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Variable annuities versus mutual funds: a Monte-Carlo analysis of the options

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Abstract

Mutual funds and variable annuities are similar instruments that differ mainly in their tax treatment. Their relative appeal is the subject of intense debate in the industry. This paper contributes to the literature by quantifying the impact of investment return uncertainty when comparing the two. We focus on the embedded tax options using Monte-Carlo simulations. We conclude that although low-cost variable annuities are superior to low-cost mutual funds over long time horizons, the critical threshold is at least 10 years for typical levels of risk aversion. If, however, one ignores the tax options, the erroneous break-even horizon drops to 5 years. © 2001 Published by Elsevier Science Inc.

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1. Introduction

Mutual funds and variable annuities are very similar savings and investment instruments that differ mainly in their tax treatment. While the former is treated akin to a portfolio of individual securities—which primarily consists of capital gains—the latter is taxed entirely as ordinary income, but with tax-deferred investment gains. The relative appeal of one versus the other, is the subject of intense debate in the industry and ultimately depends on subtle assumptions regarding investment horizons, management expenses and investment returns, as well as pre- and post-retirement tax rates. However, we believe that most of the existing debate between the defenders and detractors of non-qualified variable annuities are ignoring

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the crucial impact of *return* uncertainty when making their comparisons to fully-taxable mutual funds.

A priori, one would think that the *stochasticity* of returns—conditional on a particular growth rate—would not affect the relative superiority of one product versus the other. But this is not the case. The possibility of an investment loss endows the holder of the mutual fund with a ‘real tax option’ to harvest those losses. The strategic investor can re-establish a similar position at a lower tax-basis, and deduct any current losses against comparable gains. *De facto*, this creates a tax refund, which supplements the return from the mutual fund. Indeed, the recent market decline during the 2000–2001 period has generated much tax-loss selling activity. This type of strategy cannot be easily employed within a variable annuity (VA). Since, despite the favorable ordinary income treatment on losses, which can be netted against ordinary interest gains, lapsing or selling, the VA will most likely induce surrender charges on the order of 5–10%. Furthermore, this ‘real tax option’ to use losses against other gains would be even more valuable when comparing variable annuities to a portfolio of individual securities in which losing positions can mitigate the tax liability of winners, or with highly tax-efficient index funds.

Indeed, during any given year investment returns can be extremely volatile, even if the long-term growth rate of the market is relatively stable. We will demonstrate that, *ceteris paribus*, while the expected after-tax return of a variable annuity is higher, the after-tax returns from a variable annuity are more volatile than the after-tax returns from a mutual fund. Therefore, consumers might rationally shun the higher expected (after-tax) return from the variable annuity simply because of the higher volatility. One therefore, requires a risk adjustment mechanism for choosing amongst the two alternatives.

Similarly—although we do not pursue this directly—if the holder of the investment faces the (albeit) small possibility of having to lapse, surrender or sell the investment, prior to his or her intended time horizon, then the possible stochasticity of time horizon should also be factored into the comparison. In other words, even if the holder has a *declared* 10-year horizon, the possibility of early termination introduces another dimension of stochasticity. This is especially important given the 10–15% annual lapsation rates that are built into industry pricing models for variable annuities.

Therefore, to account for and value this uncertainty, we introduce a risk-adjustment mechanism. Namely, we compare non-qualified variable annuities to mutual funds by computing the certainty equivalent of utility (CEU). This methodology will be explained in detail, but is commonly used in the economics literature when comparing the welfare implications of various product designs, investment strategies or policy changes. Knight and Mandell (1992) used a similar method to analyze the costs and benefits from dollar-cost averaging. In our framework, a variable annuity is preferred to a mutual fund—conditional on a particular investment horizon—*only* if its Certainty Equivalent of Utility is greater. This method of ranking risky alternatives is quite common in the finance and economics literature, and can be traced to work by Aboudi and Thon (1995), Thistle (1993), Levy (1992), and Hadar and Seo (1988).

Analytic techniques aside, our main practical observation is that although we find that low-cost variable annuities are indeed superior to low-cost mutual funds for investors with a long-time horizon, the critical threshold is at least 10 years for typical levels of risk aversion.

If, however, we ignore the embedded real tax options, the erroneous break-even horizon drops to 5 years. We can summarize the main message of this paper with one sentence. *The uncertainty of investment returns increases the break-even horizon.*

The remainder of this paper is organized as follows. In Section 2, we review some of the existing academic and practitioner literature on the topic of 'real tax options' and the choice between variable annuities and mutual funds. Section 3 develops the concept of the CEU and how it can be used to compare and contrast risky alternatives. Section 4 goes back to basics and presents a deterministic model of the trade-off between variable annuities and mutual funds. This framework serves to confirm and calibrate existing models as well as to set the background for the stochastic simulation model, which is presented in Section 5. We then generate a statistical distribution of after-tax wealth at the terminal horizon, assuming that either mutual funds or variable annuities have been employed. The mutual fund investor is assumed to utilize all tax-timing strategies at his or her disposal. We then rank the distribution—and the value to the consumer—by computing their CEU. This allows us to determine which of the two strategies is 'better' in the face of uncertain outcomes for both. As a by-product of the simulation, we can also provide a risk and return trade-off between variable annuities and mutual funds. Arguably, this type of financial recommendation is more appropriate, accurate and helpful than a deterministic either/or statement picking one type of savings vehicle over the other. Section 6 concludes the paper with a summary of the key insights.

2. The existing literature

Non-qualified variable annuities have mushroomed into a trillion-dollar market that is currently inhabited by both insurance companies and banks. Cowan, Howell, and Power (2001) provided a discussion of the benefits from selling these products.

