Assessing the Impact of Rebalancing on Equal-weighted and Value-weighted Portfolios over Five Decades

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Abstract

This study investigates the impact of transaction costs on the performance differential between equal-weighted portfolios (EWPs) and value-weighted portfolios (VWPs). Employing a comprehensive dataset of 181 stocks from 1970 to 2023, we utilize paired two-sample tests to identify statistically significant differences in turnover and risk-adjusted returns. Our findings reveal a substantial performance advantage for EWPs, with annualized return surpluses ranging from 115 to 188 basis points over VWPs, depending on the assumed transaction cost level. Notably, this outperformance persists until transaction costs reach a critical threshold of 728 basis points of portfolio turnover. The analysis further demonstrates that EWPs outperform VWPs in 94.5% of scenarios devoid of transaction costs, declining to 84% when incorporating realistic cost assumptions. These results highlight the potential of EWPs to exploit diversification benefits but also emphasize the crucial role of transaction costs in moderating their outperformance.

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Introduction

Since the inception of the S&P 500 in 1957, market capitalization weighting has stood as the dominant indexing approach, steadfastly maintaining its influence over indices. The 1970s witnessed the emergence of index mutual funds, followed by the introduction of index ETFs in the 1990s. As of the close of 2022, the combined net assets in these categories had skyrocketed to $10.9 trillion. Notably, index mutual funds and index ETFs jointly constituted 46 percent of assets in long-term funds, a substantial leap from the 22 percent recorded in 2012 (Investment Company Institute, 2023).

Aligned with the Capital Asset Pricing Model (CAPM) proposed by Sharpe (1964) and Lintner (1969), the market-cap weighted approach—also known as the value-weight (VW) approach—has been a consistent force. Drawing on the foundational principles of Markowitz (1952), Sharpe (1964), and others, the finance sector has translated these insights into trillions of dollars allocated to or measured against market indexes

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such as the S&P 500 or Russell 1000 (R. D. Arnott et al., 2005).

The Efficient Market Hypothesis (EMH) by Fama (1970) posits that a VW index signifies an optimal mean-variance efficient investment. However, debates on market efficiency, notably fueled by Roll's critique (1977), persist. Various alternatives to VW indices, including fundamental indexing (R. D. Arnott et al., 2005), smart beta (Amenc et al., 2011; Amenc & Goltz, 2013; Amenc & Sourd, 2005), and equal-weighted (EW) indices (DeMiguel et al., 2009; Malladi & Fabozzi, 2017; Plyakha et al., 2012), have been proposed. Fundamental indexing incorporates factors like earnings to capture robust financial fundamentals, while smart beta utilizes alternative factor weighting methods like volatility to enhance returns (Amenc et al., 2016).

The primary objective of this paper is to conduct a performance comparison between Equal-Weighted Portfolios (EWP) and Value-Weighted Portfolios (VWP), with a particular emphasis on the influence of transaction costs. The intentional concentration on an equal-weighted index in this study is designed to ensure clarity and specificity, facilitating a more thorough examination of the effects of equal weighting without introducing the complexities associated with smart beta or fundamental indices. Theoretically, EWP can outperform a given VW index using the same index constituents. This allows us to create an EWP from a passive investable VW index and surpass its return.

The findings of this study will be of interest to both investors and academics. Investors will benefit from a better understanding of the risks and returns of EWPs, as well as the factors that influence their performance. Academics will benefit from a more rigorous and comprehensive empirical analysis of EWPs.

The rest of the paper is organized as follows: the next section reviews the current literature. The section on data explains how 181 stock data from 1970 to 2023 are used in this study. The methodology section describes the exhaustive random sampling method deployed in this paper, followed by the results and discussion. The final section presents conclusions and identifies the scope for further research.

The Literature of Equal-weighted Portfolios

Academic studies (DeMiguel et al., 2009; Malladi & Fabozzi, 2017; Plyakha et al., 2012, 2021) suggest that portfolios employing equal-weighting (or 1/N portfolios) exhibit superior performance compared to other portfolio strategies.

The benefits of an equal-weighted portfolio (EWP) are well documented. None of the 14 models assessed in DeMiguel et al. (2009) across seven empirical datasets consistently outperforms the EWP in Sharpe ratio, certainty-equivalent return, or turnover. Malladi and Fabozzi (2017) demonstrated, using theory, simulation, and real-world data from 1926 to 2014, that EWP outperforms the value-weighted portfolio (VWP). Moreover, in a two-stock, two-period setting, they demonstrate that a significant portion of the excess return is attributable to portfolio rebalancing. Importantly, they illustrate that, despite higher turnover costs associated with equal weighting, the excess returns surpass the increased expenses, making equal weighting economically justified. Plyakha et al. (2021) find that, despite accounting for a fifty basis point transaction cost, the monthly-rebalanced EWP surpasses VWP in terms of total mean return and one- and four-factor alphas.

Three historical explanations in academic literature account for the distinctions between EWP and VWP: the noisy market hypothesis, illiquidity, and autocorrelation (Pae & Sabbaghi, 2015). The noisy market hypothesis of Arnott (2006) posits that market errors or value tilting lead to inflated market capitalizations of overvalued stocks, resulting in lower expected returns for large-cap stocks and higher expected returns for undervalued small-cap stocks. As a result, the VWP is sub-optimal (Hsu, 2006). The elevated illiquidity premium linked to small firms can lead to an EWP demonstrating superior returns and increased volatility compared to a VWP (Amihud, 2002; Malladi & Fabozzi, 2017). The heightened autocorrelation of an EWP could result in a higher return compared to a VWP (Atchison et al., 1987).

Two recent additions to the three prior explanations include size and the rebalancing effect. Plyakha et al. (2012, 2021) reported that 58% of EWP's total excess mean return over VWP is attributable to the systematic component, compensating for exposure to smaller stocks as anticipated. However, 42% is derived from the difference in alphas, primarily influenced by the rebalancing effect (i.e., monthly rebalancing to maintain constant weights in the EWP). Additionally, Malladi and Fabozzi (2017) found that 85% of EWP's excess returns can be attributed to the rebalancing effect. Divergences exist between these two studies regarding the analysis timeframe and transaction costs. The former investigated up to 300 stocks between 1967 and 2009, accounting for 50 basis points (bps) in transaction costs. Conversely, the latter study examined 500 stocks from 1926 to 2014, factoring in a transaction cost of 169 bps. Swade et al. (2023) show that EWP consistently surpasses VWP, regardless of the rebalancing frequency spanning from 1 month to 60 months. Notably, the highest level of outperformance occurs with monthly rebalancing.

While EWP generally outperforms VWP in the long term, there are instances, particularly in short-term periods, where EWP may underperform VWP (Taljaard & Maré, 2021). The existing literature on EWP broadly suggests that they generally outperform VWP in the long term. However, some important questions have not yet been fully answered, such as the statistical significance of excess returns in the presence of transaction costs, the long-term consistency of excess returns, the impact of turnover on excess returns, and the potential for additional returns by optimizing efficiency to minimize transaction costs. In this study, we aim to address these unanswered research questions:

- **Statistical Significance of Excess Returns**: Do excess returns demonstrate statistical significance in the presence of transaction costs across extended time frames?
- **Long-Term Consistency of Excess Returns**: How consistent are excess returns over decades for the financial instrument, and what factors contribute to this consistency?
- **Impact of Turnover on Excess Returns**: What are the measurable effects of turnover on the excess return, providing insights into the financial instrument's dynamics?
- **Efficiency Optimization and Additional Returns**: By optimizing efficiency to minimize transaction costs, what additional returns can a firm generate, and to what extent does this contribute to financial performance?

Data

We create EWPs and VWPs of three stocks from a pool of 181 stocks, using monthly returns from the Center for Research in Security Prices (CRSP) from January 1, 1970, to January 1, 2023, to enable comparisons and validations through mathematical proofs. However, in the subsection titled "Robustness Checks in a Larger Portfolio," we show that our findings with three stocks hold true with a portfolio of 10 stocks as well. The 181 stocks in this paper meet two criteria: 1) 636 monthly returns from 01/01/1970 to 01/01/2023, and 2) a minimum market capitalization of $1 billion on 01/01/2023 to ensure enough liquidity for monthly portfolio adjustments.

There are 971,970 unique ways to select three stocks from a pool of 181, as per the combination formula $nCr = \frac{n!}{r!(n-r)!}$, considering unordered selection. Extending Malladi and Fabozzi (2017) two-stock portfolio model, we applied it to a three-stock scenario. We randomly chose 1,000 combinations from the 971,970 possibilities through exhaustive random sampling, calculating portfolio performance metrics and summary statistics from 01/01/1970 to 01/01/2023. Tables 1 and 2 present summary statistics and a list of the 181 stocks, respectively.

Table 1. Summary Statistics for Variables in the Study

Monthly Data	***Stock Price***	***Stock Return***	***Shares Outstanding (in thousands)***	***Market Cap (in millions, $)***
Mean	47.22	0.0124	315,957	16,527
Median	36.92	0.0103	80,944	3,034
Standard Error	0.12	0.0003	2,345	122
Standard Deviation	42.39	0.0909	795,794	41,248
Kurtosis	29.65	11.4846	63.84	35,665
Skewness	4.27	0.8622	6.86	5,265
Minimum	0.38	(0.8323)	451.00	0.69
Maximum	747.63	2.1352	11,144,681	581,099
N, (181 x 636)	115,116	115,116	115,116	115,116

- The monthly data to build EWP and VWP from 01/01/1970 to 01/01/2023 is shown below.
- A total of 115,116 monthly data points (181 stocks x 636 months) are used for analysis.
- A median company in this study has a stock price of $36.92, yielded a monthly total return of 1.03% (or annualized return of 13.08%), 80.944 million shares outstanding, and a $3.034 billion market cap.
- One can observe that monthly stock returns exhibit positive skewness and excess kurtosis.

