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Probabilistic Estate Planning

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Probabilistic estate planning is based on the principle of maximizing expected net present value commensurate with the risk assumed. Rather than assuming that death occurs at life expectancy, probabilistic estate planning treats death as a random variable. Compounded to randomly chosen ages of death, estate assets are taxed and distributed to heirs. The purpose of probabilistic estate planning is to find the estate plan and asset/liability combination that maximizes the expected net present value of assets passing to heirs and to convey some idea of the risk associated with that estate plan.

INTRODUCTION

"In America, it was the tax statutes of the 19th and 20th centuries which acted as the catalysts for the tremendous interest in estate planning" (Ackerman, 1973). Prior to the existence of those taxes, estate planning, while not unnecessary, was uncomplicated. Assets were simply passed by will or by gift from one generation to another. Incorporated farms, complex trusts, and other legal fictions were unnecessary. As the burden of estate, gift, and inheritance taxes and probate fees increased, estate planning and estate planners became more sophisticated.

"For over 25 years estate planning concepts have been 'sold' to clients. Typically, the clients want to concentrate on one numerical result—the amount of tax dollars saved" (Miller, 1987). In his article on "Selling Estate Planning with a Computer Screen and Without Reams of Paper," Ralph Gano Miller (1987) describes a computer screen approach to explaining estate planning to his clients. Assumptions are made regarding the date of death of the first decedent and the number of years until the remaining spouse dies. "Based on statistics it is generally concluded that the husband will die first." A growth rate for the estate is assumed, and the computer does the tax calculations to determine how much the heir(s) will receive after the first decedent and the remaining spouse have died. Various estate planning scenarios are depicted (all to spouse, use of different trusts, gifting of

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assets), and the computer screens "show" the clients which type of estate plan, given the assumptions made, is "best" for their situation.

Whether done on a computer screen or handed to the client in a twenty pound bound paper volume, the principles underlying current estate planning are similar: choose an age of death for the husband (wife), choose a survival period for the wife (husband), choose a growth rate for the estate, analyze alternate estate plans, and determine, given the assumptions made, which estate plan minimizes the tax liability for the heir(s).

Probabilistic estate planning introduces risk into this process. Rather than assuming a specific date of death for the husband (wife) and then a survivorship period for the widow (widower), ages of death for both spouses are treated as random variables. The process involves the use of a mortality table and a random number generator. The methodology follows.

Methodology Underlying Probabilistic Estate Planning

The 1984 U.S. Life Table states that of an original population of 100,000 females age 0, at age 50 those still alive number 95,261. Similarly, for 100,000 age 0 males, 91,207 remain alive at age 50 (National Center for Health Statistics, 1987). A portion of the mortality table is shown in Table 1 (see columns 1, 2, 3, 6, 7, 8).

By dividing the number of people who survive to age 51...62...73...84...by the number alive at age 50, the resulting quotients compute the probability of survival to age 51...62...73...84. Those quotients, one for a male age 50 and one for a female age 50, underlie the random age death selection process (see columns 5 and 10 in Table 1).

A computer generates two random numbers (with a uniform distribution on the unit interval). Assume that the computer generates 0.676 for the male and 0.834 for the female. In Table 1, Male Data, column 5, the number 0.676 lies between 0.70040 and 0.67325, indicating that the male has attained age 70 when he dies; the number 0.834 indicates that the female has attained age 69 when she dies. The computer also notes that the female, in this trial, dies first.

A graph of the data (See Figure 1) in columns 5 and 10 versus age shows that, initially, the survival curves are decreasing at an increasing rate, and, as the population of persons who had attained age 50 shrinks, decreasing at a decreasing rate (note: the graphs are extended beyond the data shown in columns 5 and 10 through the end of the 1984 U.S. Life Mortality Table).

Since the random numbers are generated on the unit interval, they must fall on the vertical axes of the graphs in Figure 1. Starting from 0.676 (on the MALE curve) and 0.834 (on the FEMALE curve), go across each graph horizontally to the point where the random number generated intersects the survival curve; drop down

