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A Multicriteria Approach to Mutual Fund Selection

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In practice, when investors select a mutual fund, they take into account a number of factors. However, the most popular approach for evaluating mutual funds employs only a single criterion, the funds' mean, risk-adjusted, rate of return (Jensen's α coefficient). In the present paper a multicriteria approach to mutual fund selection is presented. The multicriteria methodology allows numerous factors to be considered, for example, the standard deviation of the funds' α's, front and back-end load fees, the level of diversification, quality of service, and so on. It also recognizes that individual investors possess heterogeneous attributes and preferences, and hence, allows investors to formulate different ratings (and consequently rankings) of the set of competing mutual funds.

I. Introduction

In the decade ended December, 1989, the number of U.S. households owning mutual fund shares increased by 495 percent to 22.8 million and the total number of shareowner accounts increased to 58.2 million. Furthermore, total U.S. mutual fund assets increased by 1,750 percent to $982 billion and the number of U.S. funds more than quadrupled to 2,918.[1] The mutual funds offered vary enormously in terms of their investment objectives, types of securities held, historical returns and risk levels, load and management fees, levels of diversification, quality of service, and so on. Similarly, fund shareholders vary enormously in terms of their wealth levels, rates of portfolio turnover, degrees of risk tolerance, understanding of financial markets, beliefs in the ability of mutual funds to outperform the market, their own portfolio's level of diversification, and so on. With this impressive growth record and the increasing complexity, diversity and competitiveness of the mutual fund industry, it is important to examine the approach adopted by investors when evaluating mutual fund managers.

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Evaluating competing money managers is an integral part of the decision-making process facing individual investors who are determining which mutual fund to select or corporations who are deciding which pension fund manager to hire. In what has become the classic article on performance evaluation, Jensen (1968) uses a single criterion approach to provide evidence that (on a risk-adjusted, net of management fees basis) mutual funds as a class do not outperform the market. It is important to note however, that his purpose was "not to evaluate the funds from the standpoint of the individual investor, but only to evaluate the fund managers' forecasting ability" (p. 404). For examining the forecasting ability of mutual fund managers' as a class, Jensen's single criterion approach may be appropriate.[2] However, if one's objective is to evaluate competing funds from the perspective of a potential investor, a multicriteria approach is required.[3]

Jensen's single criterion is the "average incremental rate of return on the portfolio per unit time" (p. 394); what is now referred to as Jensen's α coefficient. Jensen explicitly chose not to consider other criteria, such as a mutual fund's consistency in earning incremental returns, or its level of diversification (p. 415). Furthermore, Jensen omitted the fund's front-end and back-end load fees (p. 404), as well as the level of service offered to its investors. The objective of this paper is to provide an alternative to the single criterion approach adopted by Jensen (and numerous more recent studies), so that a large number of competing mutual funds can be evaluated by an individual investor.

The multicriteria approach developed in this paper is based on a model by Cook and Kress (1991). This model, which was designed for multiple criteria problems with purely ordinal data, is adapted here to incorporate the cardinal data inherent in the mutual fund problem. The model explicitly recognizes that investors possess heterogeneous attributes and preferences, and hence, in general, they formulate different ratings (and consequently rankings) of the set of competing mutual funds. In contrast to this implication of the multicriteria approach, if investors employed Jensen's single criterion approach, unanimity would exist in their fund ratings (and rankings). All investors would then select the same mutual fund, an implication which is clearly inconsistent with the large number (2,918) of funds currently sold in the U.S.

To develop a multicriteria approach to mutual fund selection by an individual investor, this paper proceeds as follows: Section II presents, for illustrative purposes, a set of six criteria which could be used by the investor. Section III first introduces the general concept used for combining multiple criteria, and presents three general forms of investor preference regarding the mutual funds and the criteria against which the funds are to be judged. Then, from the three general forms of investor preference, five constraints on the weights chosen by the model to rank the various funds are derived. Next, the general multicriteria approach for ranking mutual funds is presented, along with a simple illustrative example. Section IV provides an example of the selection procedure using data from ten mutual funds evaluated on the basis of the six suggested criteria. Importantly, it is demonstrated that a very different ranking of the ten funds is derived using this paper's multicriteria approach compared with Jensen's single criterion approach. Furthermore, it is demonstrated that, using the multicriteria approach, investors with heterogeneous attributes and preferences, do obtain different fund rankings. Conclusions are presented in Section 5.

II. The K Criteria

A set of **M** mutual funds $i = 1,2,...,\mathbf{M}$ are to be ranked according to a set of **K** criteria $k = 1,2,...,\mathbf{K}$. For each of the **M** funds an observation (either cardinal or ordinal) for each of the **K** criteria must be provided by the investor. For illustrative purposes, assume there are six criteria:

i). α_i, the mean, risk-adjusted rate of return, net of management fees, observed for fund i during the last **T** periods (i.e., Jensen's α coefficient).[4] Using prior rates of return to select a mutual fund could be considered a questionable practice in view of evidence that past performance contains little or no predictive value. However, Allerdice and Farrar (1967), Friend, Blume and Crocket (1970), Smith (1978) and Woerheide (1982) provide evidence that past rates of return are positively correlated with a fund's net sales. Furthermore, survey evidence presented by Lewellen, Lease and Schlarbaum (1977) and the Investment Company Institute (1987) indicate that prior returns are one of the primary variables used by investors when choosing mutual funds. Investors' beliefs regarding the ability of mutual funds to outperform the market will be a primary determinant of the importance they accord to α_i.

ii). σ_i, the standard deviation of the **T** α_i's observed for mutual fund i.[5] Investors who are less risk tolerant will place relatively greater importance on σ_i.

iii). $\mathbf{D}_i$, mutual fund i's degree of diversification (that is, how closely the fund's portfolio approximates the market portfolio).[6] Investors who are less risk tolerant or whose portfolios are less well diversified, will accord relatively more importance to $\mathbf{D}_i$.[7]

iv). $\mathbf{F}_i$, fund i's front-end load fee. Normally $\mathbf{F}_i$ is a decreasing function of the quantity purchased and hence, will be a relatively less important criterion for investors who are wealthier and purchase larger quantities.

v). $\mathbf{B}_i$, fund i's back-end load fee. Normally $\mathbf{B}_i$ is a decreasing function of the investor's holding period and hence, will be a relatively less important criterion for investors with longer expected time horizons or equivalently, lower rates of portfolio turnover.

vi). $\mathbf{S}_i$, fund i's service level.[8] Investors whose understanding of financial markets is limited will generally accord relatively more importance to $\mathbf{S}_i$.

