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Effective teaching and use of the constant growth dividend discount model

Thomas H. Payne[a], J. Howard Finch[b,*]

[a]*Department of Finance, The University of Tennessee at Martin, Martin, TN 38238, USA*
[b]*Department of Finance, The University of Tennessee at Chattanooga, 615 McCallie Avenue, Chattanooga, TN 37403, USA*

Abstract

The appropriate application of the constant growth dividend discount model (DDM) requires an understanding of the fundamental nature of the model and its parameters. It is important that students not only be able to mechanically "plug and chug" the formula, but that they also understand the model's assumptions, inputs, sensitivity to error and practical limitations. This paper demonstrates that the valuation measure derived from using the DDM is very sensitive to the relationship between the required return on investment (K_s) and the assumed growth rate (g) in earnings and dividends. Examples show that the valuation error increases at an increasing rate when the values of K_s and g converge in the formula. Classroom experience has indicated that students believe and strive to compute a single "correct" valuation of the share price. They should realize that the goal of valuation analysis is to estimate a reasonable range for the intrinsic value of a share price, rather than a single point estimate as often implied by end-of-chapter and exam-type problems using the DDM. © 1999 Elsevier Science Inc. All rights reserved.

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1. Introduction

The theoretical soundness and practical simplicity of the constant growth dividend discount model (DDM) (Gordon & Shapiro, 1956; Gordon, 1962) have led to its extensive

* Corresponding author. Tel.: +1-423-755-5250; fax: +1-423-785-2329.
E-mail address: Howard-Finch@utc.edu (J.H. Finch).

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application for common stock valuation. Students are usually introduced to the formula and its conceptual framework in their first finance course. In more advanced courses, they apply the model to security analysis, to cases involving security issuance and mergers and acquisitions, and to other valuation related problems. While most students eventually become comfortable with mechanically "plugging and chugging" the formula, they often have little understanding of its practical limitations. This paper provides a method for illustrating the nature of estimation and a means of demonstrating the nonlinear sensitivity of the DDM to variations in required rate of return (K_s) and the growth rate (g) estimates.

Nearly all texts discuss the basic assumption that K_s must be greater than g for the model to hold. This requirement is predicated on the practical limitation that a stock's price must be non-negative. Similarly, K_s must be greater than g since equivalence would result in an infinite value. Together, the economic constraints that stock prices are non-negative and finite make the model's assumption of $K_s > g$ fairly easy for students to grasp. What is not necessarily intuitive is how the relationship between K_s and g affects the estimate of the stock's intrinsic value.

Once the mathematical and economic constraints of the model have been considered, students should realize that proper implementation of the DDM requires more than the calculation of a single point estimate. Students, expecting to arrive at a "right answer," often fail to recognize the distinction between an estimate and a known price. This is not a trivial point since students often have the misconception than any answer arrived at mathematically represents absolute truth. For all applications of the model, but particularly in case studies, students should be required to provide an in-depth explanation of all assumptions, parameter estimation methods, and conclusions. Only in this way will they develop a true grasp of the model and its limitations.

2. Background

Valuation error resulting from implementation of various forms of the DDM has been addressed in numerous studies. Jacobs and Levy (1988) note that the DDM expected returns are not generally predictive and sometimes negatively correlated with actual returns. Hickman and Petry (1990) find that dividend discount approaches produce errors averaging 88% of the actual price, and 4.21 times those of price-earnings methods. In addition, a high degree of error in growth and required return estimates is found irrespective of specific modeling assumptions. Gehr (1992), noting that price estimation bias in the DDM is the result of required return and growth prediction error, proposes application of a probability weighted range of the parameter estimates. Finally, Good (1989) points out that, since the next period dividend is largely a known quantity, the reliability of the DDM is primarily dependent on the estimation of required return and growth rates. It is this very point which requires increased emphasis during classroom discussions of the DDM.

3. Sensitivity analysis

Formally, the constant growth dividend discount model is given by:

$$V_0 = D_1/(K_s - g) \tag{1}$$

Table 1
The constant growth dividend discount model: the effect of changes in $z = K_s - g$ on value estimation ($D_1 = \$1.50$)

$z = K_s - g$	Value estimate (V_0)	Absolute difference (percentage difference)	
		Incremental	Cumulative[a]
14%	\$10.71	–	–
12%	\$12.50	\$1.79 (16.7%)	\$1.79 (16.7%)
10%	\$15.00	\$2.50 (20%)	\$4.29 (40%)
8%	\$18.75	\$3.75 (25%)	\$8.04 (75%)
6%	\$25.00	\$6.25 (33%)	\$14.29 (133%)
4%	\$37.50	\$12.50 (50%)	\$26.79 (250%)
2%	\$75.00	\$37.50 (100%)	\$64.29 (600%)

[a] Cumulative differences are computed relative to the initial ($z = 14\%$) value. Incremental differences are relative to immediately preceding values.

where V_0 is the estimated intrinsic value per share, K_s is the required rate of return, g is the forecasted growth rate in earnings and dividends (assuming a constant payout ratio), and D_1 is the forecasted next period dividend. The assumptions of this model are that g will be at a constant rate for the foreseeable future, and that $K_s > g$.

