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International equity diversification and shortfall risk

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Abstract

International equity diversification benefits Canadian investors very substantially by reducing shortfall risk, as shown by results of a model that minimizes the risk of shortfall from a desired consumption level for a retired investor with an unknown date of death and stochastic investment returns. It does not benefit American investors materially. The United States equity market is a large proportion of the international equity market that is available to individual investors, and United States returns are highly correlated with other markets. © 1999 Elsevier Science Inc. All rights reserved.

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1. Introduction

The benefits of international portfolio diversification have received considerable attention in the investment literature. Using data on major international markets, numerous studies have documented the benefits from the viewpoint of reducing total risk without sacrificing expected returns. A smaller number of papers have questioned the benefits of international diversification. In this paper we use a different technique to try to resolve this conflict.

Specifically, we use minimization of shortfall risk as the choice criterion for an individual investor who is retired, and hence has no further labour income to support consumption, only an endowment from which he or she can consume both principal and income. Our model

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allows the individual to allocate this endowment among various assets classes to determine which combination will provide the smallest risk of falling short of a pre-determined desired level of consumption. We compare the minimum shortfall risk portfolios for different possible choice sets. We find that a retired Canadian investor can do much better, where "better" means lower shortfall risk, by investing in a portfolio that includes international equity, compared with a portfolio that allows only domestic assets. By contrast, an American investor in the same situation (i.e. same age, gender, wealth and desired consumption) cannot reduce shortfall risk materially by investing in international equity. Our approach allows both the rate of return and the date of death of the individual to be stochastic.

2. Literature review

In one of the earliest studies, Grubel (1968) finds that international diversification pays off from the viewpoint of an American investor—a diversified portfolio of international stock indices dominates the United States index in terms of risk and return. In two related studies, Levy and Sarnat (1970) and Grubel and Fadner (1971) confirm this result and show that the Japanese and South African indexes play an important role in the efficient portfolios. Errunza (1977) extends the work to diversification in less-developed countries.

More recent studies confirm and extend these results: Adler and Dumas (1983), Bailey and Stulz (1990), Cosset and Suret (1995), Grauer and Hakansson (1987), Hunter and Coggin (1990), Jorion (1985, 1989), Levy and Lerman (1988) and Obstfeld (1994). Some studies have considered the situation of investors outside the United States. McDonald (1973) and Solnik (1979) find that international diversification is attractive for investors whose home countries are not the United States.

On the other hand, however, there are studies which question the benefit of international diversification. Sinquefield (1996) concludes that the empirical evidence for the 1970–94 period does not support the belief that the international equity market has higher expected returns than the American equity market, nor that it can substantially diversify American portfolios. Odier and Solnik (1993) show that the benefits depend in part on whether one expects cross-country correlations, market volatilities and currency risk to change. While currency risk remains a small, although significant, component of total equity risk, there is no easy formula for determining whether to hedge currency risk, or how much of it to hedge. Goldberg and Heflin (1995) suggest that increasing the degree of international involvement decreases systematic risk but increases total risk. Haavisto and Hansson (1992), in a study of Nordic stock markets, conclude that the optimal portfolios are extremely concentrated and include in general only Finnish and Swedish assets, and that a long-term investor would have done very well by keeping an unhedged and diversified Nordic portfolio.

These studies use the techniques of modern portfolio theory. They calculate the correlation coefficients of returns in various markets, and the returns of combined portfolios. The criterion for benefits of international diversification is related to the degree to which standard deviation of a combined portfolio is lower than a single country portfolio. Some studies apply a utility function explicitly.

3. Minimizing shortfall risk as a criterion

We use a different method to investigate the benefit of international diversification. We postulate that individual investors are likely to wish to minimize the probability of falling short of predetermined goals for wealth and consumption. The shortfall approach thus combines both an investor's objective and a measure of risk. Other papers have also recommended this method, including Liebowitz and Kogelman (1991), Milevsky et al. (1997) and Tse et al. (1993).