Table 1 summarizes key statistics related to the study's variables. It covers monthly data collected from 01/01/1970 to 01/01/2023, totaling 115,116 data points from 181 stocks observed over 636 months. The median company featured in the study has a stock price of $36.92, delivering a monthly total return of 1.03% (equivalent to an annualized return of 13.08%), with 80.944 million shares outstanding and a market capitalization of $3.034 billion. An important observation from this data is that monthly stock returns exhibit a statistical distribution characterized by positive skewness and excess kurtosis, indicating certain asymmetry and heavy-tailedness in the return data.

Table 2 displays a roster of the 181 stocks employed in this study. The criteria for selecting these stocks are uncomplicated and revolve around two conditions: Firstly, these stocks needed to exhibit uninterrupted monthly returns from 01/01/1970 to 01/01/2023. Secondly, as of 01/01/2023, they were required to possess a market capitalization of at least $1 billion. This latter condition was essential to guarantee ample liquidity for the monthly portfolio rebalancing process, a critical component of the study's analytical framework. Previous academic studies (DeMiguel et al., 2009; Malladi & Fabozzi, 2017; Plyakha et al., 2012, 2021) also established a minimum market capitalization threshold to prevent the inclusion of companies with very small market capitalizations that pose challenges for the rebalancing process.

Table 2. List of 181 Stocks Used in This Study

Company Name	Ticker	Company Name	Ticker	Company Name	Ticker	Company Name	Ticker
ALCOA INC	AA.3	DANAHER CORP	DHR	INTL PAPER CO	IP	PPG INDUSTRIES INC	PPG
ABM INDUSTRIES INC	ABM	HF SINCLAIR CORP	DINO	ITT INC	ITT	PPL CORP	PPL
ABBOTT LABORATORIES	ABT	DIODES INC	DIOD	JOHNSON CONTROLS INTL PLC	JCI	PVH CORP	PVH
ARCHER-DANIELS-MIDLAND CO	ADM	DISNEY (WALT) CO	DIS	JOHNSON & JOHNSON	JNJ	RYDER SYSTEM INC	R
AUTOMATIC DATA PROCESSING	ADP	DOVER CORP	DOV	JPMORGAN CHASE & CO	JPM	ROLLINS INC	ROL
AMEREN CORP	AEE	DTE ENERGY CO	DTE	KELLOGG CO	K	RAYTHEON TECHNOLOGIES CORP	RTX
AMERICAN ELECTRIC POWER CO	AEP	DUKE ENERGY CORP	DUK	KIMBERLY-CLARK CORP	KMB	STEPAN CO	SCL
AEROJET ROCKETDYNE HOLDINGS	AJRD	DXC TECHNOLOGY CO	DXC	KENNAMETAL INC	KMT	SHERWIN-WILLIAMS CO	SHW
ALLETE INC	ALE	CONSOLIDATED EDISON INC	ED	COCA-COLA CO	KO	SKYLINE CORP	SKY.1
ALASKA AIR GROUP INC	ALK	EDISON INTERNATIONAL	EIX	KROGER CO	KR	SCHLUMBERGER LTD	SLB
ALEXANDER'S INC	ALX	EMERSON ELECTRIC CO	EMR	LOEWS CORP	L	SOUTHERN CO	SO
APA CORP	APA	EQT CORP	EQT	L3HARRIS TECHNOLOGIES INC	LHX	SPECTRUM BRND HLDG INC	SPB
AIR PRODUCTS & CHEMICALS INC	APD	EVERSOURCE ENERGY	ES	LOCKHEED MARTIN CORP	LMT	S&P GLOBAL INC	SPGI
ASHLAND INC	ASH	EATON CORP PLC	ETN	MASCO CORP	MAS	SPX TECHNOLOGIES INC	SPXC
ATI INC -NEW	ATI	ENTERGY CORP	ETR	MCDONALD'S CORP	MCD	SPIRE INC	SR
AVISTA CORP	AVA	EVERGY INC	EVRG	MDU RESOURCES GROUP INC	MDU	STANLEY BLACK & DECKER INC	SWK
AVNET INC	AVT	EXELON CORP	EXC	MARSH & MCLENNAN COS	MMC	SKYWORKS SOLUTIONS INC	SWKS
AVERY DENNISON CORP	AVY	FORD MOTOR CO	F	3M CO	MMM	STANDEX INTERNATIONAL CORP	SXI
BOEING CO	BA	FIRSTENERGY CORP	FE	ALTRIA GROUP INC	MO	TELEFLEX INC	TFX
BAXTER INTERNATIONAL INC	BAX	FOOT LOCKER INC	FL	MERCK & CO	MRK	TEGNA INC	TGNA
BRUNSWICK CORP	BC	FMC CORP	FMC	USX CORP-CONSOLIDATED	MROX.CM	TARGET CORP	TGT
BRINKS CO	BCO	FEDERAL SIGNAL CORP	FSS	MOTOROLA SOLUTIONS INC	MSI	TJX COS INC (THE)	TJX
BECTON DICKINSON & CO	BDX	GATX CORP	GATX	MURPHY OIL CORP	MUR	TIMKEN CO	TKR
BANK OF NEW YORK MELLON CORP	BK	GENERAL DYNAMICS CORP	GD	MEADWESTVACO CORP	MWV	TOOTSIE ROLL INDUSTRIES INC	TR
BRISTOL-MYERS SQUIBB CO	BMY	GENERAL ELECTRIC CO	GE	NEXTERA ENERGY INC	NEE	TRANE TECHNOLOGIES PLC	TT
BP PLC	BP	GRIFFON CORP	GFF	NEWMONT CORP	NEM	TEXAS INSTRUMENTS INC	TXN
CATERPILLAR INC	CAT	GENERAL MILLS INC	GIS	NEWMARKET CORP	NEU	TEXTRON INC	TXT
CABOT CORP	CBT	CORNING INC	GLW	NATIONAL FUEL GAS CO	NFG	TYCO INTERNATIONAL PLC	TYC
CROWN HOLDINGS INC	CCK	GENUINE PARTS CO	GPC	NISOURCE INC	NI	UGI CORP	UGI
COLGATE-PALMOLIVE CO	CL	GOODYEAR TIRE & RUBBER CO	GT	NORTHROP GRUMMAN CORP	NOC	UNION PACIFIC CORP	UNP
CLEVELAND-CLIFFS INC	CLF	HALLIBURTON CO	HAL	NUCOR CORP	NUE	UNIVERSAL CORP/VA	UVV
CLOROX CO/DE	CLX	HAWAIIAN ELECTRIC INDS	HE	NEW YORK TIMES CO -CL A	NYT	VF CORP	VFC
COMMERCIAL METALS	CMC	HESS CORP	HES	OGE ENERGY CORP	OGE	VULCAN MATERIALS CO	VMC
CUMMINS INC	CMI	HECLA MINING CO	HL	ONEOK INC	OKE	VORNADO REALTY TRUST	VNO
CONSUMERS ENERGY CO	CMS1	HONEYWELL INTERNATIONAL INC	HON	OLIN CORP	OLN	WALGREENS BOOTS ALLIANCE INC	WBA
CENTERPOINT ENERGY INC	CNP	HELMERICH & PAYNE	HP	OXFORD INDUSTRIES INC	OXM	WEC ENERGY GROUP INC	WEC
CONOCOPHILLIPS	COP	HP INC	HPQ	OCCIDENTAL PETROLEUM CORP	OXY	WELLS FARGO & CO	WFC
CAMPBELL SOUP CO	CPB	BLOCK H & R INC	HRB	PG&E CORP	PCG	WHIRLPOOL CORP	WHR
CRANE HOLDINGS CO	CR	HORMEL FOODS CORP	HRL	POTLATCHDELTIC CORP	PCH	WILLIAMS COS INC	WMB
CARPENTER TECHNOLOGY CORP	CRS	HOST HOTELS & RESORTS INC	HST	PUBLIC SERVICE ENTRP GRP INC	PEG	WEIS MARKETS INC	WMK
CARLISLE COS INC	CSL	HERSHEY CO	HSY	PEPSICO INC	PEP	WEYERHAEUSER CO	WY
CTS CORP	CTS	INTL BUSINESS MACHINES CORP	IBM	PFIZER INC	PFE	XCEL ENERGY INC	XEL
CVS HEALTH CORP	CVS	IDACORP INC	IDA	PROCTER & GAMBLE CO	PG	EXXON MOBIL CORP	XOM
CHEVRON CORP	CVX	INTL FLAVORS & FRAGRANCES	IFF	PARKER-HANNIFIN CORP	PH	XEROX HOLDINGS CORP	XRX
DILLARDS INC -CL A	DDS	IMPERIAL OIL LTD	IMO	PERKINELMER INC	PKI		
DEERE & CO	DE	INTL PAPER CO	IP	PINNACLE WEST CAPITAL CORP	PNW		

The selection criteria are straightforward with two conditions: 1) Stocks have continuous monthly returns from 01/01/1970 to 01/01/2023; 2) Stocks have $1 billion market capitalization on 01/01/2023 (to ensure enough liquidity for monthly portfolio rebalancing).

Methodology

This section explains the steps for the EWP and VWP portfolio construction. For illustration, we construct two portfolios (EWP and VWP) comprising three stocks. We randomly picked these three stocks from our 181 stock pool. As shown previously, 971,970 unique ways of picking three stocks from this pool exist. The first subsection below demonstrates EWP, and the second one displays VWP.

Equal-weighted Portfolios (EWP)

Consider V_0 as the starting portfolio value (in this case, $100) at month $t = 0$. If m is the number of stocks in the portfolio, the initial value invested in a stock i, where $i = 1,2,\ldots,m$, is $V_{i,0} = V_0 / m$. The maximum value of m is 181 in our dataset.

