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Target-date and balanced funds: Latest market offerings and risk-return analysis

Gaobo Pang, Mark Warshawsky*

Towers Watson, 901 N. Glebe Road, Arlington, VA 22203, USA

Abstract

This analysis looks at the latest market offerings of target-date funds (TDFs) and balanced funds (BFs) and examines their risk-return characteristics through stochastic simulations. The simulation model includes standard asset market shocks, rare economic disasters, and random labor earnings correlated with macroeconomic shocks. The data suggests that some TDFs are reducing the risky equity exposure from past levels for investors near retirement. The simulation results show that glide path designs are important determinants of wealth levels and volatilities. TDFs as the sole vehicle for retirement wealth accumulation must be considered risky, particularly when the possible occurrences of large economic disasters are considered. Nonetheless, TDFs have less risk than comparable BFs close to retirement and therefore are more suitable for investors with greater priority on wealth protection.

JEL classification: G11; G23; D14; D81

Keywords: Target-date funds; Balanced funds; Defined contribution plan

1. Introduction

Target-date funds (TDFs) have recently become one of the most popular investment vehicles for 401(k) and 403(b) defined contribution (DC) retirement plans. This is in part attributable to the (partial) fiduciary relief to plan sponsors that was provided by the Pension Protection Act of 2006 and the subsequent Department of Labor's regulation, featuring TDFs as one of the three qualified default investment alternatives (QDIA). In terms of investment

* Corresponding author. Tel.: +1-703-258-7636; fax: +1-703-258-7492.
E-mail address: mark.warshawsky@towerswatson.com (M. Warshawsky).

strategy, the major appeal of TDFs lies in the automatic rebalancing and portfolio shift with age. A TDF holds a mix of stocks, bonds, cash and other assets. As the worker approaches retirement (the preset target date) and the value of her presumably low-risk human capital ebbs away, the TDF assets gradually shift from the high-risk stocks to relatively low-risk fixed-income securities. This design intends to capture good capital market outcomes over the life cycle and provide a financially smooth transition to retirement for plan participants. The diversification of asset mix and the pace of portfolio changes, however, vary significantly across TDFs. The eventual wealth accumulations and risk profiles thus differ.

This analysis looks at the latest market developments of asset allocations in actual TDF offerings. It then examines the stochastically simulated risk-return characteristics of a few select TDFs, with differing equity-bond-cash glide paths, and focuses on the trade-offs between wealth creation and security. The simulated TDF outcomes are also compared to those of balanced funds (BFs), another popular QDIA. Stochastic simulations of asset returns and yields include standard market shocks in normal times and low-probability large-magnitude rare economic disasters. Simulations of labor earnings (which produce retirement plan contributions) capture the common lifecycle age-earnings profile of workers, idiosyncratic (random) individual income variations, and broad correlations of wage levels with macroeconomic shocks.

2. Asset allocations in target-date funds

As the basic design of TDFs, the share of risky but higher return equity starts high and moves to less risky securities as the investor gets older. TDFs differ by the composition of assets and the speed of changes. Fig. 1 plots the glide paths of three representative TDFs for young investors, designated TDF1E through TDF3E. They are respectively at the 95th, 50th, and 5th percentiles, according to equity share, of the 20 largest 2050 TDFs by asset size on the market as of April 30, 2010. These TDFs are targeted to long-horizon young workers with high equity positions upon entry. Over time, the portfolios are shifted to bonds and cash. The glide paths are constructed by connecting all TDFs within a provider's fund family, with linear interpolations for allocations between target dates.

Contrasting these TDFs to ones offered in the past hints at the evolution of thinking and market offerings. The first observation is that the entry equity share has been maintained at fairly high levels for young workers. The average equity share of the longest-horizon TDFs was about 88% in March 2005 (Poterba et al., 2009), 86% in May 2009 (Pang and Warshawsky, 2010), and 88% in this current sample.