TABLE 1.
1984 U.S. Life Mortality Data

MALE DATA					FEMALE DATA				
Age [1]	*number alive* [2]	*number dying* [3]	*Probability of dying** [4]	*Probability of survival to next age* [5]	*Age* [6]	*number alive* [7]	*number dying* [8]	*Probability of dying** [9]	*Probability of survival to next age* [10]
50	91207	570	0.00625	0.99375	50	95261	330	0.00346	0.99654
51	90637	628	0.00689	0.98687	51	94931	364	0.00382	0.99271
52	90009	692	0.00759	0.97928	52	94567	400	0.00420	0.98852
53	89317	762	0.00835	0.97092	53	94167	437	0.00459	0.98393
54	88555	837	0.00918	0.96175	54	93730	478	0.00502	0.97891
55	87718	916	0.01004	0.95170	55	93252	521	0.00547	0.97344
56	86802	999	0.01095	0.94075	56	92731	567	0.00595	0.96749
57	85803	1085	0.01190	0.92885	57	92164	617	0.00648	0.96101
58	84718	1176	0.01289	0.91596	58	91547	672	0.00705	0.95396
59	83542	1270	0.01392	0.90204	59	90875	731	0.00767	0.94628
60	82272	1367	0.01499	0.88705	60	90144	795	0.00835	0.93794
61	80905	1467	0.01608	0.87096	61	89349	861	0.00904	0.92890
62	79438	1569	0.01720	0.85376	62	88488	931	0.00977	0.91913
63	77869	1672	0.01833	0.83543	63	87557	1002	0.01052	0.90861
64	76197	1777	0.01948	0.81595	64	86555	1076	0.01130	0.89731
65	74420	1881	0.02062	0.79532	65	85479	1154	0.01211	0.88520
66	72539	1986	0.02177	0.77355	66	84325	1237	0.01299	0.87221
67	70553	2099	0.02301	0.75053	67	83088	1327	0.01393	0.85828
68	68454	2222	0.02436	0.72617	68	81761	1425	0.01496	0.84333
69	66232	2351	0.02578	0.70040	69	80336	1531	0.01607	0.82725
70	63881	2476	0.02715	0.67325	70	78805	1641	0.01723	0.81003
71	61405	2595	0.02845	0.64480	71	77164	1756	0.01843	0.79159
72	58810	2706	0.02967	0.61513	72	75408	1878	0.01971	0.77188
73	56104	2808	0.03079	0.58434	73	73530	2005	0.02105	0.75083
74	53296	2898	0.03177	0.55257	74	71525	2138	0.02244	0.72839
75	50398	2977	0.03264	0.51993	75	69387	2275	0.02388	0.70451
76	47421	3041	0.03334	0.48659	76	67112	2417	0.02537	0.67913
77	44380	3089	0.03387	0.45272	77	64695	2562	0.02689	0.65224
78	41291	3119	0.03420	0.41852	78	62133	2709	0.02844	0.62380
79	38172	3128	0.03430	0.38422	79	59424	2860	0.03002	0.59378
80	35044	3116	0.03416	0.35006	80	56564	3014	0.03164	0.56214
81	31928	3079	0.03376	0.31630	81	53550	3171	0.03329	0.52885
82	28849	3015	0.03306	0.28325	82	50379	3330	0.03496	0.49390
83	25834	2921	0.03203	0.25122	83	47049	3493	0.03667	0.45723
84	22913	2797	0.03067	0.22055	84	43556	3658	0.03840	0.41883

*Given survival until age 50.

vertically to the MALE AGE axis or FEMALE AGE axis to read the attained age of the male and the female at death.

For all ages there exist curves similar to those shown in Figure 1. The vertical axis, computed by dividing the number of survivors to some future age by the

