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The Optimal Choice of Index-Linked GICs: Some Canadian Evidence

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Indexed Linked Guaranteed Investment Certificates (ILGICs), also known as equity linked term deposits, have become quite popular over the last few years. From the consumer's point of view, there are two basic categories of Indexed Linked GIC, the Capped ILGIC and the Participating ILGIC. This paper compares the relative value and appeal of the two ILGIC products from the perspective of the individual investor, using valuation techniques from option pricing theory. We compute the Value per Premium Dollar invested in an ILGIC. Our main conclusions are that ILGICs become less attractive the longer the time horizon of the investor. In addition, participating ILGICs are preferable to capped ILGICs for short-term maturities and vice versa for longer maturities. A detailed analysis of two particular Canadian products is provided as an application of the basic concepts. We conclude by demonstrating that Values per Premium Dollar range from 94% - 98%, corresponding to a 2%-6% total expense ratio on Index Linked GICs.

I. INTRODUCTION

With the general decline in the level of interest rates, Indexed Linked Guaranteed Investment Certificates (ILGICs) have become extremely popular over the past few years. Every one of the six (major) Canadian banks offers a variant on this product. An ILGIC (in the U.S., an Equity Enhanced Certificate of Deposit or Equity Indexed Annuity) is a retail savings vehicle, similar to a term deposit, with an interest rate that is linked to a diversified stock index. Upon maturity of the product, the total is determined based on the performance of the underlying index over the pre-specified term.

Within the family of products available to the consumer, there are two basic structures of ILGICs, namely the Capped ILGIC (C-ILGIC) and the Participating ILGIC (P-ILGIC). The C-ILGIC allows the holder to participate in any upward movement in the underlying stock index, up to a pre-specified annual percentage cap. The P-ILGIC allows the holder to

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participate in a fixed fraction of any upward movement in the underlying stock index, with no pre-specified cap. Both C-ILGICs and P-ILGICs have an implicit guarantee of principal plus minimal interest. There are, however, some products available that are both capped *and* have partial participation. We will not address them in this paper, since they constitute a very minor part of the market.

Here is an example: **Bank C** offers a 5-year ILGIC whose total return at maturity will be the greater of 5% *and* the net return (excluding dividends) on the SP500 Index, up to a maximum of 60%. This product is a C-ILGIC where the floor is 5% and the cap is 60%. Likewise, **Bank P** offers a 5-year ILGIC whose total return at maturity will be the greater of 3% *and* 80% of the net return (excluding dividends) on the SP500 Index. This product is a P-ILGIC where the floor is 3% and the participation rate is 80%.

At maturity, if the SP500 increases by 40% over the five year term, **Bank C** will allow full participation and credit the account interest in the same amount. On the other hand, **Bank P** will only credit the account 80% of the 40% increase, resulting in a total return of 32%. In a similar fashion, if the SP500 increases by 100% over the five year term, **Bank C** will cap the return at 60%, while **Bank P** will credit the account 80% of the 100% increase, resulting in a total return of 80%.

Figure 1 is a graphical representation of the payoff structure from both types of ILGICs. The horizontal axis is the value of the underlying index at maturity, denoted by S_T. The vertical axis represents the total payoff from the ILGIC. The C-ILGIC has a maximum possible payoff of $exp(cT)$, where c is the continuously compounded *annualized* cap rate and T is the maturity of the contract. The minimum guaranteed payoff from the C-ILGIC is $exp(g_c T)$, where g_c is the continuously compounded *annualized* floor. In contrast, the P-ILGIC has no maximum payoff, but the slope of the payoff diagram is reduced in propor-

