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An Index of Portfolio Diversification

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A recurring question in the literature concerning diversification is what is the minimum number of securities required to achieve adequate diversification. The problem is that studies on this topic assume equally distributed holdings. In reality, portfolios are not evenly divided. The purpose of this paper is to evaluate the ability of five different measures of diversification to provide meaningful information about the degree of diversification of an unevenly distributed stock portfolio. The complement of the Herfindahl index was found to be the best of the five measures and its explanatory power was deemed to be adequate for general use.

I. INTRODUCTION

A critical question for any investor is how many securities does one have to hold in order to achieve adequate diversification. Complete diversification would be achieved if one held a share of the "market" portfolio, defined as the portfolio of all assets. Adequate diversification is achieved when the variability of one's portfolio is not significantly different than that of the market portfolio. The two classic studies which define the "minimum" portfolio size to be adequately diversified are Evans and Archer (E&A) (1968) and Fisher and Lorie (F&L) (1970). The word "minimum" is used in the sense that diversification beyond this size has little economic value in terms of risk reduction and may contain significant costs in terms of transaction fees and monitoring activity.

A significant problem with these two studies and the rest of the literature that looks at the topic of portfolio size and diversification is that, as a practical matter, they do not actually answer the simple question of whether a specific portfolio is adequately diversified. Virtually the entire literature on the question of portfolio size and diversification is based on portfolios that are evenly distributed. In reality,

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it would be incredibly rare for an investor to have his wealth evenly divided among securities. Thus, all of our knowledge about adequate diversification is irrelevant unless we could tell investors whether an unevenly distributed portfolio of 15 securities is as adequately diversified as, say, an evenly distributed portfolio of 10 securities.

Furthermore, even if an investor held an evenly distributed portfolio at a point in time, such a portfolio would become unbalanced for two reasons. One reason is that prices of different securities change at different rates. Another reason is the changes in portfolio composition that result from the addition and removal of cash (an event that occurs often in practice, and rarely in empirical research!).

What investors need is a spot measure of diversification based on nothing more than the distribution of the weights representing the proportion of the portfolio invested in each security. This paper shows that an index of portfolio diversification can be constructed, which is as good in indicating the degree of diversification of an unevenly distributed portfolio as is the number of securities in indicating the degree of diversification of an evenly distributed portfolio. Such an index could be used by individuals, financial advisors, and others to evaluate the degree of diversification of any stock portfolio.

Section II provides a review of the literature on the topic of portfolio size and diversification. Section III looks at five measures of diversification used in the industrial organization literature. Section IV tests the quality of these measures in distinguishing portfolio diversification and recommends the complement of the Herfindahl for future application. Section V presents standards for diversification using the recommended index, and Section VI provides a summary along with limitations and extensions.

II. Literature Review

Portfolio Size

As indicated above, the two classic studies on the topic of portfolio size and diversification are those by E&A and F&L.[1] As this research is modelled directly on that of E&A, we would like to provide a rather detailed description of their results. E&A compute the mean of the standard deviations for 60 equally weighted portfolios at each size level ranging from 1 to 40 securities. They then regressed the mean portfolio standard deviations against the inverse of the number of securities in the portfolio and obtained the following estimate:

$$Y = .08625/N + .1191$$

where

Y = mean portfolio standard deviation, and
N = number of securities in the portfolio.

E&A concluded that a ten-security portfolio provided adequate diversification. A graph of their results, or a variation of it, is presented in nearly all corporate finance and investments textbooks today.

F&L measure risk by examining various measures of dispersion for wealth ratios over various time periods for portfolios of sizes 1, 2, 8, 16, 32, and 128 securities. For our purposes, their most significant result probably is the observation that approximately 80 percent of the achievable reduction in dispersion can be attained by holding eight stocks (the reductions range from 65 to 91 percent).

In a follow-up study to E&A, Upson, Jessup, and Matsumoto (1975) looked at the standard deviation of the standard deviations, and concluded that portfolio managers should diversify among more than 16 stocks, and that diversifying among even 30 or more stocks can be worthwhile in terms of risk reduction.