However, this growth has not been without controversy. Financial commentators and practitioners alike criticized a study by Price Waterhouse Coopers (PWC, 1997) on the impact of the 1997 Taxpayers Relief Act on the relative appeal of VA contracts. Geer (1998) and Veres (1998) offered criticism of the assumptions that went into the PWC analysis. Furthermore, a variety of class action lawsuits have been filed against life insurance companies that sell VAs both within, and outside of, tax-qualified plans. In addition to allegations of fraud and inappropriate sales practices, one of the critical issues in these cases is the proper level of mortality and expense (insurance) fees. Panko (2000) offered a discussion of the companies and products that have been targets of this type of litigation. In response, some vendors have introduced products with insurance fees on the order of 10–30 basis points per year. This is in contrast to the historical industry average of 120–130 basis points in additional insurance fees. Indeed, if the only concern with the VA structure is the added fees, this may soon become a non-issue with the widespread availability of no-load low-tee products.

In terms of existing literature, Reichenstein (2000) and Toolson (1991) have both done admirable work in analyzing the effect of time horizons, investment returns, added fees and marginal tax rates on the relative appeal of variable annuities compared to mutual funds. Both papers conclude that ceteris paribus VAs are better when net-returns are higher and time horizons are longer. Likewise, the greater the gap in marginal tax brackets, pre- and post-retirement, the more appealing the VA becomes. And, indeed, their conclusions resonate with practitioner wisdom in the matter. However, the critical assumption of constant annual returns, which underlies all research work in this area, is most suspect when time horizons are longer. *Even if the long-term annualized rate of return from equity is exactly 12%, the pattern in which these returns are achieved is quite important from a tax point of view.*

From a methodological perspective, our work is similar in spirit to the early paper by Burgess and Madeo (1980) in which simulation techniques were used to analyze the costs and benefits of particular retirement savings plans. More recently, The Conning Company (1997) and Daily (1996) have used simulations to examine the after-tax payoff from VAs. However, neither of these reports is very rigorous in their assumptions and risk preferences, nor do they analyze the embedded 'real tax option' *a la* Constantinides (1984) or Dammon, Dunn and Spatt (1989). This particular option endows the investor with an incentive to realize certain losses, and perhaps even gains, in order to reduce ones overall tax liability. Stated differently, when investing in fully taxable mutual funds, one can easily deduct losses. This is not so with variable annuities. Scott (1997) applied the concept of 'real tax options' to the value of making 401(k) contributions. Lewellen and Mauer (1988) modeled the tax-timing options associated with the opportunities for investors to tax manage their portfolios by deferring gains and taking losses. Dickson and Shoven (1994) examined the effect of tax options on after-tax returns to shareholders within an equity mutual fund. This paper examined the feasibility of managing open- and closed-end Standard and Poor's 500 index funds, which defer net capital gains realizations. Arnott, Berkin, and Ye (2001) illustrated the substantial benefits from the loss-harvesting strategy within a fully taxable portfolio.

Please see the work by Cram and Austin (1997), Cunningham (1994) as well as the classic book by Scholes and Wolfson (1992) for addition research on the interplay between taxes and investment returns.

3. The certainty equivalent of utility (CEU)

Under the minimum set of conditions for consistent and rational behavior and the assumption that all investors always prefer more wealth to less, a utility function *exists* and can be used to analyze investors' choices under uncertainty. Under these assumptions, investors will maximize their expected utility and we can use expected utility to rank risky alternatives. Stated differently, if the expected utility of the risky outcome X (read: the random after-tax return from a variable annuity) is greater than the expected utility of the risky outcome Y (read: the random after-tax return from a mutual fund), then X is preferred to Y. In algebraic terms, we write that $E[U(X)] > E[U(Y)]$, when investors prefer X to Y.

Using the utility concept, we can establish definitions of risk premium and certainty equivalents and then investigate the relationship between the utility function and investor's risk aversion.

The CEU is measured in dollars and cents and it captures the amount that makes $E[U(W)] = U(\text{CEU})$. In plain English, CEU represents the amount of money that a rational consumer would be willing to pay for a given risky outcome. In abstract terms, CEU provides the same level of utility (comfort, satisfaction and pleasure) as the risky outcome W.

So, e.g., W might represent the random monetary outcome from a lottery ticket, stock portfolio or a night at the casino. The non-random quantity CEU would represent the sum of money that a consumer would be willing to give, or receive, in exchange for W. It is a method for comparing alternatives that differ in two dimensions, using a one-dimensional metric.

Naturally, the CEU of any given W is dependent on a person's risk preferences and tastes. Some people might place a very high CEU on a particular lottery ticket, and others might think it has no value at all. The important point is that each and every rational consumer has a (theoretical) CEU for each and every random financial event in their life.

The risk premium, P, is the difference between $E(W)$ and CEU, where $E(W)$ is the investor's expected wealth. When an investor is risk averse and strictly prefers more wealth to less, then the CEU is less than the expected wealth; in other words, the risk premium is positive. Stated differently, nobody takes on risk, unless they expect to benefit.

In this paper, we adapt this concept to compare two risky alternatives random variables X and Y by calculating the CEU that make people indifferent between the two choices. Given two strategies with the same initial investment, the CEU is an amount of money that must be deducted from (or added to) one of them in order to make its expected utility of after-tax terminal wealth equal to the other.

For instance, the certainty equivalent of utility, denoted by CEU(MF:VA), between MF (the mutual fund) and VA is the amount of money that must be deducted from (or added to) the initial investment in the MF to make:

$$E[U(W(\mathrm{MF}) - \mathrm{CEU}(\mathrm{MF} : \mathrm{VA}))] = E[U(W(\mathrm{VA}))] \tag{1}$$

or, stated differently,

$$E[U(W(\mathrm{MF}) - \mathrm{CEU}(\mathrm{MF} : \mathrm{VA}))] - E[U(W(\mathrm{VA}))] = 0 \tag{2}$$

The symbol W(MF) denotes the random after-tax value of the mutual fund account, the symbol W(VA) denotes the random after-tax value of the variable annuity account. If CEU(MF:VA) is positive, it implies that the MF is better than the VA. Since, to obtain the same expected utility of after-tax wealth, the mutual fund requires a lower initial investment. Once again, we stress that the CEU of one strategy versus another is subjective in nature since it depends on attitudes towards risk, and therefore, might be positive for some, while negative for others. In the same manner, the CEU between the MF with and without the real tax option can be used to price this option in isolation.