The second observation is that TDFs now seem to be reducing from past levels the equity exposure for participants near retirement. For example, the age-65 equity share of TDF1E (the 95th percentile fund) is about 46%, while the corresponding equity share was higher at 60% back in May 2009. This development perhaps results from the revelation of higher equity risks in the recent financial crisis. Regulatory attention may also have had some influence on the portfolio changes. For instance, the Department of Labor and the Securities and Exchange Commission held a joint hearing on TDFs on June 18, 2009, with almost 40 testimonies. The underlying concerns were the investment uncertainty and lack of wealth

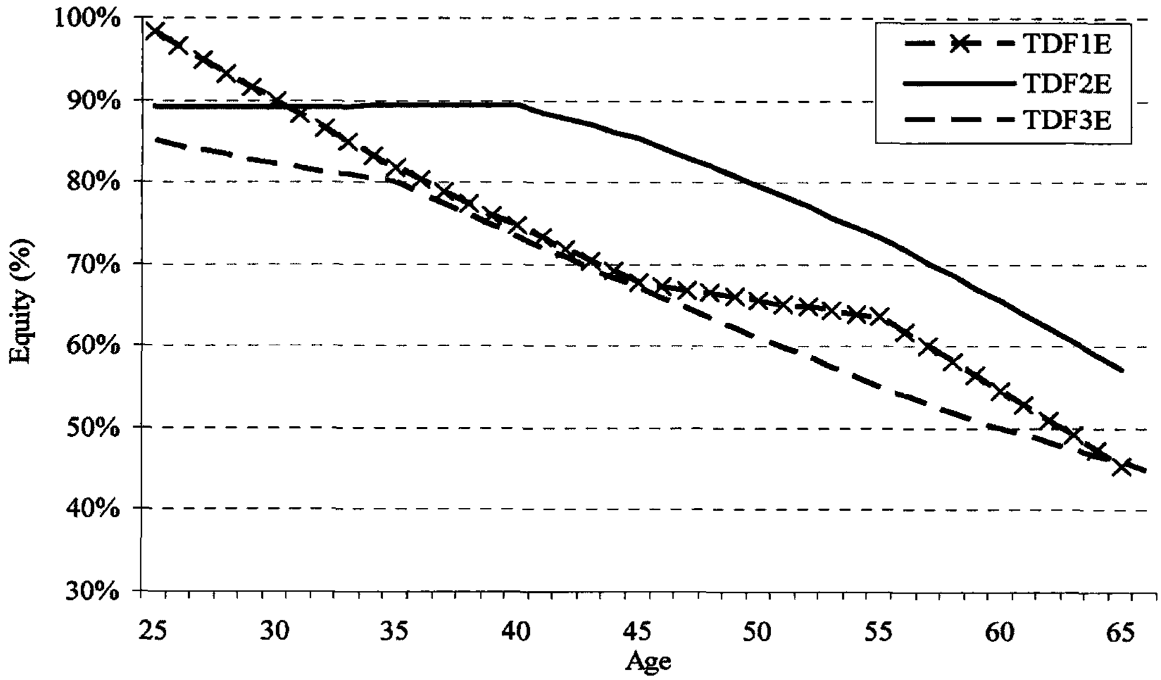

Fig. 1. Target-date fund asset allocations for early career investors. (*Source:* Authors' data collection from Morningstar and TDF providers' websites as of April 30, 2010).

protection for retiring DC plan participants and an apparent widespread lack of understanding by retirement plan participants.

The recent downward adjustment in equity exposure is also observed in TDFs that are mainly targeted to midcareer workers who are likely to retire in 15 years. Fig. 2 shows the glide paths, which are formed by connecting 2025 through 2010 TDFs. TDF1M, TDF2M, and TDF3M (the 95th, 50th and 5th percentiles for initial equity shares) start with about 85%, 76%, and 61% of assets in equity, with the remainder mainly in bonds. They end with about 52%, 49%, and 46% in equity, respectively. By contrast, the exit equity shares were about 69%, 60%, and 50%, respectively, at the same percentiles in May 2009. Upon exit at age 65, TDF1M, TDF2M, and TDF3M, respectively, have about 48%, 37%, and 45% in bonds and 0%, 14%, and 9% in cash.

For a comparison of wealth accumulations, we later also look at results assuming that workers instead invest in balanced funds. Balanced funds usually maintain a constant mix of assets, catering to a certain appetite for risk and capital appreciation independent of age and target retirement date. According to Morningstar's categorizations and as shown in Fig. 2, the average equity share is about 63% for the 20 largest balanced funds with moderate risk allocations (BFmA) and 35% for the 20 largest with conservative allocations (BFcA). The bond and cash shares are 31% and 6%, respectively, for BFmA and 49% and 16%, respectively, for BFcA. The overall risk profile of BFmA is closer to the TDFs. Asset allocations of BFmA here are almost identical to those in 2006 as documented by Pang and Warshawsky (2008).

