



Social Security reform: the effect of investing in equities

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Abstract

Several proposals have been developed to reform the Social Security System to ensure that it is fully funded. The investment of a portion of Social Security funds in equities has often been proposed as a means to avoid increasing payroll taxes. This paper develops a general equilibrium model to demonstrate that investing Social Security funds in equities will decrease the return on equities and increase interest rates on bonds, which also leads to an increase in general income taxes. Thus, investing Social Security funds in equities simply shifts a potential increase in payroll taxes to an increase in income taxes. © 2000 Elsevier Science Inc. All rights reserved.

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1. Introduction

A recent report of the Advisory Council on Social Security (Advisory Council, 1997) shows that the U.S. Social Security system will not be fully funded over the next 50 years. Forecasts show that with current and projected contributions and outflows, the Social Security Trust Fund will be completely depleted by the year 2034 and benefits would have to be reduced beyond that date by 29%. Therefore, there are several proposals to reform the Social Security system, many of which suggest investing of a portion of the Social Security Trust Fund into the equities market in order to obtain a higher return on accumulated funds. In this way, the Social Security system could supposedly become fully funded without raising payroll taxes.

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There is some debate concerning the impact of this type of Social Security reform on asset returns. Clearly, Social Security reform would have no effect on asset returns if the following two assumptions are true: (1) individuals perceive Social Security as part of their savings (or at least individuals must take into consideration the investment of Social Security when making their own allocation decision) and (2) no individuals are constrained in terms of their desired allocation, that is, every individual should be able to invest their entire retirement account (including Social Security) in bonds and/or equities according to their preference. Serious questions as to the validity of these assumptions exist. First of all, under the current defined benefit system, individuals should not care about the investment of Social Security funds because the benefits are fixed regardless of the allocation decision. Secondly, Geanakoplos, Mitchell, and Zeldes (1998) point out that if households are constrained in their investment choices, there would be macroeconomic consequences in terms of changes in asset returns due to the increase in demand for equities. This could occur if some individuals want all their retirement funds in equities (extreme risk takers) or in bonds (highly risk averse investors) or if some individuals do not have access to credit markets. If either of these two assumptions are violated, then asset returns will be affected to some degree by Social Security reform, and specifically by shifting funds to the equities market. The intuition behind this effect on interest rates and asset returns is that, if either of the two above assumptions is violated, then a shift in funds to equities is equivalent to a change in preferences. In our view, these two assumptions are likely to be violated, and this forms the basis for the model in this paper. In this paper, we formally model the impact of Social Security reform on asset returns in the presence of constrained individuals. The model shows unambiguously that interest rates on bonds will increase and the return on equities will decrease as Social Security funds are invested in the equities market. We find that the magnitude of the change in asset returns could be greater than one percentage following a forty percentage shift of Social Security funds to the equities market.

The topic of Social Security reform has attracted a large literature and has been the topic of numerous conferences (Federal Reserve Bank of Boston, 1997, American Economic Association 1996, and the National Academy of Social Insurance, 1998). Several researchers have commented, without the use of a formal model, that such a reform might affect the overall economy in terms of changing interest rates and equity returns. Diamond (1996), Stein (1997), Stiglitz, Munnell, and Frankel (the Council of Economic Advisers) in their Economic Report of the President (1997), and Geanakoplos et al. (1998) mention the potential impact of reform on asset returns. In addition, it is interesting that the Advisory Council's analysis of the effect of three different reform programs did not take into account the effect on interest rates from the implementation of the reform itself. Other researchers have developed formal models to study the impact of Social Security reform, but their focus is on topics other than the impact on asset returns. These studies generally use the life-cycle framework developed by Auerbach and Kotlikoff (1987) (e.g., Kotlikoff, Smetters, and Walliser, 1998; Bohn, 1997; Smetters, 1998; and Diamond, 1997).

The familiar life-cycle approach might seem to be the natural framework for analyzing Social Security since the very essence of a pay-as-you-go social security system is the transfer of assets from generation to generation. This is especially true if one is interested in a welfare analysis of the social security system, for which a life-cycle framework is ideal. In

contrast, the primary purpose of this paper is to examine the instantaneous effects of social security reform on asset returns. To this end, the benefits of using a life-cycle model (e.g., Bohn, 1997 and Diamond, 1997) are not obvious and can actually cloud the issue. This is because a major component of the life-cycle models is the saving-consumption and/or labor-leisure decision, which may be altered by (or may alter) returns. The empirical evidence is ambiguous concerning the relationship between returns and these decisions (e.g., Hall, 1988 and Campbell and Mankiw, 1989). The strength of our results is that they do not depend on any changes in these decisions.