4. A deterministic analysis of VA versus MF

This section provides a deterministic example of the trade-off. We assume the initial investment is \$10,000, which is critical for the analysis since only \$3,000 of losses can currently be netted against other gains. For a base case, an investor is assumed to pay a combined federal and state tax rate of 39.6% on ordinary income before retirement, and a tax rate of 31.6% during retirement. Furthermore, the investor pays the applicable capital gains tax rate of 23.6% both before and after retirement. These tax assumptions are taken directly from Reichenstein (2000), and do not reflect the recently proposed changes in the tax-law. Recently, the U.S. implemented the largest tax cut in almost 20 years. Starting from 2001, all tax-bracket rates above 15% will be reduced. The stochasticity of future tax rates and structures creates yet another problem in comparing the two alternatives, which is beyond the scope of this paper.

For the purpose of this deterministic analysis, we assume that the annual gross return of the equity mutual fund is 10% every year. Except, at year 5, we assume that the mutual fund loses 40% of its value in a severe market correction. This latter assumption is meant to show the benefit of the loss-harvesting strategy, or the real tax option. According to current tax law, the investor can use up to $3,000 of net capital losses to offset ordinary income in any year. Losses that exceed $3,000 in 1 year can be carried forward. For the mutual fund with the real tax option, we assume that at the end of year 5, the investor sells his or her investment to realize losses and to obtain a tax refund. After receiving the tax refund, the investor reinvests in (similar, to avoid wash sale issues) the mutual fund at the beginning of year 6.

We assume that 30% of total return each year is distributed to the investor as short-term capital gains and dividends, which will be taxed at the ordinary income tax rate. In addition, 59% of long-term capital gains are realized and distributed to the investor every year and will be taxed at the capital gain tax rate. These assumptions are from Poterba, Shoven and Sialm (2000). They are the average proportion of short-term distribution and the average proportion of long-term capital gains realization of 12 actively-managed equity funds during 1962–1998. Moreover, we assume that if there are capital losses, no dividend or capital gains are distributed to the investor. This assumption is for the model's simplicity and may be too strict. Some would argue that when a mutual fund has capital losses, it does not imply that each component company in the fund lost money as well. Thus, some companies in the fund may still pay dividends even in a year where the mutual fund has capital losses. The impact of this dividend payment issue on the break-even horizon for the VA versus the MF is not examined in this paper. However, this issue will not change the main point of the paper—the benefit of the loss-harvesting strategy or the real tax option (RTO).

For the base case, we assume that the total management expenses for the mutual fund are 1.41% while the total expenses for the variable annuity are 2.07%, with a contract charge of 30. These assumptions are also from Reichenstein (2000). They are average total expenses for the equity mutual funds and for the equity variable annuities.

Naturally, a large part of our analysis is driven by assumptions, and one might quibble with each and every one of them individually. However, our objective is to demonstrate the effect of ignoring the uncertainty as opposed to capturing an individual's precise circumstance. Later, we shall examine the low-cost annuity and mutual fund and change a variety of assumptions to examine the impact on our results.

According to these assumptions, Table 1 displays the investor's after-tax terminal wealth from the investment in the MF with, and without, the real tax option and from the investment in the VA for various investment horizons.

The category (column) of "mutual fund without real tax option" means that the investor does not harvest losses, but rather engages in a buy-and-hold strategy until the end of the time horizon.

Table 1
After-tax terminal wealth from investing $10,000 using three alternative strategies, for a deterministic return case

Investment horizon	After-tax terminal wealth from investing in		
	Mutual fund with real tax option	Mutual fund without real tax option	Variable annuity
10	11,771	11,203	10,806
15	15,906	14,806	14,238
20	21,494	19,674	19,265
23	25,749	23,381	23,341
24	27,346	24,773	24,921
25	29,043	26,252	26,626
30	39,246	35,140	37,407
34	49,933	44,451	49,540
35	53,031	47,151	53,198
40	71,659	63,216	76,324

This table reports the after-tax terminal wealth from investing $10,000 in the VA and MF, both with and without the real tax option for various time horizons under the following assumptions: an initial investment is assumed to be $10,000. An investor is assumed to pay a combined federal and state tax rate of 39.6% on ordinary income before retirement, and a tax rate of 31.6% during retirement. Furthermore, the investor pays the applicable capital gain tax rate of 23.6% both before and after retirement. The annual gross return of an equity mutual fund is assumed to be 10% every year. Except, at year 5, the mutual fund is assumed to lose 40% of its value in a severe market correction. This latter assumption is meant to show the benefit of the loss-harvesting strategy, or the real tax option. The 30% of total return each year is assumed to be distributed to the investor as short-term capital gains and dividends. which will be taxed at the ordinary income tax. In addition, 59% of long-term capital gains are realized and distributed to the investor every year and will be taxed at the capital gain tax rate. We assume that the total management expenses for the mutual fund are 1.41% while the total expenses for the variable annuity are 2.07%, with a contract charge of $30.

Table 1 shows that it takes 24 years for the investment in the VA to outperform the investment in the MF without the real tax option, however, it takes 35 years for the investment in the VA to outperform the investment in the MF when the real tax option *is* utilized.

Moreover, Table 1 shows that for all investment horizons, the MF with the real tax option always results in a higher amount of after-tax terminal wealth compared to the MF without the real tax option. This result is obvious and is consistent with Arnott et al. (2001), as well as Dammon et al. (1989), who find that loss harvesting is robust and adds substantial value over time.

Table 2 shows the details of how to calculate the after-tax wealth from the MF with (MF + RTO) and without the real tax option. All formulas used to calculate the after-tax wealth and detailed calculations are available from the authors upon request. According to our assumptions, the initial investment is $10,000, thus, the beginning market value of the mutual fund in year 1 is $10,000.