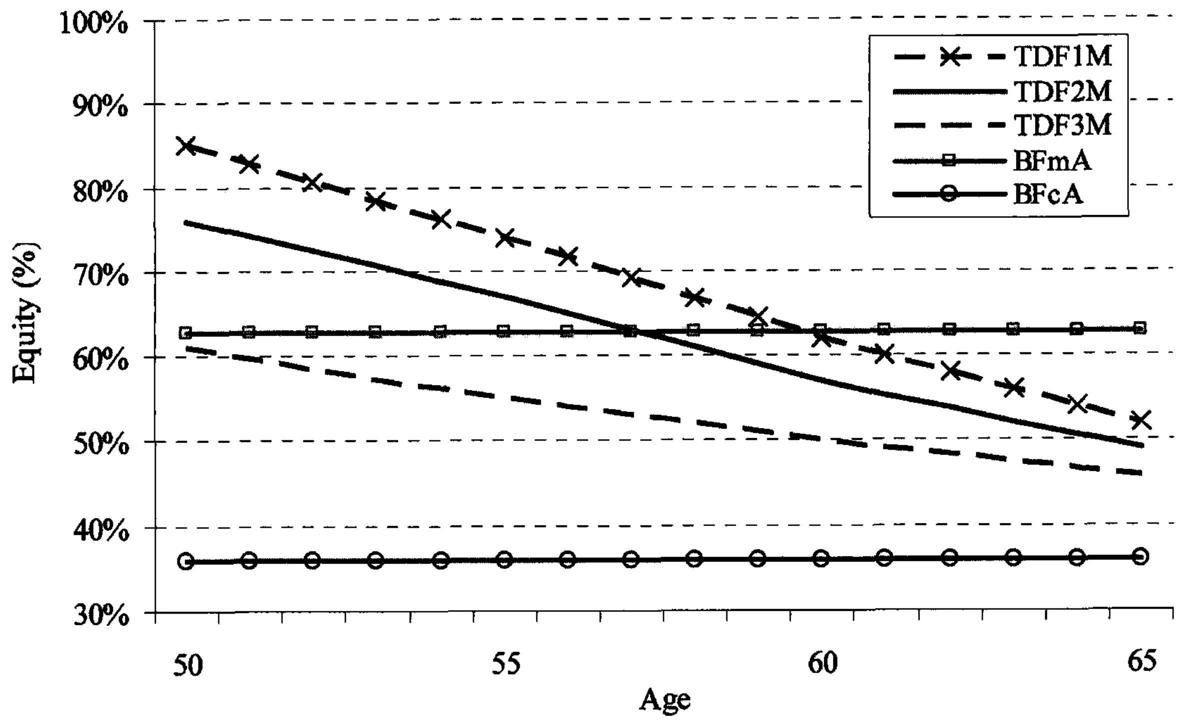

Fig. 2. Asset allocations of target-date and balanced funds for midcareer investors. (*Source:* Authors' data collection from Morningstar and TDF providers' websites as of April 30, 2010).

3. Wealth accumulations of target-date funds: Stochastic simulations

We simulate the range of possible investment outcomes according to the TDF equity-bond-cash allocations, assuming that the investors make uninterrupted contributions to the retirement plan and follow the asset glide paths throughout the years. Combined employer and employee contributions are set to be six percentage of a worker's labor earnings. We evaluate TDFs by levels and variations of wealth upon retirement. All terminal values are adjusted into real terms by simulated inflations.

3.1. Asset returns with normal shocks and rare disasters

Asset rates and returns are simulated based on a stochastic model that allows for standard market randomness (also called "shocks") in normal times and low-probability large-magnitude rare economic disasters ("fat tails"). Rates and returns in normal times are modeled as a vector autoregressive (VAR) system, following Campbell and Viceira (2004, 2005). The VAR specifies current asset returns as a function of lagged returns and current-period shocks. Rare economic disasters are simulated based on the framework of Barro (2006). As advantages over conventional models, this comprehensive modeling captures the persistence of market shocks and both the contemporaneous and serial correlations of asset classes.[1] Details of the estimation and simulations are described in the appendix.

Table 1 Statistics of simulated rates and returns (%)

	Equity return	Bond return	Money market	Inflation
	Real rates and returns			
No disasters				
Mean	3.8	2.0	0.9	—
SD	17.8	4.5	2.1	—
With disasters				
Mean	2.5	1.2	0.4	—
SD	22.3	10.4	6.1	—
	Nominal rates and returns			
No disasters				
Mean	7.4	5.6	4.5	3.6
SD	17.5	4.3	2.9	2.7
With disasters				
Mean	6.7	5.3	4.6	4.2
SD	19.6	6.0	2.9	6.3

Source: Authors' simulations, based on 1962–2009 data.