The six examples of criteria provided above are used throughout this paper to illustrate the multicriteria approach to mutual fund selection.[9]

III. A Model for Combining Multiple Criteria

1. The General Concept

In general, it is assumed that the investor can express three forms of preference regarding the funds and the criteria against which the funds are to be judged.

Preferences Among Funds by Criterion. It is assumed that the investor can rank order the **M** mutual funds according to each criterion. That is, for each criterion, the investor can decide which fund ranks in first place, which in second, and so on.[10] To rank order the **M** funds according to criterion k, define a binary $(0-1)$ matrix $\mathbf{A}^k$ where

$$\mathbf{A}^k = (a_{il}^k)$$

$$a_{il}^k = \begin{cases} 1 \text{ if fund } i \text{ is ranked } l\text{'th on criterion } k\text{, and} \\ 0 \text{ otherwise.} \end{cases} \quad (1)$$

For the moment, assume that the investor is only interested in the ordinal ranking of the funds along each of the **K** dimensions. While this approach does not immediately take advantage of the information in the cardinal data available, the cardinal data is used in the formal model presented below when choosing the weights to be applied to the rank positions (see (3a) below).

To illustrate how a binary matrix is constructed, assume there are five mutual funds to be ranked according to α_i, the k'th criterion, with $i = 1,2,\ldots,5$ used to index the funds. Then, if $\alpha_1 = 10$ percent, $\alpha_2 = 6$ percent, $\alpha_3 = 12$ percent, $\alpha_4 = 9$ percent and $\alpha_5 = 8$ percent, the ordinal ranking can be represented by the matrix

		RANK				
	FUND	1	2	3	4	5
	1	0	1	0	0	0
	2	0	0	0	0	1
$\mathbf{A}^k =$	3	1	0	0	0	0
	4	0	0	1	0	0
	5	0	0	0	1	0

Ranking the Criteria by Importance

The second assumption is that the investor can rank the **K** criteria themselves in order of importance.[11] For convenience, let $k = 1$ denote the most important criterion, $k = 2$ the next most important, and so on, with $k = \mathbf{K}$ denoting the least important criterion.[12]

Ranking the Criteria by Clearness

Criteria "clearness" is a third concern in rank ordering the M mutual funds.[13] It is an issue because the ability of the investor to distinguish between funds on the basis of, say, α_i may be greater than his ability to distinguish between funds on the basis of $\mathbf{D}_i$ or $\mathbf{S}_i$ (where $\mathbf{S}_i$ may be especially difficult to quantify). To incorporate criteria clearness into the model, the investor must be able to rank the criteria from the most to the least clear. This is a ranking over and above the "criteria importance" ranking.

Once the three forms of investor preference are incorporated into the model, the investor's objective is to obtain, for each mutual fund i, an overall rating, $\mathbf{R}_i$, which reflects the fund's aggregate standing over the set of criteria. To accomplish this, a set of weights $\{\omega_l^k\}$, for $k = 1,...,\mathbf{K}$ and $l = 1,...,\mathbf{M}$, is determined, where ω_l^k is the level of importance or weight accorded the l'th rank position for criterion k. The rating for fund i is then

$$R_i = \sum_{k=1}^{\mathrm{K}} \sum_{l=1}^{\mathrm{M}} a_{il}^k \omega_l^k. \tag{2}$$

For example, if fund #1 is ranked first, fourth, third, fifth, first and third on criteria $k = 1, k = 2, ..., k = 6$ respectively, then its rating would be

$$\mathbf{R}_1 = \omega_1^1 + \omega_4^2 + \omega_3^3 + \omega_5^4 + \omega_1^5 + \omega_3^6$$

The model's weights must be chosen in such a manner that the three forms of preferences outlined above are adhered to. Specifically, for a given criterion, a higher ranked fund should be accorded more weight than one ranked at a lower level. More important criteria should be weighted more heavily than less important criteria. Similarly, clearer criteria should be weighted more heavily than less clear criteria. In the subsections to follow, these ideas are formalized, and a model for determining a set of weights and hence, a rating $\mathbf{R}_i$ for each mutual fund i, is presented.

2. Constraints on the Choice of Weights, ω_l^k

Constraint #1 (Discriminating Among Criteria): Once the investor has rank ordered the **K** criteria (from the most important criterion, $k = 1$, to the least important, $k = \mathbf{K}$), it is necessary that the weight accorded to criterion k, ω_l^k, be at least as large as that accorded to criterion $k + 1$, ω_l^{k+1} (for a given rank position l). Letting the variable v denote the minimum gap between these two weights, the constraint $\omega_l^k - \omega_l^{k+1} \geq v$ must be imposed.

A further consideration in discriminating among the criteria, is that the investor may wish to distinguish more strongly between some pairs than others, by specifying a vector of relative criteria importance parameters, $\kappa = (\kappa^1, \kappa^2, ..., \kappa^{K-1})$. The parameter κ^k reflects the "relative" positioning of criteria k and $k + 1$, or the degree to which the investor believes that criterion k is more important than criterion $k + 1$. For example, if the investor specifies that $\kappa^1 > \kappa^2, ..., > \kappa^{K-1}$, then he wishes to discriminate (relatively) strongly between criteria 1 and 2, less strongly discriminate between criteria 2 and 3, and so on, with a relatively weak distinction made between criteria $\mathbf{K} - 1$ and $\mathbf{K}$.

Formally, the constraints on criteria importance are then given by

$$\omega_l^k - \omega_l^{k+1} - v\kappa^k \geq 0 \qquad \text{for } k = 1,2,...,K - 1 \text{ and } l = 1,2,...,M. \tag{3}$$

In this format it is clear that, while the parameter κ^k reflects the relative gap between the weights on criteria k and k+1, the product $v\kappa^k$ provides the absolute gap between the weights. The role of v will become clearer when the full model is presented below.