In this paper we focus on the asset allocation decision faced by a person who is retired and must live solely on the capital saved during the working life, and the income from it. The person has an unknown time remaining until death. The date of death is distributed according to standard mortality tables. The person has wealth, or capital, of *W*, and wishes to consume a fixed real amount, *C*, every year in retirement. The investment returns are uncertain and he or she will take this *C* first from income, and then from capital if income is insufficient. This person wishes to avoid shortfall in consumption every year, and therefore the objective is minimize the probability of being able to consume less than *C* in any year prior to death.

You could think of the problem in two equivalent forms. The retiree has a sum of money that fluctuates each year as it earns income and he consumes part of it. What is the probability that the money will run out before the date of death, given a distribution of investment returns? Alternatively, you could think of it as a stochastic present value problem. *W* is the initial investment. The future consumption is the uncertain future cash flow, with an uncertain number of years. What is the probability that the present value of the future consumption stream will be greater than *W*? In either case, the probability specified is the shortfall probability. A solution for the probability for any given distribution would also permit identification of the minimum shortfall risk portfolio for any number of assets.

This is a problem in stochastic optimization with two random variables—date of death and investment return. Milevsky et al. (1997) provide a solution in a form which requires simulation. Milevsky and Robinson (1999) derive an analytic solution using insights from the pricing of Asian options. The solution provides the probability of shortfall for any allocation among *n* investment assets, given:

- initial wealth, *W*
- the periodic amount to be withdrawn (*i.e.* principal + income) for consumption, *C*
- a mortality table or mortality function for the person
- the distribution of continuously-compounded investment returns for each class of assets.

The details appear in Milevsky and Robinson (1999). We include a summary of the derivation in Appendix 1. One useful feature of the solution is that the result is identical for all ratios of *W/C*, regardless of the size of *W* and *C*. Accordingly, we will refer to *W/C* in the rest of the paper, rather than separate values. The reader should remember that this refers to initial wealth, and fixed real consumption every year until death. It is not a constant ratio every year, because *W* is variable.

To visualize what a wealth to consumption ratio might mean, and what values might be reasonable, let us consider how much a person might need to consume per year in retirement.

We are abstracting from the form of the income—taxes are ignored, pensions are implicitly capitalized, etc. Suppose you want your financial capital to provide $30,000 per annum in real dollars, until you die. If you have $360,000 now, $W/C = 12$, which is the lowest value we will provide in our empirical section. If you have $720,000, $W/C = 24$, which is the highest value we will use. The absolute level of wealth is not relevant in how investments should be allocated, but rather the wealth relative to what you require from it. To look at it another way, suppose you are exactly a millionaire at age 65. The popular view is that a millionaire can enjoy a luxurious retirement lifestyle. A modest consumption of $50,000 per annum is equivalent to $W/C = 20$, and as we shall see, this supposedly modest income for a millionaire is not all that secure.

We emphasize that the values are in real dollars, and that both income and principal are consumed—no bequest is assumed. The value of W at any future time is random, with the randomness coming from the random return function. If inflation is positive, the actual amount drawn out in nominal dollars increases every year. If you do consume approximately C, then on average the future values of W/C will decline for most situations. That is, most people will be obliged to consume part of the principal every year, and therefore as their wealth declines, so does W/C.

As a practical matter, we will work with only three assets at a time in this paper, one risk-free and two risky. Presentation of allocations among more than three assets on a two-dimensional piece of paper is very confusing and consumes far more space without adding further understanding of the questions we consider in this paper. An earlier version of this paper used the simulation method and two assets, but we reached the same conclusions.

4. International equity diversification and shortfall risk in different countries

We want to answer three related questions:

1. Does international equity diversification reduce shortfall risk for Canadians and is it material?
2. Does international equity diversification reduce shortfall risk for Americans and is the reduction material?
3. Is international diversification relatively less effective in reducing shortfall risk for Americans than it is for Canadians?

The answer to the first two questions is very likely positive to some extent, since diversification reduces total risk as long as the assets are imperfectly correlated. If the mean returns are too low on one asset, shortfall risk may not change even when it is introduced into the opportunity set.