We construct a representative-agent model to analyze the effects of Social Security reform, addressing the constraints of Geanakoplos et al. (1998). Our model uses the optimal-portfolio selection rule derived by Merton (1969) in continuous time and Samuelson (1969) within a discrete time framework in which the optimal-portfolio selection rule is a function of a given set of expected asset returns. We extend this analysis to a general equilibrium framework to endogenously determine these expected returns. It is assumed that Social Security rules determine the allocation of a portion of each individual's wealth. Furthermore, individuals with different levels of risk aversion attempt to achieve an optimal allocation of their wealth between a risky asset (equities) and a risk-free asset (bonds). Some individuals may be unable to achieve their optimal allocation, and therefore are constrained (by Social Security) to hold funds in excess of their optimal allocation in either the bond or equity market. This leads to a change in asset returns.

An interesting implication is that with an increase in the interest rates on government debt, income taxes must be raised to pay the higher interest on the national debt in order to maintain the same budget deficit position as prereform. Thus, investing Social Security funds in equities to some extent simply shifts a potential increase in Social Security taxes to an increase in general income taxes. This impact is of immense importance when discussing potential reform because it significantly alters the merits of the potential reform plans.

In Section 2, we first construct a model in which there are constrained individuals. We then analyze in Section 3 the qualitative impact on asset returns from shifting Social Security funds to the equities market and briefly report examples of the potential magnitude of the effects this reform may have on bond and equity returns. In Section 4, we explore comparative statics to determine the impact of various other parameter values. We present extensions of the model in Section 5 to include income taxes and show why income taxes would increase if Social Security funds are invested in equities. We also briefly explore the potential implications of increasing the payroll taxes in Section 6. Finally, Section 7 concludes the paper, providing a summary and possible extensions of our research.

2. The model

The model in this section assumes that individuals consider Social Security as part of their savings and therefore take into account the allocation decisions of the Social Security Trust Fund when making their own allocation decisions. Following Merton (1969), there is one risk-free asset (bonds) and one risky asset (equities). Bonds return a real risk-free rate, $r_b > 0$, while the return on equities is normally distributed with mean r_e and variance σ^2 . Suppose

that there are m different types of individuals where each individual i maximizes his utility using a power utility function of the type $U(C_i) = C_i^{\gamma_i}/\gamma_i$, where $\gamma_i < 1$ for all i (this precludes the possibility of a risk-loving individual). The widely used power utility function is not necessary for the qualitative results of this paper, but it is convenient because it offers an explicit solution to the optimal portfolio rule. Furthermore, the power utility function is consistent with the empirical results of Friend and Blume (1975). This type of utility function exhibits constant relative risk aversion with a coefficient of relative risk aversion of $1-\gamma_i$. Individuals are ordered on the interval $[1, m]$ by their degree of risk aversion where individual 1 is the most risk-averse and individual m is the least risk-averse ($\gamma_m > \gamma_1$). In addition, each individual is assumed to have a wealth level of W_i . Maximizing utility entails choosing optimal consumption levels as well as an optimal allocation of wealth between the risk-free and risky asset. It can be shown that each individual i would optimally like to allocate a fraction of his wealth, ω_i to equities and $1-\omega_i$ of his wealth, W_i , to holding bonds where

$$\omega_i = \frac{r_e - r_b}{(1 - \gamma_i)\sigma^2} \quad (1)$$

(See Merton, 1969, page 250 or Ingersoll, 1987, page 275 for a derivation of the optimal portfolio allocation.) Individuals are forced to contribute to Social Security, and the accumulated funds in the Social Security Trust Fund are allocated without any consideration for individuals' wishes. Let $\nu \geq 0$ be the prereform fraction of Social Security funds allocated to equities (note that this is the same for everyone and under the current regime is $\nu = 0$) and ν' is the post reform fraction (where future values are denoted by a'). Finally, let η_i be the fraction of an individual's wealth controlled by Social Security. Therefore, Social Security allocates $(1-\nu)\eta_i$ fraction of an individual's wealth to the bond market and $\nu\eta_i$ fraction of an individual's wealth to the equities market. Furthermore, it is also assumed that individuals cannot sell stock short. Therefore, some individuals may be unable to achieve their optimal allocation. For any given return pair, (r_e, r_b) , individuals can be grouped into three categories when $0 \leq \nu \leq 1$: the most risk-averse individuals will be constrained to hold funds in excess of their optimal allocation in the equities market, the least risk-averse individuals will be constrained to hold funds in excess of their optimal allocation in the bond market, and the remaining individuals will not be constrained in either market. We will treat each of the three different categories of individuals in turn. Given this ordering of individuals, there exists a marginal investor, represented by i_e , such that $\omega_{i_e} = \nu\eta_{i_e}$ (individual i_e 's optimal allocation to the equities market is exactly equal to the amount Social Security allocates to the equities and bond markets). Individuals of type $x < i_e$ (or equivalently, all investors more risk averse than i_e) are constrained to hold more funds in the equities market than they desire, that is, $\nu\eta_x > \omega_x$ but $(1-\nu)\eta_x < 1-\omega_x$. These individuals will allocate a total of $\nu\eta_x W_x$ to the equities market and their remaining funds, $(1-\nu\eta_x)W_x$, to the bond market. Furthermore, there is also a marginal investor, i_b , such that $1-\omega_{i_b} = (1-\nu)\eta_{i_b}$ (individual i_b 's optimal allocation to the bond market is exactly equal to the amount Social Security allocates to the equities and bond markets). Individuals of type $z > i_b$ (or equivalently, all investors less risk averse than i_b) are constrained to hold more funds in the bond market than they desire, that is, $(1-\nu)\eta_z > 1-\omega_z$ but $\nu\eta_z < \omega_z$. Individuals in this group will allocate $(1-\nu)\eta_z W_z$ to bonds and $(1-(1-\nu)\eta_z)W_z$

