FINANCIAL SERVICES REVIEW, 2(1): 51–61
ISSN: 1057-0810

Nobody Gains from Dollar Cost Averaging Analytical, Numerical and Empirical Results

John R. Knight
Lewis Mandell

Dollar Cost Averaging is an investment system that is widely advocated by brokerage firms and mutual funds. In its best known form, an investor seeking to put a lump sum into risky assets is counseled to invest the money over a period of time in equal installments in order to avoid the devastating effect of a market fall immediately after a single, lump-sum investment. Using graphical analysis, historical stock market returns, and Monte Carlo simulations, this article demonstrates that no such benefit accrues to a Dollar Cost Averaging Strategy. Two alternative strategies, optimal rebalancing and buy and hold achieve better performance in all three analyses.

INTRODUCTION

"Dollar cost averaging" refers to an investment methodology in which a set dollar amount is placed in risky assets at equal intervals over a holding period. A number of rationales have been offered for this system in the investment literature (e.g. Dodson [1989]), centering largely around the advantages of avoiding a potentially unfortunate timing of a lump sum investment and the purported benefits of buying more shares at low prices and fewer shares at high prices.

Constantinides [1979] has shown that the inherent rigidity of dollar cost averaging makes it inferior to a constantly rebalanced portfolio on an a priori basis. Offsetting this, however, may be some of the other presumed advantages of dollar cost averaging including its simplicity, its forced savings implications, and its possible reduction in transactions costs from not having to rebalance in each period. Piros [1986] has quantified some of the differences in transactions costs.

John R. Knight • Department of Finance, University of Connecticut, CT 06268-2041.
Lewis Mandell • Department of Finance, University of Connecticut.

The purpose of this paper is to demonstrate analytically, numerically and empirically the lack of any advantage accruing to dollar cost averaging relative to two other types of systematic investment strategies, rebalancing and buy and hold. The paper is organized as follows. First, we provide a graphical representation of the effect on investor utility of following each of the investment policies. Next, we compare certainty-equivalent returns generated by the three systematic investment strategies in a series of numerical simulations. Then, using historical stock market returns, we show the lower level of utility derived from dollar cost averaging as compared with alternative strategies. In all three instances, dollar cost averaging provides the least desirable performance.

Dollar Cost Averaging

In evaluating the three types of systematic investing, we assume that investors have an initial stock of wealth invested in the riskless asset.[1] We assume that they know the balance between risky (call it, for simplicity a diversified portfolio approximating the S&P 500) and riskless (short T-bill) assets that will optimize their utility given expected returns on both assets, the volatility of the risky asset, and the investor's degree of risk aversion.

To illustrate the three types of systematic investment, let us assume that an investor's optimal balance is 50–50. The investor who practices Optimal Rebalancing would invest $50 thousand in risky assets immediately and would rebalance the portfolio at the end of each period to maintain exactly half of the portfolio in risky assets. The investor who practices the Buy and Hold strategy would put $50 thousand into risky assets immediately but would never rebalance in subsequent periods. In contrast, the investor who practices Dollar Cost Averaging would take the initial wealth endowment and move it into the risky asset in equal increments over the holding period. Using our example, the investor would put $5 thousand per year into the risky asset in each of the ten years.

Graphical Analysis

Relative to the investor who uses Optimal Rebalancing both the investor using Dollar Cost Averaging and the investor who Buys and Holds may be seen to experience utility loss, regardless of the degree of risk aversion. Figure 1 illustrates the loss of the Dollar Cost Average user. Utility curve U_{BaJ} represents the Optimally Balanced investor whose balance is at point **J** on the capital market line. An investor who is Dollar Cost Averaging to reach an optimal balance of **J** over the holding period, currently has a balance of **K**. This investor's utility curve is represented by U_{DCA} which, since it must pass through point **K** on the capital market line, is lower than utility curve U_{BaJ}. If we hold variance constant at σ_{DCA} the loss in return can be measured as $\Delta E(r)$. The Dollar Cost Averager suffers lost utility in two ways. First,

