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Tax Savings Opportunities in Estate Freeze Transactions: An Application of the Black Scholes Model

James R. Hamill
Joel S. Sternberg

Transfer tax valuation rules for interests in family businesses include a minimum value to reflect the option value of "junior" equity interests transferred to family members. We examine the option features of the junior interest and use the Black-Scholes model to identify situations in which an estate planner could structure a plan of ownership succession that results in undervaluation of the transferred interest. The Black-Scholes model may also be used to identify situations in which a lifetime ownership transfer should be avoided because of the minimum value rule.

I. INTRODUCTION

It is well documented that organizations will structure their affairs to minimize costs, both explicit and implicit, and will also restructure in response to changing costs (Wilson, 1991). Prior to the 1987 Tax Act, it was common for closely held business organizations to engage in costly restructuring of ownership (estate valuation "freezes") to minimize future estate taxes resulting from the death of the senior generation owner. In response to perceived abuses, Congress, in 1987, enacted statutory rules that generally eliminated the tax advantages of such restructurings. However, the 1987 changes raised concerns that family-controlled businesses could be forced to admit outside owners to satisfy an estate tax liability and that uncertainty regarding the application of the rules could also severely restrict commonly used ownership forms that were not adopted to save taxes. In response to growing criticism, in 1990 Congress retroactively repealed the 1987 changes and replaced them with a new set of rules.

The 1990 provisions attempt to recognize the call option features of junior equity interests created by estate freeze restructurings. The option value is explicitly recognized by use of a single minimum value that applies to all transferred junior interests. The purpose of this article is to show situations where the minimum value rule is most likely to misstate the

James R. Hamill • Anderson Schools of Management, University of New Mexico, Albuquerque, New Mexico 87131. **Joel S. Sternberg** • Department of Finance, University of Arizona, Tucson, Arizona 85721.

option value of the transferred interest. We use the Option Pricing Model developed by Black and Scholes (1973) to identify fact patterns in which a business recapitalization may still minimize transfer taxes because of improper valuation of the option. Limitations on the use of the Black-Scholes model for this purpose will also be addressed.

II. Description of Estate Freeze Transactions

A significant portion of the net worth of the owner of a closely held business is the value of the business itself,[1] and taxes imposed on the transfer of ownership to younger generation family members can force the sale of business assets or equity interests to satisfy the tax liability. To minimize the effects of transfer taxes, it has been common to incur costs to recapitalize the business ownership. Typically, a family corporation will issue two classes of equity, with voting preferred retained by the older generation and nonvoting common transferred to the younger generation.[2]

The terms of the retained interest are intended to "freeze" its value at an amount approximating the value of the entity at the date of recapitalization. This would typically involve a stated redemption value for the preferred issue that would fix the amount that the estate would receive for the interest.

Rights attached to the retained interest would support an appraisal assigning a large value to the retained interest and a corresponding small value to the transferred interest, although there often was no intent that the rights would ever be exercised.[3] The estate freeze was designed to serve two purposes. First, there would be little or no gift tax attributable to the transfer to the younger generation, because the interest was assumed to have little value.[4] Second, the estate tax value of the retained interest would be deterministic, because all future increases in the value of the corporation would be captured by the transferred interest.

In 1987, Congress adopted valuation rules applicable to freeze transactions that received critical commentary from business owners and tax experts. The Joint Committee on Taxation (JCT) prepared a discussion draft describing problems with the 1987 rules and suggesting that rights retained by the senior generation be valued at zero unless the rights must be exercised within a specified time and at a specified amount.[5] The JCT noted that the interest transferred to the younger generation contained elements of a call option and that the call feature is undervalued by taxpayers. To reflect the option value, the JCT recommended that a minimum value of 20% of the total value of the interests held by the senior generation family member be assigned to the junior interest. The 1990 Tax Act adopted a 10% minimum value rule to explicitly reflect the option feature of the transferred interest.[6]

The remainder of this article evaluates the basis for treating the transferred interest as possessing the characteristics of a call option and analyzes how the option feature could be valued. The statutory minimum value rule is compared to use of the Black-Scholes Option Pricing Model (OPM), and implications for the structure of future freeze transactions are discussed.

III. Call Option Features of Junior Equity Interests

At the death of the senior generation family member following an estate freeze recapitalization, the retained preferred stock would be redeemed by the corporation,[7] "bootstrapping" the junior equity interest holders into complete ownership of the firm. If the value of the firm

declines below the stated redemption price, the junior equity owners cannot be forced to contribute to capital the excess of the redemption price over the value of firm assets to fund the redemption (as equity owners, they can simply walk away from a corporation with assets valued at less than the redemption claim of the preferred stock interest). Thus, the junior equity owners have the right, but not the obligation, to acquire full ownership by payment of a fixed sum at a determinable time. In this way, their ownership interest resembles that of a call option.[8]

The use of a minimum value may not be appropriate to value the call feature of the junior interest. While attractive for the policy reasons of objectivity and simplicity, the minimum value rule may lead to an undervaluation of the option feature of the junior interest. Identifying the situations where the transferred interest may be undervalued can be of assistance to estate planners structuring freeze transactions. We believe that the OPM may be used to develop a representative value of the call feature of the junior interest.

