FINANCIAL SERVICES REVIEW, 3(1): 59–73
ISSN: 1057-0810

The Individual's Tax-Exempt Bond Portfolio Decision Under Income Uncertainty

Amy v. Puelz

In this article, an individual's tax-exempt bond portfolio decision is investigated. A model capturing the relationship between income uncertainty and optimal portfolio choice is defined when an individual decision-maker has the opportunity to hold higher yielding private-activity bonds. The findings in this article show that in most cases risk-averse individuals will maximize the expected utility of after-tax income by holding a large proportion of private-activity bonds in their portfolio even under income uncertainty and the risk of a minimum tax liability. Those individuals who would benefit from holding private-activity bonds in a tax-exempt portfolio are identified and the magnitude of the benefit is quantified.

I. INTRODUCTION

The Tax Reform Act of 1986 created two general classes of municipal investments, essential-purpose and nonessential-purpose or private-activity bonds. Essential-purpose bonds yield interest income that is exempt from any form of federal income tax. However, interest income on private-activity bonds issued after August 7, 1986 is classified by the Internal Revenue Service as preference income and is exempt from federal income tax only if an individual is not subject to the alternative minimum tax.[1] The alternative minimum tax (or minimum tax) is a flat tax rate applied to minimum taxable income which is the total of regular taxable income plus preference income and adjustments. An individual is subject to the minimum tax if the minimum tax liability exceeds the regular tax liability.[2] The probability of an individual being subject to minimum tax increased with 1990 tax reform when the minimum tax rate was increased from 21% to 24%.

The separation of municipal bonds into two distinct classes alters individual decision-making regarding the tax-exempt portfolio because of the relatively high yields and the uncertain tax treatment of private-activity bonds. In this article, the

Amy v. Puelz • Department of Management Information Sciences, Edwin L. Cox School of Business, Southern Methodist University, Dallas, Texas, 75275.

tax-exempt portfolio decision under income uncertainty is examined given the expanded choice set that includes private-activity bonds. The tax-exempt bond literature has focused on both sides of the market. Supply-side research has dealt with the creation and issuance of municipal debt and includes such topics as structuring bond issues (Cohen & Hammer, 1966; Puelz & Lee, 1992); insurance and signalling (Kidwell, Sorensen, & Wachowicz, 1987; Hsueh & Liu, 1990; Puelz, 1991); and underwriter bids (Bierwag, 1976; Braswell, Sumners, 1982; Nauss, 1987). Demand-side research has dealt with topics such as market segmentation (Fischer, 1980; Kidwell & Koch, 1983); market inefficiencies (Speer, 1987; Kochin & Parks, 1988); risk premiums (McInish, 1980; Gehrlein & McInish, 1985); and the portfolio decision relative to tax-exempt bonds. It is this last topic that is addressed in this article. Several authors have written about the descriptive characteristics of private-activity bonds and the minimum tax (Petersen, 1987, 1988; Aalberts & Utley, 1988; Brown, 1988; Porterba, 1989; Bettner, 1990; Day, 1991). However, there has been no rigorous analysis of the portfolio decision given the new environment of post-1986 tax reform, although the portfolio decision has been addressed in the context of income certainty (Puelz & Puelz, 1991). This article provides a more general approach by examining individual decision-making under uncertain income and uncertain after-tax returns.[3]

The institutional literature addressing the allocation of private-activity bonds to a portfolio suggests that an individual who is uncertain as to their tax status should reduce or eliminate their holdings of private-activity bonds to avoid triggering the minimum tax (Brown, 1988; Bettner, 1990; Hoffman, Smith, Willis, & Raabe, 1991). The results in this article show, contrary to the conventional wisdom, that in many cases a risk-averse individual will maximize the expected utility of after-tax income by holding private-activity bonds in their portfolio even under income uncertainty and the risk of a minimum tax liability. Through a simulation procedure, those individuals who would benefit from holding private-activity bonds in a tax-exempt portfolio are identified and the magnitude of the benefit quantified. In the next section of the article the model of individual portfolio choice is developed. This is followed by a comparison of the utility maximizing portfolio allocation derived from the model presented in this article and the naive portfolio allocation of 100% to essential-purpose bonds for different individual and market characteristics. Finally, the relationship between income uncertainty and optimal portfolio allocation is explored for different income levels, bond yield differentials, and portfolio sizes.

THE MODEL

As a starting point consider the simple case of an individual with certain income who wants to select the bond portfolio that maximizes after-tax income. The decision considered is the proportion of investable wealth to allocate to private-activity bonds, with the remainder of investable wealth allocated to essential-pur-

pose bonds. To facilitate the comparison, the bonds mature in one time period, are not sold short, and in all other aspects are identical except for their tax treatment and yields.[4] The allocation in the certain income case is straight forward. An individual who is not subject to the minimum tax will allocate 100% to private-activity bonds because of their associated higher yields. An individual who is subject to the minimum tax because of preference income (other than private-activity bond income) or adjustments will allocate 100% to essential-purpose bonds.[5] However, an individual whose preference income earned on private-activity bonds could trigger a minimum tax liability will allocate a proportion to private-activity bonds such that the minimum tax liability equals the regular tax liability.[6]