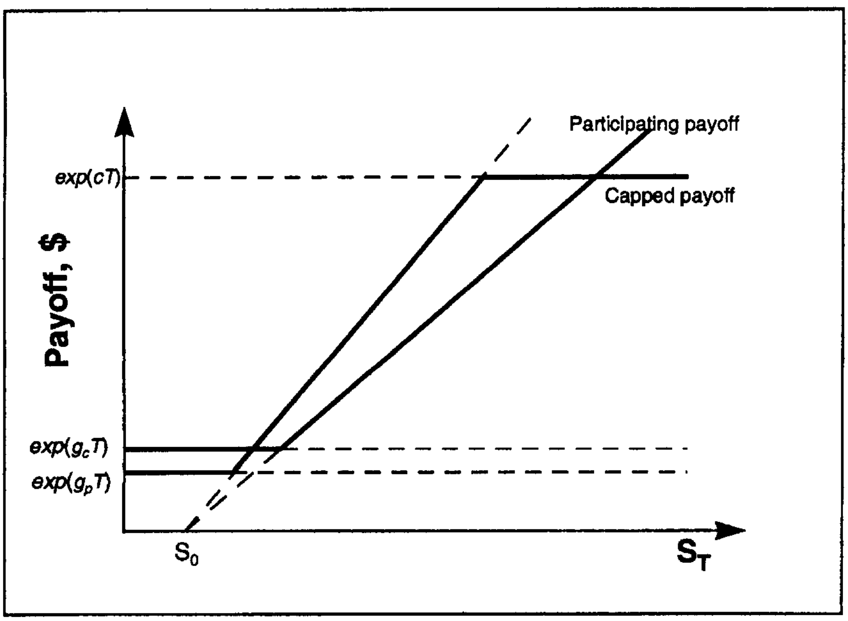

Figure 1. ILGIC Payoffs

tion to p, which is the participation rate. Once again, the minimum interest is $exp(g_pT)$, where g_p denotes the continuously compounded *annualized* floor on the P-ILGIC. Note that although the caps and floors are applicable to the entire term of the ILGIC, the figures are presented on an annualized basis in order to facilitate the comparison between products with different maturities.

With hindsight it is quite obvious which of the two basic products "was" better. In practice, however; it is quite difficult to choose the right one ex-ante. *This study compares the relative value and appeal of the two basic ILGIC products from the perspective of the individual investor. In addition we provide a methodology for extracting the implicit expense ratios of the two types of ILGIC.*

Using concepts from option pricing theory, we are able to value both types of ILGIC by decomposing their payoff into a zero-coupon bond and a suitably parameterized collection of call options on the underlying stock index. These values apply regardless of the risk preferences of the individual investor. The intuition behind this result follows from arbitrage valuation and is described at length in the body of the paper. A detailed analysis of two particular bank products is provided as an application of the basic concepts.

The remainder of this study is organized as follows: Section II conducts a brief literature review of ILGICs and then describes the particular ILGIC products available in Canada; Section III develops the pricing relationships for the two basic categories of ILGICs; Section IV presents a numerical example of the relative valuation of the two most popular bank products, thus extracting the embedded expense ratios; and Section V concludes the paper.

II. LITERATURE REVIEW

In an introductory study geared to the individual investor, Cohn and Edleson (1993) examine the basic features of ILGICs. They discuss the various products available in the U.S. market at the time and identify the embedded zero-coupon bond plus call option structure. They do not, however, focus on the relative benefits of capped vs. participating GICs, nor do they attempt to identify the best products available in the Canadian market. There are a few business press articles on the subject of Canadian-based ILGICs, see for example Bell (1997) and Croft (1997). However, there is little, if any, rigorous academic research on the specifics of the Canadian market. Baubonis, Gastineau, and Purcell (1993) provide a qualitative description, from an institutional perspective, of the embedded derivative securities in ILGICs, but avoid any discussion of pricing or relative value. Brooks (1996) provides a framework for comparing certificates of deposit that vary in their features. In particular, he uses a derivative pricing methodology to compute the value of guaranteed interest rate floors and caps. Our analysis is a natural continuation of this line of research by focusing on equity-enhanced, as opposed to interest rate-enhanced, products. We are thus able to facilitate comparison of vastly different products, using the Black and Scholes (1973) and Merton (1973) framework, as suggested by Brooks (1996).

As mentioned in the introduction, purchasing an ILGIC can be viewed as an alternative to investing in the underlying stock index and using the dividends from the portfolio to purchase put options which protect the investment against a market decline *beyond a*

