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A comparison of state university defined benefit and defined contribution pension plans: a Monte Carlo simulation

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Abstract

This paper examines investment risk in comparing defined benefit (DB) and defined contribution (DC) plans by employing a Monte Carlo simulation. Using a bivariate normal distribution, two general types of risk are associated with a DC-plan. The first is that not enough is being earned by an allocation rule to cover DB-plan outflows. Secondly the portfolio may experience runs of losses that can't be overcome by waiting for a better year because the money runs out. The general result is that higher stock allocations allow the higher earning potential of stocks, even if the losses are occasionally experienced, to accumulate enough wealth to see a DC portfolio match the promised benefits of a DB-plan. © 2001 Published by Elsevier Science Inc.

Keywords: Defined benefit; Defined contribution; Monte Carlo simulation

1. Introduction

Recently, financial planners and securities brokers have called attention to the investment risk involved with self-directed retirement portfolios, that is, the risk of not obtaining the investment result desired to sustain retirement. Recent work includes Yuh, Hanna, and Montalto (1998) on projecting retirement adequacy, Hariharan, Chapman, and Domian (2000) on risk tolerance and asset allocation choices near retirement, and Montalto, Yuh, and Hanna (2000) on the factors that determine when an individual plans to retire. However, these studies and the majority of studies in this area do not incorporate simulations of investment results and potential retirement income in their work. Simulations provide probabilistic analysis of retirement possibilities based on repeated sampling from probability distributions rather than using historical averages exclusively. Clements (2001) notes that such models are being used by Financial Engines, Morningstar's ClearFuture and T. Rowe

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Price Associates in his February 20th, 2001 *Getting Going* column entitled "Retirement Models That Let Reality Bite," in the *Wall Street Journal*.

Another rather recent development is the ability of participants to choose which type of retirement plan they prefer, defined benefit (DB) or defined contribution (DC). This trend has been, especially pronounced in state run University Systems. Forty-seven states provide university faculty and administrators with a choice of plans. Missouri, South Dakota, and Wisconsin currently do not provide a choice of plans. Hawaii has enacted the legislation providing a choice, but no defined contribution plan has been implemented at this time.

When choosing between a DB- and DC-plan, a participant is choosing who will ultimately bear the investment risk, and investment reward, associated with his or her retirement income. Under DC-plans investment risk falls on participants, with retirement income depending on the participant's investing acumen and the vicissitudes of the market. Under DB-plans the employer/sponsor assumes the investment risk, and participants in effect "buy," through service time and possibly payroll deductions, a retirement annuity.

Although, differences exist between the plans on many levels, including vesting, portability, and special features, this paper focuses on the potential retirement income and investment risk of the plans. In this paper, it is proposed that the plans be compared in the following manner: first, based on assumptions about work life and salary, the retirement income from a DB-plan is estimated. Second, a Monte-Carlo simulation of investment results from a DC-plan is conducted. Finally, the chances of matching the retirement cash flows of the DB-plan with the DC-plan are determined.

2. Cash flows

A comparison of the plans requires estimates of the contributions to be invested by a participant as if under a DC-plan, as well as the income to be received by the participant as if under a DB-plan. To begin, assumptions must be made about years of work life and the percentage of the salary, if any, to be contributed, as well as salary increases, to determine the amount and timing of the DC-plan cash flows. Assumptions must then be made about years of retirement (longevity), the benefit formula, and any cost of living adjustments (COLAs) for inflation, to determine the amount and timing of the DB-plan cash flows.

To illustrate this, assume a participant has a starting salary of $40,000, will incur a 5% salary deduction for either plan (which is matched with 5% from the sponsor under DC), will have annual increases of 3% in salary, and an expected work life covered by either plan of 30 or 35 years. Note that all plans do not require a contribution from the participant, although many do. In those that do, the matching amount by the employer varies. The framework could easily incorporate non-contributory plans using only what the sponsor provides.

These assumptions are used to generate the DC-plan cash inflows to the retirement portfolio using the formula:

$$S(1+g)^{t-1}(\mathrm{cp}) = \text{DC-plan cash inflows} \tag{1}$$

where t: current years worked, S: beginning salary, g: growth rate in salary and cp: salary deduction plus sponsor's matching percentage contribution.

