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The impact of the Pension Fund on the decision to work one more year☆

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Abstract

The decision to retire or work one more year is quite complex. One factor that plays a role in this decision is the net salary derived from working one more year. The type of pension a person has, defined benefit or defined contribution, will influence whether the net salary is larger than, equal to, or less than the stated salary. The larger the net salary relative to the stated salary, the more likely an employee is to continue working. This paper defines the net salary percentage for each type of pension plan and looks at empirical estimates for these values. © 2000 Elsevier Science Inc. All rights reserved.

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1. Introduction

A critical decision we all must make is the age at which we will retire. The retirement age is often treated in economic models as one that is known with certainty (Modigliani & Brumberg, 1954). Many financial planning models have the worker designate a retirement date, and then project what a worker's retirement income would have to be to sustain the lifestyle during retirement to which he or she is accustomed. The most popularly quoted rules

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of thumb are that a worker would need a retirement income equal to 60 to 70% of preretirement income (see, e.g., Wiatrowski, 1993). This is known as the replacement ratio. Empirical research suggests that replacement ratios are a function of the level of income and do not appear to exceed the 80% threshold (Dexter, 1984 and Palmer, 1989). The retirement decision can also be viewed as an option. That is, at the start of each year the worker decides whether to retire or to work one more year. It is this last approach that is taken in this paper.

The sources of retirement income consist of three broad categories. These are: 1) governmental retirement benefits such as Social Security, 2) income from pensions and other tax-favored retirement programs such as IRAs, and 3) income from personal investments. People retiring would be able to receive income from all three sources. People who decide to work one more year would receive their salary and their projected retirement income in subsequent years would likely be higher, but they would lose one year's worth of retirement income. Thus, anyone deciding to work one more year would be earning a "net" salary equal to his or her wage income plus the present value of the increase in future retirement income, less the retirement income he or she would have received in that year. The decision to retire or work one more year is not, of course, based solely on this net salary figure. The decision to retire requires the employee to compare the utility of the "net" salary to the utility of an additional year's worth of leisure. If the utility of the net salary exceeds the utility of leisure for the next year, the individual will work. If not, the individual will retire. The individual's utility function will be shaped by his or her wealth and income from other sources, his or her health, and many other factors.

The basic assumption of this paper is that the larger the net salary number, the more likely the individual is to work one more year. This is just another way of saying that people have a positive marginal utility of wealth, a common assumption in economic research. Nonetheless, individuals may have a net salary less than their stated salary and still opt to work, or they may have a net salary greater than the stated salary and yet opt to retire. The net salary number only influences the retirement decision, it does not define it.

The purpose of this paper is to examine how the type of pension a person has influences the retirement decision. A defined benefit (DB) pension and a defined contribution (DC) pension will have different effects on the nominal value of the "net" salary. We cannot directly observe anyone's utility function. But to the extent that utility is positively correlated with "net" salary, we can reach some conclusions about how DB and DC pensions affect a person's decision to retire now or to work one more year.

One major conclusion is that the longer a person has been on the job with a DB pension, the more likely that person is to retire. Another conclusion is that with a DC pension, the decision to retire will be extremely sensitive to the value of the pension account relative to the salary at the time of retirement, and the individual's life expectancy.

Several studies (Banker's Trust, 1980; Woerheide & Fortner, 1991; Kahl & Williamson, 1994) have shown that there may be more involved in a pension than just the monthly retirement check. A pension may include such things as survivor protection, disability protection, retiree medical benefits, early retirement incentive programs, inflation adjustments, Social Security offset, and death benefits. Also, pensions include elements of risk (McLeod, Moody & Phillips, 1992/93). This paper focuses only on the cash pay outs to a retiree and omits the nonpecuniary and nonretirement income benefits, as well as the elements of risk in the different types of pensions.

2. The defined benefit case

Let us consider the case of an employee covered in a DB plan. If this employee retires today, the present value of his or her future cash inflows related to employment is the present value of the pension payments. This can be described as

$$I_0 = \sum_{t=0}^{D} \frac{S_0 * n * c}{(1 + k)^t} \tag{1}$$

where

I_0 = present value of pension benefits if a person retires today,
S_0 = "final" salary,
D = number of years a person expects to live,
n = number of years a person has worked in the DB plan,
c = pension benefit as a percentage of "final" salary per year worked, and
k = appropriate discount rate.