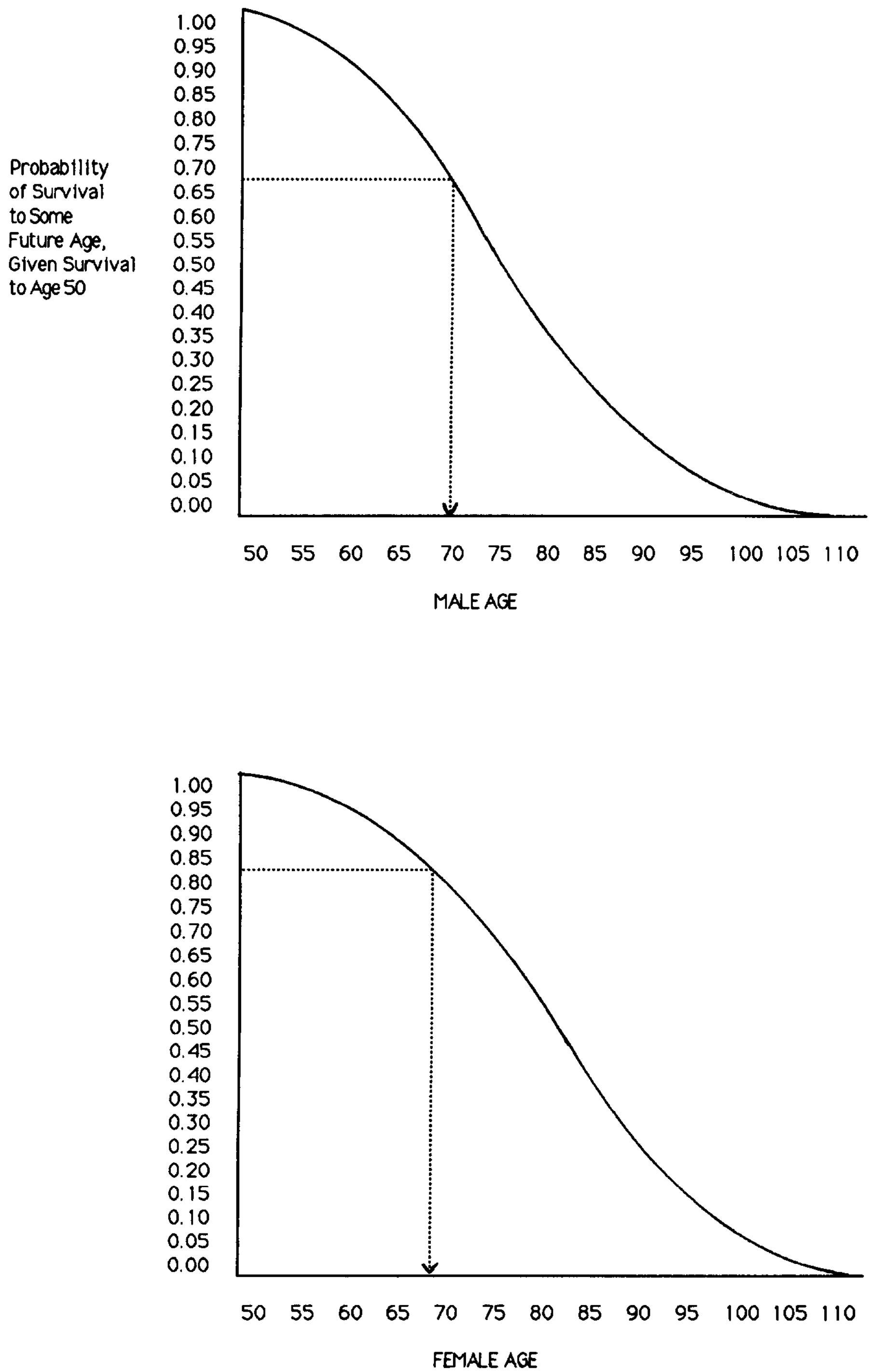

Figure 1. Probability of survival curves.

attained age of the person who is interested in probabilistic estate planning, is graphed versus attained age. For a person age 65, the series of points computed by dividing the number of survivors to age 66...75...84...to the end of the mortality table would be graphed versus age 66...75...84...to the end of the mortality table. The graph would be similar in shape to that shown in Figure 1, but the slope of the survival curve would be steeper. The older the person, the steeper the slope of the survival curve; the steeper the slope of the curve, the shorter the horizontal axis; in the limiting case (the last year in the mortality table) the survival curve would be vertical, and no matter what random number was generated, death would be predicted to occur in the upcoming year. (The only distribution assumption required for probabilistic estate planning is that time until death has a distribution defined by the 1984 U.S. Life Table. It should be noted that this distribution is not normal.)

For any pair of random numbers, three possible outcomes exist. The male dies first, the female dies first, or both the male and female die in the same future year. For example, if the computer generated 0.825 as the male random number and 0.901 as the female random number, the data in Table 1 (or the graph in Figure 1) would indicate that both the 50 year old male and the 50 year old female would die in their 63rd year of life. If the computer generated 0.585 for the male and 0.590 for the female, the male would be predicted to die in his 73rd year of life, the female in her 80th year of life. In an earlier example the female died first, the male second.

Assuming a 100 trial simulation, the computer generates two hundred random numbers, one hundred to predict the age of the male at death and one hundred to predict the age of the female at death. Sometimes the male will die first, sometimes the female will die first, and sometimes death will occur at the same age. The 200 random numbers result in 100 combinations of death for which estate taxation and distribution can be computed.

Estate growth and taxation are straightforward processes. Assume some growth rate for the assets comprising the estate. Allow those assets to grow until the death of the first person. Tax the estate. Allow the remaining estate assets to grow until the death of the second person. Tax the estate. Finally, compute the present value of the net estate passing to the heir(s). This process will yield 100 net-to-the-heir(s) numbers, and a variance associated with those numbers. Different estate planning techniques and/or asset/liability mixtures may produce different expected net present values and variances.