Since we assume that the annual gross return from the mutual fund is 10% and the total expense of the mutual fund is 1.41%, the after-fee annual net return is $10 - 1.41 = 8.59\%$. Thus, the ending before-tax market value of the mutual fund in year 1 is $(1.0859)(10{,}000) = 10{,}859$. The total return at year 1 is $10{,}859 - 10{,}000 = 859$.

Table 2
Calculation of the after-tax wealth from investing $10,000 in a mutual fund, both with and without utilization of the real tax option

Year	Annual gross return (%)	Beginning market value	Ending before-tax market value	Beginning cost base	Total return	Capital gain (loss)	Realized capital gain	Net capital gain (loss)	Dividend and short-term capital gain	Ending cost base	Ending market value	Ending after-tax market value
Panel A: with real tax option												
1	10	10,000	10,859	10,000	859	601	355	601	258	10,427	10,673	
2	10	10,673	11,590	10,427	917	642	524	888	275	10,993	11,357	
3	10	11,357	12,333	10,993	976	683	618	1,047	293	11,642	12,071	
4	10	12,071	13,108	11,642	1,037	726	662	1,155	311	12,351	12,824	
5	−40	12,824	7,514	12,351	−5,311	−5,311	0	−4,837	0	12,351	8,702	
6	10	8,702	9,449	8,702	747	523	309	523	224	9,073	9,288	
7	10	9,288	10,085	9,073	798	556	456	773	239	9,566	9,883	
8	10	9,863	10,732	9,566	849	594	538	911	255	10,131	10,504	
9	10	10,504	11,406	10,131	902	632	593	1,005	271	10,747	11,159	
10	10	11,159	12,118	10,747	959	671	639	1,083	286	11,432	11,876	11,771
Panel B: without real tax option												
1	10	10,000	10,859	10,000	859	601	355	601	258	10,427	10,673	
2	10	10,673	11,590	10,427	917	642	524	888	275	10,993	11,357	
3	10	11,357	12,333	10,993	976	683	618	1,047	293	11,642	12,071	
4	10	12,071	13,108	11,642	1,037	726	682	1,155	311	12,351	12,824	
5	−40	12,824	7,514	12,351	−5,311	−5,311	0	−4,837	0	12,351	7,514	
6	10	7,514	8,159	12,351	645	452	267	452	194	12,671	8,020	
7	10	8,020	8,708	12,671	689	482	394	667	207	13,097	8,534	
8	10	8,534	9,267	13,097	733	513	464	787	220	13,584	9,070	
9	10	9,070	9,849	13,584	779	545	512	868	234	14,117	9,636	
10	10	9,636	10,463	14,117	828	579	552	935	248	14,708	10,255	11,203

Under the same assumptions mentioned in Table 1, Panel A of this table shows the detailed calculation of after-tax wealth from the investment in the MF assuming utilization of the real tax option for a 10-year investment, while Panel B shows the detailed calculation of after-tax wealth from the investment in the MF without the real tax option.

Now, the dividend and short-term capital gain for year 1 is (0.3)(859) = 258 and the rest is long-term capital gain which is 859 − 258 = 601. Then 59% of this long-term capital gain is assumed to be distributed and taxed at capital gain tax rate. Thus, realized long-term capital gains for year 1 is (0.59)(601) = 355. The net capital gain (loss) in any year is equal to capital gain (loss) in that year plus unrealized long-term capital gain from the previous year. Thus, the net capital gain (loss) at year 1 is equal to 601. The ending market value at year 1 is equal to the beginning market value at year 1 minus tax on dividend and short-term capital gain—tax on long-term capital gain which is (10,859) − (0.396)(258) − (0.236)(355) = 10,673. The ending cost base at year 1 is equal to the beginning cost base plus dividend and short-term capital gain reinvested plus long-term capital gain reinvested which is 10,000 + (1 − 0.396)(258) + (1 − 0.236)(355) = 10,427.

Then, the beginning market value at year 2 is equal to the ending market value at year 1, and the beginning cost base at year 2 is equal to the ending cost base at year 1. We can use the same procedure to calculate the ending after-tax market value of the mutual fund investment without the real tax option at years 2–9. At the end of year 10, the investor will sell his or her investment. Thus, he or she will pay capital gain tax on unrealized capital gain or will receive the tax refund if there is capital loss. Therefore, since at year 10, the ending market value is less than the ending cost base, there is capital loss amounting to 14,708 − 10,225 = 4,483. Since capital loss exceeds 3,000, then only 3,000 can be used to offset ordinary income. Thus, the ending after-tax market value is equal to the ending market value plus the tax return which is 10,255 + (3,000)(0.316) = 11,203.

The same procedure used in the MF without the real tax option can be used to calculate the ending market value for the MF with the real tax option for any year that has net capital gain. However, for a year that has net capital losses, such as year 5 in our example, in the MF with the real tax option, we assume that the investor sells his or her investment at year 5 to realize capital loss and obtain the tax refund. Thus, the ending market value at year 5 for the MF with the real tax option is the ending before-tax value plus the tax refund. However, in our example, the net capital loss is 4,837 which exceeds 3,000, thus, only 3,000 can be used to offset ordinary income. And 4,837 − 3,000 = 1,837 will be carried forward to the next year. Thus, the ending market value at year 5 is 7,514 + (3,000)(0.396) = 8,702 compared to 7,514 in the MF without the real tax option. The beginning market value and cost base at year 6 also change to 8,702. Comparing the ending after-tax market value of the MF with and without the real tax option, it is obvious that the ending after-tax market value of the MF with the real tax option is greater than that of the MF without the real tax option.