Following Good (1989), practical application of this model requires the estimation of two key inputs, K_s and g. Because many texts first cover the valuation formula and only later the various techniques to determine these inputs, students often miss the key relationship in the denominator of the formula. For illustrative purposes, define this relationship by:

$$z = (K_s - g) \tag{2}$$

Consider a hypothetical valuation problem where the expected next period dividend (D_1) = \$1.50, $K_s = 22\%$, and $g = 8\%$. Then $z = 14\%$. Applying the DDM, students will estimate the share price by:

$$V_0 = (\$1.50)/(.22 - .08)$$

$$V_0 = \$10.71$$

Many students will instinctively stop here, assuming the problem is "finished" (and often laboring to check their "answer" with that of a classmate or solutions manual). However, it is important to convey that the value of V_0 is an *estimate* of intrinsic value and to make clear that it is not the "price" of the stock. Table 1 extends the example problem, allowing for different estimates of K_s and g. Note that, as the difference z gets smaller, the resulting estimate of share value grows larger. Specifically, as K_s and g converge, the valuation estimate increases *geometrically*. Fig. 1 illustrates this relationship graphically.

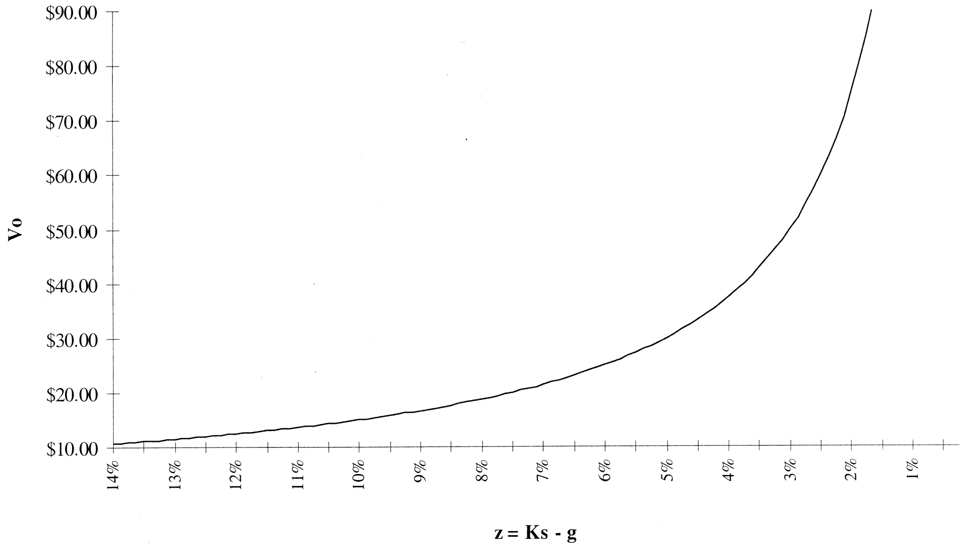

Fig. 1. Valuation estimate as K_s and g converge.

Suppose the market is in equilibrium, so that the actual value per share (which is unknown) is, in fact, the original estimate of $10.71. Fig. 1 shows that as z gets smaller, valuation estimate increases at an increasing rate. Thus, students should be aware of how sensitive the model's estimate of intrinsic value is to the relationship between K_s and g.

4. Moving from textbook examples to "real-world" applications

Investment textbooks and other resources illustrating financial analysis techniques often provide the necessary model parameters. In other words, the numbers are "given" and it is left to the "analyst" to simply perform the math. As an example, suppose a class was given the following data to use in the estimation of the intrinsic value for a firm's share price.

4.1. Information for computing the required rate of return on investment

The risk-free rate of return in the market is 5%, the expected return for the market is 14%, and this firm's beta is 1.10. The yield to maturity on its long term outstanding bonds is 8.25%, and management estimates an 8% premium for stocks over bonds.