Now consider a more general model that specifies an individual's uncertain after-tax income, $\tilde{X}$, a function of uncertain pre-tax income and uncertain tax liability, as

$$\tilde{X} = \tilde{N} + (1-\alpha)BR_e + \alpha BR_p + p\tilde{N}$$

$$- MAX[\tau\{\tilde{N}\}(\tilde{N} - \varepsilon_r\{\tilde{N}\}, T_a(\tilde{N} + \alpha BR_p + p\tilde{N} - \varepsilon_a\{\tilde{N}\})] \quad (1)$$

where,

$\tilde{N}$ = uncertain regular taxable adjusted gross income (dollars),
α = proportion of the tax-exempt portfolio allocated to private-activity bonds,
$1-\alpha$ = proportion of the tax-exempt portfolio allocated to essential-purpose bonds,
B = wealth allocated to tax-exempt bond portfolio (dollars)[7],
R_e = yield on essential-purpose bonds,
R_p = yield on private-activity bonds,
T_a = alternative minimum tax rate,
$\tau\{\tilde{N}\}$ = regular tax rate (a function of $\tilde{N}$),
$\varepsilon_r\{\tilde{N}\}$ = regular taxable income exemption (a function of $\tilde{N}$),
$\varepsilon_a\{\tilde{N}\}$ = alternative minimum taxable income exemption (a function of $\tilde{N}$),
p = proportion of income from preference items other that private-activity bonds and minimum taxable income adjustments.

The first term, $\tilde{N} + (1-\alpha)BR_e + \alpha BR_p + p\tilde{N}$, represents pre-tax income. The maximand function is the tax liability. Within this maximand an individual pays the maximum of the regular tax liability, the left hand side, or the minimum tax liability, the right hand side. The exemption and the tax functions are described in Appendix A.

The optimal allocation to private-activity bonds under income uncertainty is a function of the relationship of the after-tax returns, and the individual's financial characteristics and risk preferences. An expected utility maximizing individual will

choose the proportion of the portfolio allocated to private-activity bonds, α, that satisfies the first-order condition $\partial E[U[X]]/\partial\alpha = 0$. Since (1) is a non-differentiable, non-continuous function, simulation is employed to derive the optimal expected utility maximizing allocation to private-activity bonds, α^*.

COMPARATIVE ANALYSIS

In this section, the model derived in the previous section is simulated to derive optimal utility maximizing portfolios containing private-activity bonds and essential-purpose bonds. Model parameters are varied over reasonable ranges and the optimal portfolio strategies are compared to naive strategies of portfolios containing only essential-purpose bonds. After-tax income for this comparative analysis is calculated under the assumption of joint-filing status by a married couple. Household income, for which one individual acts as decision-maker, is assumed to follow a Pareto distribution (Quandt, 1966). Income uncertainty is measured by the dispersion factor (*DF*). Income certainty corresponds to a *DF* of one and higher income uncertainty corresponds to higher values of *DF*.[8] In addition, risk preferences of the individual decision-maker are characterized by a function displaying decreasing absolute risk aversion, $U\{X\} = \text{Log}\{X\}$. Reasonable ranges for p and B were determined to be from 0.15 to 0.25 and from 0.5 to 1.5 respectively. These ranges are based on alternative minimum tax computations from sample income tax returns (Day, 1991, p. 22). The simulation steps are presented in detail in Appendix A.

In Table 1 the portfolio decisions are listed when income uncertainty is relatively low (*DF* is 1.05). The first two columns of numbers under each income category indicate the expected utility maximizing percentage of the portfolio allocated to private-activity bonds. The next two columns of numbers in each income category are the estimated mean difference between the after-tax income when the combined utility maximizing portfolio is selected (I_c) and when the pure essential-purpose portfolio is selected (I_e). The numbers not in parenthesis are those derived when the spread between private-activity and essential-purpose bonds is 20 basis points and those numbers in parenthesis are for when the spread is 70 basis points.[9] In all cases where the estimated mean is reported, the paired t-test of the alternative hypothesis H_a: $I_c - I_e > 0$ was significant at the 0.001 level.

In almost all cases, except when p and B are both relatively high, individuals with household median income of $150,000 allocate close to 100% to private-activity bonds. This is because the risk of minimum tax is low and therefore there is a high likelihood of realizing the additional return on private-activity bonds. As median income increases, the allocation to private-activity bonds falls rapidly with increasing levels of p and B because of the higher probability of a minimum tax liability. However, when p is 20% or less, the utility maximizing portfolio contains a portion of private-activity bonds. In addition, private-activity bonds are only eliminated from the utility maximizing portfolio for median incomes and p levels

TABLE 1.