Statman (1987) argues that a well-diversified portfolio must include at least 30 to 40 stocks. Statman's analysis is based on the assumption that all investors have the opportunity to buy no-load index funds, and thus the cost of adding assets combined with the risk reduction benefits of adding these assets must be compared to the cost and risk of portfolios that combine the risk-free asset with an index fund. A variation on Statman's study by Shanker (1989) shows that the conclusions about portfolio size are dependent on the size of the benchmark portfolio used for comparison and the assumed size of transaction fees. Smaller benchmark portfolios suggest smaller optimal portfolio sizes, and smaller transaction fees imply larger optimal portfolios. A follow-up study by Murphy (1991) questions the validity of the numbers used by Statman, and concludes that portfolios of the size suggested by E&A and F&L may in fact provide the minimum necessary degree of diversification.

Much of the literature on portfolio size examines what happens to the standard deviation function in the E&A study if various conditions are placed on the types of stocks in the portfolio. In one of the most cited studies, Solnik (1974) shows that more efficient diversification is possible when one considers foreign securities, particularly if one hedges for exchange rate risk. The greater efficiency in diversification is demonstrated by the result that E&A's standard deviation curve declines at a faster rate and to a lower level when foreign securities are added to the stock population.

Wagner and Lau (1971) show that far fewer stocks are necessary to achieve a specific level of diversification when the portfolio consists of stocks rated highly by the *Standard & Poor Stock Guide* than those rated poorly. Klemkosky and Martin (1975) show that diversification can be more readily achieved with low-beta stocks than with high-beta stocks. Martin and Klemkosky (1976) show that diversification can be more readily achieved when stock classifications are considered. Their stock classifications included growth stocks, cyclical stocks, stable stocks, and oil stocks.

All of the above studies are empirical. There are some theoretical studies that have shed light on the topic of portfolio size and diversification. Goldsmith (1976) shows that not only do transaction fees limit the size of the number of securities in a portfolio, but they will also cause the optimal number of securities to hold in a portfolio a function of an investor's initial wealth. Conine and Tamarkin (1981) show that investor preference for positive skewness combined with other assumptions of perfect capital market may severely restrict the number of securities held by an individual even without transaction fees.

Measures of Diversification

Most of the literature makes no statements about how to measure the degree of diversification. As shown above, the most common measure of diversification is simply to count the number of securities in the portfolio. However, this naive measure has meaning only when the portfolio is evenly invested across all holdings, and the point of this paper is to define a more effective measure when security holdings are not evenly distributed.

The one other method suggested in the literature for measuring the degree of diversification of a portfolio is to examine the correlation coefficient between the rates of return on that portfolio and the rates of return on a surrogate for the market portfolio (see Sharpe and Alexander (1990, p. 654)). This measure obviously requires data from a period of several years and poses a serious estimation problem any time the portfolio's composition has changed.

III. Defining New Measures of Diversification

In this section of the paper, we define various indices of diversification that could be used at a point in time. To dream up such measures would be haphazard and arbitrary. Therefore, the authors have elected to examine measures of diversification that have been used in the industrial organization literature on concentration.[2] A thorough review of the literature revealed five indices that would be plausible to measure portfolio diversification. In all cases, there is no profound reason for the use of a particular functional form, other than it is mathematically different and may empirically work better than other forms.

The first one is the complement of the Herfindahl Index, perhaps the most widely used measure of economic concentration.[3] Thus, our first proposed index is

$$\mathrm{DI}(1) = 1 - \mathrm{HI} = 1 - \sum_{i=1}^{N} W_i^2$$

where

DI = diversification index,
HI = Herfindahl Index,

W_i = the proportion of portfolio market value invested in security i (in decimal form), and
N = the number of securities in the portfolio.

Our use of the complement of this index is for the stylistic purpose of altering the index value so that zero represents a portfolio with absolutely no diversification (a one security portfolio) and 1.0 would represent the ultimate in diversification.[4]

To facilitate the reader developing a feel for these various indices, index values for nine sample portfolios are shown in Table lA. (The compositions of the various portfolios are listed in Table lB.) The first portfolio (Portfolio A) contains only one security and thus has absolutely no diversification. The ninth portfolio (Portfolio I) contains 100 evenly distributed securities. Note that for our first index the portfolio with only one security has an index value of zero, and the portfolio of 100 securities has an index value of .99.