To illustrate what relative criteria importance parameters mean, assume that $\mathbf{K} = 4$ and that, regardless of what weights are actually used to represent the 4 criteria, the investor believes that the difference between ω_l^1 and ω_l^2 is twice the difference between ω_l^2 and ω_l^3 (for a given rank l). If $\kappa^1 = 6$, then $\kappa^2 = 3$ or, equivalently, $\kappa^1 / \kappa^2 = 2$. Similarly, if $(\omega_l^2 - \omega_l^3) = 3(\omega_l^3 - \omega_l^4)$ with $\kappa^1 = 6$ and $\kappa^2 = 3$, then $\kappa^3 = 1$ and $\kappa^2 / \kappa^3 = 3$. If, contrary to the above example, the investor believes that the relative positioning should reflect equal spacing, then he should choose $\kappa^1 = \kappa^2 = \kappa^3 = 1$.

Constraint #2 (Discriminating Among Rank Positions) Once the investor has ranked the **M** mutual funds according to each of the **K** criteria (from the best fund, l=1, to the worst, l=M), it is necessary that the weight accorded to rank position l, ω_l^k, be at least as large as that for rank position $l + 1$, ω_{l+1}^k (for a given criterion k). Letting the variable μ (which plays a similar role to v in Constraint #1) denote the minimum gap between these two weights, the constraint $\omega_l^k - \omega_{l+1}^k \geq \mu$ must be imposed. However, this constraint assumes that the minimum difference, μ, is the same for all rank positions, and ignores the information regarding relative rank positions that exists in the cardinal data available.[14]

The cardinal data can be used to define a vector of relative rank difference parameters. For criterion k, define a vector $\lambda^k = (\lambda_1^k, \lambda_2^k, \ldots, \lambda_{M-1}^k)$ where $\lambda_l^k = (c_l^k - c_{l+1}^k) / (c_1^k - c_M^k)$, c_l^k is the cardinal observation for the l'th ranked fund, $0 \leq \lambda_l^k \leq 1$ and $\Sigma_{l=1}^{M-1} \lambda_l^k = 1$.[15] The larger λ_l^k is, the larger is the relative difference between the l'th and l+1'th ranked security (according to criteria k). If only ordinal data is available for criterion k, then λ might be chosen as $\lambda_l^k = 1 / \mathbf{M} - 1$ for $l = 1,2,..., \mathbf{M} - 1$. For example, if there are five mutual funds and criterion 1 refers to the α_i's, with $c_l^1 = .10, c_2^1 = .08, c_3^1 = .02, c_4^1 = 0$ and $c_5^1 = -.10$, then $\lambda_1^1 = .10, \lambda_2^1 = .30, \lambda_3^1 = .10$ and $\lambda_4^1 = .50$. Note that λ_4^1, and hence $\omega_4^1 - \omega_5^1$ is relatively large because the difference between c_4^1 and c_5^1 is relatively large. If only ordinal observations are available for criterion l, then $\lambda_1^1 = .25$ for $l= 1,...,4$.

While the vector of relative rank difference parameters, λ^k, recognizes the differences among rank positions for a given criterion k, it does not reflect the relative importance of criterion k itself. For this reason, multiply each λ_l^k by a contraction factor $t^k = \Sigma_{j=k}^{K} \kappa^j / \Sigma_{j=1}^{K} \kappa^j$, which reflects the required reduction in gap size (between relative rank positions) for criterion k relative to criterion 1 (the most important criterion).

Formally, the constraints on rank position importance are given by

$$\omega_l^k - \omega_{l+1}^k - \mu t^k \lambda_l^k \geq 0 \qquad \text{for } k = 1,2,...,\mathbf{K} \text{ and } l = 1,2,...,\mathbf{M} - 1. \qquad (3a)$$

$$\omega_l^k - \omega_{l+1}^k - t^k(\omega_l^1 - \omega_{l+1}^1) \leq 0 \qquad \text{for } k = 1,2,...,\mathbf{K} \text{ and } l = 1,2,...,\mathbf{M} - 1. \qquad (3b)$$

Similar to (2) from Constraint #1, the $t^k\lambda_l^k$ reflect the relative gap between the importance attached to consecutive rank positions, while the product $\mu t^k \lambda_l^k$ provides the absolute gap. Furthermore, the gaps between consecutive rank positions are bounded from below by the constraint set (3a), and from above by the constraint set (3b).

Constraint #3 (Criteria Clearness Discrimination): To incorporate criteria clearness into the model, the investor must be able to rank the criteria from the most to the least clear. Then the variable μ in the previous set of constraints (equation (3a)), is replaced by the vector of variables $(\mu^{[1]}, \mu^{[2]}, ..., \mu^{[K]})$, where $\mu^{[1]}$ represents the relative clarity of the clearest criterion, $\mu^{[2]}$ the relative clarity of the second clearest criterion, and so on.

To reflect differences in clarity among the **K** criteria, the absolute sizes of the $\mu^{[k]}$ variables are incorporated into the model. Then, if there is a large difference in clarity between two consecutive criteria, $[k]$ and $[k+1]$, the corresponding difference, $\mu^{[k]} - \mu^{[k+1]}$, will be large, as will be the difference between the two corresponding weights. Equation (3a) (representing Constraint #2) must then be replaced by the following two sets of constraints,

$$\omega_l^k - \omega_{l+1}^k - \mu^k t^k \lambda_l^k \geq 0 \qquad \text{for } k=1,2,...,\mathbf{K} \text{ and } l = 1,2,...,\mathbf{M} - 1 \qquad (4)$$

and

$$\mu^{[k]} - \mu^{[k+1]} - \chi \geq 0 \qquad \text{for } k = 1,2,...,\mathbf{K} - 1, \qquad (5)$$

where χ is a variable measuring the minimum gap between consecutive clearness variables.

Constraint #4: The weights must be chosen in such a manner that the overall rating accorded to mutual fund i, $\mathbf{R}_i$, is less than or equal to some exogenous parameter, say 1,

$$\mathbf{R} = \sum_{k=1}^{K} \sum_{l=1}^{M} \omega_l^k a_{il}^k \leq 1. \tag{6}$$

Choosing a value of 1 is just a scaling convention; any parameter value greater than zero could be chosen and would have no impact on the overall ranking of the funds.