A Comparison of the Expected After-Tax Incomes of the Combined Expected Utility Maximizing Portfolio (I_c) and the Portfolio Containing Only Essential-Purpose Bonds (I_e) When the Dispersion Factor of Income is Set at 1.05.

p, Proportion of Income from Preference Items (Other than Private-activity Bonds) and Adjustments	*B, Tax-exempt Bond Portfolio Size (% of Median Income)*	*Median Income = 150,000*		*Median Income = 200,000*		*Median Income = 300,000*	
		α^*, *Optimal Allocation to Private-activity Bonds (%)*	*Estimated Mean* $I_c - I_e$	α^*, *Optimal Allocation to Private-activity Bonds (%)*	*Estimated Mean* $I_c - I_e$	α^*, *Optimal Allocation to Private-activity Bonds (%)*	*Estimated Mean* $I_c - I_e$
15%	50%	100 (100)	150 (525)	100 (100)	200 (700)	100 (100)	300 (1050)
	100%	100 (100)	300 (1050)	100 (100)	400 (1400)	98 (92)	588 (1923)
	150%	100 (100)	450 (1575)	82 (79)	479 (1614)	65 (61)	585 (1922)
20%	50%	100 (100)	150 (525)	92 (99)	169 (619)	53 (50)	159 (518)
	100%	100 (100)	300 (1050)	46 (50)	165 (621)	26 (25)	156 (518)
	150%	83 (82)	361 (1227)	31 (33)	163 (619)	17 (17)	153 (516)
25%	50%	95 (100)	125 (483)	0 (0)	— (—)	0 (0)	— (—)
	100%	47 (54)	119 (479)	0 (0)	— (—)	0 (0)	— (—)
	150%	32 (36)	120 (486)	0 (0)	— (—)	0 (0)	— (—)

Note: The numbers not in parentheses are when the yield differential is 20 basis points and those numbers in parentheses are when the yield differential is 70 basis points. The essential-purpose bond yield is 6.73%. In all cases where the estimated mean is reported the p-value from the paired t-test of the alternative hypothesis H_a: $I_c - I_e > 0$ was less than 0.001. The bonds are identical with the exception of yields and tax treatment. After-tax income is calculated assuming a joint filing status for a married couple. The risk preferences of the individual decision-maker are characterized by decreasing absolute risk aversion, $U\{X\} = \text{LOG}\{X\}$.

TABLE 2.

A Comparison of the Expected After-Tax Incomes of the Combined Expected Utility Maximizing Portfolio (I_c) and the Portfolio Containing Only Essential-purpose Bonds (I_e) When the Dispersion Factor of Income is Set at 1.35.

p, Proportion of Income from Preference Items (Other than Private-activity Bonds) and Adjustments	*B, Tax-exempt Bond Portfolio Size (% of Median Income)*	*Median Income = 150,000*		*Median Income = 200,000*		*Median Income = 300,000*	
		α^*, *Optimal Allocation to Private-activity Bonds (%)*	*Estimated Mean* $I_c - I_e$	α^*, *Optimal Allocation to Private-activity Bonds (%)*	*Estimated Mean* $I_c - I_e$	α^*, *Optimal Allocation to Private-activity Bonds (%)*	*Estimated Mean* $I_c - I_e$
15%	50%	100 (100)	150 (525)	100 (100)	200 (700)	100 (100)	300 (1050)
	100%	100 (100)	300 (1050)	100 (100)	400 (1400)	71 (66)	415 (1375)
	150%	100 (100)	450 (1575)	74 (80)	416 (1485)	47 (44)	413 (1371)
20%	50%	100 (100)	150 (525)	46 (95)	58 (346)	21 (23)	50 (219)
	100%	72 (100)	105 (1050)	23 (49)	49 (347)	11 (12)	48 (211)
	150%	47 (75)	83 (682)	15 (33)	56 (360)	7 (8)	50 (218)
25%	50%	0 (79)	— (17)	0 (0)	— (—)	0 (0)	— (—)
	100%	0 (40)	— (68)	0 (0)	— (—)	0 (0)	— (—)
	150%	0 (26)	— (8)	0 (0)	— (—)	0 (0)	— (—)

Note: The numbers not in parentheses are when the yield differential is 20 basis points and those numbers in parentheses are when the yield differential is 70 basis points. The essential-purpose bond yield is 6.73%. In all cases where the estimated mean is reported the p-value from the paired t-test of the alternative hypothesis H_a: $I_c - I_e > 0$ was less than 0.001. The bonds are identical with the exception of yields and tax treatment. After-tax income is calculated assuming a joint filing status for a married couple. The risk preferences of the individual decision-maker are characterized by decreasing absolute risk aversion, $U\{X\} = \text{LOG}\{X\}$.

above $200,000 and 25% respectively. Comparing the decision when the spread between private-activity and essential-purpose bonds is 20 basis points to the decision when the spread is 70 basis points there is relatively little change in α^* across all income levels. Individuals with household median incomes of $150,000 and 25% of income from preference items slightly increase their holding of private-activity bonds as the yield spread increases because they can capture the additional return without significantly increasing their risk of a minimum tax liability. In contrast, individuals with a greater risk of a minimum tax liability do not shift to private-activity bonds as the yield spread increases.

The next set of comparisons is identical to those presented in Table 1, except the uncertainty associated with income, *DF*, is increased to 1.35 from 1.05. The results are presented in Table 2. In all cases where estimated means are reported, the paired *t*-test of the alternative hypothesis H_a: $I_c - I_e > 0$ is significant at the 0.001 level.