The payment of the salary and the payment of pension benefits are assumed to be on an annual basis for simplicity of discussion. The use of monthly payments would have no impact on the results presented herein. Pension funds vary in how the "final" salary is computed. Common variations are that it is defined as the salary during the last year of employment, the average salary during the last three years of employment, and the average salary during the last five years of employment. In this paper, "final" salary and the salary during the prior year are treated as synonymous.

Note that the summation in Eq. (1) starts at zero. This means, of course, that the first pension payment would be received immediately (i.e., the pension is an annuity due).

By working one more year at a salary that has grown by the rate "g" and then retiring, today's value of the worker's income stream becomes the current year's salary plus the present value of pension benefits received during his or her retirement. This is expressed as

$$I_1 = S_0 * (1 + g) + \sum_{t=1}^{D} \frac{S_0 * (1 + g) * (n + 1) * c}{(1 + k)^t} \tag{2}$$

where I_1 = present value of the combination of salary and benefits if the employee retires one year from today, and
g = percentage increase in "final" salary for next year.

In Eq. (2), the first S_0 represents the salary in the most recent year, and the second S_0 represents the salary number upon which the pension benefit is computed. As pointed out earlier, these two could be different, and thus the associated growth rates would also be different. If the salary growth rate for the coming year is greater than the growth rate of the "final" salary, the net salary coefficient to be derived would be lower, and vice versa.

The "net salary" or marginal economic benefit of working one more year is the difference between I_1 and I_0:

$$I_1 - I_0 = S_0 *(1 + g) + \sum_{t=1}^{D} \frac{S_0 *(1 + g)*(n + 1)*c}{(1 + k)^t} - \sum_{t=0}^{D} \frac{S_0 *n*c}{(1 + k)^t} \tag{3}$$

This difference can be "simplified" to the following form

$$I_0 - I_0 = S_0 *(1 + g)*\left[1 - \frac{n*c}{1 + g} + \sum_{t=1}^{D} \frac{c + c(c*n*g/(1 + g))}{(1 + k)^t}\right] = S_0 *(1 + g)*\delta \tag{4}$$

where $\delta = 1 - \frac{n*c}{1 + g} + \sum_{t=1}^{D} \frac{c + (c*n*g/(1 + g))}{(1 + k)^t}$.

The decision to retire depends on whether the value of Eq. (4) is enough to offset the loss of utility achieved from substituting one more year of work for an extra year of leisure. As indicated earlier, there is no one single value of δ that will guarantee a person would retire or would opt to work another year. Although the current salary (S_0) and the salary increase (g) are important variables, the salary coefficient term δ is the most important term in Eq. (4). The term δ could be negative, but the more likely scenario is that it would be positive.

Some employers place a cap on the number of years that can be counted in computing the DB pension benefit. Thus, although the salary used in computing the pension benefit increases, the employee may not receive the benefit of getting another year's credit in computing the pension payment. This procedure produces a simpler and smaller value for the salary coefficient term computed above.

If the coefficient term δ were negative, then the economically rational person would presumably retire as he or she is actually paying to work. The coefficient would be negative in cases where the product of years covered in the pension plan (n) and the pension percentage per year worked (c) exceeds the value of 1.0 by an amount large enough to offset the present value of the increase in future pension benefits. The latter would be low whenever life expectancy is low. For example, if $n = 30$, $c = 0.04$, $g = 0.03$, $k = 0.10$, and $D = 2$, then the financial benefit to working one more year (i.e., $I_1 - I_0$) is equal to -3.50% of next year's salary. It should be noted that even in a situation as extreme as this, where c is incredibly large, if the person expects to live at least three more years, then the coefficient becomes positive and stays positive for increasing life expectancies.