Use of OPM to value the call option feature of junior equity interests relies on the ability to reasonably estimate the model parameters in relation to an estate freeze recapitalization.

The following form of the OPM was used in this article:

$$C = SN(D_1) - Ke^{-rt}N(D_2) \tag{1}$$

where $D_1 = \ln(S/Ke^{-rt}) / \sigma\sqrt{t} + 1/2\sigma\sqrt{t}$

$D_2 = D_1 - \sigma\sqrt{t}$

C = value of call option

S = company value at the date of the recapitalization

N = normal density function

K = strike or exercise price

σ = annual logarithmic standard deviation of company returns

t = time to expiration of option right

r = logarithmic risk-free interest rate

A call option on an asset represents a long position in that asset, together with riskless borrowing because the option holder may earn interest on the strike price until he exercises the option. $N(D_2)$ is the probability that the option will finish in-the-money and therefore be exercised, and this probability increases as the underlying asset value increases. The final term in Equation (1) is the product of the strike price and the probability of the option being exercised, discounted, and represents the effective amount being borrowed at any point in time. $N(D_1)$ is the fractional shares of stock represented by the option at any time and is also known as the hedge ratio or delta of the option. As the stock price rises, changes in the option value more closely match changes in the value of the stock and $N(D_1)$ increases.[9] The first term in Equation (1) is the dollar amount of stock represented by the option at any point in time.

The subsequent sections discuss how the parameters of the OPM could be estimated in a freeze transaction. We then provide an example of the use of OPM to identify situations where a freeze transaction leads to the most favorable or unfavorable tax law valuation. Future estate freezes may be structured to take advantage of situations where the minimum value rule undervalues the call feature of the transferred interest.

A. Value of the Asset Subject to the Option Right

In an estate freeze transaction, the junior equity owners can acquire ownership of the entity by redeeming the senior equity interest. Although valuation of a closely held business is difficult, firm value must otherwise be determined at the time of the recapitalization transaction, and OPM would add no complexity.[10]

B. Exercise Price of the Option Right

The exercise price of the option right is the redemption price of the retained interest of the senior generation family member. Typically, this amount will approximate the value of the firm at the date of recapitalization. In any event, the amount is generally readily determinable by reference to the stated terms of the redemption agreement for the retained interest.

C. Risk-free Rate of Return

The value of the call option is, in part, a function of the interest rate which enters the second term, Ke^{-rt}, of the OPM. This reflects the fact that the holder of a European call option will not exercise the option, nor pay the strike price, until expiration of the option right. Therefore, the higher (lower) is r, the lower (higher) will be the present value of the strike price and the higher (lower) will be the value of the call option. The yield on a U.S. treasury security is typically used as a surrogate for the risk-free rate of return. A treasury yield for an instrument with a term comparable to that of the redemption agreement could be used as the risk-free rate.

D. Variance of Asset Returns and Time to Maturity of the Option

The value of an option is an increasing function of both annual return variance and time. The greater is the annual return variance or the time to maturity of the option, the greater will be the variance of the asset return distribution, $\sigma\sqrt{t}$, over the life of the option. Since a call option is a contingent claim, losses can never exceed the amount paid to acquire the option, regardless of the actual return distribution from the asset. Positive returns, however, occur whenever the value of the asset exceeds the sum of the exercise price and the amount paid to acquire the option. Thus, the option holder benefits from the entire right tail of the distribution of returns, while his losses from returns in the left tail of the distribution are truncated because they are limited to the amount paid to acquire the option. As time to maturity lengthens, the option holder can also collect interest on money that would otherwise have been used to exercise the option right.

The value of a closely held firm, at any point in time, is not supported by equity traded on an active exchange. Firms may rely on use of an appraisal to determine value. However, because appraisals can be very costly, many closely held businesses planning for succession within the family define value by reference to financial statements (that is, book value). Book value and market value are not the same, but the purpose of our analysis is to identify situations in which the transfer tax value of the firm is too low. Book value is accepted for purposes of the transfer tax and is often used by family businesses. If book value is used in a buy-sell agreement, then the variance of the value may be readily determined. If value is to be set by appraisal, the firm must have an appraisal conducted at the time of the initial

recapitalization to support the values assigned to the retained and transferred interest. If the appraiser uses capitalization of earnings, then earnings variance may be an appropriate proxy for annual return variance. The same may be said for use of cash flow valuation or any other approach favored by a competent appraiser. The insights and analysis used by the appraiser at the time of the recapitalization may be used to develop a return variance proxy. The results of use of the OPM may also be readily analyzed for sensitivity to different specifications of the return variance, as shown in our example.