Constraint #5: Finally, constraints on the minimum values of the variables v and χ are added,

$$v \geq z \qquad \text{and} \tag{7}$$

$$\chi \geq z \tag{8}$$

where z is a non-negative parameter.

3. Ranking The Mutual Funds

Given any set of weights $\{\omega_l^k\}$, the investor can obtain a rating $\mathbf{R}_i$ for each of the **M** mutual funds. By ranking the funds in descending order according to $\mathbf{R}_i$, an optimal fund is identified. Effectively, by choosing a set of weights, an additive utility function $\mathbf{R}_i(\omega)$ is defined, where ω denotes an $(\mathbf{M} \times \mathbf{K})$ matrix (ω_l^k is the (l,k) element of ω) and any set of weights satisfying Constraint #1–Constraint #5 is feasible. Rather than simply, and arbitrarily, choosing any set of weights that is feasible, an approach that has been applied in efficiency analysis (see, for example, Charnes, Cooper and Rhodes (1978)) can be adopted. This approach, applied in the present context, determines for each mutual fund i, the set of weights $\{\omega_l^k\}$ that provides the most favorable rating $\mathbf{R}_i$. Specifically, solve the following maximization problem for each fund i (where C#i denotes Constraint #i),[16]

$$\text{Max}_{\{\omega\},v,\{\mu\},\chi}\mathbf{R}_i = \text{Max}_{\{\omega\},v,\{\mu\},\chi} \sum_{k=1}^{K} \sum_{l=1}^{M} \omega_l^k a_{il}^k$$

subject to:

$$\omega_l^k - \omega_l^{k+1} - v\kappa^k \geq 0 \qquad \text{for } k = 1,2,\ldots,\mathbf{K}-1 \text{ and } \forall l \qquad \text{(C\#1)} \tag{2}$$

$$\omega_l^k - \omega_{l+1}^k - t^k(\omega_l^1 - \omega_{l+1}^1) \leq 0 \qquad \text{for } l = 1,2,\ldots,\mathbf{M}-1 \text{ and } \forall k \qquad \text{(C\#2)} \tag{3b}$$

$\omega_l^k - \omega_{l+1}^k - \mu^k t^k \lambda_l^k \geq 0$ for $l = 1,2,...,\mathbf{M} - 1$ and $\forall\, k$ (C#2,#3) (4)

$\mu^{[k]} - \mu^{[k+1]} - \chi \geq 0$ for $k = 1,2,..., \mathbf{K} - 1$ (C#3) (5)

$$\sum_{k=1}^{K} \sum_{l=1}^{M} \omega_l^k a_{il}^k \leq 1 \qquad \text{(C\#4)} \quad (6)$$

$\nu - z \geq 0$ and $\chi - z \geq 0$ (C#5) (7),(8)

$\omega_l^k, \mu^k, z \geq 0$ $\forall\, l,k$ (Non-negativity)

A Simple Example: A simple example, using one criterion and two funds, illustrates how the maximization problem is solved. While the example is trivial in terms of identifying the best fund, it is useful in illustrating the geometry of the maximization procedure. Let the single criterion be α_i, with $\alpha_1 = 0.30$ and $\alpha_2 = 0.05$, so that $a_{11} = 1$, $a_{22} = 1$ (see Figure 1) and $\lambda_1^1 = \lambda = 1.0$ (the k superscript is omitted throughout this example to simplify the exposition). From (1), $\mathbf{R}_1 = \omega_{11}$ and $\mathbf{R}_2 = \omega_{22}$, where an additional subscript is introduced to denote the fund being rated.[17] Next, observe that discriminating among criteria and criteria clearness are not issues in this example. Hence, Constraints #1 and #3 are not relevant, and the vector μ in (4) can be represented by a single variable. Finally, add $\mu \geq z$ to Constraint #5 and let $z = 0.25$.

The constraint set for the two maximization problems, $\text{Max}_{\{\omega\},\mu}\mathbf{R}_1$ and $\text{Max}_{\{\omega\},\mu}\mathbf{R}_2$, is then

$\omega_{i1} - \omega_{i2} - \mu \geq 0$ (Constraint #2)

$\omega_{i1} \leq 1$ and $\omega_{i2} \leq 1$ (Constraint #4)

$\mu \geq 0.25$ (Constraint #5)

$\omega_{i1} \geq 0$ and $\omega_{i2} \geq 0$, (Non-negativity)

for $i = 1$ and $i = 2$ respectively. The constraint set for this problem is represented by the hatched region in Figure 2. In solving "Max $\mathbf{R}_1$" one obtains $\omega_{22}^* = 1$ and ω_{21}^* is any value in the closed interval [0,0.75]. For example, take $\omega_{12}^* = 0$ and call

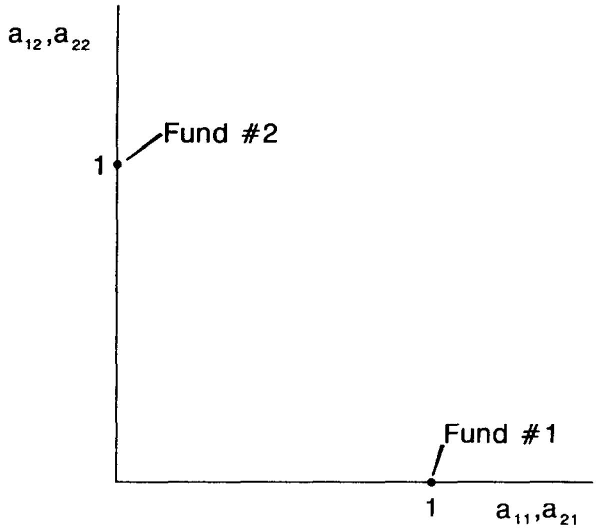

Figure 1. Requirements Space.