4.1. The data

For Canada, in Canadian $:

- T-bills: annualized rate of return of 91-day Canadian treasury bills

Table 1
Continuously compounded annual real rates of return[a]

	Canadian		American	
	Mean	Standard deviation	Mean	Standard deviation
T-bills	0.026	0.030	0.013	0.022
Bonds	0.039	0.105	0.019	0.112
TSE	0.051	0.156		
S & P	0.076	0.163	0.062	0.161
World	0.070	0.170	0.059	0.169
EAFE	0.071	0.214	0.065	0.213

Correlation Matrices

Canada	Bonds	TSE	S & P	World
TSE	0.27			
S & P	0.55	0.79		
World	0.50	0.80	0.84	
EAFE	0.32	0.69	0.60	0.92

United States	Bonds	S & P	World
S & P	0.37		
World	0.47	0.87	
EAFE	0.34	0.64	0.93

[a] The returns and correlation coefficients are 1970–1997 for World and EAFE indexes; 1957–1997 for the others. Scotia McLeod, Inc. and Morgan Stanley Capital International, Inc. provided the raw returns from which the distributions are calculated.

- Bonds: Scotia McLeod long bond index, total rate of return
- TSE: Toronto Stock Exchange 300 index, total rate of return
- S&P: Standard and Poor's 500 index, total rate of return
- World: Morgan Stanley Capital International (MSCI) global portfolio total rate of return.
- EAFE: MSCI's index portfolio for Europe, Australia, Far East, total rate of return.

For the United States, in United States $:

- T-bills: annualized rate of return of three month United States treasury bills
- Bonds: 30 year federal bonds, total rate of return
- S&P: Standard and Poor's 500 index, total rate of return
- World: Morgan Stanley Capital International (MSCI) global portfolio total rate of return.
- EAFE: MSCI's index portfolio for Europe, Australia, Far East, total rate of return.

We use annual rates of return and convert them to real rates using the Consumer Price Index of the respective countries. We show the continuously-compounded mean real rates of return, standard deviations and correlations for the period 1957–1997 in Table 1. Although the standard deviation is non-zero for T-bills, we will still use it as the risk-free rate in our analysis, since it is the closest proxy we have. The international indexes offer much higher returns than Canadian equity, and somewhat higher volatility.

The distribution of the date of death is drawn from the 1996 Individual Annuity Mortality—Basic (IAM) tables, which insurance companies in both the United States and Canada use when pricing individual life annuities. We have smoothed it using a Gompertz distribution, as developed in Milevsky and Robinson (1999). We use separate tables for males and females. Mortality tables exist for many finer distinctions like smoker vs. non-smoker and different provinces. If we used different tables we would get different estimates of the shortfall probabilities, but the conclusions of the paper would be unchanged, unless we were using a table for some group with very short life expectancies, relative to the general population. The IAM tables assume adverse selection and hence the mortality function assigns higher probabilities to longer life than do standard tables for the entire population in either Canada or the United States.

4.2. Empirical results

Using the shortfall model and the data already described, we estimated the shortfall probabilities for a variety of the parameters: *W/C*, age and gender. The asset allocations were in five percentage point units. Thus, for one person we would calculate shortfall for all the possible asset allocations of three different assets adding to 100%, with each asset having in turn allocations by five percentage increments from 0 to 100%. This procedure generates an enormous amount of data very quickly. The results are quite consistent in the evidence they provide on the three questions posed earlier. We present the results in 20% increments for each asset and for each country for a single situation: a female aged 65 with $W/C = 16$. Then, to summarize a wider range of situations, we present the minimum shortfall risk probability (MSP, hence) and the allocation that yields that probability, for a variety of parameters.

Leaving aside the question of which equity portfolio to choose and which country the investor lives in, we first summarize some results concerning the choice among T-bills, bonds and equity (whether domestic or foreign). These results are similar to those in Milevsky et al. (1997), which looked at Canadian equity, T-bills and bonds. The risk premium of equity over T-bills was somewhat higher in the 1997 paper, and yet the general results are comparable, showing that the conclusions are fairly robust to changes in the estimates of the return distribution.

