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Asset allocation, human capital, and the demand to hold life insurance in retirement

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Abstract

Much research has been done on the topic of asset allocation, human capital, and life insurance demand. Some researchers attempt to explain existing patterns of each within U.S. households; others propose optimal amounts of insurance and investment given a wide variety of life-cycle assumptions. We take a case by case approach to (1) warn against the dangers of applying general rules of thumb in the investment decision making process, and (2) to demonstrate how financial planners can use simulation based risk models to help investors answer the asset allocation question and the demand to hold life insurance question simultaneously.

JEL classifications: G11; G17; G22

Keywords: Retirement portfolio; Simulation; Asset allocation; Human capital; Life Insurance

1. Introduction

Following on the pioneering works of Nobel Prize winners Robert Merton (1969), Franco Modigliani (1963), and others (Browning and Crossley, 2001), economists have been keenly interested in the life-cycle interrelations between wealth held in the form of human capital and wealth held in financial assets. For expositional convenience, we define human capital as a nontradable asset representing the present value of expected future wages. Assuming that the variability of labor market opportunities is less than the variability of the stock market,

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and further assuming a low correlation between human capital value and stock returns, economists often model the human capital asset as if it were a bond. Whether or not you agree with this characterization, it is easy to see how it leads to the investment suggestion that young wage earners should own primarily stocks in their financial asset portfolio. Most young wage earners own a modest amount of financial assets compared with the large value of their expected future paychecks. That is to say, they are overweighted in bonds by virtue of the 'coupons' from their wage income and underweighted in stocks. However, because the stream of expected future compensation is not a tradable asset, it is precariously fragile—you cannot sell your expected earned income in the capital markets. To avoid a catastrophic and irreversible economic shock to the family's consumption plan, a common recommendation is to acquire a life insurance policy—that is, an actuarial rather than a financial hedge against income loss.[1]

Given that most previous literature adopts a form of the life-cycle model, it is not surprising that the demand to hold insurance ceases at retirement as the value of human capital reduces to zero. Nevertheless, empirical studies demonstrate that most U.S. households voluntarily retain ownership of life insurance programs. Many policies held by retirees are permanent cash value contracts. Are these policies legacy assets that continue to exist merely because of indifference or inertia (Bernheim, 1992)? Are they a form of precautionary savings (Liebenberg, Carson, and Dumm, 2010)? Can they make a positive contribution to a portfolio's asset allocation? Should the retired investor retain life insurance coverage or surrender it?

This article considers the demand to hold life insurance during retirement from the perspective of a retired couple wishing to allocate financial resources optimally to provide consumption over their joint life span. The extensive research on the topic of an investor's demand for life insurance can be loosely categorized as either empirical or theoretical. Empirical research generally attempts to indicate and explain existing patterns of life insurance demand within U.S. households. For example, Lewis (1989) observes negative correlation between household demand for life insurance and net worth. Studies by Gandolfi and Miners (1996) and Gokhale (2002) note that wives are underinsured relative to husbands. Babbel's study (Babbel, 1985) of life insurance pricing confirms the intuitive notion that demand is lower for higher cost insurance plans. The extensive work of Auerbach and Kotlikoff (1985, 1989, 1991) points out that, upon becoming widowed, wives suffer an economically significant fall in living standards, suggesting that couples are under insured. Lin and Grace (2005) assert that household demand for term life insurance is a positive function of a financial vulnerability index;[2] while Liebenberg et al. (2010) note that retirees "...are more likely to drop their term coverage. because the need to insure the potential loss of future income no longer exists in retirement."

On the other hand, theoretical research generally focuses on determining the optimal amount of insurance an investor should hold. Yaari (1965) is among the first to model the demand for insurance in the context of maximizing an expected utility of consumption function over an uncertain lifetime. Following Yaari, a large body of research focused on an optimization problem: how much term life insurance should an investor hold to maximize utility. Not surprisingly, the answer depends largely on the functional form of the variables, parameters, and constraints of the model.[3] Yaari's work focuses on the uncertainty of life and takes for granted the demand function for insurance. Fischer (1973) later describes characteristics of the insurance demand function under various assumptions on the functional form of investor utility, consumption preferences, wages, and so forth. Fischer is solidly in the mainstream of academic thought as he concludes: "... an individual who lives off the proceeds of his wealth is unlikely ever to purchase life insurance. An individual who receives labor income is likely to purchase life insurance early in his life. In all simulations the individual tends to sell life insurance late in life." Campbell (1980) derives insurance demand equations within a multiperiod analysis of utility functions. Although, he acknowledges the importance of a bequest motive in the demand to hold life insurance, he, too, concludes: "viewed in a lifecycle context... as households become older the proportion of human capital that is insured should probably decrease even if their level of relative risk aversion stayed constant."

Buser and Smith (1983) solve for the optimal amount of life insurance demand by incorporating insurance instruments into a single-period, mean-variance analysis framework. The goal of the investor is to create an efficient portfolio for total end-of-period wealth given wage income, insurance policy proceeds, investments in both risky and risk-free assets, and the cost per dollar of insurance coverage. The efficient portfolio is described by two parameters: (1) the expected value of total terminal wealth, and (2) the variance of total wealth. Although insurance is seen as an important variance reduction instrument, the benefit of the variance reduction occurs because of an insurance policy's ability to replace earned income: "... insuring against the loss of a claim on future earnings as a result of the wage earner's death may be modeled as a portfolio problem in which the return on a life insurance contract is negatively correlated with the return on the claim."