to equities. Finally, there is also a group of individuals, $y \forall [i_e, i_b]$ which are not constrained to be in either market, that is, $(1-\nu)\eta_y \leq 1-\omega_y$ and $\nu\eta_y \leq \omega_y$. Individuals in this group will allocate $\omega_y W_y$ to the equities market and $(1-\omega_y)W_y$ to the bond market.

It is assumed that there is a fixed quantity of bonds and stocks, Q_b and Q_e respectively, and changes in the return of a given asset are determined by the current prices, P_b and P_e respectively. Also by assumption, there is general agreement (i.e., homogeneous expectations) on the expected future prices of bonds and stocks, $E[P'_b]$ and $E[P'_e]$, based on the expected growth in the real assets of the economy, which means that the expected-future prices are constants. Conceptually, one can think of bonds being paid off with certainty at face value at maturity and the real assets of each firm being liquidated to pay off the stockholders at a known expected value. Therefore, for $\nu > 0$, the market clearing conditions for the bond and equity market are respectively given by

$$\sum_{x=1}^{i_e-1} n_x(1 - \nu\eta_x)W_x + \sum_{y=i_e}^{i_b} n_y(1 - \omega_y)W_y + \sum_{z=i_b+1}^m n_z(1 - \nu)\eta_z W_z = Q_b P_b \tag{2}$$

$$\sum_{x=1}^{i_e-1} n_x \nu \eta_x W_x + \sum_{y=i_e}^{i_b} n_y \omega_y W_y + \sum_{z=i_b+1}^m n_z(1 - (1 - \nu)\eta_z)W_z = Q_e P_e \tag{3}$$

Note that $P_e = E[P'_e] \exp(-r_e)$ and $P_b = E[P'_b] \exp(-r_b)$, where $\exp(-r)$ represents the exponential function raised to the $-r$ power, which is the present value factor using continuous discounting. Substituting in (2) and (3) yields

$$\sum_{x=1}^{i_e-1} n_x(1 - \nu\eta_x)W_x + \sum_{y=i_e}^{i_b} n_y(1 - \omega_y)W_y + \sum_{z=i_b+1}^m n_z(1 - \nu)\eta_z W_z = \frac{Q_b E[P'_b]}{\exp(r_b)} \tag{4}$$

$$\sum_{x=1}^{i_e-1} n_x \nu \eta_x W_x + \sum_{y=i_e}^{i_b} n_y \omega_y W_y + \sum_{z=i_b+1}^m n_z(1 - (1 - \nu)\eta_z)W_z = \frac{Q_e E[P'_e]}{\exp(r_e)} \tag{5}$$

This is the most general form of the model. However, the model can be simplified by assuming there are only 3 categories of individuals rather than m . With this simplification, it is easier to demonstrate the effects on bond and equity returns. Suppose, for example, that each category of individuals discussed above is represented by a single type of individual, that is, n_x individuals with the same η_x are constrained to hold excess funds in the equities market, n_y individuals with the same η_y are not constrained to hold excess funds in either market, and n_z individuals with the same η_z are constrained to hold excess funds in the bond market. Thus, the total number of individuals is $n_x + n_y + n_z = n$. For simplicity assume that $W_x = W_y = W_z = W$ and $\eta_x = \eta_y = \eta_z = \eta$. Then the market clearing conditions for the bond and equity markets, (4) and (5), can be rewritten as

$$n_x(1 - \nu\eta) + n_y(1 - \omega_y) + n_z(1 - \nu)\eta = \frac{B}{\exp(r_b)} \tag{6}$$

$$n_x \nu \eta + n_y \omega_y + n_z (1 - (1 - \nu) \eta) = \frac{E}{\exp(r_e)} \quad (7)$$

where $B = Q_b E[P_b'] / W$ and $E = Q_e E[P_e'] / W$ are standardized values for bonds and equities (relative to total wealth), respectively.