- Women have higher shortfall probabilities than men, all else constant.
- The allocation to equity that minimizes shortfall decreases as age and *W/C* rise. Men and women still require at least some equity in their portfolios unless they are very old and/or have a high *W/C* ratio.
- A 100% allocation to T-bills or bonds has a higher shortfall risk than a 100% allocation to equity for all but the oldest people or higher values of *W/C*.
- Changes that move a portfolio towards the MSP reduce shortfall risk at a declining rate. If a portfolio has proportions that are close to the MSP, then the difference in shortfall probabilities between it and the MSP portfolio is quite small. For example, if your minimum risk occurs at 50% equity, and your current portfolio is 100% T-bills, every

Table 2
Asset allocation and shortfall risk for a Canadian woman, aged 65, with *W/C* = 16[a]

TSE	0%	20%	40%	60%	80%	100%
A: Bonds[b]						
0%	0.488	0.428	0.393	0.383	0.388	0.405
20	0.455	0.401	0.374	0.369	0.378	
40	0.433	0.386	0.365	0.363		
60	0.421	0.381	0.364			
80	0.419	0.384				
100	0.423					
B: World equity[c]						
0%	0.488	0.428	0.393	0.383	0.388	0.405
20	0.378	0.346	0.339	0.348	0.368	
40	0.311	0.306	0.316	0.337		
60	0.283	0.293	0.314			
80	0.278	0.297				
100	0.288					
C: S & P 500[d]						
0%	0.488	0.428	0.393	0.383	0.388	0.405
20	0.361	0.330	0.324	0.334	0.355	
40	0.280	0.276	0.289	0.312		
60	0.241	0.254	0.277			
80	0.229	0.250				
100	0.232					

[a] The column headings at the top of each panel show the allocation to the TSE 300. The row headings are the allocations to bonds, World Equity index and S & P 500, respectively, in Panels A, B, and C. The allocation to T-bills is 100% − column heading% − row heading%. The entries are the shortfall probabilities for the given allocations.

[b] Minimum shortfall probability = 36.1% TSE 50%; bonds 45%; T-bills 5%.

[c] Minimum shortfall probability = 27.8% TSE 0%; World 75%; T-bills 25%.

[d] Minimum shortfall probability = 22.9% TSE 0%; S & P 500 85%; T-bills 15%.

5% you switch into equity up to 50% will decrease your risk, but at a decreasing rate. The change from 100% T-bills to 95% T-bills and 5% equity will reduce shortfall risk by a greater amount than will a change from 55% T-bills, 45% equity, to a 50%–50% portfolio.

Table 2 summarizes some results for a female aged 65, with *W/C* = 16. In panel A she is a Canadian choosing among Canadian T-bills, bonds and equity (TSE 300). The column headings are the percentage of the portfolio allocated to equity; the row headings are the allocation to bonds. The allocation to T-bills is (100%–equity%–bonds%). Each entry in the table is the probability of shortfall for that particular asset allocation. The MSP and the allocation that produces it, to the nearest 5% increment, are in the lower right hand. For example, a portfolio invested 40% in the TSE 300, 40% in bonds and 20% in T-bills has a shortfall probability of 36.5%.

Panel B replaces bonds with the World equity index. All the shortfall probabilities for allocations that include world equity are lower. The MSP is reduced from 36.1% (50% TSE, 45% Bonds, 5% T-bills) to 27.8% (75% World, 25% T-bills). This is a material change in both asset allocation and shortfall probability. Panel C provides an even greater shortfall risk reduction when the S & P 500 is substituted for Canadian bonds and the MSP falls to 22.9%. In both international substitutions, the TSE is dominated for a 65 year old woman with W/C = 16.

We have not shown the result if the choice were among EAFE, TSE and T-bills, but it lies between the others. The MSP is 32.9% for an allocation of TSE 15%; EAFE 45%; T-bills 40%.

The reader may suspect that the results are partly because we dropped the long bonds out of the picture. In fact, the MSP allocation is identical when the S & P 500 is the international index and either long bonds or the TSE is included. That is, the MSP is 22.9% with an allocation of: 0% bonds *or* TSE; 85% S & P 500; 15% T-bills.