However, many assumptions underlying the life-cycle model's analytical framework represent simplifications required to achieve closed-form mathematical solutions to the utility maximization issue.[4] As the theoretical models incorporate more realistic assumptions, explicit algebraic answers become elusive. In these circumstances the research must turn to other methods of calculating approximate answers. For example, Purcal (1999) provides a numerical solution to the Richard (1975) model by using a Markov Chain approximation approach for an investor with a constant relative risk aversion function. Huang and Milevsky (2008) develop an even more advanced model by allowing wages to be stochastic, incorporating correlation between wages and risky asset returns, and maximizing a more appropriate hyperbolic absolute risk aversion function. Pliska and Ye (2007) use a dynamic programming approach to derive the optimal insurance and consumption control variables for a wage earner under an uncertain life span. Their model maximizes utility by solving a multiperiod value function expressed in the form of a Hamilton Jacobi Bellman (HJB) equation.[5] Breaking away from typical models where longevity is exogenous, Ehrlich (2000) examines the insurance demand question by assuming that longevity can be controlled at the margin by investing in what he calls "self-protection." Although the models used in previous research sometimes result in conflicting findings,[6] nevertheless they provide a good framework for thinking normatively about the demand for life insurance. That is, they suggest how much life insurance a generic investor *should* hold to optimize resource allocation.

2. Methodology and motivation

This article assumes that the retired couple is concerned with allocation of resources to provide consumption over the joint life span. It attempts to give a more positive analysis of an investor's demand to hold life insurance during retirement. Rather than offering a generalized algebraic model of the demand function, it examines cases based on specific investor circumstances. The question of how much life insurance to hold is reduced to a simpler Boolean question of whether to hold pre-existing life insurance or to let go of, that is, surrender, it. By examining a binary choice question and by looking at specific cases, the complexity of the analysis is reduced, but, at the same time, the tighter focus permits examination of both term life and permanent life insurance. The case study approach also provides a real world context in which investors can frame their unique questions about the value of privately owned life insurance, and it illustrates how a simulation of credible risk models can provide investment guidance on an investor-specific basis.

There are various types of insurance products and associated ways to surrender them. For term life insurance, the most basic way is to simply let the insurance policy lapse by stopping premium payments. Whole life insurance products can be surrendered for cash or for other non-forfeiture options, such as an annuity income at a guaranteed annuity pricing factor. In certain cases, both term and permanent policies can be sold to life settlement firms. The analysis here focuses primarily on a whole life policy that can be converted for immediate cash and a term life policy that may be allowed to lapse.[7] Furthermore, in addition to hedging against the loss of income, there are other reasons for holding onto life insurance. These may include planning to discharge anticipated estate tax liabilities, funding business liquidity needs, prepaying final expenses, and so forth.[8] As stated, this study limits the analysis to investors concerned with providing joint-life consumption. Later, we briefly extend the analysis to funding intergenerational bequests.

Further motivation for this study comes from an article by Chen, Ibbotson, Milevsky, and Zhu (2006). Their research takes a case study approach to the optimization problem, uses simulation to model portfolio evolutions, and suggests that further work be conducted on a model of shortfall probabilities. Rather than a traditional utility maximization approach assuming a lognormal return distribution, we use a shortfall probability approach.[9] We define *shortfall probability* as the *bankruptcy rate*, which is the chance that either the insured or the spouse beneficiary is alive and that their financial asset portfolio is fully depleted. The focus on risk in terms of the sustainability of income over the joint life span reflects the propensity for the majority of married individuals with life insurance policies to name their spouse, instead of children, as beneficiary.[10] This suggests that the purpose of the insurance is to provide consumption for the beneficiary spouse. Under this definition of shortfall risk, holding life insurance is pointless in the event that the beneficiary spouse predeceases the retired insured. Although, in all likelihood, an investor would never allow an auto-pilot distribution policy to bring him to a zero-resource threshold, we select a portfolio-depletion benchmark because the bankruptcy rate is a measure of *longevity risk*–the risk that either the investor or spouse outlives their money given their current retirement income preferences. Of course, another aspect of shortfall probability is the failure to discharge a *bequest objective*. This is the probability that the retired investor's *terminal wealth value* (the value of the portfolio when both spouses are no longer alive) is less than the amount of money they wish to bequeath. In general, the shortfall probability approach is a more convenient decision making tool because most investors do not know how to maximize, let alone know or understand, their utility of wealth function. Indeed, an investor may consider shortfall probability to be a useful risk measure, but may use other portfolio preferencing criteria for making investment strategy decisions.

Shortfall probabilities reflect simulated outcomes of a risk model. A *simulation* is a set of random trials, where each trial represents a potential future feasible outcome.[11] A trial ends when the investor and spouse are no longer alive. The method of generating individual and joint lifetimes is similar to the method Crabb (1991) utilizes, but we use the Society of Actuaries annuity table of "white collar" retirees from Defined Benefit Pension Plans instead of the U.S. Life Table. Survival rates using the annuity table are higher than those of the U.S. Life Table. Each case models two alternatives: the investor's decision to hold life insurance and the investor's decision to surrender the life insurance and invest the surrender proceeds, if any, into a diversified portfolio of financial assets. The simulation generates 5,000 random trials each of which reflects investor longevity as well as inflation environments and market evolutions for six diversified portfolio asset allocations. The results presume that once the decisions are made, the investors do not deviate from them. That is, the investors do not invest dynamically by opting to modify initial decisions based on future simulated outcomes as they would in a dynamic retirement withdrawal planning model (Stout and Mitchell, 2006) or a midcourse adjustment model (Spitzer, 2008).[12] Furthermore, the results do not assume an age phased reduction in the equity allocation for the six allocation options. Instead, each portfolio is rebalanced annually to its original allocation target. The model further assumes that, at the insured's time of death, the full amount of the life insurance benefits are invested in the portfolio as prescribed by the asset allocation. Portfolio values are adjusted for inflation but not for taxes. Both taxes and transaction costs are ignored for the purposes of isolating the economic tradeoffs in the insurance and asset allocation decisions.[13] The model measures shortfall probabilities by dividing the number of trials that fail the shortfall criterion by the total number of trials (5,000) in the simulation. The retirees can compare the results from the two simulations to make an informed decision on whether to hold an insurance policy or surrender it, based on the shortfall probabilities and the terminal wealth values that best reflect their economic goals, risk preferences, and constraints. Thus, a preference for a specific asset allocation/insurance configuration defines implicitly the investors' utility function.[14]

Some argue that insurance products (mortality contingent claims) are asset classes in the portfolio theory sense (Weber and Hause, 2008). Such contracts do not meet the conventional criteria for defining an asset class. For example, Sharpe, Chen, Pinto, and McLeavey (2007) assert that "... the investor needs a logical framework for examining the not infrequent claims by sponsors of new investment products that their product is a new asset class ..." They provide five criteria for specifying asset classes:

1. Assets within an asset class should be relatively homogeneous;
2. Asset classes should be mutually exclusive;
3. Asset classes should be diversifying;

4. The asset classes as a group should make up a preponderance of world investable wealth; and,
5. The asset class should have the capacity to absorb a significant fraction of the investor's portfolio without seriously affecting the portfolio's liquidity.