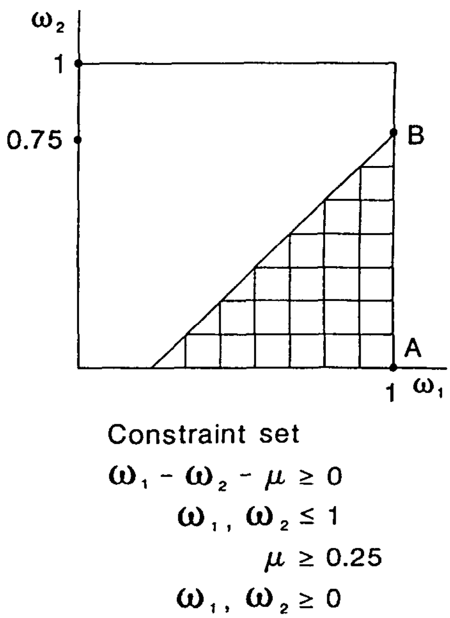

Figure 2. Solution Space for $\lambda = 1.0$, $z = 0.25$.

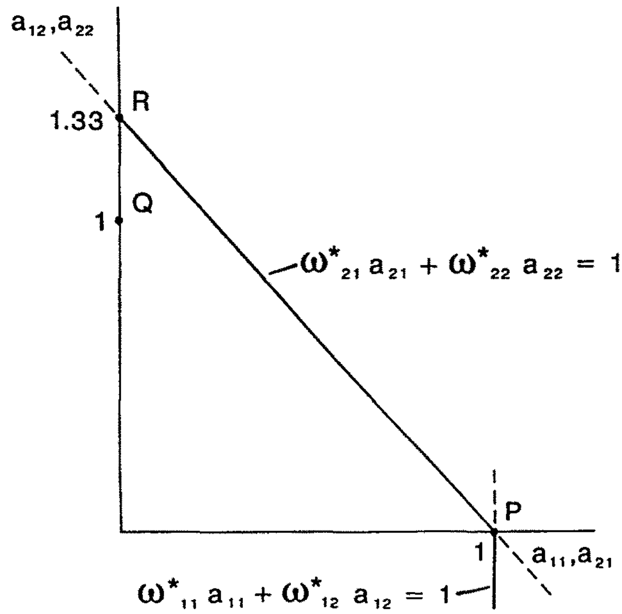

Figure 3. Requirements Space for $\lambda = 1.0$, $z = 0.25$.

this point **A**. Similarly, in solving "Max $\mathbf{R}_2$", one obtains $\omega^*_{22} = 0.75$ and $\omega^*_{21} = 1$; call this point **B**. Next, observe that Constraint #4 defines two hyperplanes (one for both Max $\mathbf{R}_1$ and Max $\mathbf{R}_2$,

$$\omega^*_{11}a_{11} + \omega^*_{12}a_{12} = 1 \text{ and}$$

$$\omega^*_{21}a_{21} + \omega^*_{22}a_{22} = 1,$$

which are drawn in Figure 3. Note that the solution to the second maximization problem (Max $\mathbf{R}_2$) is given by the ratio of OQ / OR = 1 / 1.33 = 0.75, while the solution to the first maximization problem (Max $\mathbf{R}_1$) is given by OP = 1.

In the literature on efficiency measurement (see, especially, Charnes, et al (1978)), the line segment **RP** and the vertical segment emanating downward from **P**, are referred to as the efficient frontier. Points on the frontier are said to be efficient, while points beneath the frontier (toward the origin) are inefficient. Consequently, in Figure Three, mutual fund #1 (Point **P**) is efficient, while fund #2 (Point #**Q**) is inefficient. Solving the maximization problem for a specific mutual fund is then equivalent to determining the position of the fund with respect to the efficient frontier.

Note that, if in this example we had set $\alpha_1 = \alpha_2$ so that $\lambda = 0$ (i.e., there is no distinction between the first and second rank positions), then Constraint #2 would have been $\omega_{i1} - \omega_{i2} \geq 0$ for i=1,2. In this case, the constraint set would appear as

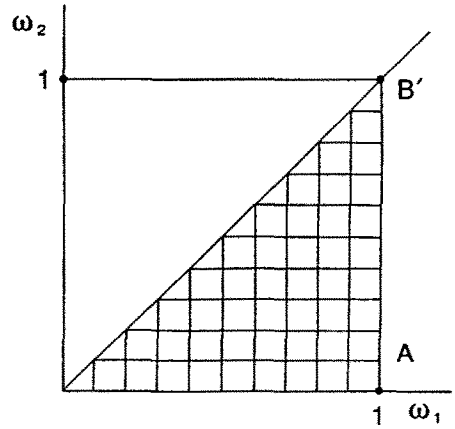

Figure 4. Solution Space for $\lambda = 0.0$, $z = 1.0$.

shown in Figure 4, and the corresponding hyperplanes would be as shown in Figure 5. In this case, both mutual funds lie on the frontier, hence both are efficient. Alternatively, if $\lambda = 1.0$ and z is increased to 0.50, the corresponding efficient frontier is shown in Figure 6. Here, $\mathbf{R}_2 = \omega^*_{22} = 0.5$. Note that OQ/OR=0.5 as well.

While it is difficult to construct a geometric image of higher dimension problems, it can be seen from this simple example that increasing a parameter such as z acts to decrease the likelihood of a given mutual fund being classed as

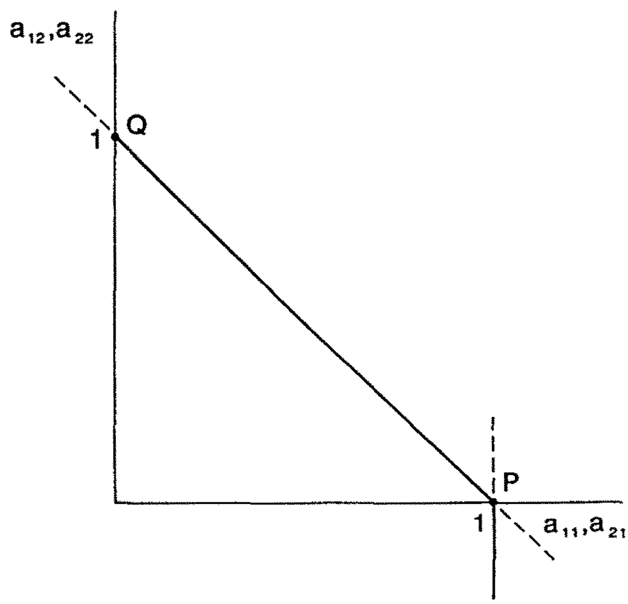

Figure 5. Requirements Space for $\lambda = 1.0$, $z = 1.0$.