2.1. Estimates of δ

Let us now consider estimates of the salary coefficient term. Table 1 shows δ as a function of the number of years the employee expects to live (i.e., D) and the number of years on the job (i.e., n). Note that because the salary and pension payments are assumed to be received

Table 1
Estimates of the salary coefficient term (δ) under a DB plan

Variable value	c 0.01	g 0.04	k 0.08	
	n			
D	5	10	20	30
0	0.952	0.904	0.808	0.712
1	0.963	0.917	0.824	0.731
2	0.973	0.929	0.839	0.750
3	0.983	0.940	0.853	0.767
4	0.991	0.950	0.866	0.783
5	1.000	0.959	0.878	0.798
6	1.007	0.968	0.889	0.811
7	1.014	0.976	0.900	0.824
8	1.020	0.983	0.909	0.835
9	1.026	0.990	0.918	0.846
10	1.032	0.997	0.926	0.856
11	1.037	1.003	0.934	0.865
12	1.042	1.008	0.941	0.874
13	1.046	1.013	0.948	0.882
14	1.050	1.018	0.954	0.889
15	1.054	1.022	0.959	0.896
16	1.057	1.026	0.964	0.902
17	1.061	1.030	0.969	0.908
18	1.064	1.034	0.974	0.913
19	1.066	1.037	0.978	0.918
20	1.069	1.040	0.981	0.923
21	1.071	1.043	0.985	0.927
22	1.074	1.045	0.988	0.931
23	1.076	1.047	0.991	0.935
24	1.077	1.050	0.994	0.938
25	1.079	1.052	0.997	0.941
26	1.081	1.054	0.999	0.944
27	1.082	1.055	1.001	0.947
28	1.084	1.057	1.003	0.950
29	1.085	1.058	1.005	0.952
30	1.086	1.060	1.007	0.954

The salary coefficient term under a DB plan is defined as

$$\delta = 1 - \frac{n^*c}{1+g} + \sum_{t=1}^{D} \frac{c + (c^*n^*g/(1+g))}{(1+k)^t}.$$

Where
D = number of years a person expects to live,
n = number of years a person has worked in the DB plan,
c = pension benefit as a percentage of ‘‘final’’ salary per year worked,
k = appropriate discount rate, and
g = percentage increase in ‘‘final’’ salary for next year.

on the first day of each year, the vertical axis, which represents the life expectancy of the employee, represents a person dying after receiving the payment for that year. Thus, the value of $D = 1$ represents a person collecting his or her salary now, and the pension benefit one year from now, and then dying before receiving the second pension payment.

Table 1 is constructed using the assumptions that the employee expects a salary increase of four percent ($g = 0.04$), has a pension of one percent of final salary for each year worked ($c = 0.01$), and believes that the appropriate discount rate is 8% ($k = 0.08$). For example, if an employee has participated in the pension plan for 20 years ($n = 20$), and expects to live 10, 20, or 30 more years, then the salary coefficients for the coming year are 92.6%, 98.1%, and 100.7% of the projected salary. The projected salary equals $S_0 *(1 + g)$.

The most subjective of the variables used in the analysis is probably the discount rate. Payments from a DB pension are relatively safe, particularly if the pension fund is fully funded and is insured by the PBGC. Therefore, payments from an insured DB pension should have a discount rate of no more than the pretax cost of debt. The selection of 8% as the discount rate represents an approximation of this value.

2.2. Sensitivity of the coefficient term δ

To test the sensitivity of the numbers presented in Table 1 to the parameter values chosen, partial derivatives or first differences are calculated as appropriate. Also, each parameter is adjusted up and down within reasonable ranges and the impact reviewed. The resulting estimates of the coefficient terms are then compared to the base example in Table 1.

The partial derivative of the coefficient term with respect to the salary growth rate (g) is positive. However, changing the growth rate to 3% and 5% has negligible impact on the coefficient numbers produced.

The partial derivative with respect to the discount rate (k) is negative. Using 6% and 10% as the discount rate had negligible impact when life expectancy was at the lower end of its range. At the upper end (e.g., $D = 30$) the results were moderately sensitive to the discount rate used.