TABLE 1
Index-linked GICs

Bank	*Term*	*Tax Treatment*	*Description*
National Bank of Canada *NatOption*	5 years	Available only as a Registered Retirement Savings Plan (RRSP)	Classified as C-ILGIC Linked to TSE 35 $g_c = 3\%$ for entire 5-year term $c = 60\%$ for entire 5-year term The TSE35 closing level on the 11th of each month of the final year is used to calculate the final index level
Toronto Dominion Bank *GIC Plus*	3 years or 5 years	If held outside the RRSP, it can be sold back to the Bank as regular interest income with no accrual	Classified as P-ILGIC as it uses some averaging Linked to TSE 100 $g_p = 0.25\%$ per annum for the 3-year product $g_p = 2.25\%$ per annum for the 5-year product In both cases, interest is received at the end of the entire term Mean closings of the TSE100 on the same day of each month of the final year are used to calculate the final index level.
Laurentian Bank	3 years or 5 years	The minimum guaranteed interest is received years and calculated for income tax purposes as interest income	Classified as a P-ILGIC Linked to the TSE 35 $g_p = 1\%$ per annum for both terms, which is received yearly and later deducted if the index's final yield is greater than 1% p.a. $p = 75\%$ for the 3-year product $p = 100\%$ for the 5-year product The index's final level is calculated as the mean of the closings on the final 3 days prior to maturity
Bank of Nova Scotia	2 years	If held outside an RRSP, gains are calculated as interest income	Classified as a C-ILGIC Linked to the TSE 35 $c = 20\%$ for the entire 2 year term The final level of the index is the close level of the TSE 35 on the maturity date
Canadian Imperial Bank of Commerce Canadian Index Fund	3 years or 5 years	If held outside an RRSP, gains are calculated as interest income in the year in which it is received	Classified as P-ILGIC as it uses some averaging Linked to the TSE 35 $g = 0\%$ The TSE 35's final level is calculated as the mean closings of the index in the last 11 months of the term

certain level. This strategy can also be implemented in the form of portfolio insurance with an arbitrary floor.

Before continuing, we note an issue related to ILGICs that we will not discuss in detail, but is still relevant. From a utility based perspective, there is some question as to why an individual investor would want to insure a portfolio beyond a certain floor. Indeed, it appears that most individual investors exhibit decreasing, or at least constant, relative risk aversion (RRA). See recent empirical work by Schooley and Worden (1996) for evidence and references. However, Benninga and Blume (1985), Brennan and Solanki (1981), and many others show that even in markets that are highly incomplete, rational investors with decreasing relative risk aversion will avoid portfolio insurance strategies. Their research questions the need for products with arbitrary guarantees in a fully rational market place. In plain English, why are investors exhibiting preferences that are discontinuous at the guaranteed floor?

Nevertheless, these products do exists in the market place, and the remainder of this paper will attempt to shed light on their *relative* value. We are thus able to answer the question: "If I *do* want to buy an ILGIC, which gives me the best value?"

1. Details of the Canadian Market

The Canadian banking industry is very concentrated and dominated by six national banks and two or three deposit-taking trust companies and regional credit unions. Within the last two years, every one of these institutions has started to offer and promote a variant of the Index Linked GIC.

Table 1 provides a brief summary of the details of the most popular ILGICs in Canada together with a description of their embedded *options*. The same table also identifies the tax consequences of purchasing the product outside of a self-directed pension plan. (In Canada they are called Registered Retirement Saving Plans similar to IRAs and 401(k)s in the United States.) We also note that some of the products have embedded averaging features which reduce the final payoff from the ILGIC compared to the performance of the market. These products would fall under the broad category of P-ILGICs since their final payoff is proportionally reduced. The actual decomposition would involve *Asian options* which would be bounded from above by the corresponding vanilla options. Our numerical examples, however, focus on products offered by Laurentian Bank and the Bank of Nova Scotia that do not contain this added complication.

III. PRICING RELATIONSHIP

This section will present a formula for the market value of an ILGIC, *as a percent of par*, in a Black and Scholes (1973) and Merton (1973)framework. By *percent of par* we mean the ratio of the theoretical price to the initial investment in the ILGIC. Thus, a value equal to one denotes a *fairly* priced ILGIC, a value less than one denotes an *unfairly* priced ILGIC, and a value greater than one denotes a *super-fairly* priced ILGIC. A super-fairly priced ILGIC will admit arbitrage and is thus very unlikely to exist.

Our main goal is to calculate Values per Premium Dollar (VPDs) for the various products available in the Canadian market place. We achieve this by computing the risk-neutral expected payoff from the C-ILGIC and P-ILGIC and then discounting by the appropriate risk-free rate to obtain the present value.