Note that the assumption of the initial investment of $10,000 is critical and cannot be scaled up or down arbitrarily. Given the assumption of 40% loss in a severe market correction at year 5 and an initial investment of $10,000, there is net capital loss of 4,837 at year 5. However, only 3,000 can be used to offset ordinary income and 1,837 will be carried forward. In this case, the benefit of the real tax option is equal to a difference between the ending market value of the MF with the real tax option shown in Panel A and that of the MF without the real tax option shown in Panel B, which is 8,702 − 7,514 = 1,188. However, if we scale down an initial investment to 5,000, then net capital loss at year 5 is 2,418 and the benefit of the real tax option reduces to 958. If we scale up an initial investment to 20,000, then net capital loss is 9,674. Again only 3,000 can be used and 6,674 will be carried forward. In this case, the benefit of the real tax option is 10,312. In conclusion, the larger the initial investment, the higher the benefit of the real tax option and the higher the unused losses carried forward.

In addition, we perform the sensitivity analysis of minimum investment horizon needed for investment in the VA to outperform investment in the MF with and without the real tax option for various annual gross returns assumptions. The results shows that, as in the previous table, given the annual gross return, the minimum investment horizon needed for investment in the VA to outperform investment in the MF with the real tax option is longer than that needed to outperform investment in the MF without the real tax option.

Moreover, we also find that the higher the annual gross return, the shorter the investment horizon needed for investment in the VA to outperform investment in the MF. This arises from the fact that return in the VA grows tax-deferred. Thus, the higher the annual gross return, the larger the accumulation grows tax-deferred.

5. The stochastic simulation

In the past, Milevsky and Posner (2000a, 2000b) used simulation techniques to analyze variable annuities. However, previous research has focused on the limited value of the Guaranteed Minimum Death Benefit (GMDB) *vis a vis* the added insurance charges. Yet, the simulation methodology is quite robust and can help answer a much wider set of questions.

Our simulation makes the following set of assumptions. As in the deterministic example, we assume that an initial investment is \$10,000. For a base case, an investor pays a combined federal and state tax rate of 39.6% on ordinary income before retirement, and a tax rate of 31.6% during retirement. The applicable capital gains tax rate is assumed to be 23.6% for both before and after retirement. The total expense on the mutual fund is 1.41% while the total expense on the variable annuity is 2.07%, and the contract charge is \$30.

Unlike the deterministic example, in the Monte-Carlo simulation, we assume that the annual gross return of the mutual fund is stochastic with mean = 12.68% and standard deviation = 17.07%. In the simulation, each of the return relatives (one plus the rate of return) is assumed to be generated by a lognormal distribution. In so doing, our simulation generates sequences of return relatives with mean = 1.1268 and standard deviation = 0.1707. Moreover, we assume that the proportion of short-term distribution (dividend plus short-term capital gain) is uniformly distributed between 15% and 45%, and the proportion of long-term capital gains realization is uniformly distributed between 35% and 80%. All these assumptions are from Poterba et al. (2000), which are based on information of 12 actively-managed equity funds during 1962–1998.

For the base case, we assume that the investor's utility function is $W_T^{1-\gamma}/1-\gamma$, where $\gamma = 2$, which is consistent with the recent estimate by Feldstein and Ranguelova (2001). This number is used in a variety of other studies in the economics literature that 'peg' the coefficient of relative risk aversion at approximately 2. Later, we will examine the sensitivity of our results to this parameter.

Likewise, we believe that our methodology is immune to the recent criticism by Nawrocki (2001) of Monte Carlo techniques in personal finance, since one cannot obtain analytic expressions for the after-tax level of wealth, nor does the possible serial correlation of investment returns impact the relative appeal of one strategy versus the other.

We now run each simulation a total of 5,000 times. For instance, if the investor's investment horizon is T years, we generate 5,000 sequences of T-year returns. This, in turn, generates 5,000 series of after-tax terminal wealth at the end of year T from investing in a X instrument, $W_i(T/X)$, where T is the investment horizon, $i = 1, 2, \ldots, 5{,}000$ and $X = \text{MF}$ when the investor invests in the mutual fund, $X = \text{MF} + \text{RTO}$ when the investor invests in the mutual fund and uses the loss-harvesting strategy (real tax option), $X = \text{VA}$ when the investor invests in variable annuity. This after-tax terminal wealth can be used to calculate the expected value of after-tax terminal wealth, $E[W(T/X)]$ and the expected utility from investing in the X instrument, $E[U(W(T/X))]$ as follows:

$$E[W(T/X)] = \sum_{i=1}^{5,000} W_i(T/X) \tag{3}$$

$$E[U(W(T/X))] = \sum_{i=1}^{5,000} \frac{[W_i(T/X)]^{1-\gamma}}{1-\gamma} \tag{4}$$

We calculate the CEU using a two-step process. First, we randomize the future returns to compute a distribution function, then we perturb the initial investment in the mutual fund until the expect utility of both are the same. The final perturbation amount is the CEU. In other words, for two investment instruments with the same initial investment, the CEU is the amount of money that must be deducted (or added) from (or to) an initial investment to make the expected utility of after-tax terminal wealth equal.

For instance, the certainty equivalent of utility (CEU(MF:VA)) between mutual fund and variable annuity investments is the amount of money that must be deducted (or added) from (or to) an initial investment to make:

$$E[U(W(T/\text{MF}) - \text{CEU}(\text{MF}:\text{VA}))] = E[U(W(T/\text{VA})] \tag{5}$$

In other words,

$$E[U(W(T/\text{MF}) - \text{CEU}(\text{MF}:\text{VA}))] - E[U(W(T/\text{VA})] = 0 \tag{6}$$

Let $\text{Dif}U(\text{MF}:\text{VA}) = E[U(W(T/\text{MF}) - \text{CEU}(\text{MF}:\text{VA}))] - E[U(W(T/\text{VA}))] = 0$, to find CEU(MF:VA), we use an optimization routine that determines an amount of money (CEU(MF:VA)) that must be deducted (or added) from (or to) an initial investment in the mutual fund to make Dif$U$(MF:VA) closest to zero. For optimization stopping conditions, we choose to stop the simulation when the improvement in the value of Dif$U$(MF:VA) in the last 100 iterations is less than 1%. One iteration consists of 5,000 simulations.