We are not aware of any metric for comparing the relative benefits of different portfolios in reducing shortfall risk; so we rely upon common sense in determining what is a material improvement. Our judgement is that international equity diversification provides a material benefit to Canadian investors. The shortfall probabilities would be reduced somewhat more for at least some situations (i.e. for different ages, gender and values of *W/C*) if we included more than three assets. The expansion would be quite difficult to present and would not change the answer to the questions we are asking.

Table 3 displays the same three panels as Table 2, but now calculated for an American woman, aged 65, with *W/C* = 16. Panel A is the domestic portfolio choice among T-bills, long-term bonds and the S & P 500. Panel B replaces the bonds with the World portfolio and Panel C replaces the bonds with the EAFE portfolio.

The allocations in Panel A that include a substantial amount of bonds and/or T-bills have higher shortfall probabilities than the comparable Canadian entries. The real return to bonds and T-bills in the United States has been significantly lower than it has in Canada, and the standard deviation on the bonds is higher in the United States than in Canada. The S & P 500 has done much better than the TSE 300, however, whether you measure it in Canadian $ or United States $, and so the risk premium in the United States is much higher than in Canada. The MSP for the domestic allocation is slightly lower than in Canada at 32.7%, and it requires 100% invested in the S & P 500.

When we turn to Panels B and C, we find that international diversification has an effect, but it is material only when the person holds a portfolio very distant from the minimum shortfall risk portfolio. For example, in Panel B, a woman investor who holds 40% S & P, 40% bonds and 20% T-bills faces a shortfall probability of 44.5%. By replacing the bonds with the World equity index, she reduces the shortfall probability to 34.3%. However, the MSP is 32.6% with an allocation of: S & P 90, World 10%; T-bills 0%. This portfolio is scarcely distinguishable from the domestic case where the MSP is 32.7% with 100% invested in the S & P 500.

EAFE offers higher return and standard deviation than the S & P 500 or the World portfolio, and is less-correlated with the S & P 500 than is the World portfolio, because it does not include American equity, as does the World index. It provides only a small improvement in the MSP to 31.9% for an allocation of S & P 500 80%, EAFE 20%.

If we take minimizing shortfall risk as the objective, then international diversification appears to have minimal benefits for an American woman at age 65, with moderate wealth

Table 3
Asset allocation and shortfall risk for an American woman, aged 65, with $W/C = 16$[a]

S & P	0%	20%	40%	60%	80%	100%
A: Bonds[b]						
0%	0.661	0.540	0.438	0.373	0.339	0.327
20	0.648	0.531	0.437	0.379	0.349	
40	0.638	0.529	0.445	0.392		
60	0.633	0.534	0.458			
80	0.632	0.544				
100	0.637					
B: World[c]						
0%	0.661	0.540	0.438	0.373	0.339	0.327
20	0.548	0.445	0.377	0.341	0.327	
40	0.456	0.386	0.348	0.331		
60	0.400	0.359	0.340			
80	0.374	0.351				
100	0.367					
C: EAFE[d]						
0%	0.661	0.540	0.438	0.373	0.339	0.327
20	0.537	0.435	0.368	0.333	0.319	
40	0.450	0.382	0.343	0.325		
60	0.411	0.368	0.346			
80	0.404	0.378				
100	0.419					

[a] The column headings at the top of each panel show the allocation to the S & P 500. The row headings are the allocations to bonds, World equity index and EAFE index, respectively, in Panels A, B, and C. The allocation to T-bills is 100% − column heading% − row heading%. The entries are the shortfall probabilities for the given allocations.

[b] Minimum shortfall probability = 32.7% S & P 100%; bonds 0%; T-bills 0%.

[c] Minimum shortfall probability = 32.6% S & P 90%; World 10%; T-bills 0%.

[d] Minimum shortfall probability = 31.9% S & P 80%; EAFE 20%; T-bills 0%.

relative to desired consumption, but substantial benefits for an identical Canadian woman. In Table 4 we focus on MSP portfolios for a range of age and *W/C* and both males and females to show that this observation holds more generally.