1. C-ILGIC

An investor places an amount, denoted by I, in a C-ILGIC. The payoff from the C-ILGIC, at maturity, is:

$$\begin{cases} I\,exp\{cT\}, & if \quad exp\{cT\} \le \frac{S_T}{S_0} \\ I\frac{S_T}{S_0}, & if \; exp\{g_cT\} \le \frac{S_T}{S_0} \le exp\{cT\} \\ I\,exp\{cT\}, & if \quad \frac{S_T}{S_0} \le exp\{g_cT\} \end{cases} \tag{1}$$

where $c > 0$ is the annualized cap rate, $g_c \geq 0$ is the annualized guaranteed minimal interest floor, T is the time to maturity, S_0 is the initial level of the stock index and S_T is the (stochastic) value of the stock index at maturity. By definition we assume that $c > g_c$ for the ILGIC to make sense. From a qualitative perspective, if the one-plus total return from the stock market index, S_T/S_0 over the time period $[0,T]$, is less than the minimal guarantee $exp\{g_cT\}$, the payoff from the C-ILGIC is simply $I\,exp\{g_cT\}$. On the other hand, if the one-plus total return from the stock market index, S_T/S_0 over the time period $[0,T]$, is *greater* than the minimal guarantee $exp\{g_cT\}$, but *less* than the total cap $exp\{cT\}$, the payoff from the C-ILGIC is $I\,(S_T/S_0)$. Finally, if the one-plus total return from the stock market index, S_T/S_0 over the time period $[0, T]$, is *greater* than the total cap $exp\{cT\}$, the payoff from the C-ILGIC is (capped at) $I\,exp\{cT\}$. The three branches of the payoff structure can be combined using the *max*[.] operator. See the appendix for details.

2. P-ILGIC

Upon investing I in the Participating ILGIC, the payoff is:

$$\begin{cases} I\left(\frac{S_T}{S_0} - 1\right)p + I, & if \; \left(\frac{S_T}{S_0} - 1\right)p + 1 \ge exp\{g_pT\} \\ I\exp\{g_pT\}, & if \; \left(\frac{S_T}{S_0} - 1\right)p + 1 \ge exp\{g_pT\} \end{cases} \tag{2}$$

where $p > 0$ is the participation rate, $g_p \geq 0$ is the annual guaranteed minimal interest floor, T is the time to maturity, S_0 is the initial level of the stock index and S_T is the (stochastic) value of the stock index at maturity. Intuitively, if one-plus p% of the total return from the stock market over the time period $[0, T]$, is less than the minimal guarantee $exp\{g_pT\}$, the payoff from the C-ILGIC is simply $I\,exp\{g_pT\}$. On the other hand, if the one-plus p% of the total return from the stock market, over the time period $[0, T]$, is greater than the minimal guarantee $exp\{g_p T\}$, the payoff from the C-ILGIC is $I\left(\frac{S_T}{S_0} - 1\right) p + I$, which is simply the original investment plus the proportional participation. Once again, the two branches of the payoff structure can be combined using the *max*[.] operator. See the appendix for details.

3. Valuation of C-ILGIC

We refer the reader to the appendix where we derive the fair value of the C-ILGICs using the standard Black and Scholes (1973) and Merton (1973) methodology for pricing contingent claims. In this section we simply reproduce the valuation formulas.

Given a set of product parameters (c, g_c), where c is the cap rate and g_c is the guaranteed floor, and capital market parameters (r, q, σ), and maturity T, the Value per Premium Dollar (VPD) of the C-ILGIC is:

$$V_c(c,g_c) = exp\{(g_c - r)T\} + exp\{-qT\}N(b_1) - exp((g_c - r)T\}N(b_2) - exp\{-qT\}N(b_3) + exp((c - g_c - r)T\}N(b_4) \quad (3)$$

where $N(\cdot)$ denotes the cumulative density function of the normal distribution, r is the risk-free interest rate in the market, q is the dividend yield on the underlying stock index, σ is the volatility of the underlying stock index and

$$b_1 = \left(\frac{r-q-g_c}{\sigma} + \frac{\sigma}{2}\right)\sqrt{T}, \qquad b_2 = \left(\frac{r-q-g_c}{\sigma} - \frac{\sigma}{2}\right)\sqrt{T}, \quad (4)$$

$$b_3 = \left(\frac{r-q-c-g_c}{\sigma} + \frac{\sigma}{2}\right)\sqrt{T}, \qquad b_4 = \left(\frac{r-q-c-g_c}{\sigma} - \frac{\sigma}{2}\right)\sqrt{T} \quad (5)$$

Notice that equation (3) does not involve the variable S_0, since all that is relevant is the percent increase. We explicitly parametrize V_c by the two important variables (c) and (g_c). Obviously, the actual value will depend on the implicit parameters σ, q, r, T as well.