The initial quantity of stock i in the portfolio at the beginning of the month $t = 0$, $Q_{i,0} = V_{i,0} / P_{i,0}$, where $P_{i,0}$ is the price of stock i at month $t = 0$.

The value of *t* ranges from 0 to 636 months. The portfolio value before transaction cost (TC) at the end of month *t*, denoted as $V_t[No\ TC]$, is computed with Equation (1).

$$V_t[No\ TC] = \sum_{i=1}^{m} \left(V_{i,t-1}[No\ TC] \times (1 + R_{i,t})\right) \quad \textbf{(1)}$$

where $R_{i,t}$ is the monthly return of stock *i*, between months *t-1* and *t*.

$R_{i,t}$, return between *t* and *t-1*, is obtained from the column *RET* in the *MSF* table in the *CRSP*. $P_{i,t}$ is obtained from the column *PRC* in the *MSF* table in the *CRSP* database. In the *CRSP* database, returns (*RET*) are already adjusted for stock splits and dividends— however, prices (*PRC*) and shares outstanding (*SHROUT*) are not.

The EWP monthly return without TC at the end of month *t*, denoted as $R_t[No\ TC]$, is computed with Equation (2).

$$R_t[No\ TC] = \frac{V_t[No\ TC]}{V_{t-1}[No\ TC]} - 1 \quad \textbf{(2)}$$

The number of stocks in the EWP at the end of month *t*, $Q_{i,t}$, is computed with Equation (3). If there is a stock split or buy-back during the month, the $Q_{i,t}$ is adjusted accordingly. The portfolio is rebalanced after the end of each month (before the next month's cycle begins) so that an equal amount of portfolio value is invested in each stock, i.e., $V_{1,t} = V_{2,t} = \cdots = V_{m,t}$.

$$Q_{i,t} = \frac{V_t[No\ TC]}{m \times P_{i,t}} \quad \textbf{(3)}$$

The EWP monthly turnover (TO) at the end of month *t*, denoted as TO_t, is computed with Equation (4).

$$TO_t = \frac{\sum_{i=1}^{m} \left(abs(Q_{i,t} - Q_{i,t-1})\right)}{\sum_{i=1}^{m} Q_{i,t-1}} \quad \textbf{(4)}$$

Portfolio turnover incurs transaction costs (TC) and diminishes portfolio returns. Nevertheless, there remains no consensus on the exact TC. For instance, Plyakha et al. (2021) apply a 50 basis point (0.50% bps) TC, citing French's (2008) claim that trading costs for U.S. equity decreased from 0.55% of total market cap in 1980 to just 0.21% in 2006. Malladi and Fabozzi (2017), however, opt for a more conservative TC of 169 bps, based on Edelen et al.'s (2013) finding that the average TC is 1.69% for large funds (averaging $2.88 billion in assets) and 1.19% for small funds (averaging $164 million in assets), based on data from 3,799 open-end U.S. equity mutual funds, using quarterly portfolio holdings from Morningstar spanning 1995 to 2006. Our study employs the conservative 169 basis point transaction cost and demonstrates that if EWP outperforms VWP at 169 bps TC, it will thus also outperform at the lower 50 bps. The EWP value with TC at the end of month *t*, denoted as $V_t[TC]$, is computed with Equation (5).

$$V_t[TC] = V_t[No\ TC] \times (1 - TC \times TO_t) \quad \textbf{(5)}$$

The $V_t[TC]$ is recursively incorporated into Equation (1) starting from the first month (i.e., *t* = 1) to calculate the monthly portfolio value and return of EWP for all following months until 01/01/2023. The initial condition property that $V_{t=0}[TC] = V_{t=0}[No\ TC] = V_0$ is useful in this recursive process. The EWP monthly return with TC at the end of month *t*, denoted as $R_t[TC]$, is computed with Equation (6). When TC = 0, $R_t[No\ TC] = R_t[TC]$.

$$R_t[TC] = \frac{V_t[TC]}{V_{t-1}[TC]} - 1 \quad \textbf{(6)}$$

Value-weighted Portfolio (VWP)

We assume that the initial VWP value, denoted as $\hat{V}_0$, is identical to the starting portfolio value, V_0, in EWP, which stands at $100 at month *t* = 0. We employ hat-accented notation (ˆ) for VWP to differentiate it from EWP. In addition, VWP incorporates an additional variable, denoted as *W*, representing the value-weight of each stock. The weight of stock *i* in VWP (based on the market cap) at the beginning of month *t*, denoted as W_t, is computed with Equation (7).

$$W_{i,t} = \frac{P_{i,t} \times Q_{i,t}}{\sum_{i=1}^{m} P_{i,t} \times Q_{i,t}} \quad \textbf{(7)}$$

where $Q_{i,t}$ is the number of stocks outstanding for stock *i*, in month *t*. It is obtained from the column *SHROUT* in the *MSF* table in the *CRSP* and represents the unadjusted number of publicly held shares recorded in 1000s. It represents the actual, undiluted value, so fractional shares are possible.

Considering *m* is the number of stocks in the portfolio, the initial value invested in a stock *i*, *(i* = 1,2,…,*m)*, is $\hat{V}_{i,0} = W_{i,0} \times \hat{V}_0$. The maximum value of *m* is 181 in our dataset. The initial number of stocks in the portfolio at the beginning of the month *t* = 0, $\hat{Q}_{i,0}= \hat{V}_{i,0}$ / $\hat{P}_{i,0}$, where $\hat{P}_{i,0}$ is the price of stock *i* at month *t* = 0. The value of *t* ranges from 0 to 636 months. The portfolio value before transaction cost (TC) at the end of month *t*, denoted as $\hat{V}_t[No\,TC]$, is computed with Equation (8).

$$\hat{V}_t[No\,TC] = \sum_{i=1}^{m}\left(\hat{V}_{t-1}[No\,TC] \times W_{i,t} \times (1 + R_{i,t})\right) \qquad \textbf{\textit{(8)}}$$

The VWP monthly return without TC at the end of month *t*, denoted as $\widehat{R_t}[No\,TC]$, is computed with Equation (9).

$$\widehat{R_t}[No\,TC] = \frac{\hat{V}_t[No\,TC]}{\hat{V}_{t-1}[No\,TC]} - 1 \qquad \textbf{\textit{(9)}}$$

The stock quantity of stock *i* in the VWP at the end of month *t*, $\hat{Q}_{i,t}$, is computed with Equation (10). The portfolio is rebalanced after the end of each month (before the next month's cycle begins) such that the weighted amount invested in each stock equals the market cap weights, i.e., $\frac{\hat{V}_{i,t}}{\hat{V}_t} = W_{i,t}$. If there is a stock split or buy-back during the month, the $\hat{Q}_{i,t}$ is adjusted accordingly.

$$\hat{Q}_{i,t} = \frac{\hat{V}_t[No\,TC] \times W_{i,t}}{P_{i,t}} \qquad \textbf{\textit{(10)}}$$

The VWP monthly turnover (TO) at the end of month *t*, denoted as $\widehat{TO}_t$, is computed with Equation (11) and the VWP value with TC, denoted as $\hat{V}_t[TC]$, is computed with Equation (12).

$$\widehat{TO}_t = \frac{\sum_{i=1}^{m}\left(abs(\hat{Q}_{i,t} - \hat{Q}_{i,t-1})\right)}{\sum_{i=1}^{m}\hat{Q}_{i,t-1}} \qquad \textbf{\textit{(11)}}$$

$$\hat{V}_t[TC] = \hat{V}_t[No\,TC] \times \left(1 - TC \times \widehat{TO}_t\right) \qquad \textbf{\textit{(12)}}$$

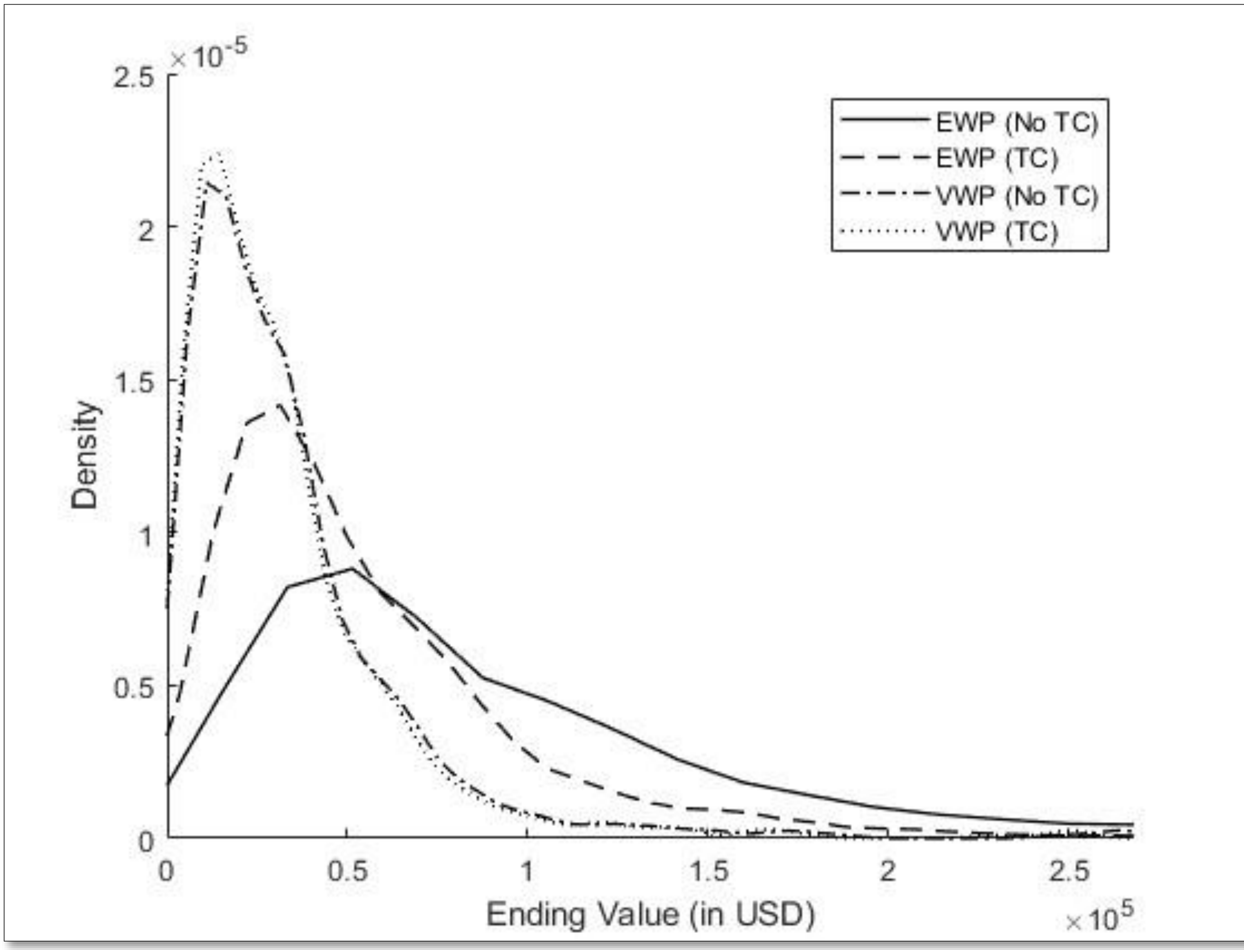
The $\hat{V}_t[TC]$ is recursively incorporated into Equation (8) starting from the first month (i.e., *t* = 1) to calculate the monthly portfolio value and return of VWP for all following months until 01/01/2023. The initial condition property that $\hat{V}_{t=0}[TC] = \hat{V}_{t=0}[No\,TC] = \hat{V}_0$ is useful in this recursive process. The VWP monthly return with TC at the end of month *t*, denoted as $\hat{R}_t[TC]$, is computed with Equation (13). When TC = 0, $\hat{R}_t[No\,TC] = \hat{R}_t[TC]$.