The entries in Table 4 display the MSP allocations with the MSP in parentheses underneath each allocation. We provide results for males and females aged 65 and 75, with *W/C* ratios of 12, 16, 20 and 24. The investors can choose between two sets of assets in each country. In Canada, the choices are either (1) domestic equity, bonds, bills, or, (2) domestic equity, S & P 500, bills. In the US, the choices are either (1) domestic equity, bonds, bills, or, (2) domestic equity, EAFE index, bills. In each case we chose the non-domestic portfolio that gives the best diversification results, but other choices would not materially affect our conclusions.

Table 4 shows the same pattern as Tables 2 and 3 showed. The Canadian investor changes his or her portfolio allocation a great deal and reduces shortfall risk significantly when the S & P 500 replaces bonds in the available assets. The American investor makes almost no change to the allocation and the shortfall risk falls very little, when the EAFE index replaces bonds. This is despite the fact that the Canadian bond index has outperformed the American

Table 4
Asset allocations yielding minimum shortfall probability (MSP)[a]

	Canada		United States	
	TSE-Bonds-Bills in % (MSP)	TSE-S & P-Bills (MSP)	S & P-Bonds-Bills (MSP)	S & P-EAFE-Bills (MSP)
Age 65				
W/C = 12				
Male	60-40-0 (0.48)	0-100-0 (0.34)	100-0-0 (0.43)	75-25-0 (0.43)
Female	85-15-0 (0.64)	0-100-0 (0.45)	100-0-0 (0.57)	75-25-0 (0.56)
W/C = 16				
Male	40-40-20 (0.24)	0-75-25 (0.16)	90-0-10 (0.23)	75-20-5 (0.22)
Female	50-50-0 (0.36)	0-85-15 (0.23)	100-0-0 (0.33)	80-20-0 (0.32)
W/C = 20				
Male	30-25-45 (0.11)	0-60-40 (0.07)	70-0-30 (0.12)	55-20-25 (0.12)
Female	30-30-40 (0.17)	0-60-40 (0.10)	80-0-20 (0.18)	65-20-15 (0.18)
W/C = 24				
Male	25-20-55 (0.05)	0-55-45 (0.03)	60-0-40 (0.06)	50-10-40 (0.06)
Famale	25-25-50 (0.07)	0-55-45 (0.04)	60-0-40 (0.09)	50-15-35 (0.09)
Age 75				
W/C = 12				
Male	40-35-25 (0.22)	0-80-20 (0.16)	90-0-10 (0.21)	75-15-10 (0.21)
Female	50-50-0 (0.35)	0-90-10 (0.25)	100-0-0 (0.33)	80-20-0 (0.32)
W/C = 16				
Male	30-30-40 (0.09)	0-65-35 (0.06)	75-0-25 (0.09)	60-15-25 (0.09)
Female	30-30-40 (0.15)	0-70-30 (0.10)	80-0-20 (0.15)	65-15-20 (0.15)
W/C = 20				
Male	25-25-50 (0.04)	0-55-45 (0.03)	60-0-40 (0.04)	50-15-35 (0.04)
Female	25-25-50 (0.06)	0-55-45 (0.04)	60-0-40 (0.07)	50-15-35 (0.07)
W/C = 24				
Male	25-25-50 (0.02)	0-50-50 (0.01)	60-0-40 (0.02)	50-10-40 (0.02)
Female	25-25-50 (0.03)	0-50-50 (0.02)	55-0-45 (0.04)	45-15-40 (0.03)

[a] The four columns display the asset allocations that yield MSPs for four different choice sets for an individual investor. The individual entries show the allocations to the asset classes in the column heading, in percent, with the shortfall probability in parentheses underneath it. The first two columns are Canada: domestic equity, bonds, and bills; domestic equity, S & P 500, and bills. The second two columns are the United States with the same domestic choice and with the EAFE index replacing bonds for the last column. The rows are different investors. The first set are aged 65, male and female, with different *W/C* ratios. The second set are aged 75.

bond index (in terms of domestic currency) very substantially, and therefore the international substitution for an American investor is replacing a very low return asset in the choice set.