4. Valuation of P-ILGIC

In the same manner, the VPD of the P-ILGIC is:

$$Vp(p, g_p) = exp\{(g_p - r)T\} + pexp\{-qT\}N(a_1) - pexp((g_p - r)T\}N(a_2) \quad (6)$$

where p is the participation rate and g_p is the guaranteed floor and

$$a_1 = \left(\frac{r-q-g_p}{\sigma} + \frac{\sigma}{2}\right)\sqrt{T}, \qquad a_2 = \left(\frac{r-q-g_p}{\sigma} - \frac{\sigma}{2}\right)\sqrt{T}. \quad (7)$$

5. Analysis

A few stylized facts emerge from our analysis. From a qualitative point of view, we can make the following statement without need for much algebra.

$$V_c(\infty, g) \geq V_p(h, g) \qquad \forall p \leq 1$$

For a given floor g, no cap is better than partial participation. Likewise

$$V_c(c, g) < V_p(1, g) \qquad \forall c < \infty$$

for a given floor g, total participation is better than a cap, no matter how large the cap. Also,

$$V_c(\infty, g) = V_p(1, g)$$

can be verified by plugging $c = \infty$, $p = 1$, $g_c = g_p = g$ into equation (3) and (6), respectively.

Figure 2 displays the VPD for both the C-ILGICs and P-ILGICs as a function of the maturity time horizon T, for an arbitrary set of capital market and product parameters.

The first point that is evident from the picture is that both curves decrease as a function of time. This means that the consumer gets less, per dollar invested, the longer the maturity of the ILGIC. The intuition behind this result is simple. The price of the zero coupon bond, used to fund the minimal interest, decreases at a faster rate than the price of the call option increase, which is used to fund the equity participation. Therefore, all else being equal the longer the time horizon, the more the investor looses. This result can be rigorously obtained by taking the derivatives of equation (3) and (6) with respect to the time horizon, T, and showing that it is negative. Of course, the rate at which the VPDs decline, or the

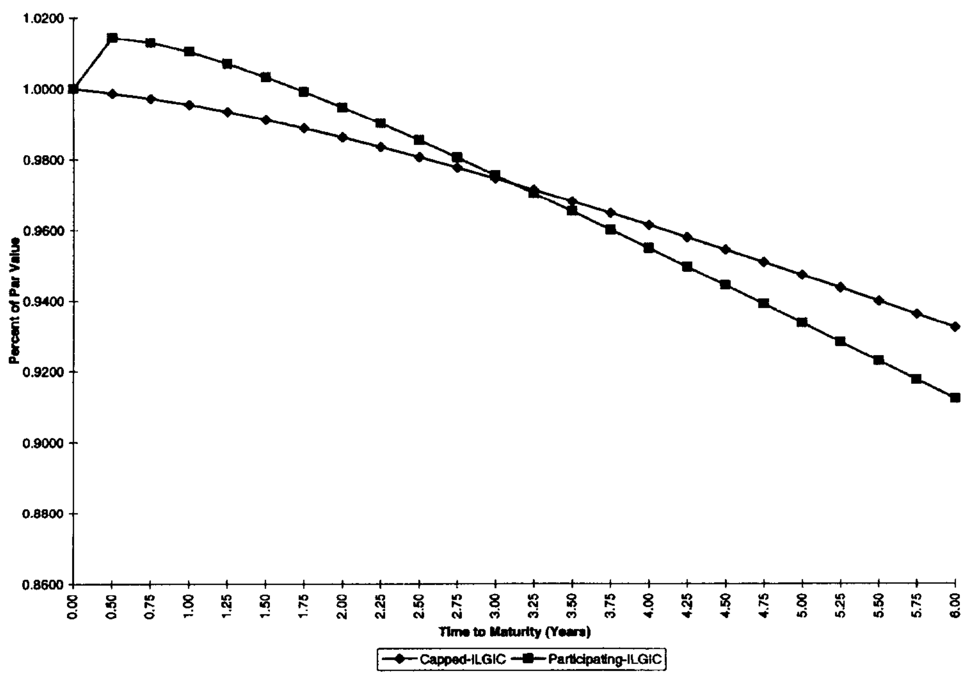

Figure 2. Value per Premium Dollar (VPD) of Index Linked GIC.
Capped-ILGIC vs. Participating ILGIC. Cap = 9.12% p.a., Participation = 75%
No Minimal Interest Guarantee
Risk Free Interest Rate = 4.3%, Volatility = 12%, Dividend Yield = 2.0%

slope, will depend on the capital market parameters (r, q, σ) *vis á vis* the magnitude of the cap and the participation rate.