A much more interesting calculation is the partial derivative of the coefficient term with respect to the pension benefit per year worked (c). The sign of the derivative is ambiguous as it depends on how long one expects to live after retirement. If one expects a “short” life, then the derivative is negative. Thus, the larger the pension benefit per year worked, the lower the salary coefficient term becomes and the more likely the employee is to retire. If one expects a “long” life, then the larger the pension benefit per year worked the larger the coefficient term and the more likely the employee is to work another year. The point at which life expectancy (D) changes from “short” to “long” is a function of k, g, and n. Although the derivative is relatively insensitive to differences in k and g over relevant ranges, it is quite sensitive to n. As n becomes shorter, the changeover value of n becomes shorter. For example, if $k = 0.10$, $g = 0.05$, and $n = 10$, then the change from a positive to a negative derivative occurs at $D = 11$. If $k = 0.10$, $g = 0.05$, and $n = 20$, then the derivative changes from positive to negative at $D = 39$. Thus, the longer one has worked at a job, the more likely the derivative is to be positive and hence higher values of c would make a person more likely to continue working. The briefer one has worked at a job, the more likely the derivative is to be negative and hence higher values of c would actually make a person more likely to retire.

The size of the coefficient term for values of c other than 0.01 are easily computed as

$$\delta = 1 - p^{*}(1 - TV) \tag{5}$$

where $p =$ a multiple of 0.01, and

$TV =$ the number in Table 1 that represents the combination of the number of years one has worked and the number of years one expects to live.

Thus, if one wanted to know the salary coefficient term for c = 0.02 when $n = 30$ and D = 20, then it equals 1 - 2 x (1 - 0.923), or 0.846. It is clear from Eq. (5) that when the Table 1 value is near 1.0, the changes in the value of c will have minimal effect. But, where the Table 1 value is substantially different than one, differences in c will have a substantial effect on the salary coefficient.

The partial derivative of the coefficient term with respect to n is negative provided one assumes $g < k$. Virtually all actuarial work in the area of pensions assumes that prospective salary growth rates are less than the expected return on the invested assets of the pension fund (Wyatt Company, 1981). Thus, it is also likely that the salary growth rate would be less than the discount rate. The negative effect of n can also be verified by looking across any row in Table 1 and noting that the coefficient term gets smaller as one moves to the right (i.e., as n gets larger).

We cannot take the partial derivative of the coefficient term with respect to life expectancy as life expectancy defines the number of terms in the summation. However, we can take the first difference between the coefficient terms when the life expectancy is D+1 years and when it is D years. This difference is always positive. This is verified by looking down any column in Table 1. Thus, the longer one expects to live, the larger the salary coefficient term and the less likely one is to retire because of the greater number of increased pension payments that will be received. Note that life expectancy has a greater impact when n is large (e.g., 30 years), than when it is small (e.g., 5 years). In other words, people who physically feel like they could continue working would have the most fiscal incentive to do so.

3. The defined contribution case

Let us now consider the case of an employee covered by a DC plan. When this employee retires, he or she has two choices with regard to the pension account. One is to roll it over into an IRA account, and the other is to annuitize it. If the roll over option is elected, then the employee may have a number of choices depending on his or her age. We will not consider these options as they take us well beyond the scope of this paper.

If the employee opts to annuitize, then the assets in the account are used to purchase a life annuity with an insurance company. The portfolio manager of the assets in which the pension account money is invested and the insurance company may be part of the same company. When the employee is ready to retire, the DC pension account may be annuitized with the same company, or the employee may roll the account to another insurance company that promises a higher monthly payment. A valid comparison of the effects of the pension plan type upon the retirement decision requires that we assume that the employee annuitizes his or her account upon retirement.

If the employee retires today, the present value of pension payments is

$$I_0 = \sum_{t=0}^{D} \frac{V/AF_{Q,j}}{(1+k)^t} = \sum_{t=0}^{D} \frac{m*S_0/AF_{Q,j}}{(1+k)^t} \tag{6}$$

where

V = the accrued value of the DC pension account,

$AF_{Q,j}$ = annuity due factor used by the pension provider, based on the provider's expectations the person will live Q more years and that the provider uses a discount rate of j, and

m = the ratio of V to $S_{0.}$

The actual calculation used for the payment is more complicated than simply the present value interest factor of an annuity due based on the life expectancy of the individual. (See, e.g., Chapter 9, Cissell, Cissell & Flaspohler, 1990.) However, the actual calculation is immaterial to our results here. As in the DB case, note that the pension payments start at time zero.