The second index we consider is the complement of one originated by Rosenbluth (1961) and described in Marfels (1971). In this index, security holdings are ranked in descending order by size with the i-th firm receiving rank i.[5] The index value is

$$DI(2) = 1 - 1/(2 \times \sum_{i=1}^{n} i(W_i) - 1).$$

Note that in Table lA these first two indices and the next one to be introduced provide identical values for portfolios G, H, and I. It is easily shown that all three indices are mathematically equivalent to the term [1 – 1/N] when security holdings are evenly held.

The third index is defined by Marfels (1971) as the "exponential of the entropy measure" and is computed as

$$DI(3) = 1 - \prod_{i=2}^{N} W_i\text{^}W_i.$$

A fourth measure of diversification is the complement of one offered by Horvath (1972) and named by him the "comprehensive concentration index." It is

$$DI(4) = 1 - W_1 - \sum_{i=2}^{N} W_i^2\,[1 + (1 - W_i)]$$

where W_1 = the largest single portfolio holding. The index value for a single-security portfolio (Portfolio A) is zero, and for an evenly weighted portfolio of 100 securities

TABLE 1A.
Values of Diversification Indices for Various Portfolios

Diversification Index	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
DI (1): Herfindahl	0	.44	.61	.70	.76	.79	.88	.90	.99
DI (2): Rosenbluth	0	.40	.57	.67	.73	.77	.88	.90	.99
DI (3): Exp. of Entropy	0	.47	.64	.72	.77	.81	.88	.90	.99
DI (4): CCI	0	.15	.26	.36	.43	.49	.67	.73	.97
DI (5): Entropy	0	.64	1.01	1.28	1.49	1.66	2.08	2.30	4.60

TABLE 1B.
Composition of Portfolios Used in Table 1A

	% Distribution in Portfolio					
Security No.	*A*	*B*	*C*	*D*	*E*	*F*
1	100%	66 2/3%	50%	40%	33 1/3%	28.6%
2		33 1/3	33 1/3	30	26 2/3	23.8
3			16 2/3	20	20	19.0
4				10	13 1/3	14.3
5					6 2/3	9.5
6						4.8

Portfolio G: 8 securities, evenly distributed, (12 1/2% each).
Portfolio H: 10 securities, evenly distributed, (10% each).
Portfolio I: 100 securities, evenly distributed (1% each).

(Portfolio I) is .97. This and the next index are the only indices we are considering that, for evenly distributed portfolios, do not reduce to the value of [1 – 1/N].

Our final index is the entropy measure. It is defined in Hart (1971) as

$$DI(5) = -\sum_{i=1}^{N} W_i \ln(W_i).$$

where ln = natural logarithm.

The entropy measure is distinct from the others in that it is not constrained between zero and one. Note that in Table 1A, the index values for all the sample portfolios except A and B are greater than one.

IV. Evaluating the Quality of the Five Indices

In order to evaluate the quality of the indices proposed in Section III, we empirically examine the relationship between the standard deviation of returns of randomly selected portfolios and the respective indices of diversification. The quality of each index is measured by the closeness of the fit (in regression terms) between portfolio risk (i.e., the standard deviation of returns) and the index number. The methodology is the same as that used by E&A, except that the distribution of securities in our portfolio is based on randomly determined weights, rather than evenly distributed weights.

A total of 1,740 portfolios are examined. There are 60 portfolios which contain two securities each, 60 with three each, and so on up to 60 portfolios which contain 30 securities each.[6] To compute the standard deviation for each portfolio we obtained monthly rates of return data from the 1985 CRSP tapes.[7] We used the rate of return series which included dividends. We selected only those companies with monthly rates of return covering the entire period from December 1965 to December 1985. This provided us with a sample size of 483 companies.[8]

We computed the monthly value relative for each portfolio as

$$\overline{R}_i = \sum_{k=1}^{N} W_i \, R_{i,k}$$

where $R_{i,k}$ = the value relative for security k during month i (the value relative is the price at the end of the month plus any dividend paid during the month, divided by the price at the start of the month).