Equity, bond, and cash returns in the VAR specification are proxied by the S&P 500 total return index, the five-year government bonds total return index and 90-day Treasury bills, respectively. The regression uses 1962–2009 quarterly data. When economic disasters strike, equities contract, government bonds can default, and cash rates may drop, with certain probabilities and by varying degrees. These probabilities and value losses are estimated in Barro (2006) based on 60 economic events (World Wars I and II, Great Depression, postwar depressions, etc.) in the 20th century in 35 countries. Table 1 summarizes the simulated rates and returns. The incorporation of rare economic disasters significantly lowers the expected asset returns and increases their volatilities.

3.2. Labor earnings

Labor earnings are simulated based on the model in Campbell et al. (2001) and Cocco et al. (2005). Uncertainty about labor earnings affects account balance and future retirement income owing to the variability of contribution flows to the DC account. Individual labor income exhibits a life-cycle age-earnings profile and is also simultaneously affected by aggregate economic shocks. The life-cycle component in logarithm is modeled as a deterministic age polynomial plus idiosyncratic permanent and transitory shocks. Macroeconomic shocks are introduced via the correlations between individual life-cycle income and the dynamics of excess equity returns, which are defined as the difference between equity return and money market rate (nearly risk-free T-bills).

We utilize the baseline estimates of parameter values from Campbell et al. (2001) and Cocco et al. (2005) and generate earnings for workers with a high school education. Specifically, the variances of permanent and transitory (log) earnings shocks are 0.0106 and 0.0738, respectively. The correlation coefficient between (log) earnings and excess equity

Table 2 Wealth balance at retirement for early-career investors (real $000)

Fund	5th percentile	50th percentile	95th percentile	Mean	SD
A. No consideration of economic disasters					
TDF1E	89.5	190.3	410.7	212.4	104.8
TDF2E	86.3	190.3	426.2	214.7	111.6
TDF3E	89.9	188.5	401.1	209.6	101.3
B. Consideration of economic disasters					
TDF1E	45.8	161.3	381.3	181.1	107.6
TDF2E	42.5	158.7	392.0	180.8	113.2
TDF3E	47.7	160.7	373.2	179.5	104.2

Source: Authors' simulations.

return is 0.3709. The simulated real earnings exhibit the well documented hump shape: they start with about $34,000 in a worker's 20s and reach a peak around $51,000 in her or his 50s. Earnings in nominal terms incorporate the stochastically realized inflations.

3.3. Results for early career investors

Table 2 reports the probability distributions of retirement wealth for early career workers who have a long investment horizon (TDF equity allocations depicted in Fig. 1). The 5th and 95th percentiles of terminal balances indicate "bad" and "good" outcomes, respectively. Outcomes will more likely come out around the median (50th percentile).

Several inferences can be made. First, investment risk is substantial across TDFs, though excellent outcomes are also possible. With about a 5% chance, terminal wealth falls to around $90,000 (Panel A in Table 2). This ruin of wealth is almost certain to mandate either a reduced living standard in retirement or a postponement of retirement. DC plans thus bears a price for the flexible investment and withdrawal advantage: the lack of a benefit guarantee, which is otherwise featured in defined benefit plans.

Second, wealth creation in TDFs is dampened severely by occurrences of rare economic disasters. Wealth balances in Panel B of Table 2 are significantly lower than in Panel A simply because economic disasters devastate wealth in the lower percentiles.

And third, the pace and composition of portfolio changes are critical determinants of final wealth levels and volatilities besides the initial asset mix. The equity position of TDF2E starts lower than TDF1E but remains substantially higher than both TDF1E and TDF3E for most of the years. The double-edge effect of equity investment follows: TDF2E has the potential to realize a greater balance ($392,000 at the 95th percentile outcome, Panel B) but may end up underperforming others ($42,500 at the 5th percentile). In other words, the more significant shift to fixed-income securities lowers the risks in TDF1E and, especially, TDF3E. The performance differences, however, are generally small across TDFs at the means and 50th percentiles because their 40-year portfolio changes follow a similar trend and even cross over at times.