If this employee decides to work one more year, then the present value of his or her income stream is the current year's salary plus the present value of subsequent pension benefits received. This is expressed mathematically as

$$I_1 = S_0 *(1+g) + \sum_{t=1}^{D} \frac{[m*S_0 *(1+i) + S_0 *(1+g)*(1+i)*p]/AF_{Q-1,j}}{(1+k)^t} \tag{7}$$

where

p = percentage of salary contributed to the pension account, and

i = rate of return the employee expects to earn on the pension account.

It is assumed that the pension contribution is made at the beginning of each year when the employee is paid.

Note that Eqs. (6) and (7) incorporate two potentially different dates of death. The first, D, represents the number of additional years the employee expects to live. The second, Q, represents the employee's life expectancy based on mortality tables used by the provider of the annuity. The size of the annual pension payment is determined in part by Q. The number of payments the employee expects to receive is defined exclusively by D. Most people believe they will outlive the life expectancy defined by the mortality at the time of retirement.

Eqs. (6) and (7) similarly allow the rate of return expected on the pension account assets (i) to be different than the rate of return used in computing the pension payments (j). Both of these rates may be different than the discount rate used by the employee in determining the present value of the future pension payments (k). The most crucial relationship among these variables is that j will be less than k. If the value of j is less than k, then the annuitization process represents a negative net present value decision.

There are two reasons to believe that j is less than k. First, the pension provider incurs expenses. It costs money to run a portfolio, to send out statements, to send out monthly checks, and so forth. If j were equal to k, then there is no profit margin for the pension provider. Thus, if j were equal to k, no company would ever sell an annuity.

The second reason is that once a person annuitizes his or her DC pension account, the pension provider takes on a risk exposure that a retiree will live longer than expected. It is true that the pension provider benefits if a retiree dies prematurely. However, the pension provider must place a premium on the risk that the majority of retirees drawing pensions live longer than expected. The premium would be that the discount rate j used to determine the monthly benefit is less than the market rate k that the pension provider expects to earn on the pension assets. Thus, because of its operating expenses and mortality risk, the discount rate j must be less than k.

As noted earlier, if j is less than k, then the decision to annuitize represents a negative net present value. The value of this negative net present value is the price the individual pays the pension provider for the management of the account and for taking the risk the employee may live beyond his or her life expectancy at the time of retirement.

It was also noted above that i (the rate of return earned on the DC account assets) could be different than j and k. However, it is likely the case that the expected value of i would be relatively close to k (the difference equaling the cost of managing the pension account), and in the following empirical estimates it is assumed these are equal.

The "net salary" or marginal economic benefit to working one more year is, as in the DB case, the difference between I_1 and I_0:

$$I - I_0 = S_0 *(1 + g) + \sum_{t=1}^{D} \frac{[m*S_0 *(1 + i) + S_0(1 + g)*(1 + i)*p]/AF_{Q-1,j}}{(1 + k)^t} - \sum_{t=0}^{D} \frac{m*S_0/AF_{Q,j}}{(1 + k)^t} \tag{8}$$

This difference can be "simplified" to:

$$I_1 - I_0 = S_0 *(1 + g)*\left[1 + \sum_{t=1}^{D} \frac{[m*(1 + i)/(1 + g) + p*(1 + i)]/AF_{Q-1,j}}{(1 + k)^t} - \sum_{t=0}^{D} \frac{(m/(1 + g))/AF_{Q,j}}{(1 + k)^t}\right.$$
$$= S_0 *(1 + g)*\delta' \tag{9}$$

where

$$\delta' = 1 + \sum_{t=1}^{D} \frac{[m*(1 + i)/(1 + g) + p*(1 + i)]/AF_{Q-1,j}}{(1 + k)^t} - \sum_{t=0}^{D} \frac{(m/(1 + g))/AF_{Q,j}}{(1 + k)^t}$$

Note that Eqs. (4) and (9) are identical except for the definition of the salary coefficient term.