$$\hat{R}_t[TC] = \frac{\hat{V}_t[TC]}{\hat{V}_{t-1}[TC]} - 1 \qquad \textbf{\textit{(13)}}$$

Results and Discussion

This section summarizes the results presented above. Figure 1, titled "Portfolio Ending Value (Kernel Density Diagram)," illustrates the kernel density for four portfolio ending values described in section (4) (i.e., $V_t[No\,TC]$, $V_t[TC]$, $\hat{V}_t[No\,TC]$, and $\hat{V}_t[TC]$). A kernel density diagram is a non-parametric way to estimate the probability density function of a random variable. It is a smooth curve that shows how likely it is to find a data point at any given value. Kernel density diagrams are useful for visualizing data distribution and comparing the distributions of different data groups.

Figure 1. Portfolio Ending Value (Kernel Density Diagram)



- This figure displays the kernel density for four portfolio ending values (VWP, EWP; with & without transaction costs, TC).
- 1,000 EWP and VWP portfolios are constructed using three randomly selected stocks. The average portfolio values are shown below. Each run examines the portfolio for the whole period using monthly returns from 01/01/1970 to 01/01/2023. TC = 1.69% of Portfolio Turnover.
- A 'shorter and right-shifted EWP kernel density' suggests a lower and right-shifted data distribution with most points concentrated on the higher values, indicating a shift in central tendency towards higher values.
- Transaction costs significantly affect EWP negatively, whereas their impact on VWP is relatively minor.

Figure 1 presents the results of constructing 1,000 EWP and VWP portfolios, each comprised of three randomly selected stocks, and then showcases the average portfolio values. These portfolios are examined over the entire period, employing monthly returns from January 1, 1970, to January 1, 2023.

The shorter and right-shifted EWP kernel density indicates a concentration of data points toward higher values, suggesting that EWP portfolios have a higher potential to generate higher returns than VWP. The kernel density diagram also shows that transaction costs significantly negatively impact EWP portfolios but only have a minor impact on VWP portfolios. This is because EWP portfolios rebalance more frequently than VWP portfolios, resulting in higher turnover and transaction costs. It is important to note that the results shown in Figure 1 are based on historical data and may not represent future performance. Investors should carefully consider their risk tolerance and investment goals before deciding whether to invest in EWP or VWP portfolios.

Table 3 provides a detailed overview of the four portfolios' monthly and annualized returns. The table shows that the median and mean annualized returns are highest for EWP portfolios without transaction costs and lowest for VWP portfolios with transaction costs. This means that EWP portfolios have the potential to generate higher risk-adjusted returns than VWP portfolios, but they may have a higher standard deviation (riskier). The table also shows that transaction costs significantly negatively impact the performance of EWP portfolios.

Table 3. Monthly and Annualized Returns of Four Portfolios

	Monthly Return and Risk (M=3, N = 635,000)			
	VWP		**EWP**	
	No TC	**TC**	**No TC**	**TC**
Mean	0.0108	0.0107	0.0124	0.0115
Median	0.0109	0.0108	0.0120	0.0113
StDev	0.0648	0.0647	0.0641	0.0640
	Annual Return, Risk, and Risk-Adjusted Return			
	VWP		**EWP**	
	No TC	**TC**	**No TC**	**TC**
Mean	0.1373	0.1363	0.1592	0.1477
Median	0.1390	0.1380	0.1533	0.1441
StDev	0.2243	0.2243	0.2219	0.2216
Sharpe Ratio	0.3891	0.3847	0.4918	0.4410
	Monthly Portfolio Turnover			
	VWP		**EWP**	
Mean	0.0042		0.0488	
Median	0.0010		0.0384	
StDev	0.0252		0.0441	

Correlations	EWP (No TC)	EWP (TC)	VWP (No TC)	VWP (TC)
EWP (No TC)	1.0000			
EWP (TC)	0.9944	1.0000		
VWP (No TC)	0.6354	0.6568	1.0000	
VWP (TC)	0.6356	0.6570	0.9998	1.0000

- 1,000 EWP and VWP portfolios are constructed using three randomly selected stocks.
- Each run examines the portfolio for the whole period using monthly returns from 01/01/1970 to 01/01/2023.
- Since each stock has 636 monthly prices (or 635 returns), one can have N = 635,000 (635 x 1,000).
- The annualized returns are derived from the monthly returns using the formula: $R_{Annualized} = (1 + R_{month})^{12} - 1$.
- The median annualized returns in descending order for the portfolios are as follows: TC = 1.69% of Portfolio Turnover.

- EWP (No TC): 0.1533, EWP (TC): 0.1441, VWP (No TC): 0.1390, and VWP (TC): 0.1380.
- The mean values are 0.1592, 0.1477, 0.1373, and 0.1363, respectively.
- The standard deviations of annual returns are 0.2219, 0.2216, 0.2243, and 0.2243, respectively.
- Assuming a 5% risk-free rate, the Sharpe ratios, computed as (mean – rf)/StDev, are 0.49, 0.44, 0.39, and 0.38, respectively.
- The correlation matrix in the lower panel shows that VWP and EWP have a monthly correlation of 0.6354 without TC and 0.6570 with TC.

To calculate the annualized returns, the formula $R_{Annualized} = (1 + R_{month})^{12} - 1$ is applied to the monthly returns. This transformation accounts for compounding over one year. The resulting median annualized returns for the portfolios, presented in descending order, are as follows: EWP (No TC) with a value of 0.1533, EWP (TC) with 0.1441, VWP (No TC) with 0.1390, and VWP (TC) with 0.1380.

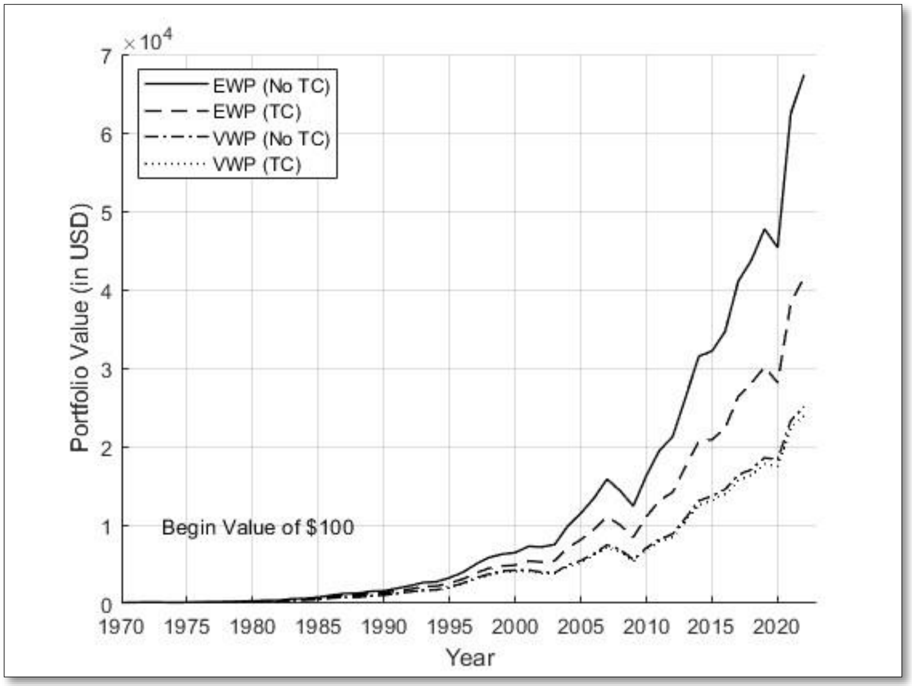
Additionally, the mean values for these portfolios are calculated, and they are as follows: 0.1592 for EWP (No TC), 0.1477 for EWP (TC), 0.1373 for VWP (No TC), and 0.1363 for VWP (TC). Furthermore, the standard deviations of the annual returns are calculated, yielding values of 0.2219 for EWP (No TC), 0.2216 for EWP (TC), 0.2243 for VWP (No TC), and 0.2243 for VWP (TC). The VWP and EWP have a monthly correlation of 0.6354 without TC and 0.6570 with TC.