Table 1
Annual cash flows

30 years work, 30 years retired		35 years work, 25 years retired	
Year	Cash flow	Year	Cash flow
1	4000	1	4000
2	4120	2	4120
3	4244	3	4244
29	9152	34	10609.34
30	9426	35	10927.62
31	−56557	36	−76493.3
32	−58254	37	−78788.1
59	−129398	59	−150966
60	−133280	60	−155495

This table presents the annual cash flows for the two scenarios. The positive cash flows represent each year's inflow to the DC-plan portfolio as defined by:

$$S(1+g)^{t-1}(\text{cp}) = \text{DC-plan cash inflows}$$

where t: current years worked; S: beginning salary ($40,000); g: growth rate in salary (3%); cp: salary deduction plus sponsor's matching percentage contribution ($5 + 5 = 10\%$).

The negative cash flows represent the promised benefits from a DB-plan as defined by:

$$(\text{FS} \times n \times p)(1+i) = \text{DB-plan cash flows}$$

where n: number of years work, FS: final average salary, p: percentage rate for defined benefit formula (2%), i: inflation rate. All numbers include a 3% COLA each year.

To determine the DB-plan benefits, assume a life expectancy at retirement of either 30 or 25 years, which corresponds with the years of retirement income, with the initial amount to be received calculated as 2% benefit per year of service multiplied by the last annual salary. Final average pay, which is the basis for calculating the DB benefit, varies widely across employers. It may be the average of the last few years, or of the highest earnings years. The last year is used here for simplicity. The annual benefit is assumed to increase by 3% a year for COLAs that represent the approximate historical rate of inflation each year. To keep the example, simple annual end-of-year contributions and payouts are used. Hence, the defined benefit plan cash flows are represented by:

$$(\text{FS} \times n \times p)(1+i) = \text{DB-plan cash flows} \tag{2}$$

n: number of years work, FS: final average salary, p: percentage rate for defined benefit formula and i: inflation rate.

Table 1 presents the annual cash flows for the two scenarios.

3. Simulation methodology

Once the relevant cash flows under each plan are specified, the next step is to model the participant's potential investment results under a DC-plan. For simplicity's sake it is assumed that there are only two asset classes available; stocks, a diversified portfolio of large capitalization stocks and bonds, a portfolio of long-term investment grade corporate bonds.

Annual returns on large cap stocks and corporate bonds, as reported by Ibbotson (2000) over the period 1950 –1999, are used to calculate estimates of the statistical properties (mean, variance and covariance) of the assets. This period, the post WWII era, is used as there are convincing arguments that the statistical properties of the returns, as well as the underlying economic processes that create them, are different after WWII compared to prior to the war.

Furthermore, it is assumed that the assets' returns are normally distributed and linearly correlated. Using the above estimates and assumptions allows the simulation to sample from populations based on the observed means, variances and covariances without limiting the observations to the historical data itself. Given the observed asset returns meet these assumptions then the joint distribution is bivariate normal.

In the simulation the stock and bond returns have means of 14.84% and 6.42%, and variances of 16.61% and 10.27%, respectively, and a covariance between annual returns of 47.27%. The variances and covariance are estimates taken from the same 1950–1999 period as the means. Monte Carlo simulations are used to generate 60 pairs of annual returns for stocks and bonds drawn from a bivariate normal distribution, the collection representing one possible way the future might unfold over the next 60 years.

In the next step an initial allocation of 20% stocks—80% bonds is applied to the simulated year-by-year return pairs. In effect this means the dollar value of the portfolio is rebalanced each year to have 20% of the wealth invested in stocks and 80% invested in bonds. The resulting portfolio return is combined with the corresponding cash flow figures presented in Table 1 (over time, DC cash inflows make the portfolio value grow and DB cash outflows make it decline) to calculate how much the portfolio is worth each year. This procedure is repeated 1,000 times, i.e., there are 1,000 iterations of the portfolio simulation, for each of two work-retirement scenarios, (i) 30 years of work, 30 years retired and (ii) 35 years of work, 25 years retired.