Under the lifecycle hypothesis, the demand to hold insurance is optimally determined simultaneously with the demand to hold other portfolio assets that, in turn, are optimized and managed in the context of total human wealth. However, the demand may arise more from a desire to hedge the risk of income loss by using an actuarial instrument rather than from a desire to mitigate diversifiable risk by using financial assets. The fact that risk can be mitigated either by an actuarial approach, an investment approach, a financial engineering approach, or by a combination thereof, does not necessarily mean that mortality contingent claims are asset classes. To argue otherwise suggests that one also has a burden of proof to show that any contingent claim contract altering a portfolio's payoff function (e.g., derivative contracts) constitute an asset class.

Mayers and Smith (1983) point out that "a sufficient condition for the separability of the demand for insurance from other portfolio decisions is that the losses of a particular type are orthogonal to the payoffs to all marketable assets, the individual's gross human capital, and the losses associated with other insurable events." Buser and Smith (1983), as noted above, argue that insuring against lost income ". . . may be modeled as a portfolio problem in which the return on a life insurance contract is negatively correlated with the return on the claim." However, according to Buser and Smith, this claim is a convenient assumption only for term insurance. Policies with nonguaranteed cash value elements (account values and dividends) rely on the performance of insurance companies investing in capital markets. In terms of modern portfolio theory, such contracts are nonzero-beta policies. The fact that (term) life insurance payoffs appear to have zero correlation to the returns of both marketable and nonmarketable assets provides some justification for asserting that an insurance contract represents a separate asset class. However, lottery tickets also provide contingent payoffs orthogonal (covariance equal to zero) to human wealth, insurance contracts, and financial assets. These observations notwithstanding, Horneff, Maurer, Stamos, (2006) and others (Horneff et al., 2007, 2008) claim that ". . . annuities define an asset class with certain age dependent return characteristics because payments are conditional on survival."

Regardless of how the insurance-as-asset-class debate resolves itself, research has shown that the asset allocation decision should be made jointly with the decision to hold life insurance (Chen, Ibbotson, Milevsky, and Zhu, 2006). Therefore, the decision to hold life insurance and the choice of asset allocation are looked at in tandem.

3. Results

3.1. Case 1

Consider a couple who are already well into retirement and have no bequest motives, but require that their assets provide income over their joint life span. Their financial assets

Table 1 Base case results

		100% equity	80–20	60–40	40–60	20–80	100% bonds
Hold	Median value	$1,770,613	$1,688,692	$1,447,576	$1,235,497	$959,166	$684,733
	Bankruptcy rate	19.1%	14.2%	10.2%	6.9%	4.1%	5.6%
Sell	Median value	$1,770,169	$1,708,587	$1,503,221	$1,187,723	$900,184	$600,390
	Bankruptcy rate	19.8%	14.5%	10.8%	7.1%	4.8%	5.0%

consist of a portfolio worth $1,000,000. The insured is a healthy 71 year old male who owns a whole life insurance policy that is paid up (no premium payments required) and is protected against inflation.[15] We assume a current death benefit value of $275,000. The insurance program assumes ownership of a fully funded participating whole life contract where dividends purchase paid-up additions. The model assumes that additions provide an increasing death benefit that approximates the path of stochastic inflation rates. The insurance can be sold for $160,000 where this amount represents the higher of a life settlement offer or the non-forfeiture cash surrender value contract option. The policy beneficiary is a healthy 68 year old female. Their portfolio must provide an inflation adjusted annual distribution of $40,000 (a common 4% distribution of initial value rule of thumb) to support the annual lifestyle target. To simplify the distribution assumption, projected annual consumption will remain the same regardless of the number of living spouses. The investors must make two decisions jointly given their current $1,000,000 portfolio value, their insurance policy provisions, their cash flow needs, and their health status:

- Should they hold or sell their insurance policy and
- How risky should their portfolio be?

Results of simulation analysis for each case are provided in the corresponding tables and figures. Each column of Table 1 represents a portfolio asset allocation option, with the first number representing the percentage allocated to equity and the number after the dash the percentage allocated to fixed income securities.[16] The portfolio allocations become more conservative when reading the table left to right. The top two rows of Table 1 and the light gray line of Fig. 1 provide the results for the simulation where the investors hold the insurance policy. The row titled "Median Value" presents the median (50th percentile) terminal portfolio value. The row titled "Bankruptcy Rate" is the percentage of trials in which at least one spouse is alive without resources. The bottom two rows and the darker line illustrate results if the investor sells the policy.