Eqs. (6) and (7) can be solved for r_b and r_e , the bond and equity returns, by substituting Eq. (1) for the optimal portfolio allocation, ω_y .

3. Qualitative effects of shifting funds on asset returns

Using Eqs. (6) and (7), it is possible to analyze the qualitative impact of shifting Social Security funds on asset returns. Eqs. (6) and (7) can be rearranged to show that r_e is a function of r_b and ν and that r_b is a function of r_e and ν .

$$r_e = f(r_b, \nu) \quad (8)$$

$$r_b = g(r_e, \nu) \quad (9)$$

Taking the total derivative of (8) and (9) with respect to ν results in

$$\frac{dr_e}{d\nu} = f_{r_b} \frac{dr_b}{d\nu} + f_\nu \quad (10)$$

$$\frac{dr_b}{d\nu} = g_{r_e} \frac{dr_e}{d\nu} + g_\nu \quad (11)$$

from which it is possible to show that

$$\frac{dr_e}{d\nu} = \frac{g_\nu (f_{r_b} - l)}{l - f_{r_b} g_{r_e}} < 0 \quad (12)$$

$$\frac{dr_b}{d\nu} = \frac{g_\nu (g - l)}{l - f_{r_b} g_{r_e}} > 0 \quad (13)$$

where $f_{r_b} = 1 + (B(1 - \gamma_y)\sigma^2)/(n_y \exp(r_b)) > 1$, $g_{r_e} = 1 + (E(1 - \gamma_y)\sigma^2)/(n_y \exp(r_e)) > 1$, and $g_\nu = (n_x \eta + n_z \eta)(1 - \gamma_y)\sigma^2/n_y > 0$. (See Appendix A for complete details of the proof.) Thus, as the Social Security Trust Fund shifts its investment from Government securities to equities, r_e decreases and r_b increases. It is possible to think of the chain of events following reform as (1) Social Security reallocates some of the Trust Fund to the equities market, (2) unconstrained, type y individuals reallocate their portfolio in order to maintain their prereform allocation, (3) constrained individuals now have more funds in the equities market (and fewer in the bond market) and therefore the equity return falls and the bond return rises, (4) type y individuals then reallocate their portfolio (moving some funds back to the bond market because $r_e - r_b$ has decreased) which offsets some of the changes in r_e and r_b in step 3, and finally (5) steps 3 and 4 are repeated until a new equilibrium is achieved.

It is interesting that the change in r_e and r_b following Social Security reform is not dependent upon the level of risk aversion of the constrained individuals. The intuition is straightforward. It is assumed that individuals that are constrained in a given market prior to reform remain constrained following reform. Whether an individual is constrained is dependent upon his risk aversion (as shown in Appendix B). The amount of funds that a constrained individual invests in a given market is by definition fixed by the constraint, regardless of the degree of constraint. Therefore all individuals constrained to hold funds in the bond (equities) market will invest the same amount of funds regardless of the degree of risk aversion.

It is also straightforward to understand why there are no effects on r_e or r_b if no one is constrained. This can be shown by noting the bond and equity market-clearing conditions under the situation in which no one is constrained. Prereform, these conditions are respectively

$$\sum_{i=1}^m \omega_i = \frac{E}{\exp(r_e)} \quad (14)$$

$$\sum_{i=1}^m (l - \omega_i) = \frac{B}{\exp(r_b)} \quad (15)$$

which is exactly the same as the post reform conditions, indicating that no change in either r_e or r_b occurs following reform. If no one is constrained, then all individuals can simply reallocate the fraction of their wealth which they have control over in order to fully achieve their prereform allocation (hence leaving returns unchanged). Therefore, effects on r_e and r_b occur only if there are individuals who are constrained. If individuals become unconstrained then that individual's action no longer has an effect on r_e and r_b (but r_e and r_b could still be changing if others continue to be constrained or if new individuals become constrained). This is consistent with the results given in (12) and (13) by noting that the "no constrained individuals" case is equivalent to $n_x = n_z = 0$ (in which case $g_\nu = 0$ and $dr_e/d\nu = 0$ and $dr_b/d\nu = 0$). The actual magnitude of these effects is an empirical question. However, we can use Eqs. (6), (7), (12), and (13) to calculate the potential magnitude of changes in interest rates and equity returns for a wide range of parameter values for ν , n_y , n_z . The changes in r_b and r_e , following a shift of forty percentage of the Trust Fund into the equities market, range from $dr_e/d\nu = -1.21\%$, $dr_b/d\nu = 1.30\%$ ($n_y = 1$, $n_z = 1$, $\nu = 0.4$) to $dr_e/d\nu = -0.03\%$, $dr_b/d\nu = 0.03\%$ ($n_y = 10$, $n_z = 1$, $\nu = 0.1$). This range of estimates is consistent with the estimated increase in interest rates of 1.44% in Elder and Holland (1999), which uses a different approach to empirically estimate the effects on interest rates.