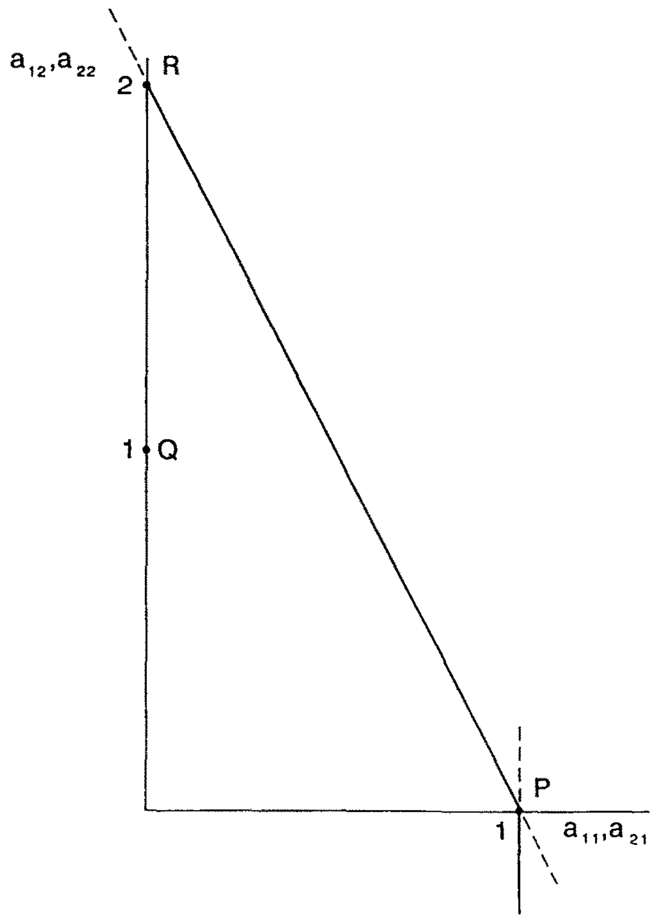

Figure 6. Requirements Space for $\lambda = 1.0, z = 0.5$.

"efficient". The same is true, in general, if κ^k, λ_i^k, or λ are increased. Thus, as z is increased, the set $\mathbf{R}^* = \{i|\mathbf{R}_i^* = 1\}$ of efficient funds becomes progressively smaller, until z reaches its maximum (beyond this value of z, there is no feasible solution to the constraint set).

The number of funds in the set $\mathbf{R}^*$ is therefore determined, to a great extent, by the value of the parameter z. If, for example, $z = 0$, then ν and χ may both be zero, and the maximization problem enjoys tremendous flexibility in its choice of weights, $\{\omega_i^k\}$. In order to decrease the number of funds in the set $\mathbf{R}^*$, the degree of flexibility allowed must be reduced. This can be accomplished by increasing incrementally the value of the parameter z. A faster and more efficient method of accomplishing this is now presented.

The maximization problem at the beginning of this subsection can be modified slightly so that it identifies the largest value of z for which the set of constraints is feasible. That is, solve for the maximum value of z, subject to the set of constraints listed above, for which the set **R*** is not empty.

$$\text{Max } z \tag{10}$$

subject to:

$$\omega_l^k - \omega_l^{k+1} - \nu\kappa^k \geq 0 \qquad \text{for } k = 1,2,\ldots,\mathbf{K}-1 \text{ and } \forall l \tag{2}$$

$$\omega_l^k - \omega_{l+1}^k - t^k(\omega_l^1 - \omega_{l+1}^1) \leq 0 \qquad \text{for } l = 1,2,\ldots,\mathbf{M}-1 \text{ and } \forall k \tag{3b}$$

$$\omega_l^k - \omega_{l+1}^k - \mu^k t^k \lambda_l^k \geq 0 \qquad \text{for } l = 1,2,\ldots,\mathbf{M}-1 \text{ and } \forall k \tag{4}$$

$$\mu^{[k]} - \mu^{[k+1]} - \chi \geq 0 \qquad \text{for } k\ 1,2,\ldots,\mathbf{K}-1 \tag{5}$$

$$\sum_{k=1}^{\mathrm{K}} \sum_{l=1}^{\mathrm{M}} \omega_l^k a_{il}^k \leq 1 \tag{6}$$

$$\nu - z \geq 0 \quad \text{and} \quad \chi - z \geq 0 \tag{7),(8}$$

$$\omega_l^k,\ z,\ \mu^k \geq 0 \qquad \forall\ l,k$$

At the optimum z^*, Constraint #4 will be binding for at least one fund i^*, i.e.,

$$\mathbf{R}_{i*} = \sum_{k=1}^{\mathrm{K}} \sum_{l=1}^{\mathrm{M}} \omega_l^k a_{i*l}^k = 1.$$

Such a mutual fund i^* will be optimal for the investor.[18] Section Four now provides an example of how to use this approach to rate ten funds, evaluated on the basis of the six criteria suggested in Section 2.

IV. AN EXAMPLE

This section demonstrates the multicriteria approach to rating a set of mutual funds and illustrates that investors with different preferences do, in general, obtain different fund ratings. This section also demonstrates that, in general, the multicriteria approach will obtain a different set of ratings than that obtained by employing Jensen's single criterion approach. To demonstrate the multicriteria approach, two types of information must first be provided: 1) Data: For each of the 10 funds, an observation (either cardinal or ordinal) for each of the 6 criteria must be provided (the data is employed in the a_{il}^k and λ_i^k parameters). 2) Parameter values: The investor must specify values for the vector of criteria importance parameters (κ) and a ranking of the six criteria according to their relative clarity (recall the $\mu^{[k]}$).

For this example, ten mutual funds were randomly selected. For the required data, α_i and σ_i were calculated using thirty-six monthly observations from 1986–89 (obtained from the Mutual Fund Sourcebook). Information on $\mathbf{D}_i$, $\mathbf{F}_i$, and $\mathbf{B}_i$ was obtained directly from the selected mutual funds. Quantifying $\mathbf{S}_i$ was quite difficult and was done on a fairly arbitrary basis.