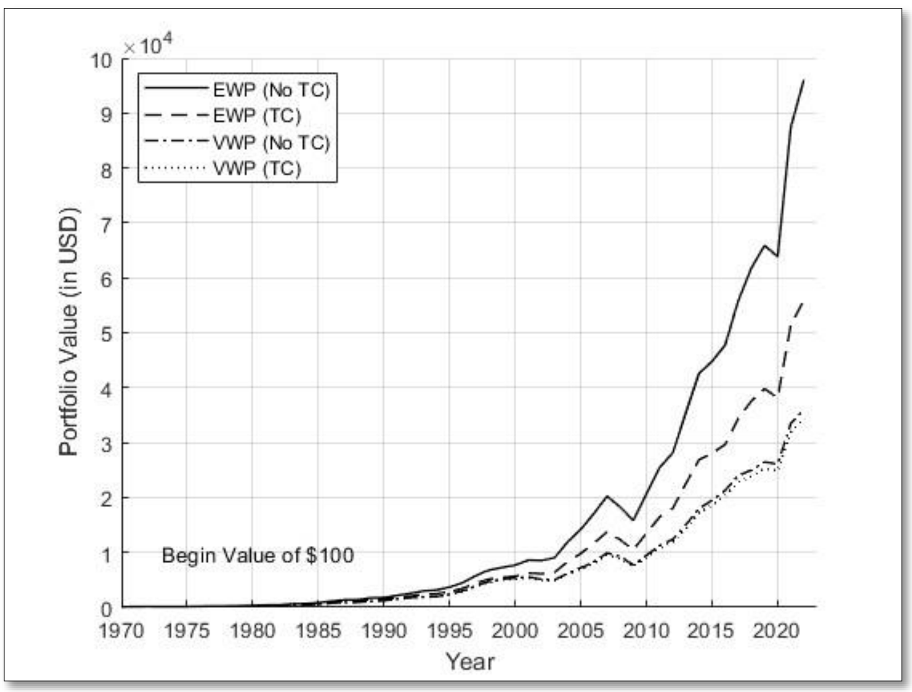
The 10-year US Treasury bond yield at the end of each year is sourced from the Federal Reserve of St. Louis (FRED) and compiled by Professor Damodaran[3]. The average yield from 1970 to 2023 is rounded to 5% and used as the risk-free rate. This rate enables the computation of Sharpe ratios to assess the risk-adjusted performance of each portfolio. The Sharpe ratio, computed as (mean annualized return – annual risk-free rate) / standard deviation of annualized return, quantifies the return achieved per unit of risk taken. The observed Sharpe ratios, denoting the risk-adjusted performance, were determined as 0.49 for EWP (No TC) and 0.44 for EWP (TC), followed by 0.39 for VWP (No TC) and 0.38 for VWP (TC), portraying a comprehensive perspective on the relative risk-adjusted performance of these portfolios.

Figure 2 depicts the four portfolios' historical median and mean values. Figure 3 displays the four portfolios' mean annualized return kernel densities. The mean portfolio values are higher than the median. Without transaction costs, EWP's ending value surpasses VWP 94.5% of the time (and 84.0% with TC). EWP's mean and standard deviation of monthly returns exceeds VWP 90.9% of the time without TC (80.5% with TC).

[3] Historical Returns on Stocks, Bonds and Bills: 1928-2023: https://pages.stern.nyu.edu/~adamodar/

Figure 2. Historical Portfolio Median (left) and Mean (right) Value

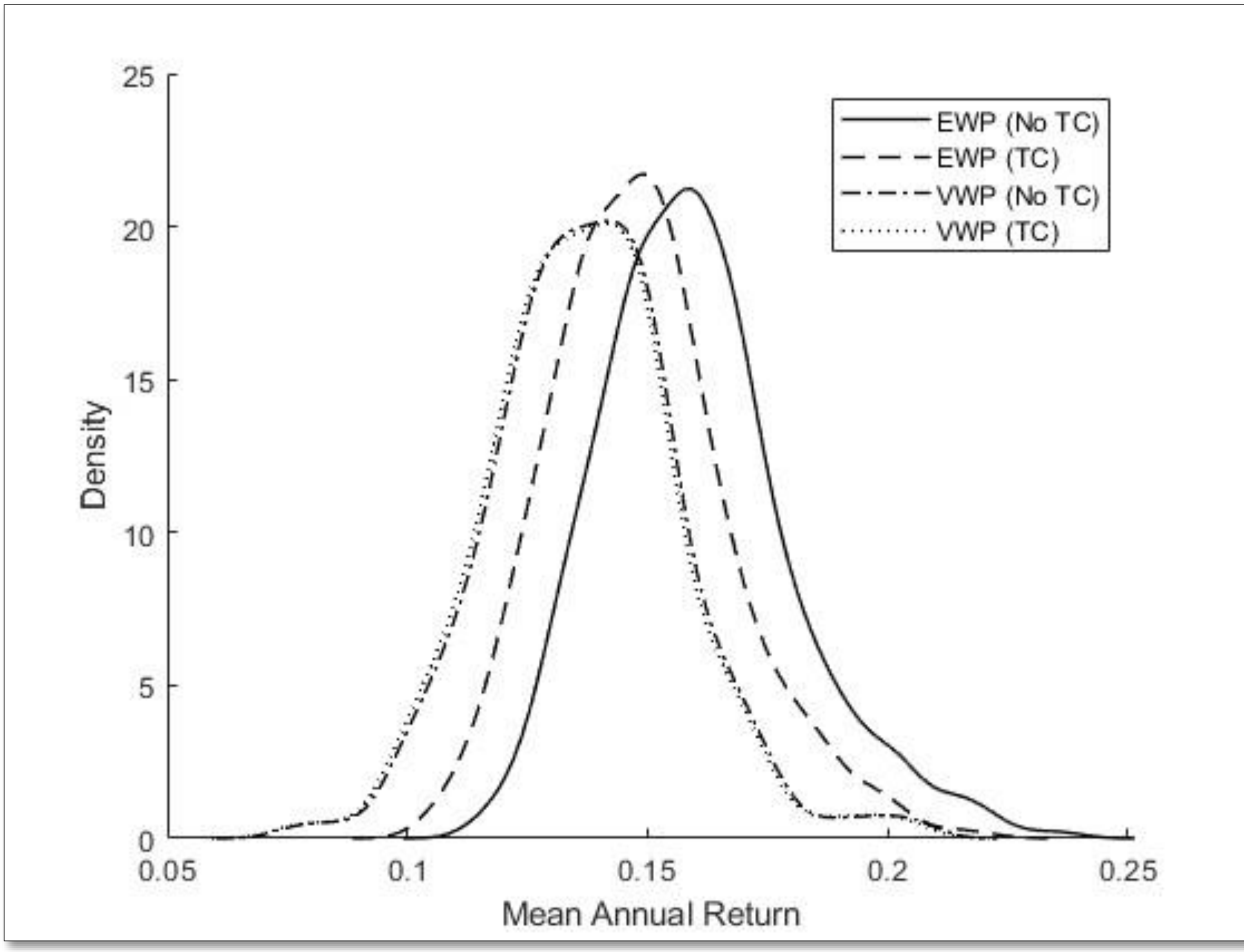




- The y-axis scale on the right chart is higher than that of the left (10×10^4 compared to 7×10^4).
- The two figures display ending values for four portfolios (VWP, EWP, with and without transaction costs, TC). TC = 1.69% of Portfolio Turnover.
- 1,000 EWP and VWP portfolios are constructed using three randomly selected stocks. The median portfolio values are on the left, and the mean is on the right. Each run examines the portfolio for the whole period using monthly returns from 01/01/1970 to 01/01/2023.

- Without TC, EWP's ending value surpasses VWP 94.5% of the time (and 84.0% with TC).
- The EWP's mean/standard deviation of monthly returns surpasses VWP 90.9% of the time without TC (80.5% with TC).
- For \$100 invested on 01/01/1970, the median portfolio ending values on 01/01/2023 are as follows: EWP (No TC): \$69,638, EWP (TC): \$43,079, VWP (No TC): \$25,543, and VWP (TC): \$24,320. The mean values are \$99,731, \$57,795, \$37,268, and \$35,488, respectively.

Figure 3. Portfolio Mean Annualized Return (Kernel Density Diagram)



- This figure shows the mean annualized return kernel density for four portfolios (VWP, EWP; with & without transaction costs, TC).
- 1,000 EWP and VWP portfolios are constructed using three randomly selected stocks. The annualized returns are derived from the monthly returns using the formula: $R_{Annualized} = (1 + R_{month})^{12} - 1$.
- Each run examines the portfolio for the whole period using monthly returns from 01/01/1970 to 01/01/2023. TC = 1.69% of Portfolio Turnover.
- Transaction costs notably harm EWP but have a minor impact on VWP.
- The peak of a kernel density diagram represents the mode or the most probable value in the dataset. The two EWPs have higher modes.
- The median annualized returns for the portfolios are as follows: EWP (No TC): 0.1533, EWP (TC): 0.1441, VWP (No TC): 0.1390, and VWP (TC): 0.1380. The mean values are 0.1592, 0.1477, 0.1373, and 0.1363, respectively. The standard deviations of annual returns are 0.2219, 0.2216, 0.2243, and 0.2243, respectively.

For an initial investment of $100 on January 1, 1970, the median portfolio ending values on January 1, 2023, are as follows: EWP (No TC) at $69,638, EWP (TC) at $43,079, VWP (No TC) at $25,543, and VWP (TC) at $24,320. The mean values for the portfolios are $99,731, $57,795, $37,268, and $35,488, respectively.