Table 3 Wealth balance at retirement for mid-career investors (real $000)

Fund	5th percentile	50th percentile	95th percentile	Mean	SD
A. No consideration of economic disasters					
TDF1M	115.9	206.4	361.6	218.7	78.1
TDF2M	118.7	203.7	343.6	214.1	71.2
TDF3M	123.2	203.3	329.4	212.0	65.0
BFmA	117.1	204.4	349.2	215.4	74.0
BFcA	129.2	197.4	297.3	203.2	52.4
B. Consideration of economic disasters					
TDF1M	59.6	190.9	353.3	198.2	87.4
TDF2M	64.9	189.3	336.2	194.9	80.4
TDF3M	68.4	190.0	322.8	193.9	74.7
BFmA	64.1	189.1	342.8	195.8	82.9
BFcA	76.4	186.8	291.6	187.9	62.4

Source: Authors' simulations.

3.4. Results for midcareer investors

For a midcareer investor aged 50, her DC account is assumed to have an existing balance of $100,000 from prior accumulations.[2] Table 3 reports the results. Standard deviations here are smaller than those in Table 2 because the shorter investment horizon for the midcareer investor means fewer years for investment outcomes to possibly diverge. Patterns of relative TDF performances here are otherwise similar to those for early career workers. That is, greater potential reward from equity exposure goes with greater risk.

TDFs in Fig. 2 maintain their relative positioning of equity exposure throughout the 15 years of investment, which makes the risk-return tradeoffs clearer. For instance, TDF1M, with the highest equity allocation, has the potential of generating about $361,600, which is about $32,200 higher than TDF3M (95th percentile outcomes, Panel A of Table 3). In down markets, the lower equity exposure of TDF3M mitigates investment loss and outperforms TDF1M by about $7,300 in real terms (5th percentile outcomes). The mitigation is more valuable with occurrences of economic disasters (a larger $8,800 outperformance of TDF3M over TDF1M at the 5th percentile, Panel B). Moreover, TDF3M has a significantly lower standard deviation, one measure of risk. These results echo the findings of Pang and Warshawsky (2010).

Also worth noting is the comparison with performances of balanced funds. First, BFs seem to have an even wider range of results than TDFs. BFmA delivers quite similar outcomes to the TDFs. BFcA, which is significantly more conservative by design and foregoes upward potential, outperforms the TDFs and BFmA in down markets, with or without consideration of occurrences of rare economic disasters, but underperforms by a lot in up markets.

And second, TDFs are sensible for the intended protection of wealth towards the end of career by moving to lower risk securities. Balanced funds like BFmA take a lower investment risk than most TDFs at younger ages when account balances are lower but stay at the higher equity positions in later years when larger account balances are at stake. For instance, the standard deviation of TDF2M balance is about 9% larger than that for BFmA balance at age 55 (results not shown), about 2% larger at age 60, but then about 3% smaller at age 65. This suggests that the investment risk of TDF2M (which is otherwise quite similar to BFmA) is relatively declining with age while BFmA is not. This is of particular importance to the retiring worker because there are fewer years of labor earnings on which to rely should large investment losses occur.

4. Conclusions and discussion

This analysis looks at asset allocations of target-date funds on the market that are being offered by leading providers. The data exhibits the common practice of high equity holdings for long-horizon younger workers. It also seems to suggest that some TDFs have recently been reducing the risky equity exposure for investors near retirement. In comparison, asset allocations in balanced funds have little changed.

As the dominant investment vehicle for DC plans, TDFs are loaded with the dual and often competing tasks of wealth growth and protection. The analysis makes quantitative comparisons of wealth accumulations and the attendant risks across TDFs with varied asset allocations. The results highlight the uncertainty in TDF outcomes owing to capital market risks and the possibility of economic catastrophes. A larger equity position is generally expected to produce a greater wealth balance at the risk of larger investment shortfalls. The pace and composition of portfolio shifts (glide path designs) are important determinants of wealth levels and volatilities. TDFs as the sole vehicle for retirement wealth accumulation must be considered risky, particularly when the possible occurrences of large economic disasters are considered. Nonetheless, TDFs have less risk than comparable BFs close to retirement and therefore are more suitable for investors who will more quickly draw upon these assets to fund their spending in retirement.

As mentioned above, the Department of Labor and SEC held a joint hearing in June 2009 on TDFs. In response to the findings there, in May and June 2010, they have issued an investor information brochure and proposed new required disclosure measures, respectively. The agencies have stated that they are concerned about the size and wide variation in losses in 2008 among TDFs with the same target date. The agencies are particularly impressed by evidence that investors were quite confused by the naming conventions of TDFs and the content of marketing materials leading to the widespread misunderstanding that a TDF would meet retirement needs, even to the extent of guaranteeing them, and that there were uniform glide path strategies in the fund industry.