Because these hypothetical investors have no bequest motives, the primary statistic of interest is the probability of bankruptcy. Of the 12 options, if the investors hold their insurance policy and invest their portfolio assets in a well diversified 20% equity portfolio and 80% fixed income, they have the lowest bankruptcy rate. Although the combination of a portfolio allocated 20–80 and an insurance policy minimizes their bankruptcy rate, it may not be the best option for every similarly situated investor. The primary objective is to adopt a "Min-Min" strategy that reduces bankruptcy risk to its lowest value. The expected cost of adopting such a strategy is the difference between the median value of the 20–80 portfolio and the competing allocation choices. For example, selecting the 20–80 allocation instead of

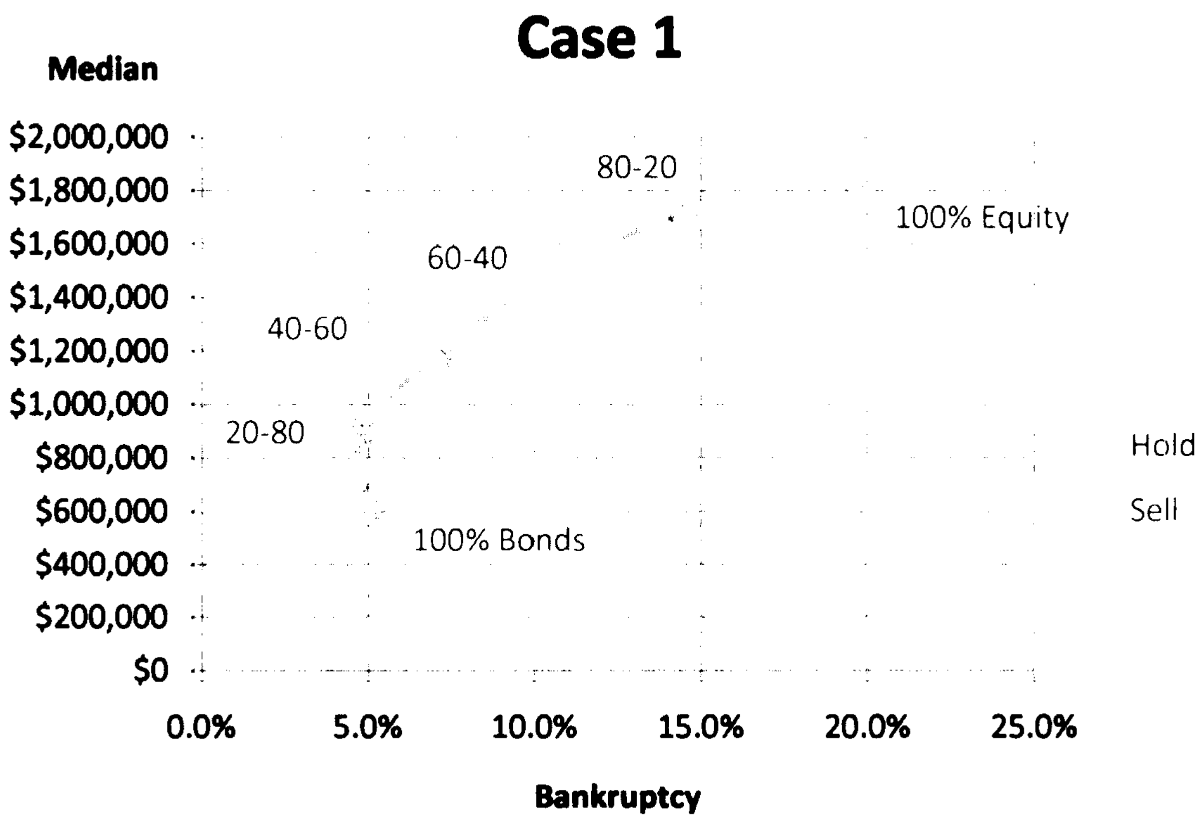

Fig. 1. Base Case Results

the 60–40 allocation reduces the risk of ruin from 10.2% to 4.1% at an expected cost of approximately $500,000. The "premium" for the risk reduction is, therefore, approximately 50% of current wealth.[17] Note that if the investors decided to allocate 100% to bonds, the decision to sell, rather than hold, the insurance policy results in lower bankruptcy rates. Although unlikely, the retirees may choose this option if they are highly sensitive to short-term changes in their portfolio's market value.

Although minimizing the chance of bankruptcy is the investors' greatest concern, they may also be interested in the expected value of their portfolio in so far as any terminal wealth value above zero has the potential of providing them a better standard of living. They would like to minimize bankruptcy risk, but would also like to spend most of their portfolio by the time both individuals are no longer alive.[18] The next case mirrors this objective.

3.2. Case 2

Suppose that after looking at the results from the Case 1 analysis, the investors conclude that although they are comfortable with the bankruptcy rates, the terminal portfolio values are high. Hypothetically, they would not mind increasing their chance of bankruptcy if it would mean increasing their living standard. They could, for example, decide to increase lifetime distributions from $40,000 to $50,000. Alternatively, they may consider a modified scenario where they enjoy a more expensive lifestyle for a few years and then gradually

Table 2 Front loaded results

		100% equity	80–20	60–40	40–60	20–80	100% bonds
Hold	Median value	$1,094,673	$1,040,790	$875,514	$721,839	$505,036	$296,652
	Bankruptcy rate	28.2%	25.5%	21.9%	20.0%	19.9%	30.1%
Sell	Median value	$1,178,710	$1,119,129	$883,744	$710,196	$457,331	$212,139
	Bankruptcy rate	28.1%	24.9%	21.5%	18.2%	17.5%	27.5%

become more conservative as they age. They are considering a "front loaded" distribution policy: $70,000 for the first 5 years, $60,000 for the next 5 years, $50,000 for years 11 through 15, then finally $40,000 for years 15 and beyond.[19] Results of this election appear in Table 2 and are illustrated in Fig. 2.

Compared to Case 1, it is not surprising that the overall bankruptcy rates increase and the overall terminal values decrease. Under a shortfall probability evaluation metric, the answer to the allocation question is still to invest in the 20–80 portfolio: it has the lowest bankruptcy rate, ignoring the decision on life insurance. In contrast to Case 1 however, the decision to sell the life insurance policy yields a lower bankruptcy rate than the decision to hold it. If the investors are still comfortable with the rate of bankruptcy, they may opt to sell their insurance policy and allocate the proceeds to a 20% equity–80% bond portfolio.