The Advantages of a Probabilistic Approach

Perhaps the most common assumptions in family estate planning are that the husband dies first, that the husband dies at his life expectancy, and that the surviving spouse dies at her life expectancy. According to the 1984 U.S. Life Table, at age 60 life expectancy for a female is 23 years, 18 years for a male (rounded to the nearest integer). However, the probability that a female age 60 dies in her 83rd year of life is 0.0387 and the probability that the male age 60 dies in his 78th year of life is 0.0379 [female probability = (3493/90144) and male probability = (3119/82272)]. Few people die in the year of their life expectancy.

Table 2, based on the 1984 U.S. Life Table, shows the probability of dying in a seven-year period centered on life expectancy. For example, a male age 50 has a life expectancy of 26 years. The probability that the male dies in the seven-year period between 73 and 79 is only 23%. [Note: sum the numbers 0.03079 + 0.03177 + 0.03264 + 0.03334 + 0.03387 + 0.03420 + 0.03430 in column (4) in Table 1 to compute the seven-year death probability; round the result of 0.23091 to 23%.] Similarly, the age 50 female expectation of death in a seven-year period centered on life expectancy is also 23%. Data for other ages for males and females show that most people do not die anywhere close to the age computed by adding their life expectancy to their attained age.

Note that the data in Table 1 can **not** be used to compute seven year death probabilities for any age except age 50. To compute the other probabilities shown in Table 2, interested readers can create six tables similar to Table 1, with columns 4 and 9 modified by dividing the number of persons dying in a given year by the number of persons who survived to ages 53, 56, 60, 63, 66, and 70. Since the number surviving decreases as age goes from 53 to 70, but the number dying at a given age (columns 3 and 8) stays constant, the probability of dying (columns 4 and 9) in a given year increases as age goes from age 53 to 70, and the sum of a seven year series (of the one year death probabilities) increases. Yet even at age 70, only one of every three decedents will die within a seven year period centered on life expectancy.

Computations based on death at life expectancy are clearly inconsistent with

TABLE 2.
Death in a Seven Year Period Centered on Life Expectancy

Age	*Male expectation of life**	*Probability of death in a seven year period***	*Female expectation of life**	*Probability of death in a seven year period***
50	26	23%	31	23%
53	23	24%	28	24%
56	21	25%	26	26%
60	18	26%	23	29%
63	16	28%	20	30%
66	14	30%	18	33%
70	12	33%	15	36%

*Rounded to nearest integer; **rounded to nearest percent.

reality and computations based on death near life expectancy are little better; and, contrary to popular belief, actuaries do not base their calculations on life expectancies. "It is popularly believed that the expectation of life is widely used in actuarial calculations. In reality, it is of interest to actuaries only because it affords an index for comparing different mortality tables" (Jordan, 1952).

In the case of life annuities, "One of the persistent misconceptions is that the present value of a life annuity at age x is equal to the value of an annuity-certain for a term equal to the life expectancy at age x.... The annuity-certain for the term of the life expectancy always exceeds the life annuity value" (Jordan, 1952). The previous statement is based on Jensen's Inequality.

When Jensen's Inequality is applied to estate planning, three possible outcomes are possible. If the growth rates of the estate assets exceed the discount rate, then the mean of the expected value(s) will exceed the expected value of the mean(s). That is, the probabilistic mean will be larger than the traditional life expectancy point estimate. If the growth rates of the estate assets equal the discount rate, then the mean of the expected value(s) will be equal to the expected value of the mean(s), and the probabilistic mean will equal the traditional point estimate. If the discount rate exceeds the growth rates of the estate assets, then the mean of the expected value(s) will be less than the expected value of the mean(s), and the traditional point estimate will be larger than the probabilistic mean.

In only one case (a highly unlikely case) is the traditional point estimate equal to the probabilistic mean. The magnitude of the error associated with the traditional estimate is directly proportional to the magnitude of the difference between the growth rates of the estate assets and the discount rate and directly proportional to the remaining lifetime(s) of the estate owner(s). The probabilistic estate planning mean is actuarially sound, and it presents a clearer picture of reality than the traditional life expectancy point estimate.

Even when spouses are the same age, and, as everyone knows, females have a larger life expectancy than males of the same age, there is a significant chance that a wife will predecease her husband. Table 3 is based on a simulation, where each cell

TABLE 3.
Probability that Wife Predeceases Husband

Age of husband	*Age of wife* 50	53	56	60	63	66	70
50	39%	43%	53%	61%	66%	73%	79%
53	30%	33%	46%	57%	61%	66%	75%
56	23%	30%	37%	49%	53%	62%	70%
60	18%	21%	27%	37%	44%	52%	60%
63	14%	20%	23%	28%	36%	44%	51%
66	11%	15%	13%	23%	30%	38%	47%
70	8%	9%	12%	17%	22%	29%	38%

is the result of choosing 1000 random ages of death for men age x and women age y, and x and y vary from 50 to 70. Holding the age of men and women equal (see the equal age diagonal of Table 3), the chance that a woman predeceases her husband varies from 39% to 33%, with a mean of 37%. For a woman six years older than her husband that probability increases to a mean of 53%. For a woman six years younger, it decreases to mean of 23%.