4.2. Information for estimating the growth rate for earnings and dividends

In 1996 the firm's earnings per share (EPS) was $2.00, and in 1999 EPS was $2.32. The firm maintains a 50% dividend payout ratio, and 1999 return on equity (ROE) was 11%. Analysts who follow the firm's stock estimate earnings growth of 6 to 8% in the next few years.

Then students, following textbook methodology, should estimate the DDM parameters as follows:

4.2.1. Estimating K_s

Method 1 – the CAPM:

$$K_s = r_f + [K_m - r_f]ß \tag{3}$$

$$K_s = 5\% + [14 - 5\%]1.10$$

$$K_s = 14.90\%$$

Method 2 – Bond Yield plus Risk Premium Approach:

$$K_s = \text{YTM} + \text{Risk Premium} \tag{4}$$

$$K_s = 8.25\% + 8\%$$

$$K_s = 16.25\%$$

So, the average estimated required return would be:

$$K_s = (14.9\% + 16.25\%)/2 = 15.58\%$$

4.2.2. Estimating g

Method 1 – Retention Growth Model:

$$g = b(\text{ROE}) \tag{5}$$

$$g = .50(11\%)$$

$$g = 5.5\%$$

Method 2 – Point to Point Estimate:

$$FV = PV(1 + g)^n \tag{6}$$

$$\$2.32 = \$2.00(1 + g)^3$$

$$g = 5.07\%$$

Method 3 – Analysts' Forecast:
Low Analyst Estimate: $g = 6\%$
High Analyst Estimate: $g = 8\%$
Average Analyst Estimate: $g = 7\%$

Giving equal weight to each technique, the average estimated growth rate in earnings and dividends is: $g = (5.5\% + 5.07\% + 7\%)/3 = 5.85\%$

Applying the DDM, we know that last year's dividend D_0 was \$2.32 (0.50) = \$1.16, giving a share price estimate of:

$$V_0 = \frac{\$1.16(1.0585)}{.1558 - .0585}$$

$$V_0 = \frac{1.2279}{.0973}$$

$$V_0 = \$12.62$$

Investment texts and other resources that outline these various methodologies generally do so in a very discrete fashion. They do not adequately address the range of estimates arrived when different methodologies and assumptions are applied to the same problem. The main point which students should be aware of is how sensitive this estimate of intrinsic value is to z, the relationship between the estimates of K_s and g. Reworking the problem, using both the highest and lowest input estimates, gives the following range of values for V_0:

$$V_0 = \frac{\$1.16(1.08)}{.1625 - .08} = \$15.19$$

$$V_0 = \frac{\$1.16(1.0507)}{.149 - .0507} = \$12.39$$

Students, working on assigned problems or an exam problem, will likely panic at these divergent results, but a practicing security analyst would be very comfortable with a conclusion such as “We feel the shares to be fairly valued in the $12 to $15 range.”

5. Non-constant dividend growth

Another application of the DDM regularly covered in finance courses is the nonconstant, or two-stage growth model. This method simply incorporates multiple growth rates into the share valuation analysis.

The nonconstant growth model involves three consecutive steps: 1) estimate the dividends in the high growth period(s) individually, 2) estimate the share price at the final growth phase using the constant growth DDM, and 3) discount the future cash flow stream back to the present at the required rate of return and sum.

Changing the previous problem slightly, suppose the firm in the example is expected to grow earnings and dividends at a 15% annual rate over the next three years, with subsequent growth slowing to the previously estimated average rate of 5.85% annually. Then the dividends for the next three years are forecasted to be

$$D_1 = \$1.16(1.15) = \$1.33$$

$$D_2 = \$1.33(1.15) = \$1.53$$

$$D_3 = \$1.53(1.15) = \$1.76$$

The estimate of share price would then be given by:

$$V_0 = \frac{1.33}{1.1558} + \frac{1.53}{(1.1558)^2} + \frac{1.76}{(1.1558)^3} + \frac{1.76(1.0585)/(.1558 - .0585)}{(1.1558)^3}$$

$$V_0 = 1.15 + 1.15 + 1.14 + 12.40$$

$$V_0 = \$15.84$$

Typically, students spend the majority of their time on the first and third steps in solving the problem. However, instructors should emphasize the contribution of the constant growth estimation of share price at the end of the high growth phase to the overall share estimate. In this problem, this step contributes

$$\$12.40/\$15.84 = 78\%$$

of the total share value. Here again, the emphasis should be on estimating a reasonable range for the price at the end of the high growth period (end of year three), given its significance to the overall share valuation estimate.