When the mean portfolio values are higher than the median, the distribution of portfolio values is positively skewed. This means there are more portfolios with lower values than those with higher values, but the few portfolios with very high values are enough to pull the mean above the median. Positively skewed distributions are often seen in financial markets like the stock market. This is because a few stocks outperform the market by a large margin. Investors should be aware of the skewness of a distribution before investing in it. Positively skewed distributions can offer the potential for high returns but also have a higher risk of loss. Such a distribution is expected and can be approximated as log-normal if the price process follows a geometric Brownian motion.

For a robustness check, as shown in Table 4, a paired two-sample test was conducted to assess the similarity of means between VWP and EWP. This test, evaluating whether the paired sample means are equivalent, consistently yielded a p-value of 0 across all three instances in the data. This outcome strongly suggests substantial divergence between the mean values of VWP and EWP in each case, consequently leading to the rejection of the hypothesis proposing their equality.

Table 4. Statistical Significance Tests

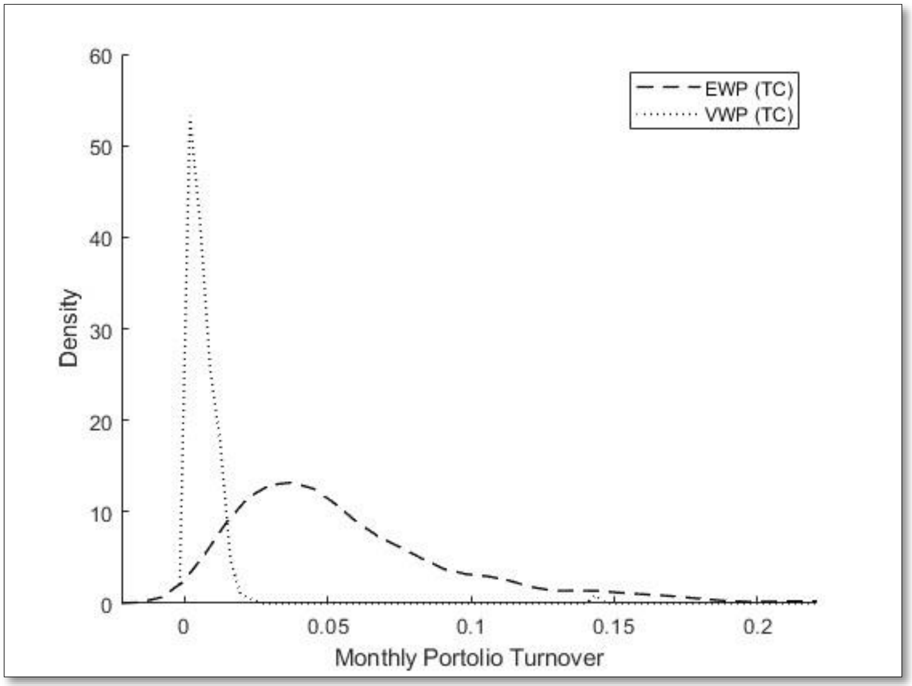
Paired Test Type, Same Mean	**Mean VWP**	**Mean EWP**	**p-value (two-sided)**	**CI of Difference**	**t-Stat**	**DF**
Portfolio Monthly Turnovers	0.0042	0.0488	0.0000 ***	0.0447 to 0.0444	807.73	634,999
Annualized Returns (No TC)	0.1373	0.1592	0.0000 ***	0.0229 to 0.0209	41.47	999
Annualized Returns (with TC)	0.1363	0.1477	0.0000 ***	0.0124 to 0.0105	23.07	999
Standard Deviation of Returns (No TC)	0.2243	0.2219	0.015 *	0.0042 to 0.0005	2.44	999
Standard Deviation of Returns (TC)	0.2243	0.2216	0.0067 **	0.0045 to 0.0007	2.72	999

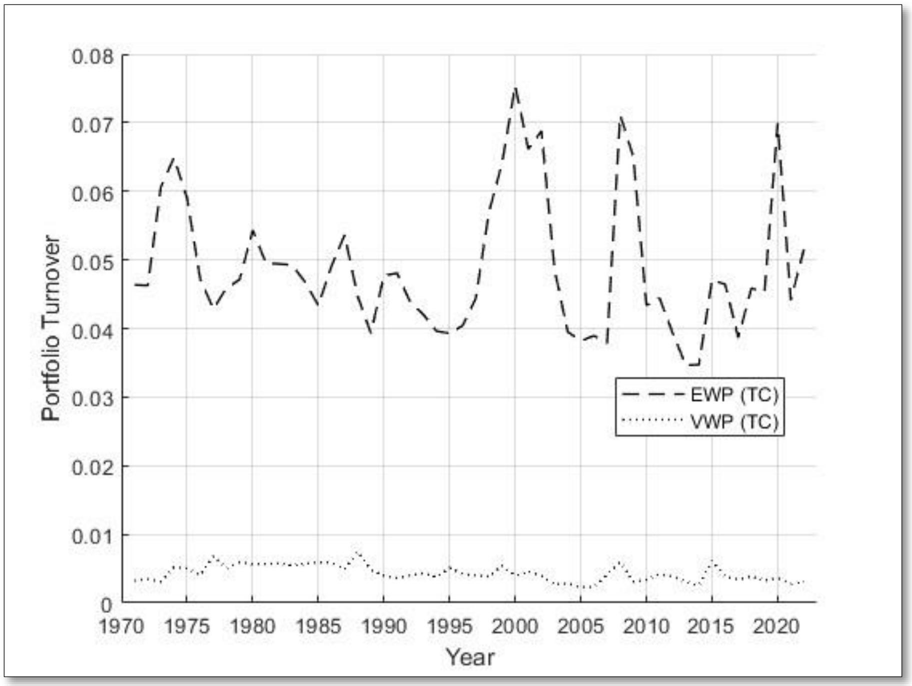
- A *Paired Samples Test* assesses the difference between two sets of paired observations or measurements. It is used when each observation in one set relates to a specific observation in the other.
- 1,000 EWP and VWP portfolios are constructed using three randomly selected stocks. Each run examines the portfolio for the whole period using monthly returns from 01/01/1970 to 01/01/2023, i.e., 636 monthly prices (or 635 returns). TC = 1.69% of Portfolio Turnover.
- In a paired t-test, when comparing two groups or conditions, the degrees of freedom (DF) are calculated as n - 1, where n is the number of paired observations. This accounts for the constraint imposed by using the paired differences to estimate the mean difference.
- The *Same Mean* test evaluates whether the averages of the paired samples (VWP and EWP) are identical or distinct. The confidence interval (CI) of the mean difference is provided for each test.
- Across all three tests in the provided data, a p-value of 0 was obtained, implying significant divergence between the means of VWP and EWP in each scenario.
- This substantiates the rejection of the hypothesis, stating that the means are equal.
- We conclude that the turnover and returns of VWP and EWP exhibit statistically significant differences in their means.

- The asterisk (*) denotes the significance level, * indicates a p-value less than 0.05, ** for a p-value less than 0.01, and *** for a p-value less than 0.001.

Figure 4 analyses monthly portfolio turnover for four portfolios. The left part of the figure indicates that EWP exhibits significantly higher monthly portfolio turnover than VWP, with a median ratio of 34.7 and a mean ratio of 11.4 of EWP monthly turnover to VWP monthly turnover. This heightened turnover increases transaction costs, primarily impacting EWP (TC) returns, which are notably lower than EWP (No TC).

Figure 4. Monthly Portfolio Turnover (Kernel Density Diagram on the left and Historical trend on the Right)