Therefore, the agencies issued an investor bulletin explaining TDFs in fairly simple language, emphasizing the need for investors to understand the riskiness of TDFs, the variety of different glide paths, and the importance of whether the TDF is intended to be used immediately upon retirement to support spending or will remain invested for some time after retirement.[3] The SEC proposed that TDFs disclose the asset allocation policy of the fund at the target date prominently in the marketing material, that is, at the first mention of the target date. The SEC is also proposing that the marketing material include a prominent table or chart clearly depicting the asset allocations among types of investments over the entire life of the fund, including past the target date. Moreover, it is proposing that the materials state that the TDF is not guaranteed, and that its use should depend on the investor's risk tolerance and personal circumstances. Finally, the SEC proposes that any investment fund could be subject to antifraud enforcement if it suggests that the investment is appropriate on the basis of a single factor, such as age, or that investing in the plan is simple and needs no monitoring.

The results of the empirical and stochastic simulation analysis here are broadly consistent with the stance and actions taken by the Department of Labor and SEC thus far. The agencies are not imposing hard and fast rules on glide paths or asset allocations. Rather they are suggesting that better information will help investors to evaluate risks and returns among TDF offerings and to also compare TDFs with alternative investment strategies. The selection of TDFs requires a deep consideration of an investor's risk tolerance, other investments, retirement and labor income, and other personal circumstances for the years before and after retirement.

Notes

1. This integrated model was first implemented in Pang and Warshawsky (2010).
2. At age 50, $100,000 is approximately the average real value of TDF1E, TDF2E, and TDF3E.
3. Also see the testimony of Mark Warshawsky at the June 2009 joint hearing, at http://www.dol.gov/ebsa/pdf/WatsonWyatt061809.pdf (accessed July 1, 2010).

Acknowledgment

Opinions expressed here are the authors' own and not necessarily those of their affiliation, and also are not intended to constitute investment advice.

Appendix: Rates and returns with normal shocks and rare economic disasters

1. Dynamics of asset returns in normal times

1.2. Asset returns

Asset returns in normal times are simulated as a vector autoregressive process (VAR). The VAR coefficients and variance matrix are first empirically estimated and then embedded in the simulations with stochastic shocks.

Following the specification in Campbell and Viceira (2005), asset classes include money market (90-day T-bills), stocks (proxied by S&P 500 Total Return index), and bonds (proxied by five-year US Government Bond Total Return Index). These rates and returns in the VAR estimation are expressed in logarithm real terms (after adjusting for inflation that

Table A.1. Augmented Dickey-Fuller unit root tests for variables

	Test statistic	1% Critical value	5% Critical value	10% Critical value	p-value
Log real 90-day T-bill rate	−8.26	−3.48	−2.88	−2.57	0.00
Log real excess equity return	−12.67	−3.48	−2.88	−2.57	0.00
Log real excess bond return	−14.52	−3.48	−2.88	−2.57	0.00
Log nominal 90-day T-bill rate[a]	−2.41	−2.35	−1.65	−1.29	0.01
Log dividend yielda	−1.32	−2.35	−1.65	−1.29	0.09
Log yield spread	−5.56	−3.48	−2.88	−2.57	0.00

Note. No lags of the variable are included.

[a] Indicates a drift included in the test.

Source: Authors' estimations.

is measured by the change in the CPI-U index), using 1962–2009 quarterly data. Additionally, three forecasting variables (state variables), which help form expectations of future rates and returns, include short-term nominal interest rate (nominal T-bills), equity dividend yield, and the slope of the yield curve (yield spread as the difference between US five-year T-note zero-coupon yield and the yield on 90-day T-bills).

Technically, let V be a vector containing the variables. The vector evolves in an autoregressive pattern:

$$V_t = \beta_0 + \sum_{k=1}^{K} \beta_k V_{t-k} + \mu_t$$

where $\mu \sim (0,\Sigma)$ denotes a vector of serially uncorrelated normal errors with $E\mu_t\mu_s = 0$, for $t \neq s$. The contemporaneous correlations of shocks are incorporated via the variance-covariance matrix Σ and serial correlations of the variables via the coefficient matrix β. The econometric regression on historical data yields estimates of the coefficients, $\hat{\beta}$'s and the variance-covariance matrix $\hat{\Sigma}$. Table A.1 reports the augmented Dickey-Full unit root tests. All variables are stationary. The SBIC selection order criterion suggests one lag of the variables in the VAR specification. The estimated coefficient and variance-covariance matrices are shown in Tables A.2 and A.3, respectively.