Estate planning based on death at life expectancy (or even near life expectancy) is not realistic. Estate planning based on the assumption that the male dies first is often incorrect. Probabilistic estate planning, using realistic mortality rates, allows an estate planner to choose a mathematically optimal plan. Since the computations are being made net of taxes (or net of costs), and since the computations are being made on a present value basis, the probabilistic estate planning methodology is, in part, essentially an application of the basic financial principle "Maximize Expected Net Present Value."

Simplicity is the primary value of life expectancy estate planning. Usually expressed on a future value basis, life expectancy estate planning simultaneously overstates the value of the tax savings, overstates the size of the inheritance of the heir(s), and ignores the riskiness associated with the estate plan. While a present value analysis (of the future taxes saved or the size of the inheritance) could be easily incorporated into traditional estate planning, variance based risk analysis cannot, and to ignore variance based riskiness is to implicitly ignore Markowitz. In his classic article on "Portfolio Selection," Markowitz (1952) stated "We saw that the expected returns or anticipated returns rule is inadequate. Let us now consider the expected returns-variance of returns (E-V) rule." Choosing an estate plan simply because it has the largest tax savings (like choosing a security with the highest expected return) without investigating the variance associated with that estate plan (security) is inconsistent with modern portfolio theory.

As Markowitz (1952) closed his classic article, he stated "I believe that what is needed is essentially a 'probabilistic' reformulation of security analysis. I will not pursue this subject here, for this is 'another story.' It is a story of which I have read only the first page of the first chapter." As defined in this paper, probabilistic estate planning permits modern portfolio theory (mean, variance tradeoffs) to be used in selecting an optimal estate plan, and presents, in this author's opinion, the first page in the first chapter of the reformulation of traditional estate planning.

THE DISADVANTAGES OF PROBABILISTIC ESTATE PLANNING

The primary disadvantages of this approach are two: computational complexity and the choice of the appropriate discount rate for computing present values. While the logic underlying the computations demanded by probabilistic estate planning is not particularly complex, the computer program required to execute that logic is both complex and lengthy. For the simple two estate plan analysis (status quo and a single change from status quo) that was presented at the Second Annual

Meeting of the Academy of Financial Services in New Orleans in the Fall of 1988, 1000 lines of BASIC code were required. For the analysis which follows this section, 500 more lines of code were added.

The problem of computational complexity pales when compared to the choosing of the appropriate discount rate. An expenditure of hard work can solve the programming problem; the conceptual problems underlying the choice of the appropriate discount rate could keep a financial philosopher in deep thought for the remainder of his/her lifetime. Is the proper rate used in discounting the expected inflation rate, the time preference rate of the beneficiary, or the long term U.S. Treasury Bond rate? What adjustment should be made when different asset/liability combinations or estate plans have different levels of risk? The inability of this author to unequivocally answer these questions does not invalidate the probabilistic estate planning model, but in practical applications these questions must be addressed and answered. Since the goal of probabilistic estate planning is to maximize the net present value of the after-tax estate passing to the heir(s), perhaps the most logical choice of discount rates is the time preference rate of the heir(s) of the estate.

A Comparison of Traditional and Probabilistic Estate Planning

Assume that a couple, both age 60, have a $2,000,000 estate. Assume that $1,000,000 is invested in husband owned real estate which is appreciating at 7% per year and that $1,000,000 is invested in wife owned bonds paying 9% (taxable at a constant 28%) per year. Assume that the estate has no debts. Assume their wills are spouse to spouse, remainder to child. Assume that the couple has a single child. Assume that the couple lives in Wisconsin. Assume that the child has a time preference for money of 5%.

Figure 2 depicts the traditional estate planning analysis for the above couple and their child. The husband is assumed to die at age 78, his wife at age 83 (their 1984 U.S. Life Table life expectancies, rounded to the nearest integer). Wisconsin, like many states, provides for an estate tax equal in amount to the allowable Federal Estate State Death Tax credit. Since that tax has no real cost to the heir (what the Federal government loses in tax revenue is equal to that paid to the State of Wisconsin), only the Total Estate Tax is shown in the analysis below and in all subsequent analyses. Assume that Administration Expenses are equal to 5% of the Gross Estate.