The two types of parameter values required would normally be supplied by the investor performing the ranking. For this example, a single ranking of the six criteria according to their relative clarity is provided. It is assumed that $\mu^{[1]} = \mu^{[2]} = \mu^{[4]} = \mu^{[5]} > \mu^{[3]} = \mu^{[6]}$, where [1] refers to α_i, [2] to σ_i, [3] to $\mathbf{D}_i$, [4] to $\mathbf{F}_i$, [5] to $\mathbf{B}_i$ and [6] to $\mathbf{S}_i$. Next, three vectors of criteria importance parameters are assumed, each representing a different (hypothetical) investor's set of preferences. This is done to, as simply as possible, demonstrate that investor's with different preferences do, in general, obtain different fund ratings.

As mentioned in the Introduction, fund shareholders vary enormously in terms of at least six variables, their: wealth levels; rates of portfolio turnover; degrees of risk tolerance; understanding of financial markets; beliefs in the ability of mutual funds to outperform the market; and their own portfolio's level of diversification. Assume that the three hypothetical investors can be represented by whether they rate high (1), medium (2) or low (3) according to each of the six variables:

	Investor #1	*Investor #2*	*Investor #3*
Wealth Level	1	2	3
Portfolio Turnover	1	2	3
Risk Tolerance	1	2	3
Understanding	1	2	3
Belief in Funds' Ability	1	2	3
Diversification	1	2	3

Consistent with the above representation, the three vectors of criteria importance parameters are assumed to be:

	Investor #1		*Investor #2*		*Investor #3*	
Rank	*Criteria*	κ^k	*Criteria*	κ^k	*Criteria*	κ^k
1	α	2	**D**	3	**F**	0
2	**B**	2	**B**	0	**S**	3
3	**F**	1	**S**	5	**D**	4
4	σ	1	σ	1	**B**	1
5	**D**	0	α	1	σ	1
6	**S**		**F**		α	

TABLE 1
Rankings of the Ten Mutual Funds For Investors #1, #2 and #3 and for Jensen's Single Criterion Procedure

Rank	*Investor #1*	*Investor #2*	*Investor #3*	*Jensen's Single Criterion Procedure*
1	1*	2	5*	3
2	3*	1	6*	1
3	8	10	1	8
4	5	6	10	6
5	6	8	9	5
6	2	3	2	2
7	10	9	8	10
8	9	7	4	9
9	4	5	3	4
10	7	4	7	7

* Denotes a tie for first place.

Using the data and parameter values described above, the maximization problem presented in (10) identifies the optimal mutual fund for each of the three hypothetical investors. The full ranking of the ten mutual funds for each of the three investors (using the multicriteria approach), as well as the ranking obtained by employing Jensen's single criteria approach, are presented in Table 1. Observe that the three hypothetical investors obtain different fund rankings from both one another and from that obtained by using Jensen's single criterion approach.

In Table 1 observe that Investor #1, for whom α is the most important criterion, obtains an almost identical ranking of the ten mutual funds as that provided by Jensen's single criterion procedure. Also observe that, while Investors #2 and #3 obtain significantly different fund rankings from that provided by Jensen's procedure, they place α very low (fifth and sixth, respectively). Four additional hypothetical investors are now introduced, to demonstrate that investor's who place α as their first, second or third most important criterion, may obtain fund rankings significantly different from that provided by Jensen's procedure. The four additional vectors of criteria importance parameters are assumed to be:

	Investor #4		*Investor #5*		*Investor #6*		*Investor #7*	
Rank	*Criteria*	κ^k	*Criteria*	κ^k	*Criteria*	κ^k	*Criteria*	κ^k
1	α	1	α	0	**B**	3	**B**	2
2	**B**	2	**B**	1	α	0	**F**	1
3	**F**	1	**F**	1	**F**	1	α	1
4	σ	1	σ	1	σ	1	σ	1
5	**D**	0	**D**	1	**D**	1	**D**	1
6	**S**		**S**		**S**		**S**	

Table 2 presents the full ranking of the ten mutual funds, for each of the four additional hypothetical investors. Note specifically that Mutual Fund #3, which

TABLE 2
Rankings of the Ten Mutual Funds for Investors #4, #5, #6 and #7 Using the Multi-Criteria Procedure

Rank	*Investor #4*	*Investor #5*	*Investor #6*	*Investor #7*
1	1	1	5	5
2	3	5	1	6
3	5	3	6	1
4	8	6	3	3
5	6	8	8	4
6	2	2	4	8
7	9	4	9	9
8	10	9	2	2
9	4	10	10	7
10	7	7	7	10

ranks first using Jensen's single criterion procedure, ranks second for Investor #4, third for Investor #5 and fourth for Investors #6 and #7. Also, note that Fund #1, which ranks second using Jensen's procedure (and places first for Investors's #1, #4 and #5), drops to second place for Investor #6 and to third place for Investor #7. Hence it is shown how the ranking of funds change as the investors' priorities change. Further, it is demonstrated that, even when investors place α relatively highly (their first, second or third most important criterion), investors may obtain fund rankings significantly different from that provided by Jensen's procedure.

V. CONCLUSIONS

This paper presented an approach which allows an individual investor to identify an optimal mutual fund, given his specific set of attributes and preferences. This multicriteria approach is based on the pure ordinal model of Cook and Kress (1991). The initial premise is that investors select mutual funds on the basis of several distinct criteria, rather than on the basis of a single criterion, such as Jensen's α coefficient (as is generally assumed in the mutual fund evaluation literature). A general procedure is developed for ranking a set of competing mutual funds, based upon the ranking of the set of funds according to each criteria and on the investor's ranking of the criteria (in order of importance) themselves.

An example, involving ten mutual funds and six criteria, demonstrated two important points. First, investors with heterogeneous attributes and preferences do, in general, obtain different fund rankings from one another. Second, the fund rankings obtained using the multicriteria approach differ from those obtained using Jensen's single criterion method. Also, Jensen's single criterion approach predicts that investors unanimously agree on their fund ratings (and rankings) and hence, they all select the same mutual fund. The implication of the multicriteria methodology that investors obtain different fund rankings and hence, select different funds to purchase is, however, much more consistent with the large number of mutual funds currently sold in the U.S.