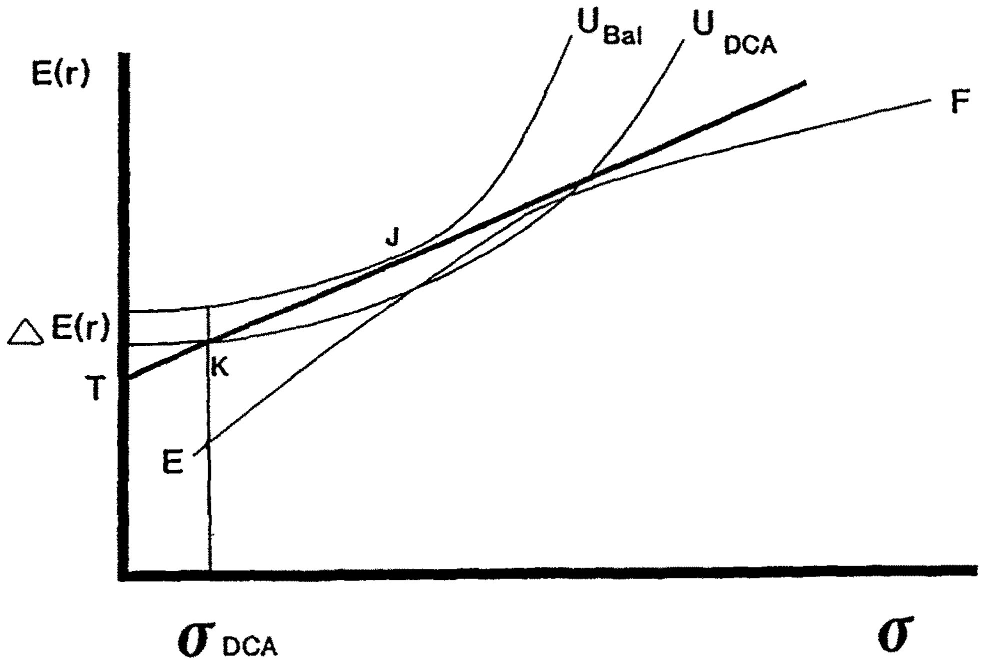

Figure 1. The Optimally Balanced (Bal) Investor Compared with the Dollar Cost Averaging (DCA) Investor.

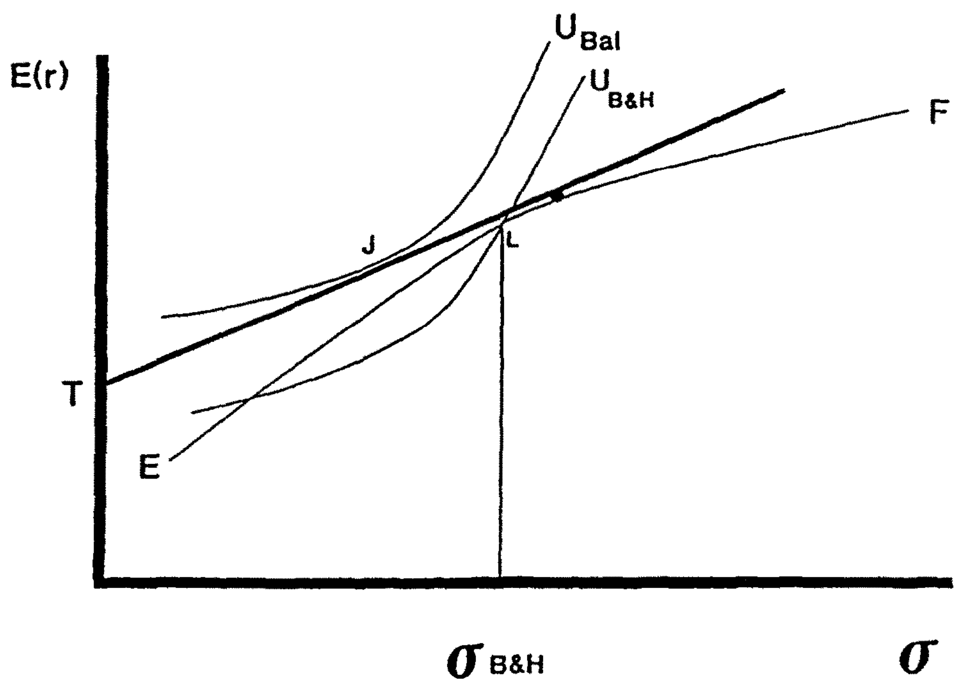

Figure 2. The Optimally Balanced (Bal) Investor Compared with the Buy and Hold (B&H) Investor.

he sacrifices the higher return he could have achieved on average with a larger proportion invested in the risky asset. Second, he suffers a utility loss from being "non-optimal" in terms of his investment balance over the entire investment horizon.