The simulations follow several steps: First, a Cholesky factorization decomposes the variance-covariance matrix to a triangle matrix. That is, the factorization finds a triangle matrix W so that $W'W = \hat{\Sigma}$. Second, a vector of random values are generated according to

Table A.2. VAR regression results

Equation	Parms	RMSE	R^2	χ^2	$p > \chi^2$
Log real 90-day T-bill rate	7	0.006	0.323	90.7	0.000
Log real excess equity return	7	0.081	0.051	10.2	0.116
Log real excess bond return	7	0.017	0.098	20.6	0.002
Log nominal 90-day T-bill rate	7	0.003	0.865	1,221.5	0.000
Log dividend yield	7	0.021	0.960	4,527.5	0.000
Log yield spread	7	0.002	0.512	199.1	0.000

Table A.2. VAR regression results (Continued)

	Coef.	SE	z	$p > \|z\|$
Log real 90-day T-bill rate				
L1.Log real 90-day T-bill rate	0.3138	0.0664	4.73	0.00
L1.Log real excess equity return	0.0048	0.0051	0.93	0.35
L1.Log real excess bond return	0.0500	0.0255	1.96	0.05
L1.Log nominal 90-day T-bill rate	0.4978	0.1008	4.94	0.00
L1.Log dividend yield	−0.0152	0.0056	−2.71	0.01
L1.Log yield spread	0.7479	0.1974	3.79	0.00
_cons	−0.0201	0.0062	−3.26	0.00
Log real excess equity return				
L1.Log real 90-day T-bill rate	0.3896	0.9116	0.43	0.67
L1.Log real excess equity return	0.1046	0.0704	1.48	0.14
L1.Log real excess bond return	0.3874	0.3498	1.11	0.27
L1.Log nominal 90-day T-bill rate	−2.0876	1.3850	−1.51	0.13
L1.Log dividend yield	0.1742	0.0773	2.25	0.02
L1.Log yield spread	−0.8175	2.7123	−0.30	0.76
_cons	0.1915	0.0848	2.26	0.02
Log real excess bond return				
L1.Log real 90-day T-bill rate	0.0465	0.1921	0.24	0.81
L1.Log real excess equity return	−0.0339	0.0148	−2.29	0.02
L1.Log real excess bond return	−0.0710	0.0737	−0.96	0.34
L1.Log nominal 90-day T-bill rate	0.7906	0.2919	2.71	0.01
L1.Log dividend yield	−0.0308	0.0163	−1.89	0.06
L1.Log yield spread	2.1539	0.5716	3.77	0.00
_cons	−0.0396	0.0179	−2.22	0.03
Log nominal 90-day T-bill rate				
L1.Log real 90-day T-bill rate	0.0002	0.0288	0.01	0.99
L1.Log real excess equity return	0.0038	0.0022	1.69	0.09
L1.Log real excess bond return	−0.0013	0.0111	−0.12	0.91
L1.Log nominal 90-day T-bill rate	0.9410	0.0438	21.50	0.00
L1.Log dividend yield	0.0024	0.0024	1.00	0.32
L1.Log yield spread	0.1555	0.0857	1.81	0.07
_cons	0.0025	0.0027	0.94	0.35
Log dividend yield				
L1.Log real 90-day T-bill rate	−0.0655	0.2373	−0.28	0.78
L1.Log real excess equity return	−0.0229	0.0183	−1.25	0.21
L1.Log real excess bond return	−0.1035	0.0911	−1.14	0.26
L1.Log nominal 90-day T-bill rate	0.3904	0.3605	1.08	0.28
L1.Log dividend yield	0.9623	0.0201	47.84	0.00
L1.Log yield spread	−0.0942	0.7060	−0.13	0.89
_cons	−0.0385	0.0221	−1.74	0.08
Log yield spread				
L1.Log real 90-day T-bill rate	−0.0038	0.0212	−0.18	0.86
L1.Log real excess equity return	−0.0009	0.0016	−0.54	0.59
L1.Log real excess bond return	0.0103	0.0081	1.27	0.20
L1.Log nominal 90-day T-bill rate	−0.0021	0.0322	−0.07	0.95
L1.Log dividend yield	0.0004	0.0018	0.23	0.82
L1.Log yield spread	0.6986	0.0630	11.08	0.00
_cons	0.0011	0.0020	0.57	0.57

Source: Authors' estimations.