3.1. The "Neutrality" Case

Eq. (9) looks complex because we have allowed the values of i, j, and k, and the values of D and Q, to differ. If the pension manager and the employee share the same life expectancy (i.e., D = Q), and all three interest rates are the same (i.e., j = k = i), then Eq. (8) reduces to

$$I_1 - I_0 = S_0 *(1 + g) + m*S_0*i + S_0 *(1 + g)*p$$

or, more simply

$$= S_0 *(1 + g)*\left[1 + \frac{m*i}{1 + g} + p\right]$$

$$= S_0 *(1 + g)*\delta'' \qquad (10)$$

where $\delta''=1+\frac{m*i}{1+g}+p$.

This derivation requires the assumption that working one more year does not alter the expected date of death. Hence, the annuity factor used by the pension provider for a person retiring today ($AF_{Q,j}$) differs from the annuity factor that would be used one year later ($AF_{Q-1,j}$) by a factor of exactly one year.

In Eq. (10), δ'' is the salary coefficient term. It is easy to see that this term will always be greater than one if either m and i are positive and p is nonnegative, or p is positive and m and i have the same sign or at least one of them is zero. Thus, when these two sets of parameters are equal, one's effective salary for the coming year (as stated in Eq. (10) will always be greater than the stated salary of S_0 * (1 + g). However, because it is unrealistic to assume that j and k are equal, we will not analyze this case any further.

3.2. Empirical estimates of δ' in a DC Plan

Table 2 provides examples of values of the net salary coefficient term for a DC pension account. In all cases, we are considering an employee with a discount rate of 8% (k = 0.08), whose contribution to a pension fund equals 10% of current salary (p = .10), and whose salary will grow by 4% (g = 0.04) if the employee opts to work one more year. The employee believes the pension account will earn 8% (i = 0.08). The discount rate and the life expectancy used in determining the pension annuity are 7% (j = 0.07) and 17 years (Q = 17). The unisex life expectancy for a 65-year old person is approximately 17 years.

The net salary coefficients are shown in Table 2 for values of m equal to 0.5, 4, 8, and 15. The rationale for these values is provided in the next section. As an example, if the employee expects to die in ten years (contrary to the pension fund's expectation), and his or her accrued pension assets equal eight times his or her salary (m = 8), then the salary coefficient (δ') in

Table 2
Estimates of the salary coefficient term under a DC plan

Variable	k	j	i	p	g	Q
value	0.8	.06	0.08	0.1	.04	17

D	m			
	.5	4	8	15
1	0.954	0.632	0.264	−0.381
2	0.969	0.681	0.353	−0.222
3	0.983	0.727	0.435	−0.076
4	0.995	0.770	0.512	0.060
5	1.007	0.809	0.582	0.186
6	1.018	0.845	0.648	0.303
7	1.028	0.879	0.709	0.410
8	1.037	0.910	0.765	0.510
9	1.046	0.939	0.817	0.603
10	1.054	0.966	0.865	0.688
11	1.062	0.991	0.909	0.767
12	1.068	1.014	0.951	0.841
13	1.075	1.035	0.989	0.909
14	1.081	1.054	1.024	0.972
15	1.086	1.073	1.057	1.030
16	1.091	1.089	1.087	1.084
17	1.096	1.105	1.116	1.134
18	1.100	1.120	1.142	1.180
19	1.104	1.133	1.166	1.223
20	1.108	1.145	1.188	1.263
21	1.111	1.157	1.209	1.299
22	1.115	1.167	1.228	1.333
23	1.118	1.177	1.245	1.365
24	1.210	1.186	1.262	1.394
25	1.123	1.195	1.277	1.421
26	1.125	1.203	1.291	1.446
27	1.127	1.210	1.304	1.469
28	1.129	1.216	1.316	1.490
29	1.131	1.223	1.327	1.510
30	1.133	1.228	1.338	1.529

The salary coefficient term under a DC plan is defined as

$$\delta' = 1 + \sum_{t=1}^{D} \frac{[m*(1+i)/(1+g) + p*(1+i)]/AF_{Q-1,j}}{(1+k)^t} - \sum_{t=0}^{D} \frac{(m/(1+g))/AF_{Q,j}}{(1+k)^t}$$

where
g = salary growth rate,
p = percentage of salary contributed to the pension account,
i = rate of return the employee expects to earn on the pension account,
V = the accrued value of the DC pension account,
$AF_{Q,j}$ = annuity due factor used by the provider, based on the expectation the person will live Q more years and that the fund uses a discount rate of j, and
m = the ratio of the value of the DC account to the ‘‘final’’ salary.