Next, the monthly mean value relative for each portfolio was computed as

$$\overline{R}_p = \exp((1/\mathrm{m}) \times \sum_{i=1}^{m} \log_e \overline{R}_i).$$

where m = 240 months. Finally, the portfolio standard deviation was computed as

$$\mathrm{SD}_p = [(1/(\mathrm{m}-1)) \times \sum_{i=1}^{m} (\log_e \overline{R}_p - \log_e \overline{R}_i)^2]^{1/2}.$$

It should be noted that implicit in these formulations is the assumption that each portfolio is reweighted at the start of each month to the original distribution of that portfolio.[9]

In order to ascertain the weights of the securities within each portfolio, we generated one random number for each security in the portfolio, summed the random numbers, and set the weights equal to the ratio of each random number to the sum. So if we were constructing a two-security portfolio and our two random numbers were 43 and 82, then our weights were .344 (= 43/125) and .656 (= 82/125).[10] Once the weights were computed, we then calculated the five index values for each portfolio.

Clearly, portfolios with the same number of securities were unlikely to produce the same index values. The means for each of the five indices for portfolios of selected sizes are shown in Table 2. Although the first four indices are theoretically

TABLE 2.
Average Index Values for Selected Portfolio Sizes

Number of Securities in Portfolio	*Diversification Index*				
	1	*2*	*3*	*4*	*5*
2	.384	.357	.421	.124	.561
3	.575	.547	.608	.238	.943
4	.676	.654	.703	.335	1.221
5	.741	.720	.762	.415	1.441
6	.781	.761	.801	.475	1.619
7	.814	.795	.830	.527	1.772
8	.836	.820	.851	.570	1.905
9	.852	.838	.866	.600	2.012
10	.868	.853	.879	.633	2.115
12	.889	.877	.899	.680	2.296
14	.904	.894	.913	.718	2.447
16	.917	.907	.924	.749	2.581
18	.926	.918	.933	.774	2.704
20	.934	.926	.940	.795	2.813
25	.947	.941	.952	.832	3.028
30	.956	.950	.960	.856	3.212

bounded by zero and one, note that indices 1, 2, and 3 jump to the upper end of the interval fairly quickly. For these three indices, portfolios with five or more securities have index values, on average, between .720 and .999. The point is that nearly three-fourths of the scale available to measure diversification has been used up in measuring the diversification of portfolios with one to five securities. This leaves approximately one-fourth of the scale to measure differences in diversification of portfolios with more than five securities. This imbalance in the use of the scale appears to be a deficiency overcome by the last two indices. In fact, the fifth index has the advantage of being open-ended on the upper end.

As the purpose of our research is to show that a diversification index could measure the degree of diversification of an unevenly distributed portfolio as well as the number of securities could measure the degree of diversification of an evenly distributed portfolio, we must first show how well the latter measures the degree of diversification. A standard can be established by repeating the E&A study, changing only the range of portfolio sizes. That is, we compute portfolio standard deviations for 60 sets of portfolios, where each set of portfolios range in size from two to 30 securities and each portfolio is equally weighted. We then compute the average portfolio standard deviation for each size portfolio and regress these average portfolio standard deviations against the inverse of the number of securities in the portfolio.

The results of this regression are shown as regression 1 in Table 3. The regression statistics are essentially the same as those obtained by E&A, the difference being that they used ten years of semiannual rates of return data and portfolios

TABLE 3.
Standard Deviations Regressed against Index Values

Regression #	*Constant (t-statistic)*	*Coefficient (t-statistic)*	*Adjusted R^2*	*F-Ratio*	*Root MSE*	*Sample Size*
1 – (1/x;equal weights, avgs.)	4.063	6.132	.980	1,398.91	.090	29
	(170.597)	(37.403)				
2 – (1/x;equal weights)	4.063	6.133	.549	2,123.56	.567	1740
	(210.141)	(46.082)				
3-DI(1)	9.880	–5.765	.548	2,110.69	.688	1740
	(89.917)	(–45.942)				
4-DI(2)	9.590	–5.505	.544	2,077.2	.691	1740
	(91.791)	(–45.576)				
5-DI(3)	10.337	–6.213	.544	2,075.8	.691	1740
	(85.637)	(–45.561)				
6-DI(4)	7.445	–3.783	.511	1,822.0	.715	1740
	(119.534)	(–42.685)				
7-DI(5)	7.306	–1.011	.479	1,602.3	.739	1740
	(116.133)	(–40.029)				

Note: This table is based on 60 repetitions where each repetition has portfolios containing 2 to 30 securities. The data for each firm is based on 240 monthly rates of return.

of one to 40 securities, whereas we use 20 years of monthly data and portfolios of two to 30 securities. The adjusted R-square is .980 and the sample size is 29.