An iteration is considered a success if the portfolio doesn't run out of money over the specified retirement horizon. The total number of successes over all 1,000 iterations is determined for the 20%/80% allocation. In the cases where an iteration is a success the end-of-retirement remaining portfolio wealth is noted. In the cases where an iteration does not succeed, the year in which the portfolio is exhausted is noted. Next, the allocation is changed to 30% stocks, 70% bonds and the entire process is repeated with the new success rate, ending portfolio values, and year exhausted being found. The underlying simulated data are not changed, just the allocation, to isolate the effect of different allocations on the portfolio results. Then the allocation is changed by another 10%, etc., continuing up to an allocation of 80% stocks, 20% bonds. The allocations are changed since Brinson, Hood, and Beebower (1986) found that more than 90% of actual investment returns are dependent on the portfolio allocation mix. Subsequent research is consistent with these findings. For example, Ho, Milevsky, and Robinson (1994) and Milevsky, Ho, and Robinson (1997) find that most retirees need to invest as significant part of their retirement funds in equities.

4. Results

The findings for the different stock/bond allocation mixes for the 30 years of work, 30 years of retirement example are reported in Tables 2 and 3.

Table 2
Successes: 30 years work & 30 years retired DC cash flows meet or exceed DB cash flows over retirement period

Allocation stocks/bonds	Success rate	Minimum ending wealth	Maximum ending wealth	Median ending wealth	Confidence intervals
20/80	15%	$22154	$12402888	$1625579	12%, 18%
30/70	31%	$99403	$28692234	$2630143	27%, 35%
40/60	52%	$3749	$59873086	$3780911	48%, 56%
50/50	69%	$25027	$116650263	$7057357	65%, 73%
60/40	80%	$94948	$217400538	$10775749	77%, 83%
70/30	87%	$151469	$396699194	$16715628	84%, 89%
80/20	90%	$407147	$749635972	$25299177	88%, 92%

The success rate indicates the percentage of cases where the DC retirement benefits meets or exceeds the DB cash flows during retirement. The minimum, maximum and median ending wealth figures represent the levels of wealth obtained by the successful outcomes of the simulation. The confidence intervals are the 99% *Z*-values for the population proportion.

Table 2 presents the findings for the iterations that are successes over 30 years of work and 30 years of retirement. It shows that as the proportion of stocks in the portfolio increases, the greater is the probability that the defined contribution plan will be able to match the outflows of the defined benefit plan (measured as a percentage of successes). For example, the results of the 50/50 allocation indicate there is a 69% success rate that the defined contribution plan will match the retirement outflows for the defined benefit plan. The ending wealth figures provide the range of possible terminal portfolio values at the end of the retirement period, in this case 30 years, for those trials that have successfully matched the defined benefit plan outflows. Note that the high median wealth figures are indicative of the substantial probability of never running out of money if the employee is able to make it through the first 30 years of retirement successfully. In the 50/50 allocation example, the minimum ending wealth is $25,027, with a maximum of $116,650,263 and a median value of $7,057,357.

Because of sampling, the success rates from repeated runs will not be identical. Scott (2000) notes that the number of successes will be a noisy estimate. Given this divergence it

Table 3
Failures: 30 years work & 30 years retired DC cash flows do not meet DB cash flows over retirement period

Allocation stocks/bonds	Failure rate	Worst case portfolio life	Median portfolio life
20/80	85%	4 yrs	13 yrs
30/70	69%	5 yrs	15 yrs
40/60	48%	6 yrs	16 yrs
50/50	31%	7 yrs	17 yrs
60/40	20%	6 yrs	17 yrs
70/30	13%	6 yrs	17 yrs
80/20	10%	5 yrs	18 yrs

The failure rate indicates the percentage of cases where the DC retirement benefits does not meet the DB cash flows during retirement. The worst and median portfolio lives represent the length of time at which the DC-plan runs out of money.

would be informative to incorporate a confidence interval. The number of successes (and failures) can be modeled as a binomial distribution (success or failure). With n as the number of trials and π as the population proportion, when both $n\pi$ and $n(1-\pi)$ are greater than or equal to 5 (Weirs, 1991, pp. 251), then the normal distribution is a good approximation to the binomial. With a simulation size of 1,000 and the asset allocations of interest, these simulations meet these criteria. With the assumption of normality, the formula for the confidence interval estimate of the population proportion is:

$$p \pm z\frac{p(1-p)^{1/2}}{n^{1/2}} = \text{confidence interval} \tag{3}$$

where p: the sample proportion (number of successes/number of trials); z: z-value that corresponds to the desired level of confidence; n: number of trials.