Eq. (9) is 0.865. That is, the employee is working for only 86.5% of his or her salary during that last year.

There are several noteworthy points about Table 2. One is how sensitive the salary coefficients are to the value of m. When m = 0.5, the coefficents range from 0.954 to 1.133. When m = 15, they range from −0.381 to 1.529. This says that when m is "small," the net salary for working one more year is about equal to the stated salary. But when m is "large," there can be substantial differences between the net salary and the stated salary. Another point is that when m = 15 and the employee expects to live less than four years (D = 1, 2, or 3), the salary coefficient term is actually negative. This means the person is paying to work. Hence, when m is large and an employee is in ill health, it not only makes physical sense for an employee to retire, it also makes fiscal sense to retire.

3.3. Estimates of plausible values of m

As indicated above, a key variable in estimating the salary coefficient term is the ratio of the pension account at the time of retirement to the salary at the time of retirement (i.e., m). It turns out that the value of m can be defined by an iterative formula. Consider the case of a person who works one year and then retires. If we continue our use of treating all payments as being made at the start of each year, then the ratio of the pension account at the end of the first year to the salary at the start of the first year (m_1) is

$$m_i = \frac{V_1}{S_0} = \frac{p^* S_0 \,^*(1 + i)}{S_0} = p^*(1 + i). \tag{11}$$

the value of m at the end of the second year (m_2) is

$$\begin{aligned} m_2 = \frac{V_2}{S_1} &= \frac{p^* S_0 \,^*(1 + g)^*(1 + i) + S_0 \,^* m_1 \,^*(1 + i)}{S_0 \,^*(1 + g)} \\ &= p^*(1 + i) + \frac{1 + i}{1 + g}{}^* m_1 \end{aligned} \tag{12}$$

For the n-th period, the value of m_n is then defined as

$$\begin{aligned} m_n = \frac{V_n}{S_{n-1}} &= \frac{p^* S_{n-1} \,^*(1 + g)^*(1 + i) + S_{n-2} \,^* m_{n-1} \,^*(1 + i)}{S_{n-12} \,^*(1 + g)} \\ &= p^*(1 + i) + \frac{1 + i}{1 + g}{}^* m_{n-1}. \end{aligned} \tag{13}$$

Note that if i = g, then m_n = n*p*(1+i).

To ascertain some plausible values for m_n, let us start with what might be considered a maximum case scenario. Let us consider an employee who starts contributing to a DC pension at age 20. This employee works for 45 years with the same company ($n = 45$), has a salary contribution rate of 10% ($p = .10$), a salary growth rate of 5% (g = 0.05), and his or her investments earn a rate of return of 8% (i = 0.08). In this situation, m_{45} = 14.47. At the other extreme, let us consider the case of an employee who does not start contributing to the pension on his or her current job until age 55. This employee works for 10 years (n = 10), has a salary growth rate of 3% (g = 0.03), and has a salary contribution rate of 5% (p = .05). In this case, $m_{10} = 0.674$. In light of the extent to which some people have multiple job changes over their working lives, do not always participate in DC pensions, and do not always even participate in pension programs, this scenario may apply in some cases. The implication from these two extreme examples is that realistic values of m may range from as low as 0.5 to as high as 15.

3.4. Sensitivity of the coefficient term δ'

As in the DB case, we can analyze the sensitivity of the coefficient term by taking partial derivatives of the coefficient term with respect to the various inputs, as well as considering actual variations in the values. The derivative of the coefficient term with respect to the salary growth rate is ambiguous. However, the impact of this parameter on the size of the term is trivial as alternative values of 3% and 5% produced negligible changes in the coefficients.

The partial derivatives of the coefficient term with respect to the percentage of salary contributed by the employer (p) and the investment yield (i) are both positive. When values of 8% and 12% are used for p, there are again negligible changes in the coefficients. However, when values of 6 and 10% are substituted for the investment yield, there are substantial changes in the coefficient terms as m becomes larger. For example, if the employee expects to live 15 years, then when m = 0.5 the coefficient changes from 1.086 to 1.096 when i is increased from 8 to 10%. However, when m = 15, the coefficient term increases from 1.030 to 1.267.