The traditional analysis demonstrates that the Total Estate Tax Payable is $3,735,233, that the net to the child is $4,569,233. Since that tax is not payable for 23 years and the net to the child is not receivable for 23 years, those numbers exaggerate both the inheritance of the child and the Total Estate Tax on the estate of the mother. In traditional analysis, the next step is usually to show how to lower the Total Estate Tax. By changing the wills and using a trust (which passes legal title) or

Spouse to spouse, remainder to child			
Gross estate of father	$3,379,932	Gross estate of mother	$8,741,543
Administration expenses	$168,996	Administration expenses	$437,077
Adjusted gross estate	$3,210,936	Taxable estate	$8,304,466
Marital deducation	$3,210,936		
		Total estate tax payable	$3,735,233
Taxable estate	$0		
Total estate tax payable	$0	Net to child	$4,569,233

Figure 2. Traditional estate planning analysis—status quo.

an outright gift (which passes both legal and equitable title) of $600,000 to the child on the death of the father, significant future tax savings (in an amount of $399,727) are possible. See Figure 3.

As an alternative to changing the wills, an estate planner might suggest the purchase of insurance and the placing of that insurance in a trust. Hence, assume that the wife takes $500,000 of her assets and purchases a single premium life insurance policy with a guaranteed face amount of $1,567,570.[1] Although tax reform has effected the investment aspects (for the policy owner who takes possession of the cash value through loans, withdrawals, or terminations) of owning single premium whole life, "the new tax treatment will in no way decrease the return to policy owners who leave the funds with the insurer and look to the death benefit as the primary benefit of the contract" (Leimberg et al., 1989). Single premium whole life insurance continues to be a viable product *if* used to provide death benefits.

Assume that this policy is on the life of the husband. Assume that the policy is held in trust for the benefit of the child, and held in such a manner (e.g., no retained interest) such that the policy proceeds are not included in the mother's estate. (Note that the gift itself will need to be included in her estate for Federal Estate Tax purposes.) Assume the trustee charges 1/2 of 1% annually to administer the trust

$600,000 to child on first death, remainder to child on second death			
Gross estate of father	$3,379,932	Gross estate of mother	$7,900,012
Administration expenses	$168,996	Administration expenses	$395,001
Adjusted gross estate	$3,210,936	Taxable estate	$7,505,011
Marital deducation	$2,610,936	Total estate tax payable	$3,335,505
Taxable estate	$600,000	Net to child, mother's death	$4,169,506
Total estate tax payable	$0		
		Net to child, father's death*	$841,531
		Total future value to child	$5,011,037

Figure 3. Traditional estate planning analysis—Revised Will. (*) Compounded to mother's death.

after the policy benefit is paid into the trust. Assume that the trustee projects a 7% rate of return on trust corpus. See Figure 4 for a visual picture of the restructured estate. The Total Estate Tax payable using the life insurance alternative is $2,931,392, significantly less than under a revised will. The net to the child is $5,235,751, significantly more than under a revised will. On a traditional estate planning life expectancy point estimate basis, the life insurance option is clearly optimal.

Initial Estate Plan

Husband Real Estate	$1,000,000
Wife Bond Portfolio	$1,000,000
Total Estate	$2,000,000

Will: Wife's Assets to Husband, Husband's Assets to Wife, Survivor's Assets to Child

Revised Estate Plan

Parent's Estate		Child's Trust Estate	
Husband real estate	$1,000,000	Life insurance trust*	$500,000
Wife bond portfolio	$481,800		
Total estate	$1,481,800	Proceeds at death of father	$1,567,570
Life insurance trust	$500,000		
State gift taxes	$17,200		
Legal and trust expenses	$1,000		
Total Estate + Trust + Expenses	$2,000,000		

*This is the initial cash value/premium for the policy. The policy proceeds will compound until the mother's death since the father is assumed to die first.

Will: Wife's Assets to Husband, Husband's Assets to Wife, Survivor's Assets to Child

Using a Life Insurance Trust Funded with the Wife's Assets

Gross estate of father	$3,379,932	Gross estate of mother	$6,522,929
Administration expenses	$168,996	Administration expenses	$326,146
Adjusted gross estate	$3,210,936	Taxable estate	$6,196,783
Marital deducation	$3,210,936	Total estate tax payable	$2,931,392
Taxable estate	$0	Net to child, mother's death	$3,265,391
Total estate tax payable	$0		
Insurance trust	$1,567,570 →	Net to child, father's death*	$1,970,360
		Total future value to child	$5,235,751

Figure 4. Traditional estate planning analysis—Life Insurance. (*) Compounded to mother's death.