According to Table 3, when considering only the average, the average of after-tax terminal wealth from the VA will be greater the MF without the real tax option when the investor's investment horizon is at least 11 years.

However, if the investor chooses the MF, and uses the real tax option, the investment horizon needed for the mean of after-tax terminal wealth from the VA to be greater than the MF with the real tax option extends to at least 14 years.

Table 3
Simulation of variable annuity and mutual fund investment strategies

Investment horizon in years	Expected wealth mutual fund + RTO	Expected wealth mutual fund	Expected wealth variable annuity	CEU: mutual fund + RTO vs. variable annuity	CEU: mutual fund vs. variable annuity	CEU: mutual fund + RTO vs. mutual fund
5	14,967 (3,889)	14,785 (3,978)	14,424 (4,041)	445 (0.0069)	273 (0.0009)	179 (0.0013)
10	21,873 (7,972)	21,266 (8,112)	21,250 (9,278)	666 (0.0099)	272 (0.0069)	397 (0.0025)
11	23,870 (9,199)	23,149 (9,363)	23,379 (11,031)	686 (0.0051)	249 (0.0006)	444 (0.0012)
13	27,839 (11,426)	26,792 (11,546)	27,652 (14,161)	707 (0.0036)	180 (0.0051)	545 (0.0005)
14	30,435 (13,784)	29,263 (13,953)	30,757 (17,919)	713 (0.0025)	130 (0.0032)	585 (0.0008)
16	35,387 (16,473)	33,774 (16,579)	36,467 (22,190)	699 (0.0004)	22 (0.0009)	682 (0.0024)
17	38,519 (19,055)	36,707 (19,204)	40,400 (26,488)	684 (0.0054)	−46 (0.0011)	729 (0.0003)
20	48,403 (25,875)	45,721 (25,933)	53,000 (38,335)	624 (0.0008)	−269 (0.0005)	877 (0.0008)
25	72,553 (44,759)	67,703 (44,419)	87,365 (75,078)	380 (0.0005)	−774 (0.0024)	1,086 (0.0058)
31	114,260 (79,932)	104,977 (78,458)	155,046 (157,290)	36 (0.0007)	−1,502 (0.0045)	1,383 (0.0018)
32	123,640 (87,024)	113,344 (85,121)	171,317 (172,965)	−57 (0.0008)	−1,558 (0.0009)	1,430 (0.0001)
35	154,067 (113,526)	140,086 (110,635)	226,608 (243,671)	−300 (0.0003)	−2,129 (0.0039)	1,531 (0.0014)

This table displays the expected value and standard deviation (in parenthesis) of after-tax terminal wealth from the strategy of investing either 10,000 in the VA, or the MF, both with and without utilization of the real tax option. It also displays CEU from selecting one strategy vs. the other. Thus, e.g., although the expected wealth from the VA strategy is greater than the expected wealth from the MF + RTO strategy after 14 years, the standard deviation (i.e., risk) is greater as well. Thus, the CEU is used as a risk-adjustment mechanism. For example, the column ‘‘CEU: mutual fund + RTO vs. variable annuity’’ displays the CEU between the mutual fund with the real tax option and the variable annuity. The positive (negative) CEU is amount of money that must be deducted from (added to) the initial investment in the mutual fund with the tax option in order to make the difference between expected utilities of after-tax wealth from two strategies equal to a number in parenthesis. Thus, it is only after 32 years that the risk adjusted MF + RTO strategy is better than the VA strategy. Note that the simulation assumes that the investor pays a combined federal and state tax rate of 39.6% on ordinary income before retirement (savings phase), and a tax rate of 31.6% during retirement (withdrawal phase). The applicable capital gains tax rate is assumed to be 23.6%, both before and after retirement. The total expense ratio for the mutual fund is 1.41% while the total expense for the variable annuity is 2.07% (the 66 basis points reflect the Mortality and Expense Risk Charge), and the contract charge is $30. The annual gross return of the mutual fund is stochastic with mean = 12.68% and standard deviation = 17.07%. Moreover, we assume that the proportion of short-term distribution (dividend plus short-term capital gain) is uniformly distributed between 15% and 45%, and the proportion of long-term capital gains realization is uniformly distributed between 35% and 80%. Finally, we assume that the investor exhibits constant relative risk aversion preferences with a coefficient of 2.

Table 3 also displays the certainty equivalent of utility. For the investor's investment horizon from 0 to 16 years, the utility of the MF is greater than the VA. Thus, under these investment horizons, some money must be *deducted* from the mutual fund's initial investment to make the utility from investing in the MF and the VA the same. When the investor's investment horizon is at least 17 years, the utility from investing in the MF is less than that in the VA. Thus, some money must be added to the mutual fund's initial investment to make the utility from both investments the same.

However, if the investor uses the loss-harvesting strategy, the investment horizon that makes the utility from the VA greater than the MF will extend to 32 years. From the last column of Table 3, we read that the mutual fund with the real tax option always outperforms the mutual fund without the real tax option by providing more utility. Moreover, the certainty equivalent of utility shown in the last column can be thought of as the value of the real tax option. Indeed, similar to standard options, the longer the time to maturity, the higher the value of the option is. Furthermore, given the assumption that an investor invests initially 10,000 into one mutual fund, the mean of unused losses carried forward is about 5,500 for 30-year-investment horizon. Thus, the value of the real tax option will be even higher if the investor has alternative investments to deduct the unused losses.