The partial derivative with respect to the discount rate is negative. But just as with the investment yield on the pension account, it is more critical when m is large, particularly when an employee expects to live a long time. For example, if the employee expects to live 30 years, then when the discount rate is increased from 8 to 10%, the coefficient falls from 1.133 to 1.104 when m = 0.5. Alternatively, this same increase in the discount rate causes the coefficient to fall from 1.529 to 1.223 when m = 15.

A more interesting case is the derivative of the coefficient term with respect to the pension fund account relative to current salary (m). The sign of the derivative is ambiguous. If one expects a "short" life, then the larger the accumulated pension account the lower the coefficient term. Conversely, if one expects a "long" life, then the larger the accumulated pension account the larger the coefficient term. This is noted by looking across the rows on Table 2. In rows 1 through 16, the coefficient term becomes *smaller* as m increases. In rows 17 and above, the coefficient term becomes *larger* as m increases.

Variations in the discount rate (j) and in the life expectancy (Q) used to determine the annuity have relatively little impact, except, again, when the value of the pension account is large relative to the final salary.

4. Summary, conclusions, and extensions

4.1. Summary and conclusions

This paper focuses on a question many people will face in their lifetime, namely, should they retire now or work one more year and redecide. The decision requires comparing the utility of one year's worth of net salary to the utility of one year's worth of leisure. The utility function incorporates the wealth and substitution effects created by all the other assets and income the employee has, as well as many other variables such as health, hobbies, and so forth. We cannot examine utility functions, so we analyzed the percentage of next year's salary one would actually earn by working under both a DB and a DC pension. It is assumed that the higher this percentage is, the more likely a person is to work another year.

Several conclusions apply regardless of the type of pension plan in which one participates. First, the longer one expects to live after retirement, the higher the net salary from working one more year. Second, the use of higher discount rates by an employee always reduces the net salary. Third, the growth rate in salary for the coming year has a minimal effect on the salary coefficient term. Thus, if you are contemplating retirement and your boss offers to change your salary increase for the next year from 3% to 4%, this is unlikely to alter your decision.

Some components in the decision to retire are unique to the type of pension plan one has. In the case of DB pension plans, the longer one is covered by the plan, the lower the net salary for working another year. In the case of DC pension plans, increases in the value of the pension account relative to a worker's current salary reduce the salary coefficient term if the worker expects to have a "short" life, and increase it if the worker expects to have a "long" life.

As shown in Table 1, the coefficient term under a DB pension plan is relatively stable. For people who have been in a job for at least 20 years, the coefficient terms are less than one, but not dramatically so. The lower the coefficient term the more likely a worker is to retire because the opportunity cost of not working becomes lower. Thus, we may infer that DB pension plans encourage people who have been on the job for a long time to retire. The numbers in Table 2 cover a larger range and are more likely to be larger than one. Because a higher salary coefficient term encourages a person to continue working, we may infer that DC pension plans are more likely to provide this encouragement.

4.2. Extensions

In order to keep the analysis manageable and to keep the focus strictly on the impact of the pension program, many simplifications were made. First, all salary numbers were on a pretax basis. If a person works one more year, the increase in his or her wealth is actually the after-tax value of the salary plus the present value of the after-tax increase in pension benefits, less the after-tax value of what the first year's pension payment would have been. The role of taxes needs to be examined.

Second, it was assumed that when the DC pension was annuitized that the annuity was a pure annuity for the employee with no survivor's benefits and no guaranteed number of payments. The introduction of either survivor's benefits or a guaranteed number of payments would likely have a significant effect on the numbers provided in Table 2.

Third, Social Security was omitted. Working one more year (or at least not starting Social Security withdrawals) has the benefit of increasing future Social Security payments. Depending on how the internal rate of return on this deferral compares with the discount rate, the salary coefficient term could be enhanced or reduced. One effort has been made at incorporating Social Security into the decision to retire (Newmark & Walden, 1995), but that presentation is not consistent with the approach offered here.

Finally, the impact of working one more year on all other financial assets, such as IRAs, 401(k)s, and 403(b)s, was ignored. A more general treatment of the retirement decision would require incorporating these assets.

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