The effect of greater uncertainty is consistent among all individuals in that private-activity bond holdings are more rapidly eliminated from the portfolio with increasing probability of minimum tax (i.e., increasing p and B). However, the only cases where individuals completely eliminate private-activity bonds from their portfolio are when p is 25% or more. Individuals with household median income of $150,000 continue to hold private-activity bonds even at high levels of p and B.

In most instances, the optimal proportion of private-activity bonds held is greater when the yield spread between private-activity and essential-purpose bonds is 70 basis points as opposed to 20 basis points. The fact that in some cases the proportion of private-activity bonds held in the portfolio drops as the yield on these bonds increases is due to the fact that the higher yield results in a greater risk of minimum tax liability. Hence, the risk-averse investor may actually reduce their holding of private-activity as the yield increases if the additional risk is too high. The yield spread is a much more significant factor in the allocation decision when income uncertainty is high.

In summary, this comparative analysis illustrates the relationship between the individual's vulnerability to the minimum tax, household income uncertainty and the market yields on private-activity bonds relative to essential-purpose bonds. Those individuals with household median incomes below $150,000 and those with low levels of preference income (other than private-activity bond income) and/or a small tax-exempt portfolios, should hold a large proportion if not all of their tax-exempt portfolio in private-activity bonds. In addition, the greater the uncertainty of income the greater the impact of market yield spreads on the allocation decision.

Private-Activity Bond Allocation

In this section the effect of uncertainty on the optimal allocation to private activity bonds is presented. As in the previous section, after-tax income is calculated under

the assumptions of joint filing status by a married couple and income following a Pareto distribution. Risk preferences of the individual decision-maker are again characterized by decreasing absolute risk aversion. The allocation decision is presented for different median income levels, different bond yield spreads, and different tax-exempt portfolio sizes.

Optimal Private-activity Bond Allocation Relative to Income

The first set of simulation results compares the optimal expected utility maximizing portfolio for different median income levels. Tax-exempt portfolio size (B) is assumed to be 100% of median income and proportion of income from preference items other than private-activity bond income plus adjustments (p) is 20%. The private-activity bond yield for the comparison is set at 6.93% or 20 basis points greater than the essential-purpose yield of 6.73.[10] Median income is varied from $150,000 to $400,000. The optimal allocations (α^*) are presented in Figure 1.

First consider the optimal portfolio for each individual under household income certainty ($DF = 1$). An individual with household income of $150,000 will not be subject to the minimum tax regardless of the allocation to private-activity bonds and will therefore allocate 100% of the portfolio to private-activity bonds. Individuals with household incomes of $200,000 and $300,000 will allocate a portion of the portfolio to private-activity bonds such that the regular tax liability equals the minimum tax liability. The individual with household income of $300,000 as compared to the individual with household income of $200,000 has a higher effective regular tax rate but also has a significantly higher effective minimum tax rate and therefore allocates a smaller portion (27% as opposed to 57% for incomes of $200,000) of the portfolio to private-activity bonds. The higher effective minimum tax rate is due to the minimum taxable income exemption phaseout that occurs for high income households. For example, a married couple filing a joint return will have a $40,000 minimum taxable income exemption that is phased out at a rate of 25% for every dollar minimum taxable income exceeds $150,000. The $40,000 exemption is phased out completely at an income of $310,000. The individual with household income of $400,000 as compared to the individual household income of $300,000 has a higher effective regular tax rate and virtually the same minimum tax rate and therefore allocates a larger portion of the portfolio (53% as opposed to 27% for the income of $300,000) to private-activity bonds.

Now consider the change in α^* relative to income uncertainty. Individuals with household median income of $150,000 have a very low probability of being subject to the minimum tax and therefore hold 100% private-activity bonds until DF is greater than 1.16. As DF increases above 1.16 the risk of a minimum tax becomes significant enough to induce the individual to reduce their holding of private-activity bonds.