5. Discussion

These results should not surprise us. The United States equity market is a substantial part of the international equity market, while Canada is only about 3%. Furthermore, the performance of American companies in the index is correlated with economic performance globally, because many of the largest American companies that make up the index have subsidiaries in other countries. Therefore, the diversification that the international equity can provide to Americans is quite limited. Canada has more of its economy based on international trade than the United States, because the Canadian economy is based on commodities. However, the Canadian companies in the TSE 300 have fewer operations outside Canada, and hence their returns are less-correlated with the equity returns elsewhere. Furthermore, the TSE 300 has a heavier weighting in natural resources (forest products, mining, oil and gas) than the S & P 500 or any other significant national or international index.

One odd, and unexpected, part of the results is the role of the debt securities. United States debt securities have much lower realized rates of return than Canadian. International debt rating agencies give American government slightly higher debt ratings than Canadian government debt, which seems reasonable. However, if debt ratings in the very long run are to accurately predict returns, there must be some difference in the realized default rate, and this has not been the case. As a result, the Canadian domestic investor has received much higher yields on domestic debt, but can also invest in United States or international equity to benefit from diversification. It seems that perhaps the research should focus on international debt diversification, but that is also problematic. The United States dollar has been generally rising against the Canadian dollar for a long time, and hence an American investor would have received much lower returns from Canadian debt.

We seem to have created a paradox in these results, therefore, since it seems that Canadians can do better than Americans, even though all could access the same choice set. The paradox is resolved if purchasing power parity holds. In that case, a Canadian retiree would require a higher *W/C* ratio than an American retiree in order to enjoy the same real standard of living. Whether this is true we leave to the economists to debate.

Our use of shortfall risk as a criterion produces results that are similar to utility maximization with increasing relative and absolute risk aversion. To see this, consider holding constant the age, gender and desired consumption, *C*. The only thing that varies is *W*, which will thus cause *W/C* to vary. Increasing wealth in this situation leads to lower allocations to equity, as the reader can see by reading down the columns in Table 4. Thus, a very wealthy person, relative to his or her desired consumption, should invest primarily in T-bills, if the criterion is to minimize shortfall risk.

In finance theory, it is generally assumed that individuals have decreasing or constant relative risk aversion. In practice, our result seems counter-intuitive to what we generally believe wealthy individuals do. If the wealthy person also has a bequest motive, then increasing equity investment is preferred, as Milevsky et al. (1997) show.

Thus, our shortfall model may not be a good normative model for persons who are very rich relative to their level of consumption. The very rich do not have to worry about survival. For the great majority of people, for whom shortfall below the desired level of consumption during retirement is a realistic possibility, the shortfall criterion appears to provide reasonable results for asset allocation.

Another way to look at the results is to focus on what income might be sustainable from a given wealth, with reasonable assurance. As long as the individual's wealth is strictly less than the risk-free discounted value of perpetual consumption, there is always a non-zero probability of ruin. Returning to Table 4, the entries for $W/C = 20$ or 24 show shortfall probabilities of 2–7%, which we might consider as fairly secure.The asset allocations in this situation are balanced, on the order of 50% debt securities, 50% equity; although the specific form of debt or equity differs.

6. Limitations of the study and directions for future research

The biggest question mark is the validity of the return distributions we use. These are historic, real rates, and they are reasonably accurate representations of what investors would have realized over long time periods in the past. Jorion and Goetzmann (1999) present results of an investigation into long run rates of return and stability of those rates for many countries, including Canada and the United States. While they characterize the United States as practically the only country with a very long-run stable series of equity returns, Canada certainly has a substantial history as well, longer than most other countries. Nonetheless, these returns histories are taken over time periods in which the underlying economies were changing. We cannot have much confidence that the past observations are drawings from the returns distribution that applies now and in the future, even in Canada and the United States.

We have no alternative, unfortunately. A forecast of long-run rates of return and the covariance matrix of them is no more valid, since it is ultimately conditioned on what evidence we have from the past. This problem affects a great deal of the research in investments and personal finance, and we simply have to be aware that the shortfall probabilities we calculate are dependent on the validity of the returns distributions that we assume.