The value (and purpose) of probabilistic estate planning becomes evident when the same estate scenarios are replayed on a probabilistic basis. A flow chart for the probabilistic estate planning process is shown in Figure 5. A 100 trial simulation of the Status Quo estate plan yielded a net present value to the child of $1,558,154

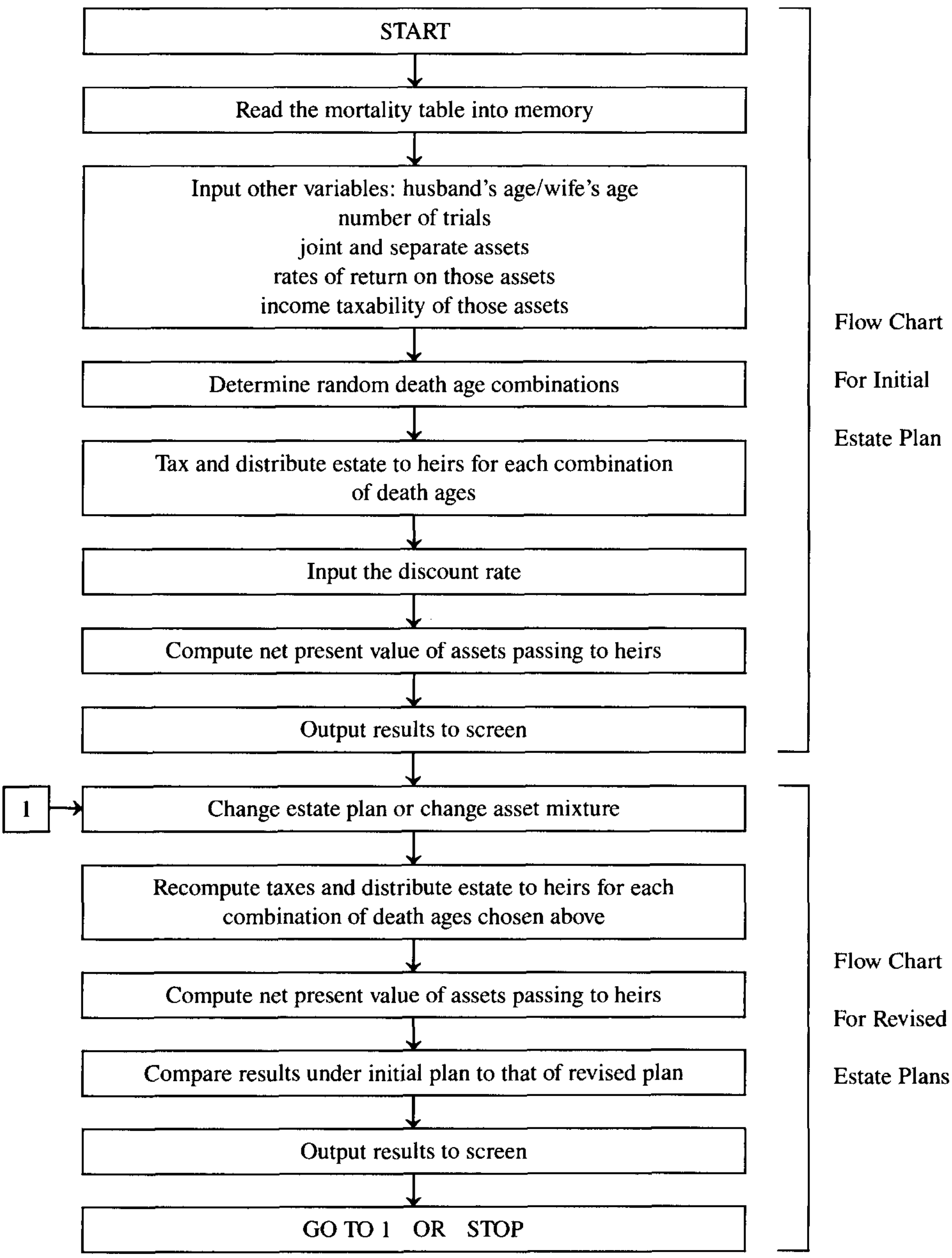

Figure 5. Computer program flow chart.

and a standard deviation of $108,438. (For comparative purposes, the present value of the traditional point estimate was $1,487,611.)

The life insurance policy option, clearly optimal on a point estimate basis, is no longer clearly optimal on a probabilistic basis. Using the same death age combinations as in the first simulation yielded a net to the child of $1,825,164, with a standard deviation of $267,547. On a probabilistic basis there is a large increase in the size of the expected estate (from a mean of $1,558,154 to a mean of $1,825,164), but accompanying that expected increase is a large rise in the level of risk (from a standard deviation of $108,438 to a standard deviation of $267,547). The decision to purchase the life insurance now depends on the risk preferences of the child.