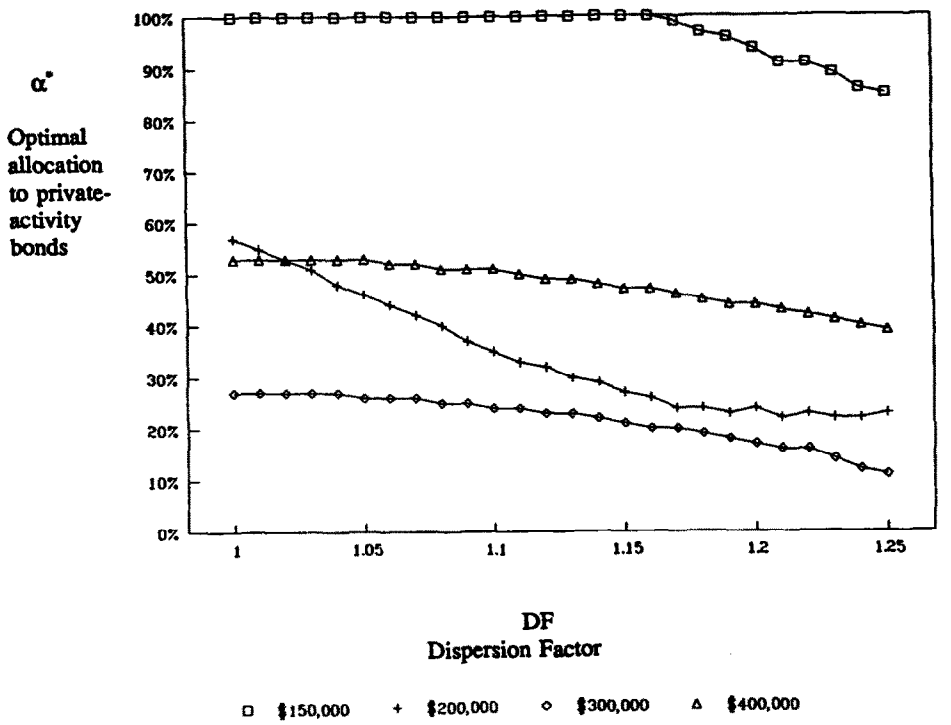

Figure 1. The optimal allocation to private-activity bonds (α^*) relative to the dispersion factor of income (*DF*) for different median incomes.

Note: Income follows a Pareto distribution and after-tax income is calculated assuming a joint filing status for a married couple. The risk preferences of the individual decision-maker are characterized by decreasing absolute risk aversion. Investable wealth (*B*) is 100% of median income. The proportion of income from preference items (other that private-activity bond income) and adjustments (*p*) totals 20%. The bonds mature in one time period, are not sold short, and are in all other aspects identical except for their tax treatment and yields. The yield of the private-activity bond and the essential-purpose bond are 6.93% and 6.73% respectively.

Individuals with household median incomes of $200,000 will experience a higher effective minimum tax rate under higher levels of uncertainty because of the minimum taxable income exemption phaseout that occurs in this example between $150,000 and $310,000. Therefore the optimal holding of private-activity bonds (α^*) decreases with increasing uncertainty (*DF*). The optimal allocation to private-activity bonds relative to uncertainty levels off at high levels of uncertainty because the probability of income falling above the upper limit of the phaseout range increases and the effective minimum tax rate is constant in income above this upper limit. Individuals with household median income levels of both $300,000 and $400,000 will slightly reduce α^* as *DF* increases because at these income levels the effective minimum tax rate is virtually constant in income.

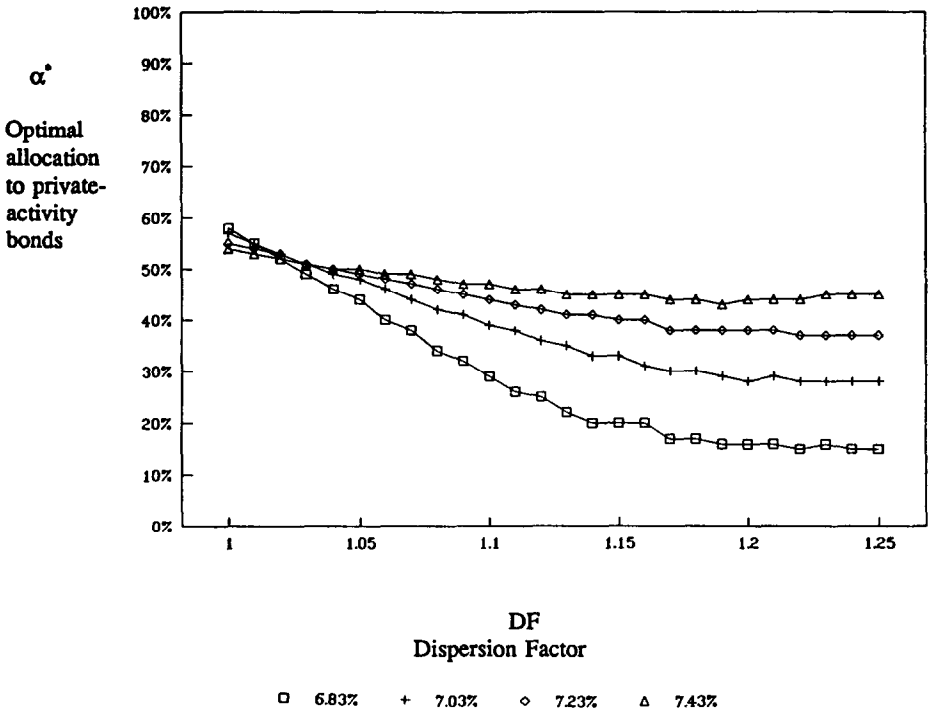

Figure 2. The optimal allocation to private-activity bonds (α^*) relative to the dispersion factor of income (*DF*) for different private-activity bond yields.

Note: Income follows a Pareto distribution and after-tax income is calculated assuming a joint filing status for a married couple. Median income is $200,000. The risk preferences of the individual decision-maker are characterized by decreasing absolute risk aversion. Investable wealth (*B*) is 100% of median income. The proportion of income from preference items (other that private-activity bond income) and adjustments (*p*) totals 20%. The bonds mature in one time period, are not sold short, and are in all other aspects identical except for their tax treatment and yields. The yield of the essential-purpose bond is 6.73%.