6. An applied example from portfolio management class

Examples from student-produced stock valuation reports highlight these issues from an individual financial management perspective. Classes are offered at the authors' respective institutions in which students manage actual securities portfolios worth over \$150,000 each. To make new stock recommendations, students are required to produce research reports that include an individual firm analysis, an industry analysis, and a quantitative valuation of the firm's share value using the nonconstant dividend growth model and other techniques. The following example illustrates how students in our classes have applied the nonconstant growth model to value a firm in 1998. The first period growth rate, g_1, was estimated using a four-year point to point estimate as 13.29%. The students' assumption was that this growth rate could be maintained for three more years. The estimated dividend stream at this growth rate for the next three years was \$0.70 per share in 1998, \$0.79 in 1999, and \$0.89 in 2000. A scenario analysis was performed using a range of required rates of return (K_s) and secondary growth rates (g_2). Following are the share valuation estimates.

Scenario 1: $K_s = 13.5\%$, $g_2 = 12\%$

Present value of g_1 stage dividends:

$$0.70/(1.135) + 0.79/(1.135)^2 + 0.89/(1.135)^3 = \$1.85$$

Estimated price at the end of year 2000:

$$(0.89)(1.12)/(0.135 - 0.12) = \$66.45$$

Present value of price 2000:

$66.45/(1.135)^3 = \$45.45$

Estimated current share price:

$\$1.85 + \$45.45 = \$47.30$

(3.91%) (96.09%) (100%)

Scenario 2: $K_s = 14\%$, $g_2 = 12.25\%$

Present Value of g_1 stage dividends:

$0.70/(1.14) + 0.79/(1.14)^2 + 0.89/(1.14)^3 = \$1.82$

Estimated share price at end of year 2000:

$0.89(1.1225)/(0.14 - 0.1225) = \$57.09$

Present value of year 2000 price:

$\$57.09/(1.14)^3 = \$38.53$

Estimated current price:

$\$1.82 + \$38.53 = \$40.35$

(4.51%) (95.49%) (100%)

Scenario 3: $K_s = 14.5\%$, $g_2 = 12.5\%$

Present Value of g_1 stage dividends:

$0.70/(1.145) + 0.79/(1.145)^2 + 0.89/(1.145)^3 = \$1.81$

Estimated share price at end of year 2000:

$0.89(1.125)/(0.145 - 0.125) = \$50.06$

Present value of year 2000 price:

$\$50.06/(1.145)^3 = \$33.35$

Estimated current share price:

$\$1.81 + \$33.35 = \$35.16$

(5.15%) (94.85%) (100%)

A couple of points are worth emphasizing in this actual application of the nonconstant growth dividend discount model. First, assuming a maturing firm, out-year growth rates for each scenario are lower than the 13.29% point-to-point estimate for g_1. Secondly, as the uncertainty about the future growth rate increases - so does the investor's required rate of return. Since the assumed growth rate after year 2000 was less certain, the required rate of return was increased in scenario 2 to reflect this higher risk. Finally, following Gordon and Gould (1978), this uncertainty generally makes K_s increase with g. Based in part on more optimistic industry and economic assumptions, higher growth rate and required return estimates were used in scenario 3. The student analysts can readily see that iterations have a profound effect on the value of the estimated share price. In this case, the overall price ranges from $35.16 to $47.30.

As the proficiency with the models and their underlying assumptions improves, it is very useful to place the staged growth model and parameter estimate formulas into an Excel spreadsheet. By doing this, estimates for K_s and g derived from the various models can be directly referenced in the constant or staged growth DDM. This eliminates duplicative calculations and allows students to change basic model assumptions and immediately see the impact on the price estimate. Students in the portfolio management classes regularly express their surprise (and occasional frustration) at how small changes in their assumptions for K_s and g can result in a wide range of current price estimates.

7. Summary

While a very powerful tool for estimating value, proper application of the constant growth dividend discount model requires an understanding of the fundamental nature of the model and its parameters. The ease of calculation makes the model intuitively appealing for finance students. However, they should be able not only to "plug and chug" the DDM formula, but also to understand the model's inputs and sensitivity to the relationship between the required rate of return and the growth rate. Students should be aware that the sensitivity of the DDM to error increases geometrically when the estimates of K_s and g converge.

Since estimates for K_s and g can vary widely depending upon the estimation methods used, it is both insufficient and impractical to merely calculate a value based on a single set of narrow assumptions. Rather, students need to recognize that implementation of the model means estimating a set of feasible values to arrive at a range of intrinsic value per share for which the student (or analyst) can feel confident.

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