In Figure 2, the Buy and Hold investor $U_{B\&H}$ may also be seen to be out of balance, but for a very different reason. The risk premium on the risky asset will cause the expected value of the risky asset to grow more rapidly than the riskless asset, leaving the initially balanced Buy and Hold investor with a continually growing proportion of the risky asset. The added return moves the balance above **J** to **L** and the utility curve that intersects the capital market line at **L** is $U_{B\&H}$ which is below U_{Bal}. The Buy and Hold investor starts with a utility curve coincident with that of the Optimally Balanced investor, but over time moves to lower curves reflecting the lost utility from bearing more than his optimal level of risk.

NUMERICAL AND EMPIRICAL COMPARISONS

We approach the problem of comparing the three investment strategies described above with numerical simulations. Within the simulation framework, we examine a wide range of attitudes toward risk by varying the risk aversion parameter γ in the direct utility function W^{γ}/γ.

In our model, following that of Merton [1969], wealth grows over the investment horizon with the following price dynamics: For the riskless asset, **x**:

$$d\mathbf{P}_0(t) = \mathbf{P}_0(t)r\,dt$$

where r is the riskfree rate of return.

For the risky asset, **y**:

$$d\mathbf{P}_1(t) = \mathbf{P}_1(t)[\mu dt + \sigma dw(t)]$$

where $\mu > r$ is the mean rate of return on the risky asset, σ the standard deviation of that return, and $dw(t)$ the increment of a Wiener process.

Wealth dynamics then are a function of 1) the stochastic evolution of stock prices and 2) the proportion invested in risky assets.

$$\mathbf{W}(t) = x(t) + y(t)$$

and

$$d\mathbf{W}(t) = \mathbf{W}(t)[(\mu - r)\alpha + r]dt + \mathbf{W}(t)[\sigma\alpha dw(t)]$$

where α is the proportion of total wealth invested in the risky asset.[2]

Our investor begins with an initial wealth $\mathbf{W}_0$. With that initial wealth, the investor can employ three strategies.

1. **STRATEGY 1** - Optimally Balanced (Bal). The investor selects the optimal balance determined by his or her degree of risk aversion. In each subsequent period, the resulting amount is rebalanced to correspond to the initial optimal balance. By rebalancing his optimal proportion each period, this investor effectively buys more of the risky asset when prices have fallen and less when prices have risen. Here, his rationale is not one of market timing, but rather one of maintaining a utility maximizing proportion of his total wealth in the risky asset.
2. **STRATEGY 2** - Buy and Hold (B&H). The investor selects the optimal balance as in **STRATEGY 1** but does not subsequently rebalance, which means that he or she will go further and further out of balance as the risky assets grow more rapidly than the riskless asset due to the higher returns on the former. This strategy creates lost utility in the gradual departure from the optimal investment proportions. As such, investors at the two ends of the risk aversion continuum would experience the smallest costs from this departure. Extremely risk averse investors optimally hold a very low proportion of their total wealth in risky assets, and this small percentage results in a very slow rate of departure from the "optimum." At the other extreme, while departure from the "optimum" is at a fast rate, the limit, 100 percent in the risky asset, is not far from "optimal" so there is little potential for loss in utility even when the limit is reached.
3. **STRATEGY 3** - Dollar Cost Averaging (DCA). The investor puts a proportion of the initial wealth into risky assets each period so that the optimal balance is achieved at the end of the time horizon. This investor is under-invested in risky assets over the entire horizon and therefore loses utility from the "non-optimal" allocation of his wealth. Worse still, he forgoes the excess returns on the percentage of the risky asset that should be, but is not, in his investment portfolio, and he forgoes the utility that would be derived from the additional wealth.[3]

For our Monte Carlo simulation, we use familiar parameters extracted from historical New York Stock Exchange data. These data indicate an average return on stocks five percent greater than the risk free rate, and a standard deviation (our proxy for riskiness) of approximately 20 percent. The investor may choose any balance of risky to total assets from 10 to 90 percent that conforms to his coefficient of relative risk aversion (CRRA), $(1-\gamma)$. For example, investors with 10 percent in risky assets have a CRRA of 12.5 while those with 90 percent in risky assets have a CRRA of 1.3889. These coefficients are implied by the proportion of total wealth held in the risky asset and are shown at the bottom of the accompanying tables.