To summarize the results presented in Figure 1, individuals with household median income levels below $150,000 will typically maximize expected utility by holding 100% private-activity bonds. However, if income is highly variable (in this example a *DF* of 1.16 for a median income of $150,000) optimal private-activity bond holdings will fall below 100%. Individuals with median incomes near the lower limit of the minimum taxable income exemption phaseout range will reduce private-activity bond holding at a faster rate with income uncertainty than individuals with incomes near or above the upper limit minimum taxable income exemption phaseout range.

This example illustrates, as one might expect, that the portfolio decision is greatly influenced by the median income level. It is interesting to note that only individuals with relatively high income levels combined with high levels of income uncertainty will hold less that one-half of their tax-exempt portfolio in private-activity bonds. Hence, private-activity bonds should not be arbitrarily eliminated from the tax-exempt portfolio if the individual is at risk of being subject to the minimum tax.

Optimal Private-activity Allocation Relative to Bond Yields

The relationship between private-activity bond yields and essential-purpose bond yields is dependent on the characteristics of the bonds being compared. For purposes of illustrating the effect of the yield differential on the portfolio allocation decision the essential-purpose bond portfolio yield is held constant at 6.73% and the private-activity bond portfolio yield is varied from a low level to a high level. The assumptions from the previous example hold except private-activity bond yield is varied from 6.83% to 7.43%, and median household income is $200,000. The simulation results are presented in Figure 2.

When the private-activity yield is relatively low at 6.83%, α^* is 58% under income certainty ($DF = 1$). As the yield on private-activity bonds increases the individual under household income certainty holds a smaller proportion of private-activity bonds in the portfolio. Although this may seem counter-intuitive, α^* is lower for higher yields under certainty in order for the portfolio to satisfy the after-tax maximizing condition that the regular tax liability equals the minimum tax liability. However, $\partial\alpha^*/\partial DF$ is lower for lower private-activity yields. This means α^* falls at a more rapid rate with increasing uncertainty the lower the yield on private-activity bonds. When the yield on private-activity bonds is high relative to essential-purpose bonds, in this example 7.43%, the proportion of private-activity bonds held in the portfolio changes very little with uncertainty.

Optimal Private-activity Allocation Relative to the Percentage Allocated to Tax-exempt Bond Portfolio

The final set of simulations examines the allocation decision relative to the size of the tax-exempt bond portfolio. The amount invested in the tax-exempt bond portfolio will influence the allocation decision in that the larger the portfolio, all else equal, the greater the probability the individual's household will be subject to the minimum tax.[11] The assumptions made are those described in the first two examples except bond portfolio size, B, is varied from 50% to 150% of median income which is set at $200,000. The results are presented in Figure 3.

As one would expect, the larger the size of the tax-exempt portfolio the smaller the allocation to private-activity bonds. However, $\partial\alpha^*/\partial DF$ is lower for low levels of B. When B is 150% of median income, α^* is almost constant in uncertainty.

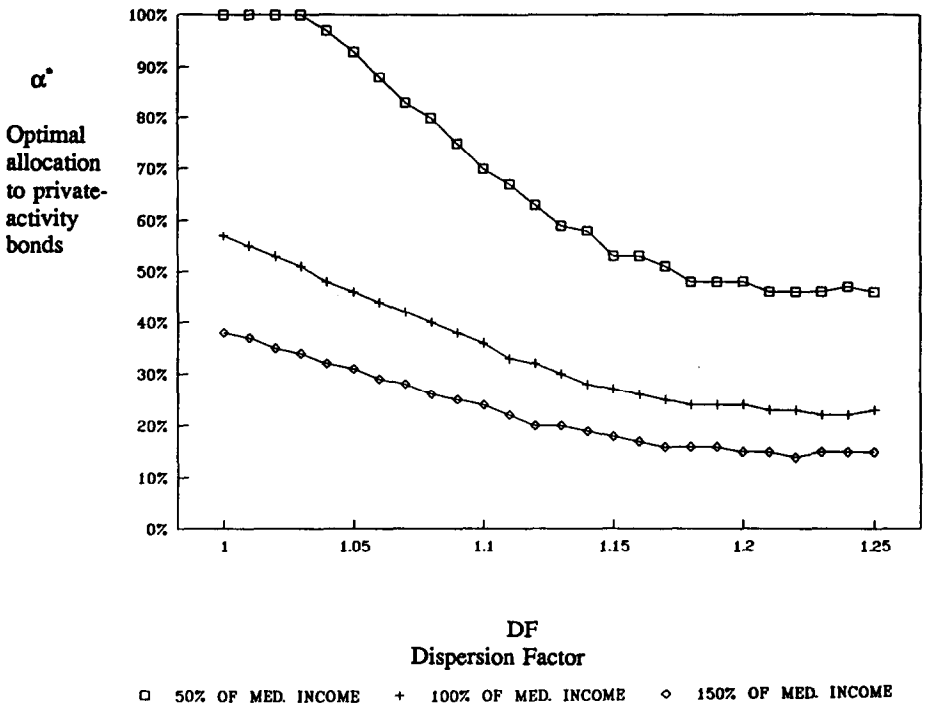

Figure 3. The optimal allocation to private-activity bonds (α^*) relative to the dispersion factor of income (*DF*) for different allocations to the tax-exempt bond portfolio size.
Note: Income follows a Pareto distribution and after-tax income is calculated assuming a joint filing status for a married couple. Median income is $200,000. The risk preferences of the individual decision-maker are characterized by decreasing absolute risk aversion. The proportion of income from preference items (other that private-activity bond income) and adjustments (*p*) totals 20%. The bonds mature in one time period, are not sold short, and are in all other aspects identical except for their tax treatment and yields. The yield of the private-activity bond and the essential-purpose bond are 6.93% and 6.73% respectively.