If the senior equity interest must be redeemed by a certain date, clearly that represents the maturity period. Generally, the redemption of the senior interest is expected to occur upon the death of the holder (since the recapitalization is intended to freeze the value of the senior interest for estate tax purposes). In the analysis presented in this article, we use the life expectancy of the senior generation family member as the maturity period. Since the life expectancy (which could also be for joint lives) is the longest period to maturity, the value of the call feature and the taxable transfer at the time the junior equity interest is created will be maximized for a given dividend payout. The maturity assumption could readily be altered for a more specific fact pattern.

IV. A Comparison Of OPM and the Statutory Minimum Value

In this section, we compare the value assigned to a junior equity interest under the new tax valuation rules with that obtained using the Black-Scholes model. The Black-Scholes model will value only the call feature of the junior interest, while the tax law requires valuation of the entire interest. However, in situations where the 10% minimum value rule of the tax law applies, the junior interest is valued strictly as a call option and OPM would provide a relevant comparison.[11]

A comparison of OPM and current tax law valuation is of interest in two situations. First, if the tax law value of the junior interest is determined to be less than 10% of the value of all interests, the minimum value rule is binding and the interest is valued strictly as a call option. OPM can then be used to determine whether the option value is appropriate (either too high or too low) given the terms of the instrument. The second situation occurs if the tax law value is not less than 10%, the minimum value rule is nonbinding, and the interest is valued using new tax law valuation principles without regard to the option feature. OPM can then be used to determine whether the option value exceeds the value of the interest as determined under tax law principles. Since the option value is only part of the total value of the junior interest, an OPM value in excess of tax law value would suggest that the true value may be substantially understated by the tax law. As discussed later, this analysis is limited by the appropriateness of the use of OPM to value the junior interest.

The tax law values the junior interest by subtracting the value of the retained (senior) interest from the total value of the firm.[12] The senior interest is valued by discounting required cash flows at a rate equal to 120% of the "applicable federal rate" (AFR) as determined under section 1274 of the Internal Revenue Code. The AFR is a Treasury security yield for an instrument of similar maturity. The only cash flows that are discounted are those that, under the terms of the instrument, must be paid with certainty (see IRC Section 2701(a)). We will consider three different situations:

1. A preferred stock issue that provides for a cumulative dividend yield equal to 120% of the AFR, and a redemption value equal to the date-of-recapitalization value of the firm, with redemption required at the death of the senior interest holder.
2. A preferred stock issue that provides for a cumulative dividend yield equal to the AFR, and a redemption value equal to the date-of-recapitalization value of the firm, with redemption required at the death of the senior interest holder.
3. A preferred stock issue that provides for no required payment of a dividend, and a redemption value equal to the date-of-recapitalization value of the firm, with redemption required at the death of the senior interest holder.

TABLE 1
Value of Call Option Feature of Junior Equity Interest Using OPM, Expressed as a Percentage of Firm Value
(dividend payout = 120% of AFR)

		Value Determined Using OPM With Log-volatility of:		
Entrepreneur Life Expectancy	*Tax Law Value (%)*	*20% (%)*	*30% (%)*	*40% (%)*
1	10.0	6.6	10.2	13.8
2	10.0	8.2	12.9	17.4
3	10.0	8.9	14.1	19.1
4	10.0	9.1	14.5	19.8
5	10.0	9.1	14.6	19.9
6	10.0	8.9	14.4	19.6
7	10.0	8.6	14.0	19.1
8	10.0	8.3	13.5	18.4
9	10.0	7.9	12.9	17.6
10	10.0	7.5	12.2	16.7
11	10.0	7.0	11.6	15.8
12	10.0	6.6	10.9	14.9
13	10.0	6.2	10.2	14.0
14	10.0	5.8	9.6	13.1
15	10.0	5.4	9.0	12.2
16	10.0	5.0	8.4	11.4
17	10.0	4.7	7.8	10.6
18	10.0	4.3	7.3	9.9
19	10.0	4.0	6.7	9.2
20	10.0	3.7	6.3	8.5
21	10.0	3.4	5.8	7.9
22	10.0	3.2	5.4	7.3
23	10.0	2.9	5.0	6.7
24	10.0	2.7	4.6	6.2
25	10.0	2.5	4.2	5.7
26	10.0	2.3	3.9	5.3
27	10.0	2.1	3.6	4.9
28	10.0	1.9	3.3	4.5
29	10.0	1.8	3.1	4.1
30	10.0	1.6	2.8	3.8
31	10.0	1.5	2.6	3.5
32	10.0	1.4	2.4	3.2

In generalized form, and assuming a constant dollar payout, the tax law would compute the value of the junior interest as:

$$FIRMVAL - [ANNUALDIV\,(1 - (1+r)^{-n} / r) + FIRMVAL / (1+r)^{n}] \qquad (2)$$

where: $FIRMVAL$ = date-of-recapitalization value of the firm
$ANNUALDIV$ = annual dividend required to be paid
r = discount rate to be applied to cash flows
n = term of analysis, equal to the entrepreneur's life expectancy at the date of the recapitalization.