Gibson (1996) suggests an heuristic method, which is to use the historical data to estimate the variance-covariance matrix, and then use a forecast of expected returns. We know of no forecast that is necessarily more accurate or reliable than using historical data for all the parameters. Our model uses very long run returns, and we have never seen forecasts for such long periods. We are dealing with periods that could be as much as 40 years (a retiree aged 65 living to the age of 105). Historical data has the advantage that it is at least objective. In addition, any return forecast must be mathematically consistent with the variance-covariance matrix, or the results from any analysis will be invalid.

There are other limitations that could perhaps be overcome:

- All the returns are before transactions costs. Transactions costs would reduce the gross returns of every portfolio, but perhaps not by equal percentages.

- All returns are before income tax. Taxes have no effect inside sheltered retirement portfolios (IRA or 401k in the United States; RRSP and RRIF in Canada). For investment outside a shelter in Canada, interest income is taxed at a higher rate than international equity, which in turn is taxed at a higher rate than Canadian equity. This would favour domestic equity even more strongly, but might reduce or even eliminate the benefit of international diversification.

While we do not believe that the omission of transactions costs and taxes would change the conclusions of the paper, a formal modelling of these two factors is the obvious direction for future research on this question.

7. Conclusion

The evidence is very strong that international equity diversification reduces shortfall risk for Canadians significantly. It does not appear to benefit Americans materially, because their equity portfolio is already closely-related to the international equity portfolio.

Finally, we note that our results apply to investors atomistically in Canada. If every Canadian individual investor read this paper and immediately shifted his or her portfolios into international equity, we have no idea how much effect that might have on Canadian markets, if any.

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Appendix: derivation of the probability of shortfall

The probability of shortfall (ruin) is derived in Milevsky and Robinson (1999). However, for the sake of completeness, we reproduce here the formula and refer the interested reader to the above-referenced paper. (Also available for download at the website: www.yorku.ca/academics/milevsky)

The probability is computed in three stages. First, we must compute the present value of a life annuity, denoted by $A(x)$, where the discounting is done at the rate of x. This calculation must be conducted using the appropriate insurance mortality table.

We then define the two new variables:

$$M_1 = A(\mu - \sigma^2), \quad M_2 = \frac{A(\mu - \sigma^2) - A(2\mu - 3\sigma^2)}{\frac{\mu}{2} - \sigma^2} \tag{1}$$

Where μ, σ are the portfolio mean and standard deviation respectively. The technical term for these quantities is the first and second moment of the stochastic present value.

The next step is to compute:

$$\alpha = \frac{2M_2 - M_1^2}{M_2 - M_1^2}, \quad \beta = \frac{M_2 - M_1^2}{M_2 M_1} \tag{2}$$

Finally, the probability of shortfall is:

$$P(Ruin) \approx G\left(\frac{c}{w} \;\middle|\; \alpha,\beta\right) \tag{3}$$

where c/w is the consumption to wealth ratio, and the $G(y|\alpha,\beta)$, denotes the cumulative distribution function (CDF) of the Gamma random variable. This function is readily available in all commercial spreadsheets such as Excel or Lotus and can be easily implemented in practice. In fact, the G(.) term is quite similar to the N(.) function, which should be well known to the users of the Black-Scholes equation. Where N(.) represents the area-under-the-curve for the standard normal distribution, G(.) represents the same concept for the Gamma distribution. In our case, the Gamma density is parameterized by two variables (similar to a mean and variance), which we denote by the greek letters alpha and beta. The Gamma distribution can take on only positive values and can be implemented also in terms of the Chi-square distribution. Once again, from a slightly more technical perspective, the inverse of the stochastic present value (SPV) of lifetime consumption is taken to be Gamma-distributed. Therefore, the probability of ruin is the probability that the SPV is greater than the initial wealth that is available to support the consumption.

It is important to note that the above formula is only an *approximation* to the true probability of shortfall. The exact expression can not be obtained analytically—to the best of the authors' knowledge. However, the approximation is quite accurate when benchmarked against Monte Carlo simulations. Furthermore, one can actually demonstrate that as the time horizon of the investor increases, the expression actually *converges* to the true probability of ruin. We refer the interested reader to Milevsky and Robinson (1999).

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