Another interesting fact to note about Figure 2 is that for maturity horizons less than approximately 3 years, the Participating ILGIC is worth more than the Capped ILGIC. At around 3 years they are both worth the same, and then for longer horizons the situation is reversed and the C-ILGIC is preferable to the P-ILGIC. This is not a spurious result of the input parameters, but rather a fundamental insight into the structure of C-ILGICs vs. P-ILGICs. Basically, as in all option pricing models, it all comes down to probabilities. For short time horizons, the C-ILGIC is worth less than the P-ILGIC because the cap is a strong constraint on the performance of the product. Indeed, it is quite likely that the market will earn more than c percent annualized, and the gains will be truncated. On the other hand, as the maturity of the product is extended, the probability that the cap is constraining, on an annualized basis, decreases and thus the *relative* value of the C-ILGIC increases. In other words, it is more likely that the SP500 will increase by more than 30% in one year compared to 30% annualized over 5 years. Therefore, the cap becomes less of an issue as time increases. Of course, the fact remains that the *absolute* value of the C-ILGIC declines with maturity.

Finally, one should not lose sight of the main conclusion that the Values per Premium Dollar *on available products* are uniformly *less than one*, which means these products are unfairly priced for the consumer. (The anomaly in the short horizon for the C-ILGIC and P-ILGIC is an artifact of the algebra and simply reflects the fact that no institution is likely to offer this kind of C-ILGIC with maturity less than three months and a P-ILGIC with maturity less than 1.6 years.)

In sum, a VPD of X% implies that $(1 - X)$% of the initial investment is paying for commissions, transaction costs and bank profits. Consequently, if the individual investor can obtain the same exact payoff structure, using basic options, but incur transaction costs that are less than $(1 - X)$%, it is cheaper to replicate the product than to purchase it from the bank. This will obviously depend on the size of the investment *vis á vis* the transaction costs incurred in self replication. From a practical point of view, the bank as an intermediary is likely to take advantage of its economies of scale in the market and incur lower transaction costs when purchasing the necessary derivatives.

IV. NUMERICAL EXAMPLE: B.N.S VS. L.B

In this section, we compare the ILGIC offered by the Bank of Nova Scotia (BNS), a C-ILGIC, to that offered by Laurentian Bank (LB), a P-ILGIC. As mentioned in Section II and

TABLE 2
Value per Premium Dollar of the Bank of Nova Scotia 3 year Capped-ILGIC as a function of risk-free interest rate (r) and volatility (σ).

$r \backslash \sigma$	*−1%*	*vol.*	*+1%*
−1%	99.224	99.3844	99.5153
int.	97.0285	97.1939	97.329
+1%	95.5944	95.6792	95.7432

Note: The middle number is the point estimate for the VPD based on the market parameters provided by Bloomberg on November 11, 1997.

TABLE 3
Value per Premium Dollar of the Laurentian Bank 3 year Participating-ILGIC as a function of risk-free interest rate (r) and volatility (σ).

$r \backslash \sigma$	*-1%*	*vol.*	*+1%*
-1%	94.7062	95.1545	95.6082
int.	92.7801	93.6980	93.6260
+1%	90.9344	91.3008	91.6861

Note: The middle number is the point estimate for the VPD based on the market parameters provided by Bloomberg on November 11, 1997.