In no event can the junior interest be valued at less than .10 *FIRMVAL*. The bracketed terms in Equation (2) represent the present value of the dividend stream received by the senior generation and the present value of the redemption payment. Equation (2) holds for the fact pattern assumed in this analysis. For more complex recapitalization terms, the value of the junior interest would be modified, although the value would not be difficult to model.

The analysis that follows assumes that 120% of the AFR is 10%. Thus, r = .10 and the AFR = .0833. The dividend yield will be 10% of firm value, 8.33% of firm value, and zero, in the respective situations under review. The analysis will examine life-expectancy from 1–32 years with log volatility in the Black/Scholes model set at 20%, 30%, and 40%.

A. Dividend Yield Equal to 120% of the AFR

When the dividend yield equals the discount rate used for tax law valuation, and the redemption value of the senior interest equals the initial value of the firm, the value of the junior interest is zero using the subtraction method. (The senior interest is equivalent to a

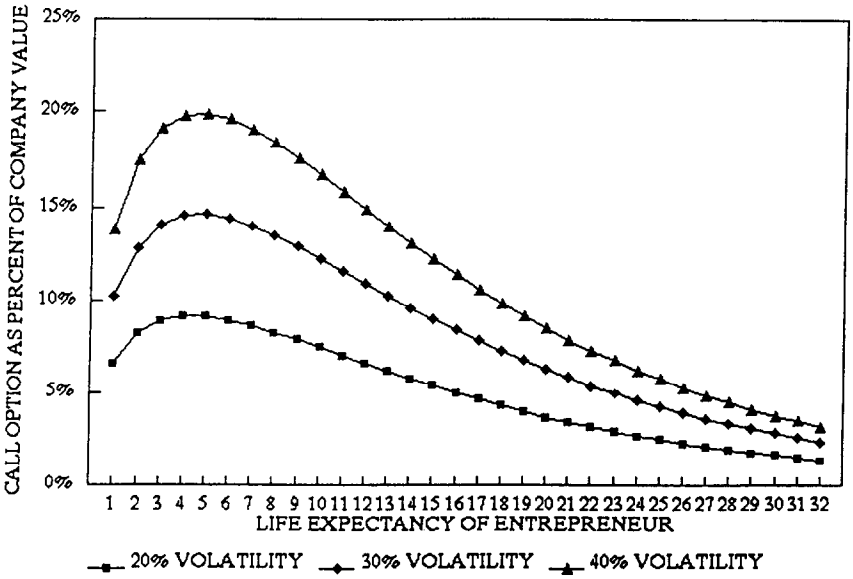

Figure 1. Value of option feature of junior equity interest (10% dividend, 8.33% AFR)

TABLE 2
Value of Call Option Feature of Junior Equity Interest Using OPM, Expressed as a Percentage of Firm Value
(dividend payout = 100% of AFR)

		Value Determined Using OPM With Log-volatility of:		
Entrepreneur Life Expectancy	*Tax Law Value (%)*	*20% (%)*	*30% (%)*	*40% (%)*
1	10.0	7.4	11.1	14.7
2	10.0	9.8	14.5	19.2
3	10.0	11.2	16.5	21.7
4	10.0	12.1	17.7	23.2
5	10.0	12.7	18.5	24.1
6	10.0	13.1	18.9	24.5
7	10.0	13.4	19.2	24.7
8	10.0	13.5	19.2	24.6
9	10.0	13.6	19.2	24.4
10	10.2	13.7	19.1	24.1
11	10.9	13.7	18.9	23.8
12	11.3	13.7	18.7	23.4
13	11.8	13.7	18.5	23.0
14	12.3	13.6	18.3	22.5
15	12.7	13.6	18.1	22.1
16	13.0	13.6	17.9	21.7
17	13.4	13.6	17.6	21.3
18	13.7	13.6	17.4	20.9
19	13.9	13.6	17.2	20.5
20	14.2	13.6	17.1	20.2
21	14.4	13.6	16.9	19.8
22	14.6	13.6	16.8	19.5
23	14.8	13.7	16.6	19.2
24	15.0	13.7	16.5	19.0
25	15.2	13.8	16.4	18.7
26	15.3	13.8	16.3	18.5
27	15.5	13.9	16.2	18.3
28	15.6	13.9	16.2	18.1
29	15.7	14.0	16.1	17.9
30	15.8	14.1	16.1	17.8
31	15.8	14.2	16.0	17.7
32	15.9	14.3	16.0	17.5

