidity.
9. The liquidity parameter could be a percentage penalty required to immediately liquidate an asset or a measure of the number of years required to liquidate an asset without penalty. Since short term consumption goals are already incorporated into the model the liquidation of the portfolio should only occur in the event of an emergency or unforseen event which would require

immediate liquidation. Therefore, it is more appropriate to measure liquidity by the percentage penalty to immediately liquidate.

10. *Expert Choice* software was used to carry out the pairwise comparison and solve for the relative weights.
11. In this case an integer math programming algorithm would be required to solve the GP model.

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