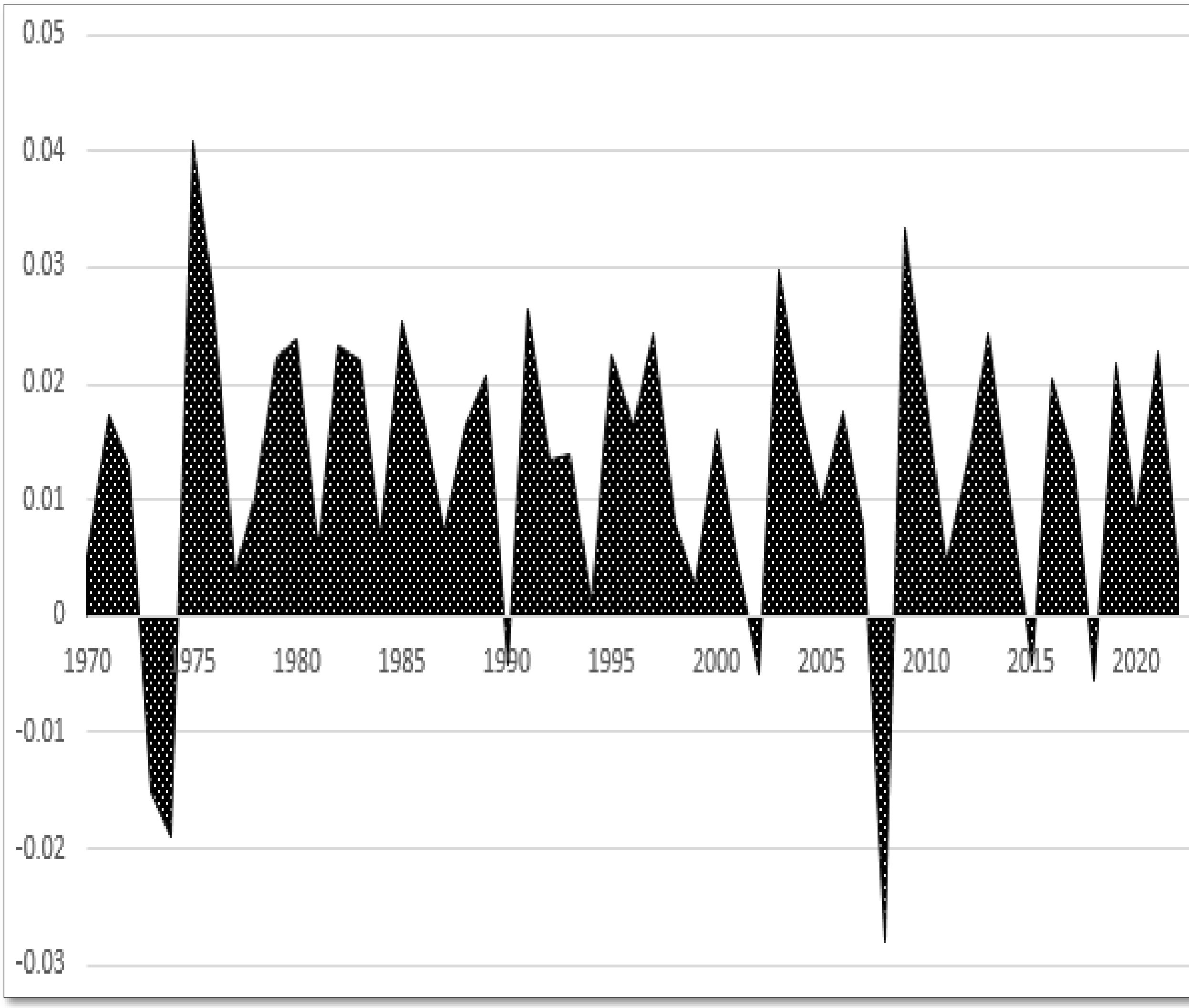


- These figures show four portfolios' monthly portfolio turnover (VWP, EWP; with & without transaction costs, TC). TC = 1.69% of Portfolio Turnover.
- 1,000 EWP and VWP portfolios are constructed using three randomly selected stocks. Each run examines the portfolio for the whole period using monthly returns from 01/01/1970 to 01/01/2023.
- The figure on the left shows that EWP has an order of magnitude higher portfolio turnover than the VWP.
- The ratio of EWP monthly turnover to VWP monthly turnover has a median of 34.7 and a mean of 11.4.
- High turnover results in high transaction costs, primarily causing EWP (TC) returns to be significantly lower than EWP (No TC) returns.
- The median annualized returns for the portfolios are as follows: EWP (No TC): 0.1585, EWP (TC): 0.1476, VWP (No TC): 0.1374, and VWP (TC): 0.1365. The mean values are 0.1600, 0.1485, 0.1376, and 0.1366, respectively. The annual returns exhibit standard deviations of 0.2198, 0.2195, 0.2232, and 0.2232, respectively.
- The figure on the right shows the relative turnover of EWP compared to EWP since 1970. EWP turnovers increase when returns are large (either positive or negative). The next Figure (5) shows the monthly stock return trend.

Figure 4 shows that EWP portfolios are more expensive to maintain than VWP portfolios. This is because EWP portfolios need to be rebalanced more frequently, which results in higher transaction costs. The figure on the right suggests that EWP portfolio managers are more active traders when market returns are high. On the right side of the figure, the relative turnover of EWP compared to VWP since 1970 is displayed. Notably, EWP turnovers increase during periods of large positive or negative returns. This relationship between turnover and returns is further explored in Figure 5, which examines the monthly stock return trend.

Figure 5 presents the average monthly stock returns for all 181 stocks included in the study. The chart highlights substantial stock return fluctuations, particularly in 1974, 2008, 2000, and 2018. Interestingly, these same years coincide with a pronounced spike in EWP turnover, as observed in Figure 4. This alignment suggests a relationship between significant fluctuations in stock returns and corresponding increases in EWP turnover, shedding light on the interplay between market dynamics and portfolio turnover during these particular years.

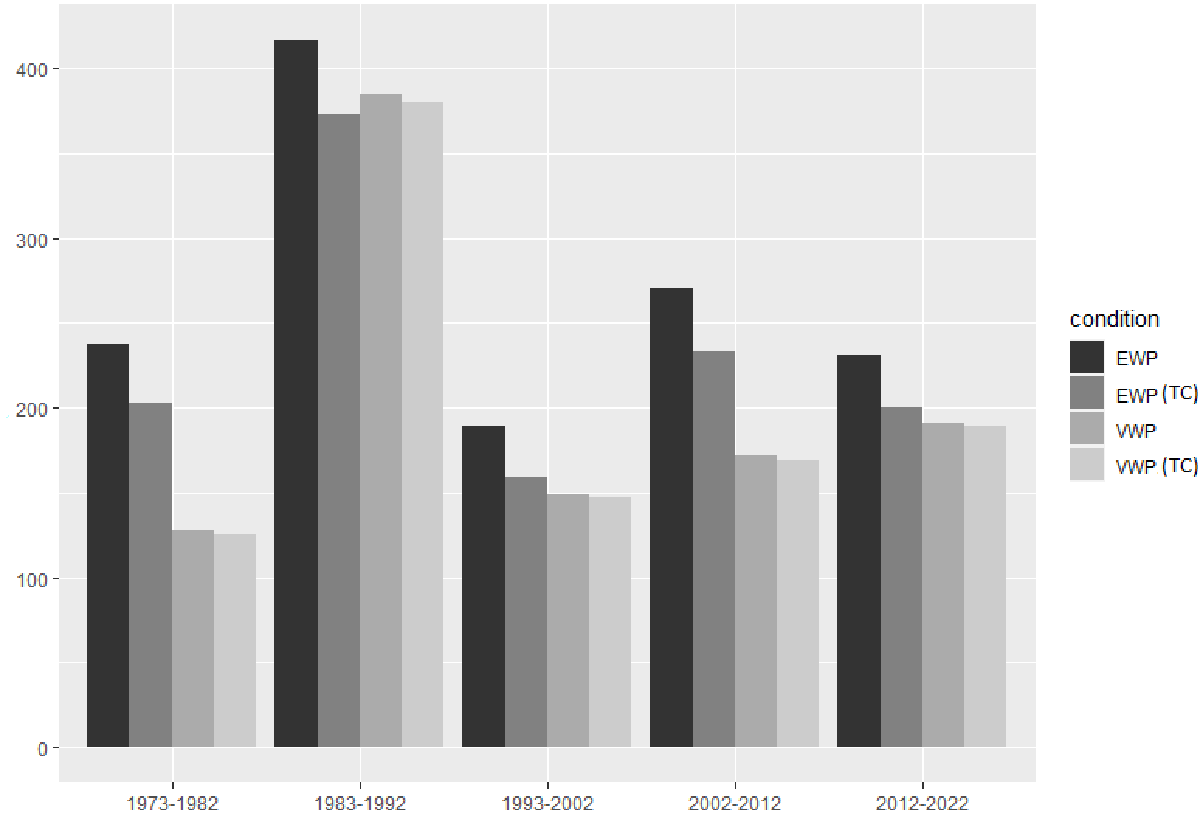
Figure 5. Average Monthly Stock Returns



- This chart displays the monthly average return of all 181 stocks in this study from 01/01/1970 to 01/01/2023.
- This chart reveals large stock return fluctuations in 1974, 2008, 2000, and 2018.
- The same years exhibit a noticeable EWP turnover spike (Figure 4, right).

Figure 6 presents the Portfolio Mean Annualized Returns by Decade, showcasing the culmination value of a portfolio after a $100 investment made at the initiation of each decade. When considering instances lacking transaction costs, the EWP consistently outperforms, yielding a higher ending value in each of the five decades. However, when transaction costs are factored in, the EWP yields a higher ending value in four out of five decades, except from 1983 to 1992.

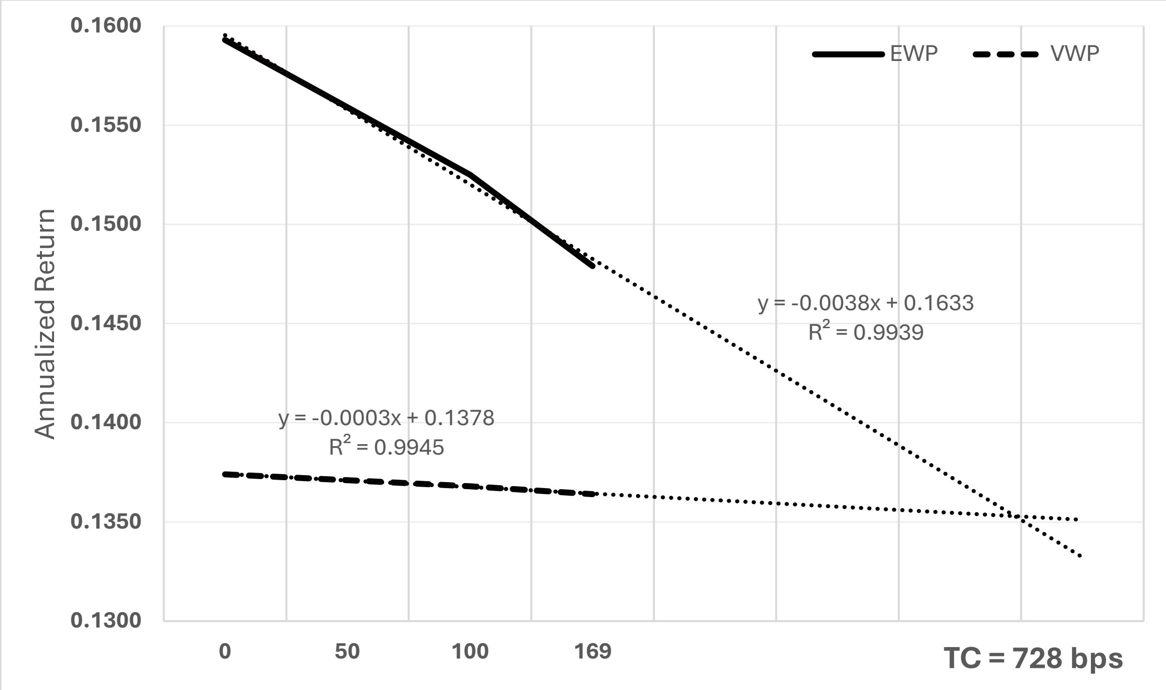
Figure 6. Portfolio Mean Annualized Returns by Decade



- The provided chart illustrates the decade-end value of a portfolio following a $100 investment at the start of each decade.
- EWP consistently yields a superior ending value across all five decades in scenarios without transaction costs.
- EWP demonstrates a higher ending value in four out of five decades, excluding the 1983-92 period. TC = 1.69% of Portfolio Turnover.