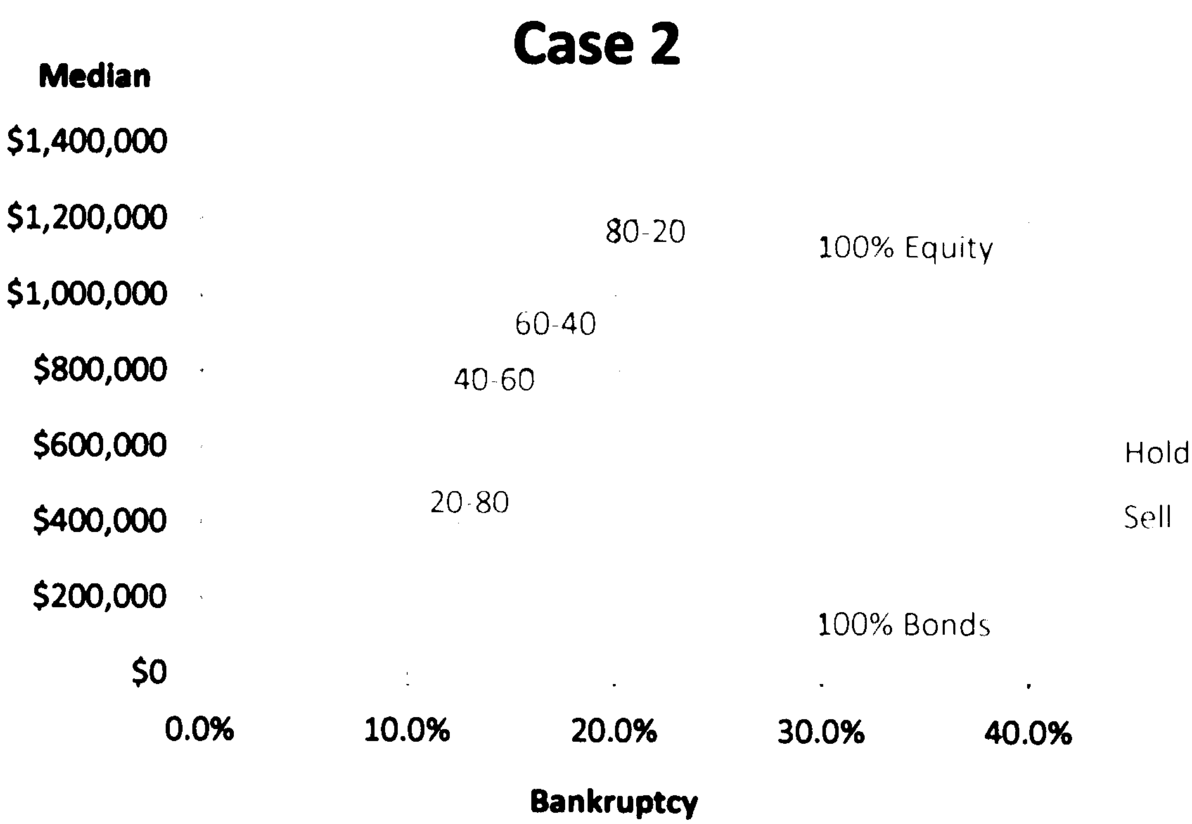

Fig. 2. Front Loaded Results

Table 3 Younger spouse results

		100% equity	80–20	60–40	40–60	20–80	100% bonds
Hold	Median value	$1,922,735	$1,781,769	$1,518,032	$1,245,520	$926,511	$585,291
	Bankruptcy rate	21.8%	17.3%	13.0%	9.3%	6.9%	9.6%
Sell	Median value	$1,861,714	$1,755,733	$1,505,592	$1,168,749	$858,727	$509,743
	Bankruptcy rate	23.7%	18.7%	14.7%	10.6%	8.3%	10.8%

3.3. Case 3

Referring back to the base case of a constant $40,000 annual portfolio distribution, suppose that the age of the female insurance beneficiary is 63 instead of 68. Case Three examines the affects of the beneficiary's age on the joint decisions (retain or sell/vector of asset allocation weights) under consideration. Results appear in Table 3 and in Fig. 3.

Not surprisingly, the value of the insurance policy increases as the age difference between the male insured and the female beneficiary increases. In Case 1, the difference is three years; here the difference is eight. Even in the 100% bond allocation, it is beneficial to hold the insurance policy. The results arise from the longer joint life span. The chance of a policy payout increases because the beneficiary has a lower probability of dying before the policy holder.