The estimates from these calculations are based on several assumptions. The relative size of n_y and n_z is analogous to the size of the population which is unconstrained relative to the amount of the population which is constrained. These results set $n_x = 0$ because currently there are no Social Security funds invested in the equities market so there cannot be anyone constrained to hold excess funds in the equities market. The assumption that n_x remains equal to zero as funds are shifted into the equities market is rather conservative. If we were to allow

initially unconstrained individuals to become n_x type individuals the results below would be magnified. The calculations set the relative size of the bond and equities market to match the actual relationship; the Flow of Funds Accounts of the United States (1999) shows that in 1998 the market capitalization of the equities market was \$15,438 billion and the market capitalization of the bond market (including Treasury and Government agency securities, corporate and foreign bonds, and municipal bonds) was \$12,387 billion.

4. Other comparative statics

It is also interesting to examine how the changes in r_b and r_e are affected by changes in n_x , n_z , and γ_y . Qualitatively, these effects are found by taking the cross derivatives of (12) and (13). First of all, it can be shown that with respect to n_x or n_z that $dr_e/(d\nu dn_x) = dr_e/(d\nu dn_z) < 0$ and $dr_b/(d\nu dn_x) = dr_b/(d\nu dn_z) > 0$, meaning that as more individuals are constrained the effects of reform on r_e and r_b are magnified. To understand this, note that as more individuals are constrained (regardless of where they are constrained), a larger amount of funds is not adjusted in each market (following the reallocation of the y individuals in step 2). This leads to a larger initial change in returns (step 3 above) and finally, a larger final effect of reform on returns. The other interesting part of these results is that the effect of reform on returns is not dependent upon where the constrained individuals are constrained, but just dependent on the *total* number of constrained individuals. For every 1% of the Trust Fund that Social Security shifts to the equities market, the allocation of z individuals (most risk-averse) moves closer to their optimal allocation by $\eta\%$ while the allocation of x individuals (least risk-averse) diverges from their optimal allocation by $\eta\%$. Regardless of whether the constrained individuals are moving closer to their optimal allocation or not, the same amount of funds are being moved to the equities market in step 3 above; hence the effect of reform on returns is only dependent on the number of constrained individuals and not where they are constrained.

Another factor determining the magnitude of the change in returns is the level of risk aversion of the unconstrained individuals, since these individuals can partially (or completely) undo the changes made by Social Security. This result is obtained by taking the cross derivative with respect to γ_y . It can be shown that $dr_e/d\nu d\gamma_y > 0$ and $dr_b/d\nu d\gamma_y < 0$. This means that when unconstrained individuals are more risk-averse, the effects on r_b and r_e will be larger following reform. To understand this, first note that the allocation decision of less risk-averse individuals is more sensitive to changes in $r_e - r_b$. If the risk premium rises, then less risk-averse individuals increase their allocation to equities much more than do more risk-averse individuals. It follows that the less risk-averse the y individuals are in step 4 above, the larger will be the amount of funds shifted back to the bond market by those individuals, hence offsetting more of the initial change in returns.

Analogous results obtain if individuals do not perceive Social Security funds as a part of their wealth (see Appendix C for details) which may be relevant if individuals do not expect to receive any benefits from Social Security in the future.

5. Implications of changing asset returns

Some of the Social Security Reform proposals suggest that it is possible to avoid an increase in the Social Security tax rate by simply shifting Trust Fund money into the equities market. Social Security investments in the Trust Fund will benefit from a movement of funds to the equities market in two ways: (1) Higher interest rates will increase the return on funds invested in the bond market, and (2) Funds moved to the equities market will have a higher expected return than the bond market. However, this is not the complete story. The increase in interest rates also has substantial implications for the federal government and fiscal policy. With a rise in interest rates, the cost of servicing the total federal debt will increase. The total government debt in 1999 (\$5,606 billion) is currently substantially larger than the Trust Fund (\$855 billion) and is forecasted by the CBO (2000) to be about twice as large over the next decade (government debt of \$6,300 billion and Trust Fund of \$3,325 billion in 2010). Therefore, the cost to the federal government will outweigh the benefits that accrue to Social Security because higher interest payments are made to all bondholders, not just Social Security. This increase in interest costs will necessitate an increase in other tax revenues relative to taxes without reform.