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NOTES

1. In January, 1990, mutual fund assets surpassed the one trillion dollar mark, with equity funds accounting for 26 percent of the total, bond funds 31 percent, money market funds 36 percent and short-term municipal bond funds 7 percent. Also, note that not all mutual fund shares are held directly by individual investors. In addition to the 51.1 million individual accounts, there are 7.1 million institutional accounts (a majority, 3.7 million, of these are fiduciary accounts), holding 25.4 percent of all stock, bond and income fund shares and 42.5 percent of all money market shares. Statistics from the Investment Company Institute (1990).
2. While Jensen's measure is the most widely used measure in academic empirical studies of forecasting ability, it has been the subject of numerous criticisms. For a good survey of the criticisms, see Grinblatt and Titman (1989).
3. Treynor (1965) also employed a single criterion approach, asserting that "The comprehensiveness of this rating is a question for the reader to decide for himself....Most readers are likely to agree, however, that at least one dimension — and a critical one — of the quality of the investment management is analyzed by this new method."
4. Using prior rates of return to select a mutual fund could be considered a questionable practice in view of evidence that past performance contains little or no predictive value. However, Allerdice and Farrar (1967), Friend, Blume and Crocket (1970), Smith (1978) and Woerheide (1982) provide evidence that past rates of return are positively correlated with a fund's net sales. Furthermore, survey evidence presented by Lewellen, Lease and Schlarbaum (1977) and the Investment Company Institute (1987) indicate that prior returns are one of the primary variables used by investors when choosing mutual funds.
5. Including σ_i to measure a fund's performance consistency was suggested by C. Poll, Managing Director, Micropal. He evaluates funds by observing their annual rank (by that year's return) over, for example, each of the last five years, and then selecting the fund which has most consistently exhibited a high annual rank.
6. If the mutual fund shares are to be held within a perfectly diversified portfolio, then the fund's level of diversification is irrelevant. However, the evidence is quite strong that a very large majority of households do not hold well diversified portfolios (Blume and Friend (1975)). Further, survey data presented by Lewellen, Lease and Schlarbaum (1977) and the Investment Company Institute (1987) demonstrate the importance of the fund's level of diversification to the individual investor's selection decision.
7. To calculate $\mathbf{D}_i$, sum the squared deviations of the proportions invested in each security in the market portfolio from the corresponding proportions in the mutual fund. Since the weight of each security in the market portfolio is very small, $\mathbf{D}_i$ can be approximated by the sum of the squares of the proportions invested in each security in the mutual fund. Note that $0 \le \mathbf{D}_i \le 1$, with lower values representing higher levels of diversification. For example, $\mathbf{D}_i = 1$ for a one stock portfolio and $\mathbf{D}_i = 0$ for the market portfolio. This measure is used by Blume and Friend (1975) and is a special case of the diversification measure suggested by Sharpe (1972); the two measures are equivalent if all securities possess the same unsystematic or residual variance.

8. For a description of the various services offered by mutual funds, see Investment Company Institute (1990; p. 28–30). For survey results on the importance investors place on different fund services, see Investment Company Institute (1987; p. 33,34).
9. Jensen's performance index can be viewed as a special case of the multicriteria model presented here. He places a weight of 1 on α_i, and a zero weight on σ_i, $\mathbf{D}_i$, $\mathbf{F}_i$, $\mathbf{B}_i$ and $\mathbf{S}_i$.
10. This may be a weak ordering, containing ties if the investor views two or more funds as being equally important according to a particular criterion.
11. This may also be a weak ordering.
12. To illustrate, recall the six criteria presented in Section Two. The investor may view α_i as the most important criterion followed by, in decreasing order of importance, σ_i, $\mathbf{D}_i$, $\mathbf{F}_i$, $\mathbf{B}_i$ and $\mathbf{S}_i$.
13. To illustrate what is meant by clearness, recall the six criteria presented in Section II (α_i, σ_i, $\mathbf{D}_i$, $\mathbf{F}_i$, $\mathbf{B}_i$ and $\mathbf{S}_i$) and observe that there are measurement problems associated with each of the six criteria. Regarding α_i and σ_i, Roll (1978) demonstrates that, unless the exact composition of the true market portfolio is known, a fundamental ambiguity exists when α_i (and hence σ_i) is measured using the security market line (for a demonstration of how sensitive mutual fund rankings based upon α_i are to the benchmark portfolio chosen, see Lehmann and Modest (1987)). Uncertainty regarding the exact composition of the market portfolio also imparts an ambiguity into the measurement of $\mathbf{D}_i$ (recall that $\mathbf{D}_i$ measures how closely fund i's composition approximates that of the market portfolio). Load fees, because of their dependence on the quantity purchased and the investor's time horizon, also lack clarity. Finally, the fund's service level is possibly the "fuzziest" or most vague of the six criteria.
14. Note that, if the investor specified that all the $\omega_l^k - \omega_{l+1}^k$ were exactly equal to 1 for all l, this would be equivalent to describing the rank positions as cardinal numbers, $\mathbf{M}, \mathbf{M}-1, \ldots, 1$. Many ordinal ranking models, particularly those involving consensus derivation, do in fact use such a mechanism to weight rank positions (see, for example, Cook and Seiford (1978)).
15. A large number of ways to define λ_l^k exist, hence this definition should only be viewed as an example. It is, however, the simplest method of employing the cardinal data.
16. First, two types of information must be provided: 1) Data: For each of the **M** funds, an observation (either cardinal or ordinal) for each of the **K** criteria must be provided (the data is employed in the a_{il}^k and λ_l^k parameters). 2) Parameter values: The investor must specify values for the vector of criteria importance parameters (κ), a ranking of the **K** criteria according to their relative clarity (recall the $\mu^{[k]}$) and the non-negative parameter, z.
17. When there is substantial flexibility in the choice of weights, it is generally the case that a different set of weights will arise when solving the optimization problem for one fund than for another. In this simple example, the weights determining the maximum value of $\mathbf{R}_1$ are different from those for $\mathbf{R}_2$. Hence, for this example, a second subscript is added so that ω_{il} denotes the weight attached to rank position l when rating fund i. In general, however, the second subscript can be omitted because the maximization problem presented in (10) below, by solving for the optimal value of z, minimizes the degree of flexibility allowed and hence, derives an identical set of weights for all **M** funds.
18. It is possible that, given z^*, more than a single fund satisfies (11).

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