For each time horizon, we measure the investor's expected utility of wealth as the average of the utility from the wealth provided by each of 500 draws from our simulated stock market. Having calculated the expected utility from following each

TABLE 1
Certainty Equivalent Wealth For The Three Strategies

Investment Horizon	*Proportion in Risky Asset by Percentage*								
	10	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*
A. Certainty Equivalent Wealth for Strategy 1 (Optimal Balancing)									
2	1.0061	1.0087	1.0160	1.0242	1.0235	1.0347	1.0350	1.0405	1.0431
3	1.0086	1.0143	1.0229	1.0264	1.0314	1.0452	1.0629	1.0768	1.0793
4	1.0089	1.0150	1.0364	1.0416	1.0427	1.0622	1.0912	1.0765	1.0978
5	1.0141	1.0209	1.0321	1.0528	1.0825	1.0724	1.1018	1.0819	1.1273
6	1.0141	1.0444	1.0334	1.0639	1.0504	1.1022	1.1129	1.0991	1.1252
7	1.0155	1.0345	1.0548	1.0831	1.0598	1.1220	1.1234	1.1629	1.1409
8	1.0180	1.0440	1.0614	1.0704	1.1054	1.0935	1.1531	1.1469	1.1544
9	1.0221	1.0551	1.0722	1.0821	1.1019	1.1375	1.1696	1.2426	1.2417
10	1.0278	1.0525	1.0785	1.1142	1.1191	1.1907	1.2132	1.2455	1.1881
B Certainty Equivalent Wealth for Strategy 2 (Buy & Hold)									
2	1.0060	1.0086	1.0165	1.0239	1.0242	1.0340	1.0353	1.0403	1.0433
3	1.0085	1.0138	1.0223	1.0269	1.0316	1.0436	1.0613	1.0769	1.0789
4	1.0083	1.0161	1.0368	1.0403	1.0434	1.0617	1.0924	1.0776	1.0975
5	1.0134	1.0200	1.0311	1.0526	1.0820	1.0703	1.1029	1.0818	1.1266
6	1.0140	1.0428	1.0320	1.0613	1.0480	1.0997	1.1094	1.0983	1.1245
7	1.0152	1.0305	1.0495	1.0824	1.0588	1.1191	1.1220	1.1623	1.1409
8	1.0177	1.0397	1.0568	1.0671	1.1036	1.0923	1.1530	1.1416	1.1539
9	1.0192	1.0531	1.0675	1.0809	1.0943	1.1330	1.1680	1.2390	1.2411
10	1.0254	1.0489	1.0739	1.1116	1.1168	1.1832	1.2056	1.2436	1.1864
C. Certainty Equivalent Wealth for Strategy 3 (Dollar Cost Averaging)									
2	1.0049	1.0073	1.0129	1.0198	1.0200	1.0268	1.0287	1.0332	1.0386
3	1.0061	1.0098	1.0169	1.0200	1.0255	1.0316	1.0476	1.0561	1.0606
4	1.0060	1.0120	1.0264	1.0274	1.0335	1.0479	1.0624	1.0514	1.0745
5	1.0092	1.0174	1.0226	1.0380	1.0540	1.0461	1.0758	1.0600	1.0954
6	1.0096	1.0271	1.0242	1.0406	1.0356	1.0641	1.0708	1.0692	1.0872
7	1.0106	1.0234	1.0322	1.0506	1.0465	1.0737	1.0818	1.0986	1.0994
8	1.0125	1.0269	1.0397	1.0475	1.0676	1.0658	1.0957	1.0979	1.0976
9	1.0133	1.0329	1.0455	1.0551	1.0611	1.0829	1.0987	1.1385	1.1391
10	1.0151	1.0316	1.0471	1.0706	1.0725	1.1056	1.1261	1.1452	1.1257
Note: Implied Coefficient of Relative Risk Aversion	12.5000	6.2500	4.1667	3.1250	2.5000	2.0833	1.7857	1.5625	1.3889

of the three compared strategies, we can solve for the certainty equivalent wealth implied by the level of utility.

$$U(W) = W^{\gamma}/\gamma \Rightarrow W = (\gamma U(W))^{1/\gamma}$$

This analysis permits the direct comparison of wealth differences among strategies as shown in Table 1. As anticipated, little difference can be seen between strategies 1 and 2, indicating that the gains to rebalancing each period are small if the investor