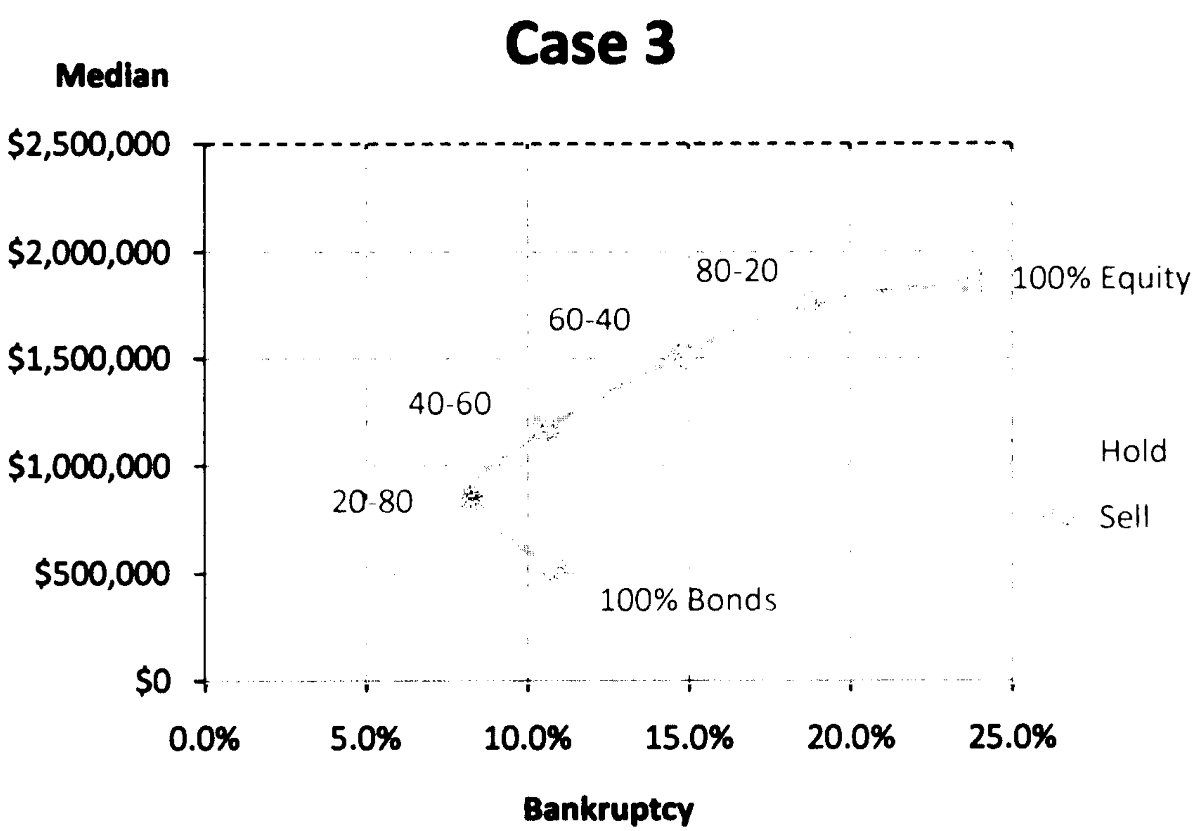

Fig. 3. Younger Spouse Results

Table 4 Older spouse results

		100% equity	80–20	60–40	40–60	20–80	100% bonds
Hold	Median value	$1,683,146	$1,592,824	$1,413,128	$1,231,163	$1,001,262	$771,600
	Bankruptcy rate	15.4%	12.2%	8.8%	5.3%	3.4%	3.8%
Sell	Median value	$1,805,835	$1,655,439	$1,431,141	$1,196,520	$945,907	$697,211
	Bankruptcy rate	16.1%	11.2%	8.5%	4.2%	2.4%	2.5%

3.4. Case 4

Now assume that the insurance beneficiary is 73, five years older than the base case of 68. This increases the probability of the beneficiary dying before the insured, so the value of holding the insurance policy should decrease. The results confirm this intuition. In fact, under a shortfall probability preferencing metric, the choice to sell is better than the choice to hold, regardless of the allocation.[20] Results appear in Table 4 and Fig. 4.

The age difference is a proxy for a more fundamental aspect affecting the demand to hold life insurance, namely the probability that the insured dies before the beneficiary, in which case the policy pays out and contributes to the sustainability of lifetime consumption. Otherwise, on the death of the surviving insured spouse, the proceeds add to terminal wealth but fail to enhance the utility of lifetime consumption.[21]