CONCLUSION

In this article, the tax-exempt bond portfolio decision is explored when an individual has the option of purchasing private-activity bonds. It is shown that the introduction of private-activity bonds affects bond portfolio decision-making when an individual's household is subject to uncertain taxable income that may result in a minimum tax liability. The results in this article show that many individuals will maximize the expected utility of after-tax income by holding a large portion of private-activity bonds in their tax-exempt portfolio even when faced with the risk of a minimum tax liability.

When applied in a portfolio planning framework, significantly greater after-tax income may be realized if private-activity bonds are included in the tax-exempt portfolio. Through simulation, the magnitude of the benefit derived from holding

private-activity bonds is quantified relative to individual household and market characteristics. Median income level, yield differential, and tax-exempt bond portfolio size are all shown to have a significant impact on the portfolio decision relative to income uncertainty.

APPENDIX A

Simulation Steps

The simulation Steps 1 through 3 are repeated 10,000 times. The α that yields the greatest average utility is selected as the optimal α^*.

STEPS:

1. Income (N) is generated by approximating a Pareto distribution. This is accomplished by generating a lognormal random variable with a median (M) and dispersion factor (DF) and setting all values less than the mode of the lognormal variable equal to the mode (Sachs, 1982. p. 111).
2. After-tax income (X) for all $\alpha = 0.0$ to 1.0 (in steps of 0.01) is derived by

$$X = N + (1 - \alpha)BR_e + \alpha BR_p + pN$$

$$-MAX[\tau\{N - \varepsilon_r\{N\}\}), T_a(N + \alpha BR_p + pN - \varepsilon_a\{N\})] \quad \text{(A.1)}$$

where,

$\varepsilon_a\{N\}$ is \$40,000 if N is less than \$150,000 or the maximum of \$0 or \$40,000 minus 25% of the difference between the minimum taxable income and \$150,000,

$\varepsilon_r\{N\}$ is \$5300 reduced by 2% for each \$2500 (or fraction of) that adjusted gross income exceeds \$150,000,

$\tau\{.\}$ follows the 1991 federal income tax schedule *YI* for a married couple filing a joint return.

3. The utility of each after-tax income is set equal to

$$U\{X\} = \text{LOG}\{X\}. \quad \text{(A.2)}$$

NOTES

1. Some private-activity bonds are qualified as tax-exempt under the tax code section 501(c)(3).
2. For a detailed discussion of private-activity bonds the reader is referred to Brown (1988), Petersen (1988), Bettner (1990), or Day (1991).
3. Piros (1987) provides a model of individual choice under uncertainty relative to taxable and tax-exempt bonds. Although this article focuses on essential-purpose and private-activity bonds, the analysis is similar in that uncertain income results in uncertain tax treatment. However, unlike the analysis by Piros, the minimum tax function is noncontinuous necessitating the use

of simulation. The security portfolio choice decision given various sources of uncertainty has been extensively addressed in the literature (i.e., Kwan & Yip (1987), Kwan (1988), Chamberlain & Cheung (1990)).

4. This simplifying comparison allows the focus of this article to be on how the allocation to private-activity bonds affects individual utility. It is not the purpose of this article to address allocation within private-activity bonds or, for that matter, within essential-purpose bonds. However, a simulation approach similar to this could be employed as a decision-making tool to address the time dependent portfolio allocation problem.
5. The relationship of after-tax returns in an efficient market where individual income is certain is

$$R_p > R_e > (1 - T_a)R_p$$

Where R_e is the yield on essential-purpose bonds, R_p is the yield on private-activity bonds, and T_a is the minimum tax rate. Refer to Puelz and Puelz (1991) for a detailed discussion of the relationship of after-tax returns given income certainty.

6. This optimality condition under certainty is illustrated in Puelz and Puelz (1991).
7. It is assumed that the investor has made all other portfolio decisions, e.g., the stock portfolio, so the only investment under consideration is the net investable wealth in the tax-exempt portfolio.
8. In the Pareto distribution the income range from median income divided by *DF* to median income multiplied by *DF* contains 68 percent of the income distribution. For example, if an individual's median income and dispersion factor are $100,000 and 1.5 respectively then the probability of income falling between $66,667 and $150,000 is 0.68.
9. The average private-activity and essential-purpose bond yields on insured bonds offered in the *Blue List* September 1, 1989 were 6.93% and 6.73% respectively. Private-activity bond yields have in the past been as much as 70 basis points higher than comparable essential-purpose bonds (Brown, 1988).
10. See Note 9.
11. These same basic results are found when p is varied.