TABLE 2

Investment Horizon	*Proportion in Risky Asset by Percentage* 10	20	30	40	50	60	70	80	90
A. Certainty Equivalent Wealth Difference: Strategy 1 (Optimal Balancing) – Strategy 2 (Buy & Hold)									
2	.0001	.0001	−.0005	.0002	−.0008	.0007	−.0003	.0003	−.0002
3	.0001	.0005	.0006	−.0005	−.0001	.0016	.0016	−.0001	.0004
4	.0005	−.0011	−.0004	.0013	−.0007	.0005	−.0011	−.0011	.0002
5	.0007	.0009	.0010	.0002	.0005	.0021	−.0011	.0001	.0007
6	.0001	.0016	.0014	.0027	.0024	.0025	.0034	.0008	.0007
7	.0002	.0041	.0052	.0007	.0010	.0028	.0014	.0006	.0001
8	.0004	.0043	.0046	.0033	.0018	.0012	.0001	.0053	.0005
9	.0029	.0020	.0047	.0012	.0076	.0045	.0016	.0036	.0006
10	.0024	.0035	.0046	.0027	.0023	.0075	.0077	.0020	.0017
Note: Implied Coefficient of Relative Risk Aversion	12.50	6.25	4.17	3.13	2.50	2.08	1.79	1.56	1.39
B. Certainty Equivalent Return From Following Strategy 1 (Optimal Balancing) Vice Strategy 2 (Buy & Hold)									
2	.00004	.00005	−.00023	.00012	−.00038	.00033	−.00017	.00013	−.00010
3	.00002	.00017	.00021	−.00016	−.00004	.00052	.00052	−.00003	.00014
4	.00013	−.00027	−.00010	.00033	−.00017	.00013	−.00028	−.00027	.00005
5	.00013	.00018	.00019	.00004	.00010	.00043	−.00022	.00002	.00015
6	.00001	.00027	.00023	.00045	.00039	.00041	.00057	.00013	.00011
7	.00003	.00058	.00075	.00009	.00014	.00040	.00020	.00008	.00001
8	.00004	.00054	.00058	.00041	.00022	.00015	.00001	.00067	.00006
9	.00032	.00023	.00052	.00014	.00085	.00050	.00018	.00040	.00007
10	.00024	.00035	.00046	.00027	.00023	.00075	.00076	.00020	.00017
Note: Implied Coefficient of Relative Risk Aversion	12.50	6.25	4.17	3.13	2.50	2.08	1.79	1.56	1.39

is initially in balance. This means that continual rebalancing yields little gain over the buy and hold strategy. However, the certainty equivalent wealth of the Dollar Cost Averager in Strategy three is consistently and substantially below that of the investors using the other two strategies.

Table 2A shows the certainty equivalent wealth difference between Strategies one and two and Table 2B shows the difference in annualized (geometric mean) return over all holding periods. The difference is tiny in all cases and, in the short run, will occasionally even go negative as a result of Monte Carlo variation acting upon small differences. With short time horizons, the investor does not venture too far from his "optimal" allocation of wealth.

Table 3A shows the certainty equivalent wealth difference between Strategies one and three and points up a considerable loss associated with Dollar Cost Averaging. While the cost of Dollar Cost Averaging varies with the proportion invested in risky assets and the number of holding periods, with 90% invested in risky assets the Optimally Balanced investor has half again the certainty equivalent