The risk to the child in the probabilistic analysis arises from the fact that if the husband dies shortly after the policy is issued, then the net present value of the future estate of the child (after his mother has died) will be significantly larger than if no insurance were purchased; conversely, if the father lives for a long time (into his late 80s or beyond), much of the interest income earned by the life insurance company will be consumed by death benefits paid to other beneficiaries, and the net present value of the estate of the child will be smaller than if life insurance had not been purchased. Since the time at which this occurrence happens is well beyond the 23 year life expectancy of the mother, this risk is not communicated to the estate owners under the traditional estate planning methodology.

To further complicate matters, when the probabilistic process (using the same death age combinations as in the first simulation) is applied to the estate plan using the revised will (to take advantage of passing $600,000 Total Estate Tax free to the child on the death of the first parent), the net present value of the estate of the child after both parents are deceased was $1,792,267, the standard deviation $140,291. While the expected value of this option is less than the expected value of the life insurance option, there is also significantly less risk. The trade-off between the expected return and the variance associated with that expected return (plotted versus the square root of the variance) for the three estate plans just discussed is depicted in Figure 6.

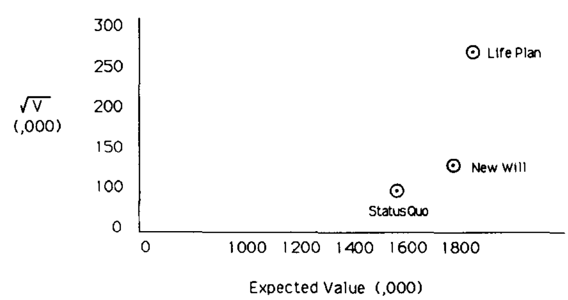

Figure 6. Three alternate estate plan E–V combinations.

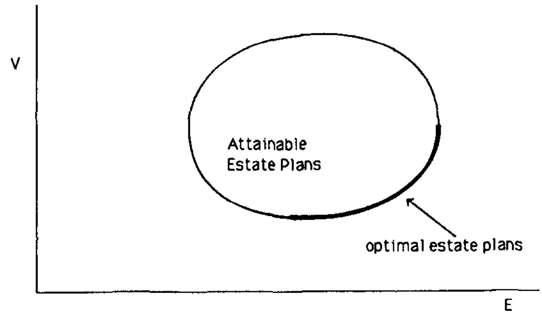

Figure 7. Attainable estate plan E–V combinations.

Now the child must decide what risk level s/he is comfortable with, and that decision will allow the child, from amongst those three estate plans, to chose the optimal one. The life insurance purchase is no longer a clearly optimal plan. Whether or not any or all of the three estate plans just examined fall on the optimal estate plan frontier or lie above and to the left of that frontier is unimportant; the object of probabilistic estate planning is now clear.

On a probabilistic basis, the net present value of assets passed to the child can be graphed versus the variance (or square root thereof) associated with that estate plan. By investigating other asset reallocation possibilities and alternative estate plans (gifting of assets without purchasing insurance, combinations of will changes, trusts, use of insurance, et al.) and plotting means and variances for different estate plans and asset/liability combinations, the estate planner can identify the optimal estate plan frontier. (See Figure 7.)

Attainable estate plans which are inferior to those on the frontier can be avoided. The plan which maximizes the expected net present value of assets passed to the child consistent with the risk level with which the child is comfortable can be implemented.

CONCLUSIONS

Probabilistic estate planning allows those decision makers who are interested in risk to make optimal estate planning decisions. If the plan which maximizes the net present value of assets passed to the heir(s) simultaneously has the smallest variance, then clearly that plan is optimal. If, however, the plan which maximizes the net present value of assets passed to the heir(s) simultaneously has the largest variance, and other plans having lower expected values have lower variances, then the E-V rule applies, and the heir(s) must choose a plan consistent with his/her/their risk level(s) from among those plans falling on the optimal estate plan frontier.

Combining the basic principle in finance of maximizing expected net present value with Jensen's Inequality and Markowitz's E-V Rule results in a process referred to by this author as probabilistic estate planning. The primary advantages of this methodology over current methodology are that it incorporates the element of risk, that the probabilistic mean is unbiased with respect to growth rates, discount rates and age(s) of the estate owner(s), and that results expressed on a present value basis (rather than on a future value basis) are more meaningful to estate owners.

In summary, probabilistic estate planning permits modern portfolio theory (mean, variances tradeoffs) to be used to select an optimal estate plan. There are other variables influencing an estate plan that could be considered as random variables. Considering time until death is a first step.

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NOTES

1. Policy issued by a major life insurance company on a no-load basis; Valedictorian I—Single Premium Life; Dec. 1987, 6% guaranteed, 9% projected.

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