bond with a coupon equal to the required market return and a redemption value equal to par.) However, the tax law requires that the value of the junior interest be equal to 10% of firm value, to reflect the call feature of the interest. As shown in Table 1, the option value of the junior interest determined using OPM varies from 1.4% to 19.9% of firm value, depending on volatility and entrepreneur life expectancy. When log volatility is low (20%),[13] the OPM valuation is below 10% for all time periods. The tax law then values the call feature of the junior interest at an amount in excess of the OPM value, with the difference generally increasing with the life expectancy of the entrepreneur (see Figure 1). The valuation difference ranges from less than 10% to in excess of 600% of the value determined using OPM.

When log volatility is 30%, the OPM call value is in excess of 10% for life expectancies less than approximately 13.3 years, and less than 10% otherwise. Depending on the term,

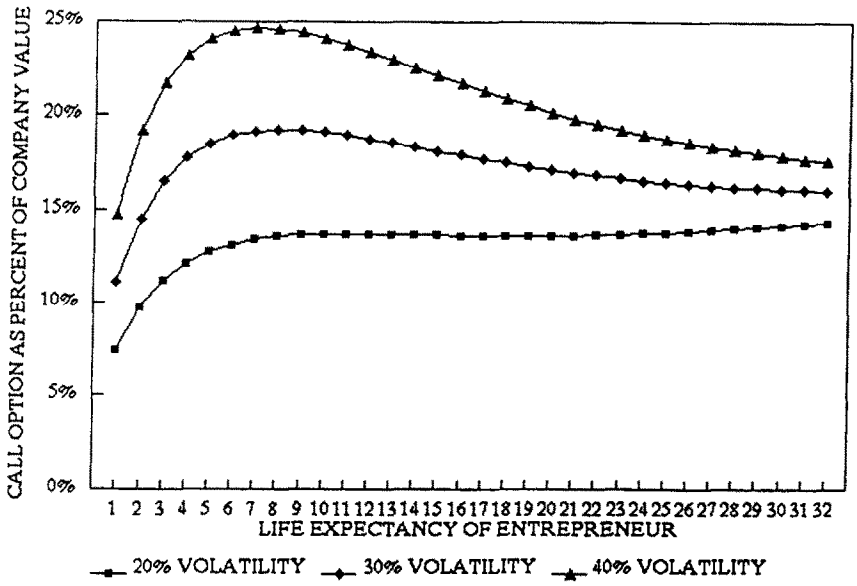

Figure 2. Value of option feature of junior equity interest (8.33% dividend, 8.33% AFR)

the minimum value rule can lead to valuations less than OPM by more than 30% (for shorter life expectancies) and valuations higher than OPM by more than 300% (for longer life expectancies). Log volatility of 40% produces similar results, with tax law undervaluations relative to OPM magnified and overvaluations reduced. The crossover point, where the minimum value rule and OPM produce similar values, occurs at 13.3 years with 30% volatility and 17.9 years with 40% volatility. Figure 1 shows the call value as a function of entrepreneur life expectancy for each level of volatility.

B. Dividend Yield Equal to the AFR

When the dividend yield equals the AFR, the value of the junior interest as determined by tax law is always positive, because the cash flows generated by the senior interest are less than the required market rate-of-return—that is, the senior interest is valued at a discount from firm value and the subtraction method results in a positive value assigned to the junior interest. For an entrepreneur life expectancy of less than 10 years, the value of the junior interest as determined by tax law is less than 10% of the firm value. The minimum value rule, therefore, is binding only for life expectancies less than 10 years and is nonbinding otherwise. In contrast, as shown in Table 2, the OPM value of the call feature of the junior interest exceeds 10% for all situations except 20% volatility with life expectancy of two years or less.

For life expectancies of less than 10 years (when the minimum value rule is binding), the tax law value is less than the OPM call value of the junior interest, except as noted in the preceding sentence, and the difference increases with increasing volatility. Figure 2 shows the call value as a function of entrepreneur life expectancy for each level of volatility.

C. Dividend Yield Equal to Zero

If the dividend yield on the senior interest is equal to zero, the value of the senior interest is equal to the present value of the stated redemption amount, paid at the death of the entrepreneur. The subtraction method assigns a relatively large value to the junior interest, and the minimum 10% value is binding only for a life expectancy of one year.[14] Thus, for tax purposes, the value of the junior interest is determined without regard to the minimum value rule. As shown in Table 3, the call value of the junior interest for the zero dividend case, as determined using OPM, exceeds 10% for all life expectancies and all volatilities. Further, the tax law valuation rules approximate the call value of the junior interest as determined using OPM, particularly for longer life expectancies. Figure 3 shows the call value as a function of entrepreneur life expectancy for each level of volatility.