TABLE 3

Investment Horizon	Proportion in Risky Asset by Percentage								
	10	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*
A. Certainty Equivalent Wealth Difference: Strategy 1 (Optimal Balancing) – Strategy 3 (Dollar Cost Averaging)									
2	.0011	.0014	.0031	.0044	.0035	.0079	.0063	.0073	.0046
3	.0024	.0044	.0060	.0064	.0059	.0136	.0153	.0207	.0187
4	.0029	.0030	.0101	.0142	.0092	.0142	.0288	.0251	.0232
5	.0049	.0035	.0095	.0147	.0285	.0263	.0260	.0219	.0319
6	.0044	.0173	.0092	.0233	.0148	.0381	.0420	.0298	.0380
7	.0048	.0111	.0226	.0325	.0134	.0483	.0416	.0642	.0415
8	.0055	.0171	.0217	.0228	.0378	.0278	.0573	.0490	.0568
9	.0088	.0223	.0267	.0271	.0409	.0546	.0710	.1041	.1026
10	.0127	.0208	.0314	.0437	.0466	.0851	.0871	.1003	.0624
Note: Implied Coefficient of Relative Risk Aversion									
	12.50	6.25	4.17	3.13	2.50	2.08	1.79	1.56	1.39
B. Certainty Equivalent Return from Following Strategy 1 (Optimal Balancing) Vice Strategy 3 (Dollar Cost Averaging)									
2	.00056	.00072	.00156	.00219	.00175	.00394	.00315	.00365	.00229
3	.00081	.00147	.00201	.00215	.00196	.00450	.00506	.00685	.00620
4	.00073	.00075	.00250	.00354	.00230	.00354	.00713	.00622	.00576
5	.00098	.00069	.00189	.00293	.00564	.00521	.00515	.00435	.00630
6	.00074	.00286	.00152	.00385	.00245	.00625	.00689	.00491	.00623
7	.00069	.00158	.00319	.00458	.00190	.00676	.00584	.00893	.00583
8	.00069	.00212	.00268	.00283	.00465	.00343	.00699	.00600	.00693
9	.00098	.00245	.00293	.00297	.00446	.00592	.00765	.01106	.01092
10	.00127	.00206	.00309	.00428	.00457	.00820	.00839	.00961	.00608
Note: Implied Coefficient of Relative Risk Aversion									
	12.50	6.25	4.17	3.13	2.50	2.08	1.79	1.56	1.39

return of the Dollar Cost Averager. And since the Buy and Hold investor does nearly as well as the Optimal Balancer, the Dollar Cost Averaging strategy is also substantially inferior to the Buy and Hold Strategy.

EMPIRICAL ANALYSIS

The theoretical and simulation results were tested empirically by using actual monthly returns from 1962 to 1992. Dividend adjusted returns of the S&P 500 and Treasury Bill returns were used as proxies for the risky and riskless assets respectively. Three disparate investors were chosen for the analysis: one who was very risk averse, corresponding to the investor with 10 percent in risky assets in Tables 1–3; a 50–50 investor of middle risk aversion; and, a 90 percent risky asset investor who represents the not very risk averse. Ten year rolling holding periods were begun

TABLE 4
Empirical Results For The Three Strategies

	Optimal Balancing	*Buy and Hold*	*Dollar Cost*
A. High Degree of Risk Aversion (10% Risky Assets)			
Averaging Mean Annualized Return	.0731	.0729	.0713
(Standard Deviation)	(.0179)	(.0184)	(.0171)
Mean Utility	−270.09	−270.29	−271.02
(Standard Deviation)	(10.15)	(10.29)	(9.83)
Number of Periods Strategy Yielded	110	82	48
Highest Return (Percentage)	(45.8%)	(34.2%)	(20.0%)
Number of Periods Strategy Yielded	114	61	65
Highest Utility (Percentage)	(47.5%)	(25.4%)	(27.1%)
B. Moderate Degree of Risk Aversion (50% Risky Assets)			
Mean Annualized Return	.0837	.0828	.0762
(Standard Deviation)	(.0296)	(.0313)	(.0234)
Mean Utility	−266.38	−267.03	−269.75
(Standard Deviation)	(14.68)	(15.15)	(11.48)
Number of Periods Strategy Yielded	87	76	77
Highest Return (Percentage)	(36.3%)	(31.7%)	(32.0%)
Number of Periods Strategy Yielded	120	44	76
Highest Utility (Percentage)	(50.0%)	(18.3%)	(31.7%)
C. Low Degree of Risk Aversion (90% Risky Assets)			
Mean Annualized Return	.0904	.0900	.0805
(Standard Deviation)	(.0426)	(.0432)	(.0296)
Mean Utility	−264.82	−265.07	−268.69
(Standard Deviation)	(20.49)	(20.64)	(13.07)
Number of Periods Strategy Yielded	84	64	92
Highest Return (Percentage)	(35.0%)	(26.7%)	(38.3%)
Number of Periods Strategy Yielded	118	29	93
Highest Utility (Percentage)	(49.2%)	(12.1%)	(38.7%)

each month from 1962 to 1982, giving us a total of 240 holding periods. During each holding period, the Optimal Rebalancer rebalanced monthly and the Dollar Cost Averager invested an equal amount of his initial stock of wealth in risky assets in each of the 120 monthly periods. The Buy and Hold investor began with the correct balance and never varied his proportion. Table 4 summarizes the empirical results.