Although comparable to that reported by E&A, regression 1 is not yet a valid standard for evaluating our indices. The reason is that our diversification indices are continuous variables, but the portfolio sizes used in the E&A study are discrete variables. Furthermore, the standard deviations used as the dependent variable in regression 1 are arithmetic averages covering 60 observations. A valid comparison with our indices could be made if we rerun the E&A regression using the individual portfolio data rather than portfolio averages. Thus, the sample size increases from 29 to 1,740. Naturally, the regression coefficients are the same, but there is a substantial decline in the adjusted R-square to .549 as reported in regression 2 in Table 3. So the relevant question becomes, can any of the diversification indices for the unevenly distributed portfolios produce an adjusted R-square as good as the R^2 in regression 2.

Regressions 3 through 7 reported in Table 3 are regressions of the portfolio standard deviations against the diversification indices for each of the portfolios. DI(l) (the complement of the Herfindahl), provides virtually the same explanatory power as the E&A study with an adjusted R-square of .548. DI(2) and DI(3) appear as good as DI(1) with adjusted R-squares of .544 for each. DI(4) and DI(5) are clearly weaker indices with adjusted R-squares of .511 and .479.[11]

Based on the results in Table 3, we conclude that at least three indices exist which explain the degree of diversification of unevenly distributed portfolios as well as portfolio size does for evenly distributed portfolios. Although we believe any of the three could be used to measure portfolio diversification, we recommend the complement of the Herfindahl (DI(1)) because we find it to be the best known of the various measures and it is a simpler mathematical computation than the other two.

V. STANDARDS OF DIVERSIFICATION

The Use of the Classical Works to Define a Standard

In this section, we seek to clarify the explicit or implicit criteria used by E&A and F&L to determine the minimum number of securities in an evenly-weighted portfolio necessary to achieve adequate diversification. We then will apply these same criteria to ascertain the minimum value of index DI(1) that is necessary to achieve adequate diversification.

As stated in Section II, E&A conclude that a ten-security portfolio provides adequate diversification. For one and 10 security portfolios, the mean standard deviations using their equation would be .20535 and .127725. The reported standard deviation for the portfolio containing all the securities in the population (the surrogate market portfolio) was .1166. Thus, E&A imply that a reasonably diversified portfolio size provides a reduction in unsystematic risk of 87.5 percent ((.20535 – .127725)/(.20535 – .1166) = .875). F&L reported that approximately 80 percent of the achievable reduction in dispersion can be attained by holding eight stocks. This 80 percent is quite close to the 87.5 percent reduction in unsystematic risk indicated by E&A's study. If we accept the criterion for the minimum size portfolio to achieve reasonable diversification is one that on average reduces unsystematic risk by 80 to 87.5 percent, we can compute the diversification index value that corresponds to each of these numbers.

The mean standard deviation for all 483 securities in our sample is 8.77 percent and the standard deviation for the portfolio consisting of all the securities (our surrogate for the market portfolio) is 4.01 percent. An 80 percent reduction in *unsystematic* risk would imply a portfolio with a standard deviation of 4.96 percent ($4.01 + .20 \times (8.77 - 4.01)$). Similarly, an 87.5 percent reduction in unsystematic risk would imply a portfolio with a standard deviation of 4.60 percent ($4.01 + .125 \times (8.77 - 4.01)$). If we use these values as the dependent variable in conjunction with the coefficients from regression 3, we obtain index values of .85 and .91.[12]

Based on the explicit and implicit criteria used by E&A and F&L, we offer as an observation that index values of less than .85 would imply a portfolio was probably not adequately diversified. Portfolios with index values greater than .91 probably are adequately diversified.