TABLE 3
Value of Call Option Feature of Junior Equity Interest Using OPM, Expressed as a Percentage of Firm Value

(dividend payout = zero)

		Value Determined Using OPM With Log-volatility of:		
Entrepreneur Life Expectancy	*Tax Law Value (%)*	*20% (%)*	*30% (%)*	*40% (%)*
1	10.0	12.1	15.7	19.4
2	17.4	19.4	24.0	28.8
3	24.9	25.8	30.9	36.3
4	31.7	31.4	36.8	42.5
5	37.9	36.6	42.0	47.9
6	43.6	41.4	46.7	52.7
7	48.7	45.8	51.0	57.0
8	53.3	49.9	54.9	60.7
9	57.6	53.6	58.4	64.2
10	61.4	57.1	61.7	67.2
11	65.0	60.3	64.7	70.0
12	68.1	63.3	67.4	72.6
13	71.0	66.0	69.9	74.9
14	73.7	68.6	72.2	77.0
15	76.1	71.0	74.4	78.9
16	78.2	73.1	76.3	80.6
17	80.2	75.2	78.1	82.2
18	82.0	77.0	79.8	83.7
19	83.6	78.8	81.4	85.0
20	85.1	80.4	82.8	86.2
21	86.5	81.9	84.1	87.3
22	87.7	83.2	85.3	88.4
23	88.8	84.5	86.4	89.3
24	89.8	85.7	87.4	90.2
25	90.8	86.8	88.4	91.0
26	91.6	87.8	89.3	91.7
27	92.4	88.7	90.1	92.4
28	93.1	89.6	90.8	93.0
29	93.7	90.4	91.5	93.5
30	94.3	91.1	92.2	94.0
31	94.8	91.8	92.8	94.5
32	95.3	92.4	93.3	95.0

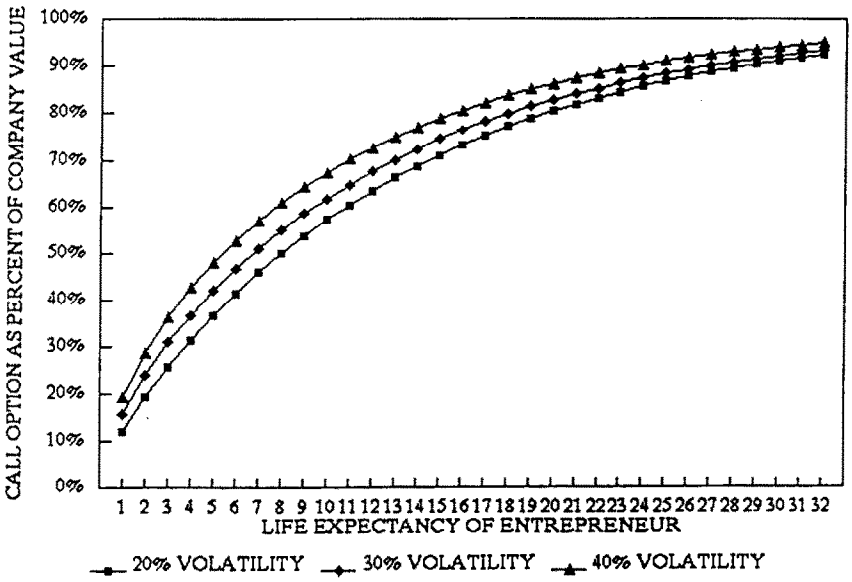

Figure 3. Value of option feature of junior equity interest (0% dividend, 8.33% AFR)

D. Analysis of Results

In Figures 1 and 2, there are two principal factors affecting the value of the option over time. On the one hand, all of the option values are enhanced by the increase in variance that occurs as the time to expiration increases, ceteris paribus. On the other hand, as time increases, more dividends are paid on the senior equity interest, reducing the value of the underlying asset and thereby the option right. For the 10% dividend case (Figure 1), at first the variance has the stronger effect, leading to increasing option values. As time passes, the negative effect of the dividend payout increases in importance, leading to a steady decrease in the value of the option towards zero. In the 8.33% dividend case (Figure 2), a point is reached where the dividend and variance effects essentially cancel, and the option value levels at approximately 15% of firm value for all volatilities. In the no dividend case (Figure 3), the dividend effect is nonexistent and the value of the option increases with time for all volatilities. Since an option can never be worth more than its underlying asset without creating arbitrage opportunities, the option value is bounded at 100% of company value and the lines in Figure 3 converge with time.