Over all levels of risk aversion, the Dollar Cost Averaging system yielded the smallest annualized return and mean utility, although differences were not significant. When the success of the three strategies was measured in terms of the number of holding periods (of the total of 240) in which the method yielded the best results, Dollar Cost Averaging also fared poorly. For the very risk averse investor, Dollar Cost Averaging produced the highest returns in only 20 percent of the cases. This increased to 32 percent and 38.3 percent for the moderate and not very risk averse investor. When measured in terms of utility, Dollar Cost Averaging showed similar results. The very risk averse investor would have done better in 27.1 percent of the cases as compared with 31.7 percent and 38.7 percent for those investors with moderate or low risk aversion.

Brokerage firms promote Dollar Cost Averaging primarily with two rationales. First, they argue that returns are augmented because more shares are purchased when prices are low and fewer when prices are high. Second, they assert that Dollar Cost Averaging enhances investor utility by preventing an ill-timed lump sum investment. Our results do not support either of these contentions.

It is reasonable to assume that transactions costs would vary inversely with the size and directly with the frequency of investment. Therefore, incorporating such costs into the model would serve to further strengthen the case against Dollar Cost Averaging. One might also speculate that reducing the frequency of investment would improve the performance of the Optimal Rebalancing strategy relative to its alternatives.

CONCLUSION

Using three separate methods of comparison, we have shown the lack of any advantage of Dollar Cost Averaging relative to two alternative investment strategies. Our numerical simulations and empirical evidence, in consonance with our graphical analysis, both favor the Optimal Rebalancing and Buy and Hold strategies over Dollar Cost Averaging. Optimal Rebalancing and Buy and Hold strategies convincingly outperform Dollar Cost Averaging on theoretical grounds as well as on the basis of numerical simulations. Historical evidence also supports these two strategies, though the empirical differences are not significant.

Our results strongly imply that the additional cost and effort associated with Dollar Cost Averaging cannot be justified for any investor, regardless of degree of risk aversion. With the possible exception of its promoters, nobody gains from Dollar Cost Averaging.

Acknowledgements: We gratefully acknowledge many beneficial conversations with Tom O'Brien on the subject of Dollar Cost Averaging.

NOTES

1. Thus, we take care not to confuse Dollar Cost Averaging with periodic investing where the investor puts aside a regular amount of savings each period to invest in risky securities. Periodic investment of savings as funds become available is a Buy and Hold strategy and has the additional positive attribute of encouraging habitual savings, a characteristic not shared by Dollar Cost Averaging.
2. Merton (1969) shows that, with costless and continous portfolio rebalancing possible, the investor with constant relative risk aversion has an optimal proportion of wealth (independent of the level of wealth) invested in the risky asset. This proportion is

$$\alpha = \frac{\mu - r}{\sigma^2(1 - \gamma)}$$

where $\gamma < 1$ is the investor's coefficient of relative risk aversion. We use this analytically optimal proportion as the benchmark for each utility function analyzed, but recognize that portfolio rebalancing is neither costless nor continous.

3. Since risky assets, on average, grow more rapidly than riskless assets, desired end of horizon results are also difficult to achieve with Dollar Cost Averaging. To use the example given earlier, a Dollar Cost Averager with $100 thousand in assets who wished to achieve a 50–50 balance at the end of 10 periods might put $5 thousand per period into risky assets. However, over the ten periods the funds invested in risky assets would be expected to increase more rapidly than the funds put into riskless assets. Therefore, on average, more than 50 percent of the portfolio would be in risky assets at the end of the ten periods.

REFERENCES

Constantinides, George M. 1979. "A Note on the Suboptimality of Dollar-Cost Averaging as an Investment Policy," *Journal of Financial and Quantitative Analysis*, 14(2): 443–450.

Dodson, Lori. 1989. "Three Dimensions of Diversification," *Journal of Accountancy*, 167(June): 131–139.

Merton, Robert C. 1969. "Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case," *The Review of Economics and Statistics*, 51: 247–257.

Piros, Christopher D. 1986. "How Bad is Dollar Cost Averaging?" Unpublished Paper.