Table 4 displays the certainty equivalent of utility for the low-cost case. We assume that the total expense of the MF is 0.2% while the total expense of the VA is 0.3%. All other assumptions are the same as those in the base case. The main result from the table is that for the low-cost case, it takes 5 years for the VA to outperform the MF without the real tax option compared to 17 years in the base case. However, it takes 10 years for the VA to outperform the MF with the real tax option compared to 32 years in the base case. In our model, the low-cost mutual fund is assumed to be tax inefficient. With a low-cost, tax-efficient mutual fund, such as an S&P 500 indexed mutual fund, investors would be able to defer taxes for a longer time horizon than they do in our analysis. This will increase investment horizon needed for the investment in the VA to outperform the investment in the MF.

Table 4
Analysis of the low-cost variable annuity vs. mutual fund, with and without the real tax option

Investment horizon in years	Expected wealth mutual fund + RTO	Expected wealth mutual fund	Expected wealth variable annuity	CEU: mutual fund + RTO vs. variable annuity	CEU: mutual fund vs. variable annuity	CEU: mutual fund + RTO vs. mutual fund
4	14,221 (3,303)	14,107 (3,374)	14,083 (3,439)	135 (0.0030)	25 (0.0006)	116 (0.0021)
5	15,501 (3,908)	15,339 (3,994)	15,409 (4,181)	138 (0.0006)	−14 (0.0012)	155 (0.0008)
9	21,952 (7,441)	21,498 (7,588)	22,653 (9,037)	19 (0.0009)	−292 (0.0011)	306 (0.0078)
10	23,973 (8,782)	23,410 (8,924)	25,111 (11,061)	−37 (0.0023)	−396 (0.0019)	352 (0.0007)

Holding all other assumptions in Table 3 constant, this table displays the mean value (and standard deviation) of after-tax terminal wealth and the certainty equivalents of utility from the investment in the low-cost VA and the low-cost MF with and without the real tax option. In the low-cost case, we assume that the total expense of the MF is 0.2% while the total expense of the VA is 0.3%. The difference is attributed to the insurance expense, also known as the Mortality and Expense (M&E) Risk Charge.

Table 5
Sensitivity analysis of the required time horizon for (average cost) variable annuity dominance over mutual funds—with and without the real tax option—as a function of the mean return, standard deviation and coefficient of relative risk aversion

Annual mean return (%)	Standard deviation (%)	Minimum investment horizon needed for investment in VA to outperform MF and MF + RTO			
		CRRA = 2		CRRA = 3	
		MF + RTO	MF	MF + RTO	MF
14.68	19.07	24	14	>40	20
	17.07	19	13	32	17
	15.07	16	12	23	15
12.68	19.07	>40	18	>40	29
	17.07	32	17	>40	26
	15.07	25	16	39	21
10.68	19.07	>40	27	>40	>40
	17.07	>40	24	>40	>40
	15.07	>40	23	>40	35

This tables reports the sensitivity of the minimum investment horizon needed for the VA to have the higher utility of after-tax terminal, holding other assumptions in Table 3 constant. Note that a lower subjective (pre expense) investment return, greater standard deviation and greater risk aversion all increase the required time horizon the for VA to dominate.

6. Comparative statics

This section determines the minimum investment horizon needed for the VA to have the higher utility of after-tax terminal wealth compared to the MF with and without the real tax option, for various parameters in our model.

Table 5 shows that, holding other parameters constant, the lower the annual mean return, the longer the horizon needed for the VA to outperform the MF. This is consistent with the

Table 6
Sensitivity analysis of the required time horizon for (average cost) variable annuity dominance over mutual funds—with and without the real tax option—as a function of the tax rate

	Pre-retirement (savings) tax rate (%)	Retirement (withdrawal) tax rate (%)					
		31.60	29.60	27.60	25.60	23.60	21.60
MF + RTO	39.6	32	28	26	23	20	17
MF	39.6	17	15	12	10	8	6
MF + RTO	31.6	36	33	30	27	23	20
MF	31.6	23	21	18	16	13	12

The sensitivity of the minimum investment horizon needed for the VA to dominate the alternative, as a function of tax rates, holding other assumptions in Table 3 constant. For example, if the marginal tax rate prior to retirement is 31.6% and the marginal tax rate during retirement—when the funds are withdrawn—is 21.6%, then a VA is better than a MF strategy if the horizon is greater than 20 years. If, however, the real tax option is not included, the VA is erroneously deemed better after 12 years. Note, that as the tax rate during retirement increases, the break-even horizon increases.

deterministic analysis. Given other parameters, the higher the standard deviation of the gross return, the longer the horizon needed for the VA to outperform the MF. Finally, if we hold constant all other parameters, the larger the coefficient of relative risk aversion, the longer the horizon needed for investment in the VA to outperform investment in the MF. *In other words, more risk averse people are less likely to purchase the variable annuity because of the larger variability in after-tax returns.*

Two of the critical parameters in our model are the pre- and post-tax rates. Clearly, the lower the tax rate in retirement, compared to the pre-retirement rate, the greater the benefit of a variable annuity. Table 6 confirms that this is the case, by maintaining all parameters constant and then gradually reducing the after-retirement tax rate. Indeed, the lower the rate, the smaller the number of years needed for break-even.

7. Conclusion and final remarks

The main objective of this paper was to quantify the impact of return uncertainty when measuring the relative benefits of variable annuities versus mutual funds. We introduced the concept of a 'real tax option' to harvest losses in mutual funds, that does not exist within a variable annuity structure. We argued that the distribution of (uncertain) after-tax wealth within a variable annuity, compared to a mutual fund, is higher, but more volatile. This fact calls for a risk-adjustment mechanism to properly compare and rank the two alternatives. To that end, we used simulation techniques to compute the certainty equivalents of utility and then value the option and properly rank the strategies.

We concluded that although low-cost variable annuities—with insurance expenses lower than 10 basis points—are superior to low-cost mutual funds for investors with a long time horizon, the critical threshold is at least 10 years for typical levels of risk aversion. And, for those who are more risk-averse than average, the break-even horizon is even larger.