The 1990 Tax Act explicitly requires that the call value of the interest be recognized through a statutory minimum value rule. The minimum value rule generally will apply only when the retained interest provides for fixed, cumulative dividend payments, and then only for certain payment terms. Payment terms that result in a binding minimum value rule should be avoided if the OPM suggests the junior interest option feature is worth less than 10% of firm value. For example, in the analysis conducted in this article, with log volatility of 20%, it is not advisable to pay a dividend equal to the discount rate used for tax law valuation. In such a case, the statutory minimum value rule is binding and OPM suggests that the call value will be overstated, with the overstatement generally increasing with the life expectancy

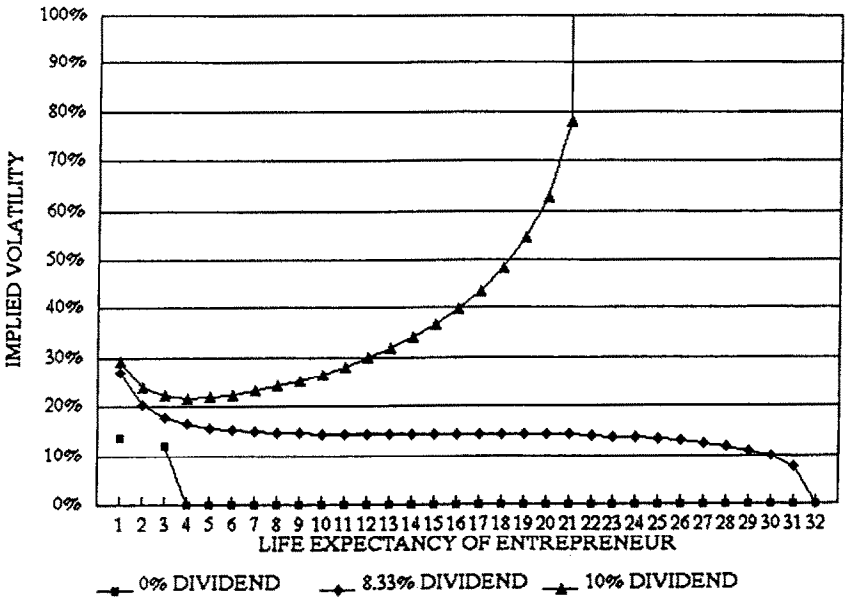

Figure 4. Implied volatility for 10% option value

of the entrepreneur. If a smaller dividend is paid and the entrepreneur's life expectancy is relatively short, the minimum value rule is binding but the call feature of the junior interest is undervalued when OPM is used as a benchmark. As volatility increases, the OPM value of the call feature can exceed the minimum value if the dividend yield equals the assumed discount rate and the life expectancy of the entrepreneur is short, but this is reversed for longer life expectancies.

The informed estate planner has an incentive to suggest terms of the senior interest to take advantage of any valuation distortions created by tax law. Figure 4 plots volatility of the firm on the y-axis and entrepreneur life expectancy on the x-axis with all points reflecting the volatility required to value the junior interest at 10% using OPM. When no dividend is paid on the senior equity interest, the underlying asset value is assumed to steadily increase and the option right becomes more valuable with time. In fact, with maturities of four years or longer there is no volatility low enough to result in a 10% (OPM) option value. As the dividend payout grows, the decrease in the value of the underlying asset reduces the value of the option. At a 10% payout, Figure 4 illustrates that the option value is less than 10% regardless of the volatility level when the time to maturity exceeds 21 years. Thus, dividend payments to the senior generation that approximate the valuation discount rate (120% of the AFR) should be avoided if the objective is to minimize transfer taxes. This is because the minimum value rule will be binding, although the call value of the junior interest is likely to be less than the 10% value assigned by law.

Figure 4 is sensitive to the dividend payout assumptions used in the first part of this analysis. However, in conjunction with Figures 1–3, it shows that the option value of the junior equity interest is sensitive to the interaction of time, volatility, and dividend payout.

E. Limitations of This Analysis

There are two important limitations of the analysis shown in this article. First, a valuation expert hired in connection with the estate freeze transaction must be able to reasonably estimate OPM parameters. As discussed above, volatility of firm value would be the most difficult parameter to estimate. However, as shown in Figure 4, it is possible to determine the volatility implied by a specified option value. If the implied volatility appears unreasonable, then the tax law option value is also likely to be unreasonable. Also, upper and lower boundaries could be specified, and the resulting option values could be examined for sensitivity to volatility misspecifications. The second limitation is whether it is appropriate to use the Black-Scholes model to value the option feature of the junior interest. The junior interest clearly has features of a call option, but the features do not literally meet all of the assumptions of OPM. For example, if life expectancy is viewed as the maturity date, the junior interest "option" is not strictly European because the senior generation owner can certainly die before "maturity" and cause an early exercise. However, the analysis is intended to be used for planning purposes only and can incorporate the effects of an exercise at some term other than life expectancy. A sensitivity analysis, similar to that shown in Figures 1–3, can be useful in determining the effects of misspecification of a model parameter.