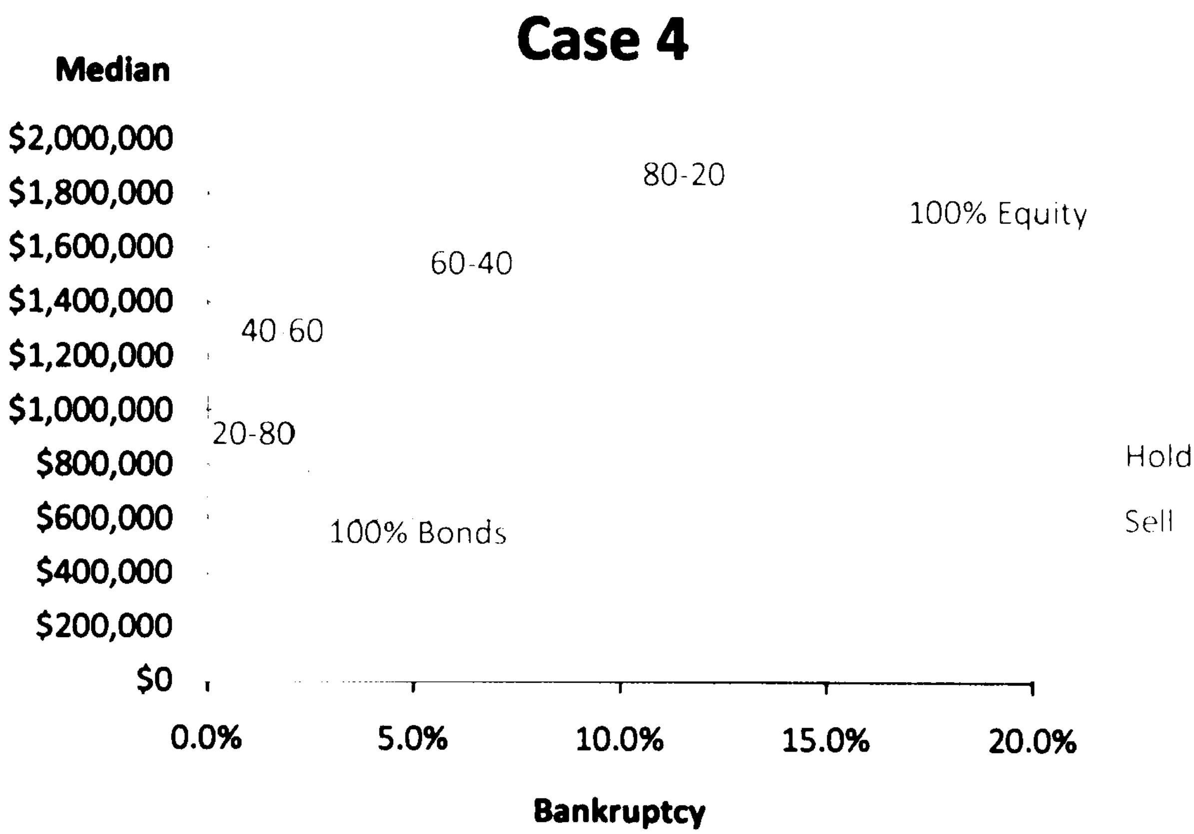

Fig. 4. Older Spouse Results

Table 5 Term insurance results

		100% equity	80–20	60–40	40–60	20–80	100% bonds
Hold	Median value	$1,351,959	$1,326,170	$1,155,177	$891,919	$641,633	$379,589
	Bankruptcy rate	22.3%	19.4%	15.4%	12.4%	10.6%	14.5%
Lapse	Median value	$1,348,962	$1,253,783	$1,061,470	$861,370	$626,544	$374,234
	Bankruptcy rate	23.7%	19.1%	15.3%	11.7%	10.0%	13.7%

3.5. Case 5

The universe of life insurance contracts is so vast that an examination of each type of product is beyond the scope of this paper. However, we examine of the potential effects of a different insurance policy type on the investor's financial objectives. The cases considered so far assume a whole life insurance policy that requires no premium payments and that pays an inflation adjusted benefit at the time of the insured's death. In this case, we assume that the insured investor from Case 1 has a term insurance policy set to expire in seven years, and that he will not renew it after it expires. Furthermore, the policy has no cash value and requires a monthly premium payment of $400 that is paid from the portfolio. Therefore, if the insured dies during the first seven years, the nominal benefit of $275,000 is paid out and premium payments from the portfolio stop. If the insured dies after seven years, premium payments have stopped, the policy expires, and no benefit is paid out at the time of the insured's death. The insurance option for the investor here is slightly different. Instead of surrendering the policy for a lump sum of money to be invested in the portfolio, the economic benefit of exercising the surrender/lapse option is to retain within the portfolio the flow of payments that would have gone to insurance premiums. The asset allocation questions, however, remain the same.

If the investors want the lowest bankruptcy rate, they would again select a 20% equity allocation weighting and let their insurance policy lapse (the series labeled "Sell" is replaced with "Lapse" in Table 5 and Fig. 5). This is in contrast to Case 1 where the best option was to hold onto the life insurance product. There is no doubt that different insurance products will affect the decision to hold or sell pre-existing insurance policies. Even within this simple case of term insurance, it is easy to see how different term structures or premium amounts can lead the investor to different decisions. Adding to the dimensionality of the problem is the fact that the optimal insurance decision is also a function of the preferred asset allocation. This is apparent for Case 5 as there is no systematic dominance of the retain or lapse option throughout the range of portfolio allocation choices.

3.6. Case 6

Now suppose that the investors in Case 5 are not only concerned with their chance of bankruptcy, but are also concerned with leaving a bequest of $300,000 to their children or charity. With the bequest objective in mind, the investors have added a new criterion to evaluate their investment options. Instead of selecting the portfolio and insurance option with the lowest bankruptcy rates, they choose the option that has the lowest probability of a

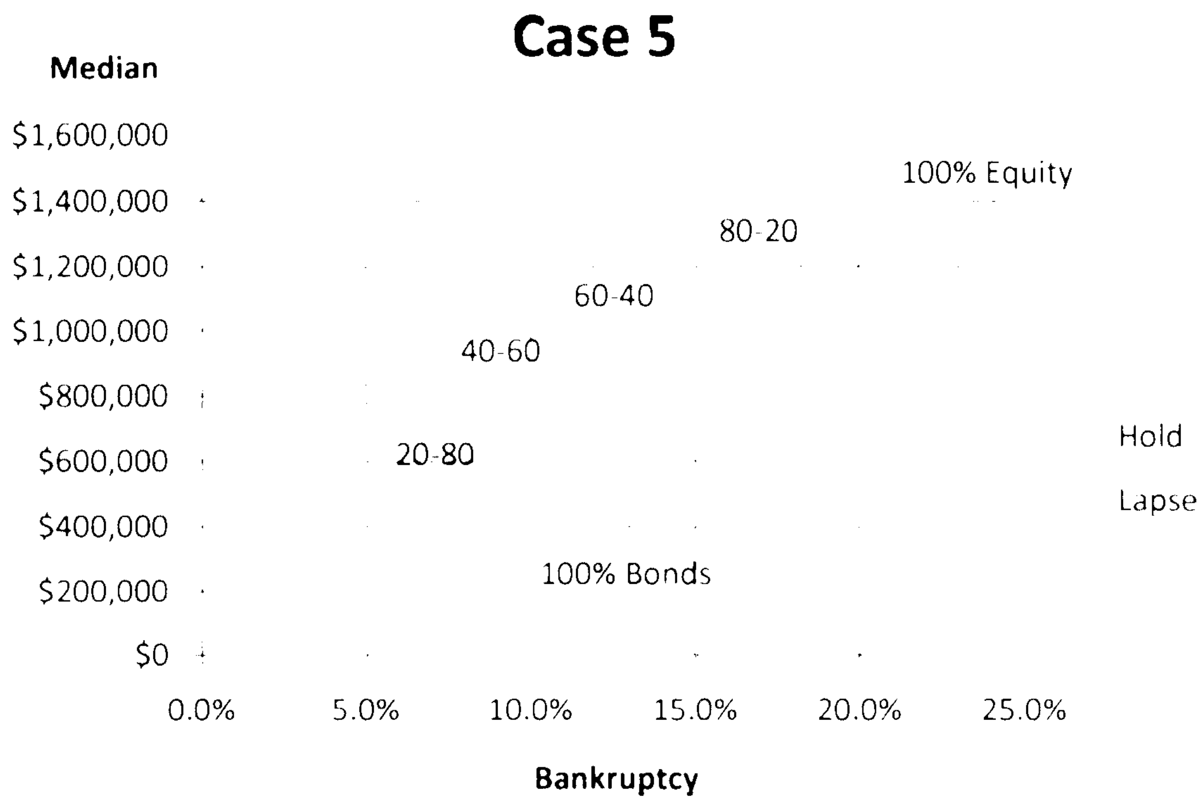

Fig. 5. Term Insurance Results

bequest value of less than $300,000. Using the results from the simulation in Case 5, we replace the bankruptcy shortfall criterion with the bequest shortfall criterion.