Various comparisons are conducted with Plyakha et al. (2021) at 50 basis points (bps) and Malladi and Fabozzi (2017) at 169 bps. Plyakha et al. note that at 50 bps TC, the Equal-Weighted Portfolio (EWP) surpasses the Value-Weighted Portfolio (VWP) with mean annual returns of 13.19% and 10.48%, respectively, equating to an excess return of 271 basis points. Similarly, Malladi and Fabozzi demonstrate at 169 bps TC that the EWP outperforms the VWP, securing mean annual returns of 13.88% and 12.87%, respectively, with an excess return of 101 basis points. Figure 7 displays the impact of TC on the EWP and VWP returns. Additionally, our findings reveal a statistically significant excess annualized return of 188 basis points for the EWP over the VWP when TC = 50 bps and 115 basis points when TC = 169 bps. Notably, Figure 7 shows that when TC reaches 728 basis points of portfolio turnover, the EWP and VWP portfolio returns converge, suggesting a pivotal threshold for these portfolios.

Figure 7. Impact of Transaction Costs on Portfolio Return



	Transaction Cost (TC, bps)			
	0	50	100	169
Annualized Returns (VWP)	0.1374	0.1371	0.1368	0.1364
Annualized Returns (EWP)	0.1593	0.1559	0.1525	0.1479
Excess Return (EWP - VWP)	0.0219	0.0188	0.0157	0.0115
Standard Deviation of Returns (VWP)	0.2215	0.2215	0.2215	0.2215
Standard Deviation of Returns (EWP)	0.2191	0.2190	0.2190	0.2189

- The chart demonstrates how TC (on the *x*-axis) affects EWP and VWP portfolio returns (on the *y*-axis).
- 1,000 EWP and VWP portfolios are constructed using three randomly selected stocks. Each run examines the portfolio for the whole period using monthly returns from 01/01/1970 to 01/01/2023. Thick lines depict exhaustive random sampling, while lighter dotted lines represent trendlines.
- Plyakha et al. (2021) show that at 50 bps, TC EWP outperforms VWP, with mean annual returns of 13.19% and 10.48%, respectively (or 271 bps excess returns).
- Malladi and Fabozzi (2017) demonstrate that at 169 bps TC, EWP outperforms VWP, with mean annual returns of 13.88% and 12.87%, respectively (or 101 bps excess returns).
- We report that the EWP achieves a statistically significant excess annualized return over the VWP of 188 bps when TC = 50 bps and 115 bps when TC = 169 bps.
- When TC reaches 728 bps of portfolio turnover, EWP and VWP returns converge.

Robustness Checks in a Larger Portfolio

So far, we have used three stocks in our analysis to facilitate easier comparisons and validations through mathematical proofs. In this subsection, we increase the number of stocks to ten to demonstrate the robustness of our results. The findings with a larger number of securities maintain the integrity of our initial results with three stocks. There are 8.074 quadrillion unique ways to select ten stocks from a pool of 181, as per the combination formula $nCr = \frac{n!}{r!(n-r)!}$, considering unordered selection. Given the impracticality of exhaustively analyzing quadrillions of portfolios, we created 1,000 portfolios of ten randomly selected stocks each for our analysis. The results of both three- and ten-stock portfolios are presented in Table 5.

The results of the analysis are robust, as evidenced by consistent patterns across different portfolio types and varying numbers of stocks (M). For both VWP and EWP, as the number of stocks increases from three to ten, the standard deviation of returns decreases significantly, as expected (e.g., annual StDev decreased from 22% to 16% for EWP and to 14% for VWP). EWP mean and median returns exceed those of VWP in both cases (i.e., M = 3 and M = 10). EWP maintained higher returns than VWP in both cases. Both strategies experienced significant risk reduction and improved Sharpe ratios with an increased number of stocks. Transaction cost impact decreased significantly for EWP as the number of stocks increased, while it remained minimal for VWP in both cases.

These results align with portfolio theory: increased diversification leads to lower risk, equal-weighting outperforms value-weighting in terms of returns and risk-adjusted returns, and the impact of transaction costs on equal-weighted portfolios decreases with more stocks, likely due to reduced rebalancing needs. The consistent decrease in returns as transaction costs increase, the consistent outperformance of EWP over VWP, and the reduction in risk as the number of stocks in the portfolio increases reinforce the reliability and robustness of the results, demonstrating that the observed trends are not anomalies but reflective of underlying market dynamics and portfolio characteristics.

Conclusions

In conclusion, the paper's results based on the methodology and data presented demonstrate several key findings. EWP portfolios exhibit the potential for higher risk-adjusted returns, particularly in the absence of transaction costs. Transaction costs significantly impact the performance of EWP portfolios. Moreover, EWP portfolios tend to have higher turnover, leading to increased transaction costs, especially during periods of significant market returns. These results highlight the trade-offs between higher return potential and increased costs associated with EWP portfolios. Investors should carefully consider their risk tolerance and investment objectives when choosing between EWP and VWP portfolios, considering the complex relationship between turnover, transaction costs, and market dynamics.

Future research in this area may encompass the following directions:

- Transaction Cost Optimization: Investigating strategies and methodologies to minimize transaction costs in EWP portfolios, potentially through advanced algorithms or trading techniques.

- Incorporating various transaction costs by periods: We kept transaction costs fixed to ensure consistency with previous studies and facilitate cross-comparison, as shown in Figure 7. Incorporating transaction costs by period is a valuable direction for future research.

- We randomly chose 1,000 combinations (of three stocks) from the 971,970 possibilities through exhaustive random sampling (from the available pool of 181 stocks). This pool could typically originate from an investment firm's stock selection methodology, an index, or similar sources. Researchers are encouraged to experiment with different pools and varying numbers of stocks in the portfolio.

- Asset Class Expansion: Extending the research to include various asset classes beyond equities, such as fixed income, real estate, or alternative investments, to understand how EWP and VWP strategies perform in diverse investment landscapes.

- Machine Learning Applications: Leveraging machine learning and predictive modeling techniques to forecast future returns, turnover, and transaction costs for both EWP and VWP portfolios, offering predictive insights into portfolio management.

- Global Market Comparisons: Conducting comparative analyses of EWP and VWP strategies in different global markets and examining the influence of cross-border investment considerations.

These research avenues can collectively contribute to a more comprehensive understanding of the dynamics and implications of EWP and VWP portfolio management in various contexts, offering valuable insights for investors and portfolio managers.

Table 5. Robustness Check in a 3-stock (left panel) and 10-stock (right panel) Portfolio

	Monthly Return and Risk (M=3, N = 635,000)			
	VWP		EWP	
	No TC	TC	No TC	TC
Mean	0.0108	0.0107	0.0124	0.0115
Median	0.0109	0.0108	0.0120	0.0113
StDev	0.0648	0.0647	0.0641	0.0640
	Annual Return, Risk, and Risk-Adjusted Return			
	VWP		**EWP**	
	No TC	**TC**	**No TC**	**TC**
Mean	0.1373	0.1363	0.1592	0.1477
Median	0.1390	0.1380	0.1533	0.1441
StDev	0.2243	0.2243	0.2219	0.2216
Sharpe Ratio	0.3891	0.3847	0.4918	0.4410

	Monthly Return and Risk (M=10, N = 635,000)			
	VWP		EWP	
	No TC	TC	No TC	TC
Mean	0.0099	0.0099	0.0123	0.0123
Median	0.0117	0.0116	0.0144	0.0143
StDev	0.0417	0.0417	0.0465	0.0465
	Annual Return, Risk, and Risk-Adjusted Return			
	VWP		**EWP**	
	No TC	**TC**	**No TC**	**TC**
Mean	0.1259	0.1249	0.1582	0.1574
Median	0.1498	0.1486	0.1873	0.1862
StDev	0.1445	0.1445	0.1610	0.1611
Sharpe Ratio	0.5254	0.5185	0.6721	0.6668

- 1,000 EWP and VWP portfolios are constructed using three (left panel) or ten (right panel) randomly selected stocks.
- Each run examines the portfolio for the whole period using monthly returns from 01/01/1970 to 01/01/2023.
- Since each stock has 636 monthly prices (or 635 returns), one can have N = 635,000 (635 x 1,000).
- The annualized returns are derived from the monthly returns using the formula: $R_{Annualized} = (1 + R_{month})^{12} - 1$.
- The results of the analysis are robust, as evidenced by consistent patterns across different portfolio types and varying numbers of stocks (M). For both VWP and EWP, as the number of stocks increases from three to ten, the standard deviation of returns decreases significantly, as expected (e.g., annual StDev decreased from 22% to 16% for EWP and to 14% for VWP). EWP mean and median returns exceed those of VWP in both cases (i.e., M = 3 and M = 10).
- Despite these reductions in returns, the standard deviations of the portfolios show minimal change, indicating that risk levels remain relatively stable regardless of transaction costs and the number of stocks in the portfolio.
- The Sharpe Ratio, a measure of risk-adjusted return, consistently decreases with the inclusion of transaction costs, highlighting the impact on overall performance.
- The consistent decrease in returns and Sharpe Ratios across different configurations (monthly and annual returns, with and without transaction costs) reinforces the reliability and robustness of the results, demonstrating that the observed trends are reflective of underlying market dynamics and portfolio characteristics.

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