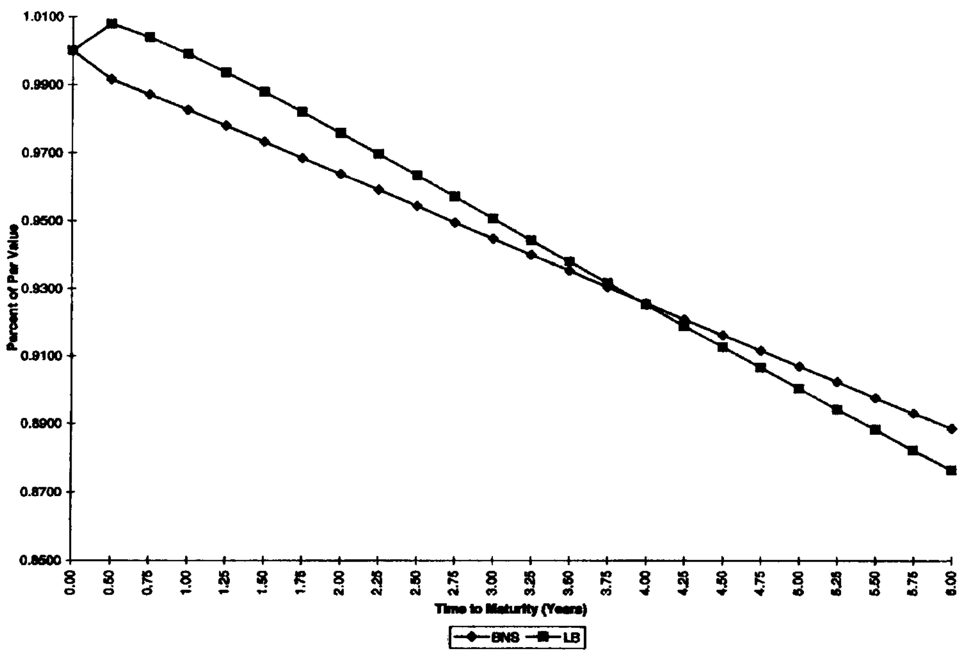

Figure 3. Value per Premium Dollar of Index Linked GIC.
Bank of Nova Scotia (C-ILGIC) vs. Laurentian Bank (P-ILGIC)
Valuation Date: November 11, 1997
Data Source: Bloombergs

displayed in Table 1, the BNS product is a 2-year C-ILGIC linked to the Toronto Stock Exchange's index of the 35 largest stocks as measured by market capitalization. (Referred to as the TSE35 Index.) The BNS product does not offer a minimum guaranteed interest rate and its payoff is capped at 20% over the 2 years. In contrast, although LB's P-ILGIC is linked to the TSE 35 Index, it offers a 1% minimum guaranteed interest per annum. We will examine the 3-year product as it is an example of a product with partial (75%) participation.

Using the pricing equations presented in Section III, we insert the relevant parameters and solve for the VPD of a C-ILGIC and P-ILGIC. We value the BNS product recalling equation (3), where the relevant product parameters are: $c = \ln\sqrt{1.2} = 0.0912$, $g_c = 0$, and $T = 2$. We arbitrarily take our relative valuation date to be November 11, 1997 which results in a dividend yield of $q = 0.015$, a 2-year volatility of $\sigma = 10.2\%$ and a 2-yr risk-free interest rate of $r = 4.64\%$. All of which were provided by Bloomberg's Information Systems which is the *de-facto* standard for market values in the derivatives trading industry. The resulting VPD for the BNS product is 97.19%, which translates into an implicit 'cost' of 2.81%.

We conducted a sensitivity analysis of our VPD's by perturbing the market volatility and interest rate figures by 1% around the point estimate provided by Bloomberg. Table 2 is a summary of those results.

Continuing, equation (6) can then be used to value LB's 3-year product, where the parameters are: $p = 0.75$, $g_p = ln(1.01) = 0.0995$ and $T = 3$. Once again, we take our relative valuation date to be November 11, 1997 which, in this case, results in a dividend yield of $q = 0.015$, a 3-year volatility of $\sigma = 9.42\%$and a 3-year risk-free interest rate of r = 4.89%. The resulting VPD is 93.69%, which translates into an implicit 'cost' of 6.31%.

Once again, we conducted a sensitivity analysis of our VPD's by perturbing the market volatility and interest rate figures by 1% around the point estimate provided by Bloomberg. Table 3 is a summary of those results.

Based on a VPD analysis, it therefore appears that the 2-year C-ILGIC offered by The Bank of Nova Scotia is more valuable than the 3-year P-ILGIC offered by Laurentian Bank, by approximately 4.50%.

Figure 3 is a graphical illustration of the VPD for both products as a function of time horizon, which an average implied volatility of 10% and incorporating the complete term structure of interest rates.

V. CONCLUSION

Every one of the six major Canadian banks offers a variant of the Indexed Linked Guaranteed Investment Certificates product. An ILGIC is a retail savings vehicle, similar to a term deposit, with an interest rate that is linked to a diversified stock index. Upon maturity of the product, the total return will be determined based on the performance of the underlying index over the pre-specified term. There are two basic structures of Indexed Linked GICs.The Capped ILGIC allows the holder to participate in any upward movement in the underlying stock index, up to a pre-specified annual percentage cap. The P-ILGIC allows the holder to participate in a fixed fraction of any upward movement in the underlying stock index, with no pre-specified cap. Both products have an implicit guarantee of principal plus minimal interest.