Applying the Index Without a Standard

It is our hope that the diversification index identified in this paper becomes a standard component of brokerage account statements. If the cutoff values for diversification are not universally or even popularly agreed to, then it is doubtful our index will come into common use. There is an alternative way to provide investors with the information provided by our index, without actually citing the index itself. The alternative is that in lieu of the index value, investors could be told the diversification equivalent of their portfolio if their portfolio were evenly distributed.[13] For example, we know that an evenly distributed portfolio of eight securities produces an index value of .88. Therefore, if the index for a portfolio rounded off to .88, the investor could be told simply that his diversification is equivalent to an evenly distributed portfolio of eight securities and the investor could decide for himself if this was sufficient diversification.

VI. Summary and Conclusions, Limitations and Extensions

The purpose of this paper has been to explore the two questions of 1) whether a diversification index could measure the degree of diversification of an unevenly distributed portfolio as well as the number of securities does when the holdings are evenly distributed, and 2) if such an index exists, what index value represents a reasonably diversified portfolio.

Variations of five measures of diversification that are frequently used in the industrial organization literature on industry diversification were identified and tested. Three of these five indices were found to be satisfactory measures of diversification. The complement of the Herfindahl index was recommended as the measure of diversification for unevenly distributed portfolios based on its simplicity and wide-spread usage. Based on the explicit and implicit criteria used by E&A and F&L, we concluded that index values of less than .85 imply that a portfolio was *probably* not adequately diversified. Portfolios with index values greater than .91 were *probably* adequately diversified. We also indicated that the index could be used to define an evenly distributed portfolio equivalency. Although this research does not provide information for the important question of whether a portfolio lies on the efficient frontier, it does provide information important to all investors.

There are obvious deficiencies to the index as proposed. The index and standards presented herein are for portfolios consisting entirely of common stock. Future work will need to focus on the impact of such non-stock holdings as options, futures contracts, bonds, Treasury securities, money market instruments, and various types of mutual funds. In addition, the impact of buying on margin will also have to be considered. Nonetheless, we think the index proposed here is a good first step to providing a critical tool for investors, but we acknowledge many more steps are necessary for practical application.

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Notes

1. For an excellent review some of the literature on diversification, the reader is referred to Alexander and Francis (1986), pp. 193–202.
2. Related to concentration ratios are inequality measures. We considered measures of inequality such as the Lorenz-curve-based Gini index, but found them to be internally inconsistent for our purposes and therefore do not include them here.
3. See, e.g., Polakoff, et al., (1981, p. 684); or Lovett, (1988, pp. 197–202).
4. The use of the complement to distinguish diversification indices from concentration indices is common in the industrial organization literature. See Acar, et al.(unpublished).
5. Hart (1971, p. 77) shows that the Rosenbluth index is a modified version of the Gini coefficient. Hall and Tideman (1967) have also proposed the same measure as Rosenbluth.
6. The decision to use a maximum portfolio size of 30 securities is somewhat arbitrary. Our early work on this paper used a maximum portfolio size of 40 securities. The 10 additional securities added at least a day to the time it took our computer program to run. More importantly, the 10 additional portfolios had the effect of making the empirical tests of our results appear stronger than they really were because our indices clearly indicate there is no significant difference in the diversification of 30- and 40-security portfolios.
7. At the time the data was collected for this research, more recent tapes were available but they included the year 1987. We wanted to avoid any potential problems associated with the dramatic market movements of that year. Although we believe our results are independent of the time period used, we are doing follow-up tests with a later time period.
8. This method of selection obviously creates a survivorship bias. As we are not aware of any evidence that nonsurvivor firms have variances and covariances different from survivor firms, it is not felt that this bias is of any significance.
9. These formulas are the same as those used by E&A except that we have adjusted them to allow for unequal weights. E&A also implicitly reweight their portfolios each month to the original (even) distribution.
10. To generate the random numbers, we used the RANDOMIZE command in BASICA. The seed number was one for the first portfolio, and was augmented by one for each successive set of weights computed.
11. At the suggestion of an earlier referee, we twice reran the last five regressions. First we added as a second independent variable the number of securities in each portfolio, and then we added the inverse of the number of securities. The change in adjusted R^2's was negligible.
12. Specifically, let $4.96 = 9.88 - 5.76 * DI(1)$, then $DI(1) = .85$. Also, let $4.60 = 9.88 - 5.76 * DI(1)$, then $DI(1) = .91$.
13. Our thanks to Lew Mandell for this suggestion.

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