V. Conclusion

This article compares the use of the statutory minimum value rule, intended to capture at least the option value of a junior equity interest, with the OPM value of the option feature. The results suggest that estate freeze transactions can still lead to transfer tax savings when the tax law understates the call value of the transferred interest. Conversely, estate freeze terms that result in a binding minimum value rule should be avoided if the call value is less than the statutory 10%. The results are limited by the ability of the valuation expert to estimate OPM parameters and the applicability of the Black-Scholes model to the junior interest created in the freeze.

Notes

1. Prior research has recognized that an entrepreneur may not be diversified to the extent suggested by portfolio theory (Leland & Pyle, 1977).

2. It is assumed in this analysis that the older generation owns 100% of the corporation prior to the recapitalization. The costs of restructuring this ownership can be both monetary and nonmonetary.

3. These rights could include rights to dividend payments, to liquidate the entity and receive assets, to put the interest to the entity, and to convert the frozen interest into an interest with appreciation potential, among others. Because a third party would pay for these rights, an appraisal could include the value of the rights.

4. The value of the transferred interest has been determined by subtracting the value of the retained interest from the total value of the enterprise. By assigning a redemption value to the preferred stock equal to the date-of-recapitalization value of the entity and attaching valuable rights to the preferred, it was argued that the preferred value approximated the total value of the entity. The subtraction method would then assign very little value to the transferred interest. Enforcement

difficulties arose because of the inherently factual nature of a valuation question and the lack of any statutory guidance.

5. The Joint Committee on Taxation is a standing committee of Congress that advises both houses on fiscal and administrative issues associated with tax legislation. It consists of members of the tax writing committees of Congress and professional staff with backgrounds in law, economics, and accounting.

6. IRC section 2701 (a)(4). The minimum value rule was initially set at 20% under H.R. 5425 (August 1, 1990), the first proposed amendment to the estate freeze rules following the Joint Committee discussion draft. The Senate eliminated the minimum value standard in S. 3113 (September 26, 1990), but it later resurfaced at 10% in S. 3209 (October 13, 1990). The final bill was approved by both houses of Congress on October 27, 1990, and retained the standard at 10%.

7. Or earlier, if the agreement so provides.

8. Technically, the junior equity owners do have an enforceable obligation to redeem the senior interest. The obligation arises from the capacity as equity owners of the corporation and is limited to the value of firm assets, resulting in an obligation to exercise the option when it finishes in-the-money. Since this is rational behavior, the obligation does not change our conclusion.

9. At one extreme, if there is absolute certainty that the option will be exercised, the value of the option will move dollar-for-dollar with the underlying stock. $N(D_1)$ will take on a value of 1 in this situation because the option holder will inevitably acquire the stock by exercise of the option. At the opposite extreme, if there is no chance of the option being exercised, the option value is not affected by changes in the stock price and $N(D_1)$ equals zero.

10. The "subtraction" method requires that the value of the retained interest be subtracted from the total value to determine the value of the transferred interest.

11. Recall that the minimum value rule exists to recognize the call value only.

12. For ease of exposition, we are again assuming that the senior generation initially owns 100% of the firm. The analysis could readily be extended to other situations.

13. One standard deviation unit of 20% log volatility means that the value of the firm in one year has the same probability of increasing by 20% (a factor of 1.2 in log form) as decreasing by 20%. That is, $Pr.\ [Value(t_1) = Value(t_0) \times 1.2] = Pr.[Value(t_1) = Value(t_0) / 1.2]$.

14. Of course, this is a function of the discount rate. In general, the minimum value rule would be binding only when $FIRMVAL - [FIRMVAL / (1+r)^n] < .10\ FIRMVAL$. As r decreases (increases), the minimum value is binding for longer (shorter) life expectancies.

REFERENCES

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy, 81*(May–June), 637–654.

Joint Committee on Taxation. (1990, April 20). *Federal transfer tax consequences of estate freezes. (JCS–13–90).*

Leland, H., & Pyle, D. (1977). Information asymmetries, financial structure, and financial intermediation. *Journal of Finance, 32*(May), 371–385.

Merton, R.C. (1973). Theory of rational option pricing. *Bell Journal of Economics and Management Science, 4*(Spring), 141–183.

IRS may upstage hill as source of big tax news in '91, FBA told. (1991). *Tax Notes,* (March 11), 1044–1045.

Wilson, P. (1991). Future research directions in taxation. *The Journal of the American Taxation Association, 13*(Fall), 64–73.