REFERENCES

Aalberts, R. J., and J. C. Utley. 1988. "The Tax-Free Municipal Bond: A Security in Decline," *Journal of Taxation and Investments*, 6: 267–285.

Bettner, J. 1990. "More People Face the Minimum-Tax Formula," *Wall Street Journal*, October 29.

Bierwag, G. O. 1976. "Optimal TIC Bids on Serial Bond Issues," *Management Science*, 22: 1175–1185.

Braswell, R. C., and D.L. Sumners. 1982. "An Analysis of the TIC Criterion for Accepting Tax-Exempt Bond Bids Under Certainty," *Decision Sciences*, 13: 88–100.

Brown, D. 1988. "Impact of the Tax Reform Act of 1986 on Municipal Bond Yields," *Municipal Finance Journal*, 9: 111–146.

Chamberlain, T. W., and C. S. Cheung. 1990. "Optimal Portfolio Selection Using the General Multi-Index Model: A Stable Paretian Framework," *Decision Sciences*, 21: 563–571.

Cohen, K. J., and F. S. Hammer. 1966. "Optimal Coupon Schedules for Municipal Bonds." *Management Science*, 3: 161–166.

Day, C. 1991. "Individual Income Tax Rates, 1987." *Statistics of Income*, 11: 13–25.

Fischer, P. J. 1980. "On the Extent of Segmentation in the Municipal Securities Market," *Journal of Money Credit and Banking*, 12: 71–83.

Gehrlein, W. V., and T. H. McInish. 1985. "Cyclical Variability of Bond Risk Premia," *Journal of Banking and Finance*, 9: 157–165.

Hoffman, W. H., J. E. Smith, E. W. Willis, and W. A. Raabe. 1991. *Individual Income Taxes, 1992 edition*, St. Paul: West Publishing Company.

Hsueh, L. P., and Y. A. Liu. 1990. "The Effectiveness of Debt Insurance as a Valid Signal of Bond Quality," *Journal of Risk and Insurance*, 57: 691–700.

Kidwell, D. S., and T. W. Koch. 1983. "Market Segmentation and the Term Structure of Interest Rates," *Journal of Money Credit and Banking*, 15: 40–55.

Kidwell, D. S., E. H. Sorensen, and J. M. Wachowicz. 1987. "Estimating the Benefits of Debt Insurance: The Case of Municipal Bonds," *Journal of Financial and Quantitative Analysis*, 22: 299–313.

Kochin, L. A., and R. W. Parks. 1988. "Was the Tax-Exempt Bond Market Inefficient or Were Future Expected Tax Rates Negative?" *Journal of Finance*, 43: 913–931.

Kwan, C. C. Y., and P. C. Yip. 1987. "Optimal Portfolio Selection with Upper Bounds for Individual Securities," *Decision Sciences*, 18: 505–523.

Kwan, C. C. Y. 1988. "Optimal Portfolio Selection and Cutoff Rates of Security Performance: A Multi-Index Case," *Decision Sciences*, 19: 682–699.

McInish, T. H. 1980. "The Determinants of Municipal Bond Risk Premiums by Maturity," *Journal of Financial Research*, 3: 129–138.

Nauss, R. M. 1987. "True Interest Cost in Municipal Bond Bidding: An Integer Programming Approach," *Management Science*, 32: 870–877.

Petersen, J. E. 1987. "Examining the Impacts of the 1986 Tax Reform Act on the Municipal Securities Market" *National Tax Journal*, 40: 393–402.

Petersen, J. E. 1988. "The Municipal Bond Market in a Changing Economy," *Public Budgeting and Finance*, 8: 22–34.

Piros, C.D. 1987. "Taxable vs. Tax-Exempt Bonds: A Note on the Effect of Uncertain Taxable Income," *Journal of Finance*, 42: 447–451.

Porterba, J. M. 1989. "Tax Reform and the Market for Tax-Exempt Debt." *Regional Science and Urban Economics*, 19: 537–562.

Puelz, A. v., and R. Puelz. 1991. "The Effect of Alternative Minimum Tax Bonds on Optimal Bond Portfolio Choice," *Proceedings of the 1991 Annual Meeting of the Decision Sciences Institute*, 1: 138–140.

Puelz, A. v. 1991. "Call Provisions and the Cost Effectiveness of Debt Insurance," *Municipal Finance Journal*, 12: 23–34.

Puelz, A. v., and S. M. Lee. 1992. "A Multiple-Objective Programming Model for Structuring Tax-exempt Serial Revenue Bonds," *Management Science*, 38: 1186–1200.

Quandt, R. E. 1966. "Old and New Methods of Estimation and the Pareto Distribution," *Metrika*, 10: 55–82.

Sachs, L. 1982. *Applied Statistics: A Handbook of Techniques*. New York. Springer-Verlag.

Speer, P. D. 1987. "Toward an Efficient Market in New Municipal Bonds," *Municipal Finance Journal*, 8: 329–331.