Furthermore, if one ignores the stochasticity and the embedded real tax option, the erroneous break-even horizon drops to 5 years. In other words, a typical *deterministic* analysis of variable annuities versus mutual funds—regardless of conservative versus aggressive assumptions—is automatically *biased* to favor variable annuities because of the constant growth assumptions. Moreover, for the average cost variable annuity with 66 basis points of insurance expenses, the risk-adjusted break-even horizon can be as high as 30 years.

Further research will apply the same technique to the payout stage of a variable annuity. Indeed, although less than 5–10% of variable annuities are annuitized, the tax treatment of the decumulation phase is more favorable than a comparable drawdown strategy. Brown, Mitchell, Poterba, and Warshawsky (1999) examined tax treatment of the payout stage. We, therefore, intend to compute the CEU with a broader range of investment strategies. Likewise, the inclusion of various guaranteed minimum death and retirement benefits in the new generation of variable annuities, might serve to increase the certainty equivalents of utility for high enough levels of risk aversion. Finally, assuming the co-existence of both products, we intend to examine the issue which investment assets belong within a variable annuity structure, and which belong outside the shelter also known as the asset location' problem—*a la* Dammon, Spatt and Zhang (2001).

In time honored tradition, we leave these questions for future research.

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References

Aboudi, R., & Thon, D. (1995). Second-degree stochastic dominance decisions and random initial wealth with applications to the economics of insurance. *The Journal of Risk and Insurance*, *62*(1), 30–49.

Arnott, R. D., Berkin, A. L., & Ye, J. (2001). Loss harvesting: What's it worth to the taxable investor? *The Journal of Wealth Management*, Spring, 10–17.

Brown, J., Mitchell, O., Poterba, J., & Warshawsky, M. (1999). Taxing retirement income: Non-qualified annuities and distributions from qualified accounts. *National Tax Journal*, *14*(3), 219–234.

Burgess, R. D., & Madeo, S. A. (1980). A simulation study of tax sheltered retirement plans. *Journal of the American Taxation Association*, *1*, 34–41.

Constantinides, G. (1984). Optimal stock trading with personal taxes: Implications for prices and abnormal returns. *The Journal of Financial Economics*, *13*, 65–69.

Cowan, A. R., Howell, J. C., & Power, M. L. (2001). Wealth effects of banks' rights to market and originate annuities. *The Quarterly Review of Economics and Finance*, in preparation.

Cram, T. L., & Austin, J. R. (1997). An analysis of the tradeoff between tax deferred earnings in IRAs and preferential capital gains. *Financial Services Review*, *6*(4), 227–242.

Cunningham, W. P. (1994). The effect of tax increases on variable annuities. *The Journal of the American Society of CLU and ChFC*, January, 50–56.

Daily, G. S. (1996). The uses of Monte-Carlo simulation. *NAPFA Advisor*, May, 14–18.

Dammon, R. M., Dunn, K. B., & Spatt, C. S. (1989). A reexamination of the value of tax options. *The Review of Financial Studies*, *2*(3), 341–372.

Dammon, R. M., Spatt, C. S., & Zhang, H. H. (2001). Optimal asset location and allocation with taxable and tax-deferred investing. *Working Paper*, Carnegie Mellon University.

Dickson, J. M., & Shoven, J. B. (1994). A stock index mutual fund without net capital gains realization. *NBER*, April.

Feldstein, M., & Ranguelova, E. (2001). Individual risk in an investment-based social security system. *American Economic Review*, *91*(4), 1116–1125.

Geer, C. T. (1998). The great annuity rip-off. *Forbes*, February, 106–110.

Hadar, J., & Seo, T. K. (1988). Asset proportions in optimal portfolios. *The Review of Economic Studies*, *55*(3), 459–468.

Knight, J., & Mandell, L. (1992). Nobody gains from dollar cost averaging: Analytical, numerical and empirical results. *Financial Services Review*, *2*(1), 51–61.

Levy, H. (1992). Stochastic dominance and expected utility: Survey and analysis. *Management Science*, *38*(4), 555–593.

Lewellen, W. G., & Mauer, D. C. (1988). Tax options and corporate capital structures. *Journal of Financial and Quantitative Analysis*, *23*(4), 387–400.

Milevsky, M. A., & Posner, S. E. (2000a). The titanic option: Valuation of the guaranteed minimum death benefit in variable annuities and mutual funds. *The Journal of Risk and Insurance*, *65*(2), March.

Milevsky, M. A., & Posner, M. (2000b). *The probability of fortune: Financial strategies with the best odds*. New York: Stoddart Publishing Co.

Nawrocki, D. (2001). The problems with Monte-Carlo simulation. *Journal of Financial Planning*, *14*(11), 13–27.

Panko, R. (2000). Can annuities pass muster? *BEST'S REVIEW*, July, 103–109.
Poterba, J. M., Shoven, J. B., & Sialm, C. (2000). Asset location for retirement savers. *NBER*, November.
Price Waterhouse Coopers (1997). *The effect of the 1997 taxpayer relief act on VA contracts*.
Reichenstein, W. (2000). An analysis of nonqualified tax-deferred annuities. *The Journal of Investing*, Summer, 1–12.
Scholes, M. S., & Wolfson, M. A. (1992). *Taxes and business strategy: A planning approach*. Englewood Cliffs, NJ: Prentice Hall.
Scott. J. S. (1997). *Determining the size and distribution of the 401(k) tax benefits: An empirical application of real options pricing theory*. Unpublished Ph.D. Dissertation, Stanford University.
CONNING Insurance Research and Publications (1997). *Variable annuities: We've only just begun*. Hartford, CT: Conning & Company.
Thistle, P. D. (1993). Negative moments, risk aversion, and stochastic dominance. *The Journal of Financial and Quantitative Analysis*, *28*(2), 301–311.
Toolson, R. B. (1991). Investing: Comparing variable annuities and mutual funds. *Journal of Accountancy*, May, 71–77.
Veres, R. N. (1998). Numbers games. *The Dow Jones Investment Advisor*, March, 39–45.