This study compares the relative value and appeal of the two basic ILGIC products from the perspective of the individual investor. Our valuation technique can be applied to any ILGIC. From an empirical point of view, our main conclusions are that ILGICs becomes less attractive, the longer the time horizon of the investor. In addition, participating ILGICs are preferable to capped ILGICs for short term maturities and vice versa for longer maturities. A detailed analysis of two particular Canadian products was provided as an application of the basic concepts. We concluded by demonstrating that VPDs range from 95% - 98%, corresponding to a 3% - 5% expense ratio on Index Linked GICs.

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APPENDIX

Derivation of Valuation Formulas

With some algebra, we can transform the payoff from the **Capped-ILGIC**, in equation (1), to:

$$I\,exp\{g_c T\} + I\left[max\left[\frac{S_T}{S_0} - exp\{g_c T\},0\right] - max\left[\frac{S_T}{S_0} - exp\{(c-g_c)T\},0\right]\right] \quad (8)$$

or

$$I\,exp\{g_c T\} + I\left(\frac{1}{S_0}\right)[max[S_T - S_0 exp\{g_c T\},0] - max[S_T - S_0 exp\{(c-g_c)T\},0]] \quad (9)$$

Given a state-contingent payoff structure, we compute the No-Arbitrage price for the ILGIC using risk neutral pricing.

$$\begin{aligned} &I\,exp\{(g_c - r)T\} + I\left(\frac{1}{S_0}\right)BS(S_0, S_0 exp\{g_c T\}, T, q, r, \sigma) \\ &-I\left(\frac{1}{S_0}\right)BS(S_0, S_0 \exp\{(c-g_c)T\}, T, q, r, \sigma) \end{aligned} \quad (10)$$

where r is the risk free rate (by definition greater than the minimal guarantee g_k) and $BS(S, X, T, q, r, \sigma)$ is the Black and Scholes price of a call option, defined by:

$$BS(S, X, T, q, r, \sigma) = S\,exp\{-qT\}N(d_1) - X\,exp\{-rT\}N\{d_2) \quad (11)$$

where

$$d_1 = \frac{\ln\left[\frac{S}{X}\right] + \left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \qquad d_2 = \frac{\ln\left[\frac{S}{X}\right] + \left(r - g - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.$$

Plugging back into equation (10), simplifying, and then dividing by the initial investment I, we obtain the C-ILGIC value as a *percent of par*, equal to:

$$\begin{aligned} V_c(c, g_c) = &\; exp\{(g_c - r)T\} + exp\{-qT\}N(b_1) - exp((g_c - r)T\}N(b_2) \\ &-exp\{-qT\}N(b_3) + exp((c - g_c - r)T\}N(b_4) \end{aligned} \quad (12)$$

where,

$$b_1 = \left(\frac{r-q-g_c}{\sigma} + \frac{\sigma}{2}\right)\sqrt{T}, \quad b_2 = \left(\frac{r-q-g_c}{\sigma} - \frac{\sigma}{2}\right)\sqrt{T} \tag{13}$$

$$b_3 = \left(\frac{r-q-c-g_c}{\sigma} + \frac{\sigma}{2}\right)\sqrt{T}, \quad b_4 = \left(\frac{r-q-c-g_c}{\sigma} - \frac{\sigma}{2}\right)\sqrt{T}. \tag{14}$$

Next, let us examine the **Participating ILGIC**. With some algebra, we can transform the payoff from the P-ILGIC, in equation (2), to:

$$I exp\{g_p T\} + Ipmax\left[\frac{S_T}{S_0} - exp\{g_p T\},0\right] \tag{15}$$

which simplifies to:

$$I exp\{g_p T\} + I\frac{p}{S_0}max[S_T - S_0 exp\{g_p T\},0]. \tag{16}$$

Once again, we compute the No-Arbitrage price for the ILGIC using risk neutral pricing to obtain:

$$I exp\{(g_p - r)T\} + I\left(\frac{p}{S_0}\right)BS(S_0,S_0 exp\{g_p T\},T,q,r,\sigma) \tag{17}$$

and, finally we obtain the P-ILGIC value as a *percent of par*, equal to:

$$V_p(p,\, g_p) = exp\{(g_p - r)T\} + p\ exp\{-qT\}N(a_1) - p\ exp((g_p - r)T\}N(a_2) \tag{18}$$

where

$$a_1 = \left(\frac{r-q-g_p}{\sigma} + \frac{\sigma}{2}\right)\sqrt{T}, \quad a_2 = \left(\frac{r-q-g_p}{\sigma} - \frac{\sigma}{2}\right)\sqrt{T}. \tag{19}$$

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