One implication of a change in interest rates and equity returns is a redistribution of income that would result depending on the particular tax system. First of all, there would be a transfer of income from equity holders to bond holders as interest rates increase. Secondly, general income taxes would increase for bond holders, equity holders, and wage earners in order to pay for the increasing cost of debt service. Overall, there would therefore be a redistribution of income from wage earners and equity holders to bondholders. This is further complicated by the fact that a substantial portion of U.S. Government debt (approximately one third of publicly held debt) is held by foreigners. This creates a political dilemma in which there is distribution of income from American taxpayers (U.S. wage earners and holders of capital) to foreign bondholders. Regardless of the tax scheme, moving Social Security funds to the equities market would necessitate higher income taxes due to the effect this type of reform has on interest rates.

6. Increase the payroll tax

One of the major alternatives to shifting funds is to increase the payroll tax and continue investing the full Trust Fund in Treasury securities. Note that this would substantially increase the size of the Trust Fund (and reduce the amount of publicly-held debt). Intuitively, we would predict that an increase in the Social Security Trust Fund would cause interest rates to fall because of the decreased amount of debt the government must market to the public. Along a similar, but qualitatively opposite, line of thought as above, the borrowing costs of the federal government would fall which would allow a decrease in overall taxes. This decrease in interest costs would allow for a tax reduction offsetting some of the increase in Social Security payroll taxes. Furthermore, in this case there would be a transfer of income from foreign bondholders to American taxpayers. Even though either reform regime (shifting funds to equities or increasing payroll taxes) is expected to save the Social Security system,

the secondary effects of the two different reforms from a change in interest rates are significantly different and should be considered before any specific reform is decided upon.

7. Summary and conclusion

Several proposals have been developed to reform the Social Security System in order to ensure that it is fully funded. The investment of Social Security funds in equities has often been proposed as a means to avoid increasing payroll taxes. The general equilibrium model developed in this paper demonstrates some of the effects of these reform proposals. We show that investing Social Security funds in equities will increase interest rates on bonds and decrease the return on equities. Furthermore, such a shift in Social Security investments will lead to a necessary increase in general income taxes (assuming the level of total federal debt remains larger than the Social Security Trust Fund) and possible income redistribution. Thus, to some extent, investing Social Security funds in equities simply shifts a potential increase in payroll taxes to an increase in income taxes. Dynamic extensions of the model may be interesting to pursue by giving more insight into the long-run effects of any reform. It would be desirable to take into account the long-run changes which may occur due to lower weighted average cost of capital and subsequent increases in real assets of the firm or distortions which may occur in the labor market due to higher Social Security payroll taxes.

Appendix A.

A.1. Proof

It is possible to rearrange (6) and (7) to get r_e as a function of r_b and ν and similarly for r_b

$$r_e = \frac{\left[n_x - n_x \nu \eta + n_y + \frac{n_y}{(1 - \gamma_y) \sigma^2} r_b + n_z \eta - n_z \eta \nu - \frac{B}{\exp(r_b)} \right]}{n_y} (1 - \gamma_y) \sigma^2$$

$$= f(r_b, \nu) \tag{16}$$

$$r_b = \frac{\left[n_x \eta \nu + \frac{n_y}{(1 - \gamma_y) \sigma^2} r_e + n_z - n_z \eta + n_z \eta \nu - \frac{E}{\exp(r_e)} \right]}{n_y} (1 - \gamma_y) \sigma^2 = g(r_e, \nu) \tag{17}$$

Taking the total derivatives of (16) and (17) with respect to ν results in

$$\frac{dr_e}{d\nu} = f_{r_b} \frac{dr_b}{d\nu} + f_\nu \tag{18}$$

$$\frac{dr_b}{dv} = g_{r_e} \frac{dr_e}{dv} + g_v \tag{19}$$

where

$$g_{r_e} = 1 + \frac{E(1 - \gamma_y)\sigma^2}{n_y(\exp(r_e))} \tag{20}$$

$$g_v = \frac{(n_x\eta + n_z\eta)(1 - \gamma_y)\sigma^2}{n_y} \tag{21}$$

$$f_{r_b} = 1 + \frac{B(1 - \gamma_y)\sigma^2}{n_y(\exp(r_b))} \tag{22}$$

$$f_v = -\frac{(n_x\eta + n_z\eta)(1 - \gamma_y)\sigma^2}{n_y} = -g_v \tag{23}$$

Using (18) and (19) we now have the following

$$\frac{dr_e}{dv} = f_{r_b} \frac{dr_b}{dv} - g_v \tag{24}$$

$$\frac{dr_b}{dv} = g_{r_e} \frac{dr_e}{dv} + g_v \tag{25}$$

which is a system of two equations in two unknowns, dr_e/dv and dr_b/dv . This system has the solution

$$\frac{dr_e}{dv} = \frac{g_v(f_{r_b} - 1)}{1 - g_{r_e}f_{r_b}} \tag{26}$$

$$\frac{dr_b}{dv} = -\frac{g_v(g_{r_e} - 1)}{1 - g_{r_e}f_{r_b}} \tag{27}$$

It can be shown that $f_{r_b} > 1$, $g_{r_e} > 1$, and $g_v > 0$ and therefore

$$\frac{dr_e}{dv} < 0 \tag{28}$$

$$\frac{dr_b}{dv} > 0 \tag{29}$$

Appendix B.