Table A.3. Estimated variance-covariance matrix of residuals

	Log real 90-day T-bill rate	Log real excess equity return	Log real excess bond return	Log nominal 90-day T-bill rate	Log dividend yield	Log yield spread
Log real 90-day T-bill rate	0.000033					
Log real excess equity return	0.00005	0.006315				
Log real excess bond return	9.20E-06	0.000175	0.00028			
Log nominal 90-day T-bill rate	1.50E-07	−0.000034	−0.000033	6.30E-6		
Log dividend yield	−0.000016	−0.001604	−0.000043	8.90E-6	0.00043	
Log yield spread	−1.30E-06	0.000015	7.00E-06	−3.60E-6	−4.00E-6	3.40E-6

Source: Authors' estimations.

IID N(0, 1). Multiplying this vector by the Cholesky factor matrix generates correlated shocks to rates and returns. Third, multiplying the VAR coefficients by previous period returns, plus the shocks, gives current period returns. The procedure is repeated forward until the end of time horizon under consideration. A large number of simulations are implemented to get numerically reliable results.

II. Asset contraction and default in rare economic disasters

The simulations consider low-probability large-magnitude disasters, such as depressions and wars. Expected returns on assets decrease with increases in the likelihood of a disaster. Negative skewness (fat tails) is thus featured in the distribution of asset returns. The framework is based on Barro (2006) who demonstrates that the possibility of rare disasters can explain the equity premium puzzle and "risk-free" interest rate behavior (for example, why expected real interest rates were low in major wars) that are not well explained by conventional models.

Rare disasters are assumed to occur with a probability of p per unit of time. With a probability of $exp(-p)$, the equity market is not affected by the disaster, but with the remaining probability of $1\text{-}exp(-p)$, the equity value shrinks by size b. Conditional on the economic disaster, government bonds default with a probability of q but otherwise deliver the specified face value with the remaining probability of $1\text{-}q$. A default reduces the bond value by size d. Simultaneously with the default of government bonds, cash rate (T-bills) is assumed to decline by size c (a "flight to quality" scenario).

The baseline parameters in Barro (2006) are calibrated based on 60 economic events (15+ percent declines in real per capita GDP during World War I and II, Great Depression, postwar depressions, and so forth) in the 20th century in 20 OECD, 8 Latin American and 7 Asian countries. Specifically, p is estimated to be 0.017 and q is 0.4. Sizes of loss and default (b, d, and c) are randomly simulated (with necessary interpolations), based on the distribution of per capita GDP contractions in Table 1 and Figure 1 of Barro (2006). Specifically, the distribution is in the range of 15–64% in the raw data and roughly 0.15–0.70% if adjusted for trend growth. Realized values may differ for b, d, and c.

III. Investment returns with normal shocks and rare economic disasters

In normal times, asset returns follow the stochastic VAR process. When disasters strike, the simulated shrinkages of b, d, and c are applied to money market, stocks, and bonds, while leaving the same-period state variables intact. Impacts of these disasters carry forward in the VAR through the antocorrelation of each asset class with its own lagged value, and more significantly, through the highly persistent autoregressive coefficients on the state variables (nominal interest rate, dividend yield, and yield spread). Disasters are simulated on an annual frequency and the shrinkages are divided into 4 equal values for the quarterly VAR simulations. Summary asset returns are reported on an annual basis below in Table A.4. The likelihood of rare economic and financial disasters, and thus large equity contraction, is about 1.7% per annum. The probability of government bond default is about 0.7%.

Table A.4. Statistics of simulated rates and returns (%)

	Equity return	Bond return	Money market	Inflation
	Real rates and returns			
No disasters				
Mean	3.8	2.0	0.9	—
SD	17.8	4.5	2.1	—
With disasters				
Mean	2.5	1.2	0.4	—
SD	22.3	10.4	6.1	—
	Nominal rates and returns			
No disasters				
Mean	7.4	5.6	4.5	3.6
SD	17.5	4.3	2.9	2.7
With disasters				
Mean	6.7	5.3	4.6	4.2
SD	19.6	6.0	2.9	6.3

Source: Authors' simulations, based on 1962–2009 data.

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