Given the simulation size (n) of 1,000 and assuming a 99% confidence interval which corresponds to a z-value of 2.58, Eq. (3) becomes to:

$$p \pm \frac{2.58(p(1-p))^{1/2}}{(1000)^{1/2}} = 99\%\ \text{confidence interval} \tag{4}$$

Looking again at the 50/50 allocation, the success rate (p) is 69% with a 99% confidence interval of 65.2%–72.8%.

Table 3 presents data for the iterations that are failures. In the 50/50 allocation case the failure rate is 31%, meaning that proportion of the iterations are unable to match the defined benefit outflows for the entire 30 years. The worst case scenario for the 50/50 allocation case is 7 years, the earliest a retiree would run at of money using a defined contribution plan. The median portfolio life shows that half the failures ran out of money prior to the 17th year. Interestingly, neither the median time nor the worst case time to failure appreciably increases with the proportion of stocks in the portfolio. That the retirement portfolio runs out of money in some iterations of all the allocation rules illustrates the investment risk involved.

Table 4
Successes: 35 years work & 25 years retired DC cash flows meet or exceed DB cash flows over retirement period

Allocation stocks/bonds	Success rate	Minimum ending wealth	Maximum ending wealth	Median ending wealth	Confidence intervals
20/80	27%	$15454	$23670438	$2246138	23%, 31%
30/70	52%	$17196	$34577528	$2942761	48%, 56%
40/60	75%	$17177	$56186155	$4900485	71%, 79%
50/50	88%	$25259	$102810932	$7784290	85%, 91%
60/40	94%	$155586	$177978466	$13031401	92%, 96%
70/30	97%	$405035	$297074637	$20997803	96%, 98%
80/20	98%	$132208	$482955774	$32445394	97%, 99%

The success rate indicates the percentage of cases where the DC retirement benefits meets or exceeds the DB cash flows during retirement. The minimum, maximum, and median ending wealth figures represent the levels of wealth obtained by the successful outcomes of the simulation. The confidence intervals are the 99% Z-values for the population proportion.

Table 5
Failures: 35 years work & 25 years retired DC cash flows do not meet DB cash flows over retirement period

Allocation stocks/bonds	Failure rate	Worst case portfolio life	Median portfolio life (yrs)
20/80	73%	4 yrs	13 yrs
30/70	48%	5 yrs	16 yrs
40/60	25%	5 yrs	17 yrs
50/50	12%	6 yrs	17 yrs
60/40	6%	6 yrs	16 yrs
70/30	3%	7 yrs	13 yrs
80/20	2%	6 yrs	11 yrs

The failure rate indicates the percentage of cases where the DC retirement benefits does not meet the DB cash flows during retirement. The worst and median portfolio lives represent the length of time at which the DC-plan runs out of money.

Tables 4 and 5 present the simulation results for 35 years of work followed by 25 years of retirement. The results show what is expected; a shorter work life with a longer retirement period fails more often to duplicate the DB-plan and it also accumulates less wealth. A longer work life and shorter retirement period generates more successes and has larger ending wealth.

At retirement the employee has an option to continue to work if their defined contribution portfolio wealth is deemed insufficient to meet projected retirement needs. A comparison of Tables 2 and 4 shows that by working five more years the success rate increases for all asset allocations. For example, the 50/50 allocation the success rate goes from 69% to 88%. The defined benefit plan eliminates this option by capping the number of years worked used in determining the retirement benefits. For example, if the state capped the number of years at 30, no additional retirement benefits will be earned if you work more than 30 years. Lastly, the median wealth figures indicate that if success is attained then there is a high probability of never running out of money.

5. Conclusion

The paper highlights investment risk in comparing DB- and DC-plans by employing a Monte Carlo simulation. The investment risk of a DC-plan is of two general types: (1) that not enough is being earned by an allocation rule to cover the DB-plan outflows, particularly with the lower stock allocation of 20/80 and 30/70 in a DC-plan, and (2) that regardless of allocation, sometimes the portfolio experiences runs of losses that can't be overcome by waiting for a better year because the money runs out. The best odds of exceeding a DB-plan's payouts are found for very high stock allocations such as, 70/30 and 80/20; allocations that allow the higher earning potential of stocks, even if losses are occasionally experienced, to accumulate enough wealth to see the portfolio through. Finally, the high median wealth figures are indicative of the substantial probability of never running out of money if the employee is able to successfully match the DB-plans benefits over the retirement horizons for each scenario.

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