In our analysis, we assumed that the most risk-averse individual remained constrained in the equities market, the least risk averse individual remained constrained in the bond market,

and the intermediate individual remained unconstrained following a change in ν . In order to ensure these results remain valid, it is necessary to find an upper bound on γ_x and a lower bound on γ_z . In order to ensure that the most risk-averse individual is originally constrained to be in the equities market, it is necessary that, initially $\nu\eta > \omega_x$ and post reform $\nu'\eta > \omega_x'$ (where $\omega_x = (r_e - r_b)/(1 - \gamma_x)\sigma^2$ and $\omega_x' = (r_e' - r_b')/(1 - \gamma_x)\sigma^2$) which implies

$$\gamma_x < \min\left(1 - \frac{r_e - r_b}{\eta\nu\sigma^2}, 1 - \frac{r_e' - r_b'}{\eta\nu'\sigma^2}\right) \tag{30}$$

where r_e' and r_b' are the post reform returns and ν' is the post reform fraction the Social Security Trust Fund allocates to equities. The least risk-averse individual is assumed to originally be constrained to hold funds in the bond market, and following reform is assumed to remain constrained. In order for this to be true, it is necessary that prereform $(1 - \nu)\eta > 1 - \omega_z$ and post reform $(1 - \nu')\eta > 1 - \omega_z'$ where ω_z and ω_z' are similar to the above. Therefore, a lower bound for γ_z can be solved for as

$$\gamma_z > \max\left(\frac{r_e - r_b}{[(1 - \nu)\eta - 1]\sigma^2} + 1, \frac{r_e' - r_b'}{[(1 - \nu')\eta - 1]\sigma^2} + 1\right) \tag{31}$$

It can be shown that $\gamma_x < \gamma_z$. Finally, type y individuals must be identified by a γ_y such that

$$\begin{aligned} \max\left(1 - \frac{r_e - r_b}{\eta\nu\sigma^2}, 1 - \frac{r_e' - r_b'}{\eta\nu'\sigma^2}\right) &< \gamma_y \\ &< \min\left(\frac{r_e - r_b}{[(1 - \nu)\eta - 1]\sigma^2} + 1, \frac{r_e' - r_b'}{[(1 - \nu')\eta - 1]\sigma^2} + 1\right) \end{aligned} \tag{32}$$

in order to remain unconstrained prior to and after reform. The above restrictions concerning γ_x , γ_y , γ_z (together with the assumption that all three individuals are risk-averse to some degree) are sufficient to show that $r_e > 0$ and $r_e - r_b > 0$.

Appendix C.

C.1. Trust-fund actions do not affect individual's decisions

If individuals do not perceive funds in the Social Security Trust Fund as a portion of their savings then the Trust Fund can be treated as just another very large investor. Therefore, an individual of type i has control over $\hat{W}_i \neq W_i$ of which they would optimally allocate $(1 - \omega_i)\hat{W}_i$ to bonds and $\omega_i\hat{W}_i$ to equities. Also, define SSF as the amount of funds that Social Security controls. If there are three groups of individuals identified by concerning γ_x , γ_y , and γ_z with respective population sizes of n_x , n_y , n_z , then we can define γ^* which solves the equation

$$n_x\omega_x + n_y\omega_y + n_z\omega_z = n\omega^* \tag{33}$$

where $n = n_x + n_y + n_z$.

The solution to this equation, γ^* , satisfies the following equation:

$$1 - \gamma^* = \frac{(n_x + n_y + n_z)(1 - \gamma_x)(1 - \gamma_y)(1 - \gamma_z)}{n_x(1 - \gamma_y)(1 - \gamma_z) + n_y(1 - \gamma_x)(1 - \gamma_z) + n_z(1 - \gamma_x)(1 - \gamma_y)} \quad (34)$$

Without loss of generality, we can assume that there is only one type of individual, identified by γ^* , which leads to the bond and equity market-clearing conditions

$$(1 - \omega^*) + (1 - \nu)ssf = \frac{B}{\exp(r_b)} \quad (35)$$

$$\omega^* + \nu(ssf) = \frac{E}{\exp(r_e)} \quad (36)$$

where $\omega^* = (r_e - r_b)/((1 - \gamma^*)\sigma^2)$, $ssf = SSF/\hat{W}$, $B = Q_b E[P'_b]/\hat{W}$, $E = Q_e E[P'_e]/\hat{W}$. By a similar methodology as above, (35) and (36) can be rearranged so that

$$r_e = F(r_b, \nu) \quad (37)$$

$$r_b = G(r_e, \nu) \quad (38)$$

Taking the total derivatives of (37) and (38) with respect to ν , the resulting two equations can be solved for $dr_e/d\nu$ and $dr_b/d\nu$ as

$$\frac{dr_b}{d\nu} = -\frac{G_\nu(G_{r_e} - 1)}{1 - F_{r_b}G_{r_e}} > 0 \quad (39)$$

$$\frac{dr_e}{d\nu} = \frac{G_\nu(F_{r_b} - 1)}{1 - F_{r_b}G_{r_e}} < 0 \quad (40)$$

where $F_{r_b} = 1 + B(1 - \gamma^*)\sigma^2/\exp(r_b) > 1$, $G_{r_e} = 1 + E(1 - \gamma^*)\sigma^2/\exp(r_e) > 1$, and $G_\nu = ssf(1 - \gamma^*)\sigma^2 > 0$. These results are similar to the results given in Section 2 with r_e decreasing and r_b increasing following a shift of Social Security Trust Fund money into the equities market.

References

- Advisory Council on Social Security. (1997). *Report of the 1994–1996 Advisory Council on Social Security, Vol. 1: Findings and Recommendations*. Washington: Government Printing Office.
- American Economic Association. (1996). *The American Economic Review*, 86, 358–377.
- Auerbach, A., & Kotlikoff, L. (1987). *Dynamic Fiscal Policy*. Cambridge: Cambridge University Press.
- Bohn, H. (1997). Social Security Reform and financial markets. In S.A. Sass and R.K. Triest (Eds.), *Social Security Reform Conference Proceedings*, Federal Reserve Bank of Boston.
- Campbell, J. Y., & Mankiw, N. G. (1989). Consumption, income, and interest rates: reinterpreting the Time Series Evidence. *NBER Macroeconomics Annual*, 185–216.
- Congressional Budget Office. (2000). *The Budget and Economic Outlook: Fiscal Years 2001–2010*. Washington, Government Printing Office.

- Diamond, P. A. (1996). Proposals to restructure Social Security. *Journal of Economic Perspectives*, 10, 67–88.
- Diamond, P. A. (1997). Macroeconomic aspects of Social Security Reform. *Brookings Papers on Economic Activity* (forthcoming).
- Elder, E., & Holland, L. (1999). Implications of Social Security Reform on interest rates: theory and evidence. Working paper; University of Arkansas at Little Rock.
- Federal Reserve Bank of Boston. (1997). Social Security Reform conference proceedings: links to saving, investment, and growth. In S. A. Sass and R. K. Triest (Eds.), *Social Security Reform conference proceedings*, Federal Reserve Bank of Boston.
- Federal Reserve Board of Governors. (1999). Flow of Funds Accounts for the United States. Washington, DC.
- Friend, I., & Blume, M. (1975). The demand for risky assets. *The American Economic Review*, 65(December), 900–922.
- Geanakoplos, J., Mitchell, O., & Zeldes, S. (1998). Would a privatized Social Security System really pay a higher rate of return?. In R. D. Arnold, M. J. Graetz, & A. H. Munnell (Eds.), *Framing the Social Security Debate* (pp.137–157). Washington, DC: Brookings Institution Press.
- Hall, R. E. (1988). Intertemporal substitution and consumption. *Journal of Political Economy*, 96, 339–357.
- Ingersoll, J. E. (1987). *Theory of financial decision making*. Savage, MD: Rowman & Littlefield Publishers.
- Kotlikoff, L., Smetters, K., & Walliser, J. (1998). Privatizing Social Security in the US: comparing the options. *Technical Paper Series: Macroeconomic Analysis and Tax Analysis Divisions*. Congressional Budget Office.
- Merton, R. C. (1969). Lifetime portfolio selection under uncertainty: the continuous-time case. *Review of Economics and Statistics*, 51, 247–257.
- National Academy of Social Insurance. (1998). Conference proceedings. In R. D. Arnold, M. J. Graetz, & A. H. Munnell (Eds.), *Framing the Social Security debate*. Washington, D.C.: Brookings Institution Press.
- Samuelson, P. A. (1969). Lifetime portfolio selection by dynamic stochastic programming. *Review of Economics and Statistics*, 51, 239–246.
- Smetters, K. (1998). Privatizing versus prefunding Social Security in a stochastic economy. *Technical Paper Series: Macroeconomic Analysis and Tax Divisions*. Congressional Budget Office.
- Stein, H. (1997). Social Security and the single investor. *The Wall Street Journal*, Wednesday, February, 5, A18.
- Stiglitz, J. E., Munnell, A. H., & Frankel, J. A. (Council of Economic Advisors). (1997). *Economic Report of the President*. Washington, DC: U.S. Government Printing Office.