In Table 6, "Bankruptcy Rate" has been replaced with "Shortfall Rate" to indicate the likelihood that the terminal portfolio value, or bequest, is less than $300,000. In this scenario, the option with the lowest probability of bequeathing $300,000 is to hold onto the term insurance policy and hold a portfolio allocated 40% to equities and 60% to fixed income. When compared to Case 5, Fig. 6 illustrates the effects of adding a bequest motive to investor objectives. The option with the lowest bankruptcy rate was to let the term insurance policy lapse and select a 20–80 allocation. The option with the greatest likelihood of achieving a bequest equal to or greater than $300,000 is to retain the insurance policy and select a 40–60 allocation.

Table 6 Bequest motive results

		100% equity	80–20	60–40	40–60	20–80	100% bonds
Hold	Median value	$1,351,959	$1,326,170	$1,155,177	$891,919	$641,633	$379,589
	Shortfall rate	31.2%	28.2%	25.5%	23.2%	25.9%	41.7%
Lapse	Median value	$1,348,962	$1,253,783	$1,061,470	$861,370	$626,544	$374,234
	Shortfall rate	32.0%	27.6%	25.7%	23.9%	25.6%	41.9%

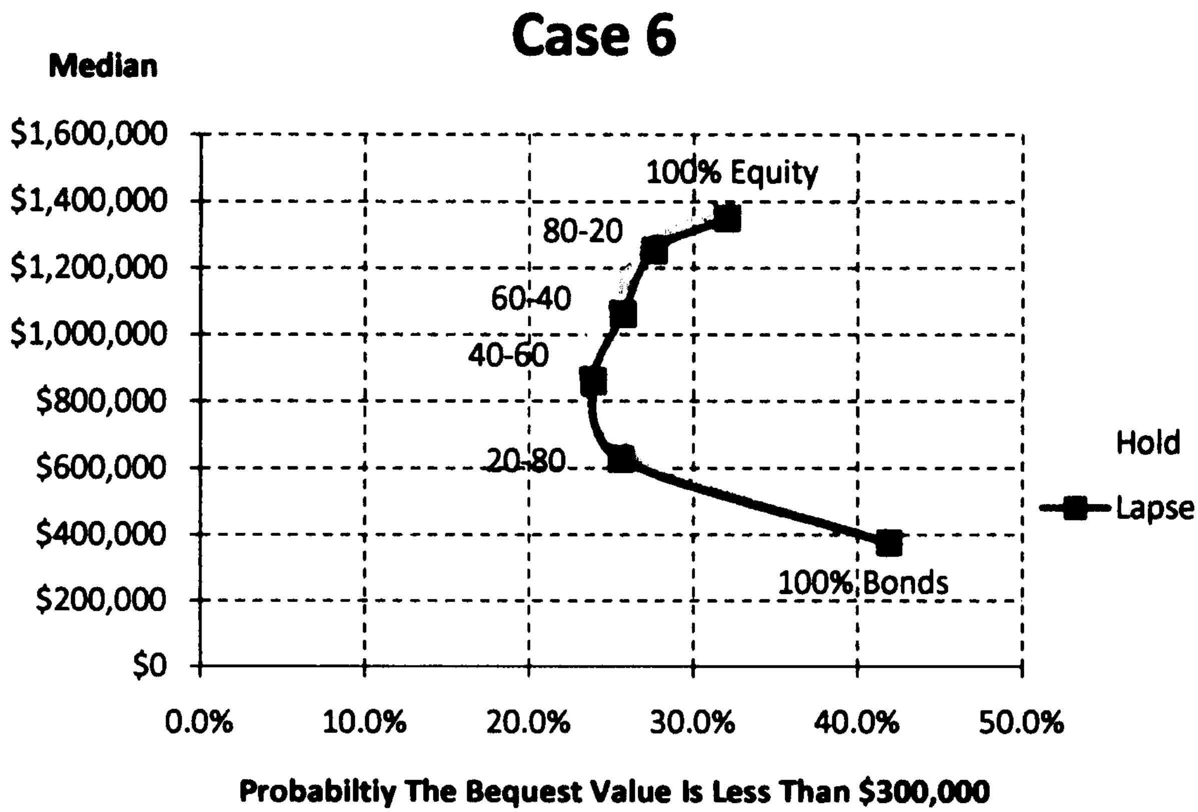

Fig. 6. Bequest Motive Results

4. Conclusion

We examined six different scenarios, under several shortfall preferencing criteria. Each case yields its own best investment/insurance options. Case 1 (the base case) introduces client circumstances. Case 2 considers a front-loaded distribution policy and how it affects the best investment/insurance options for the investors. Case 3 and Case 4 illustrate how option values are a function of age difference. Case 5 illustrates planning options for term insurance. Finally, Case 6 shows the potential impact of a bequest objective. Table 7 summarizes the preferred joint options for each fact pattern under a shortfall risk minimization preferencing criterion. Table 7 suggests the difficulty financial planners face in applying a general rule (or, a narrowly focused preferencing metric) regarding insurance advice for retirees.

Although theoretical models provide a framework for thinking about investment/insurance decisions, financial planners that blindly follow theoretical models can lead investors astray whenever the models do not account for unique risk and reward preferences. By contrast, financial planners could use output from a credible simulation based risk model tailored to client specific circumstance to jointly determine asset allocation and insurance demand. Is it better to retain or sell the life insurance policy? Will a sale enhance or detract from the ability to take more periodic income from the portfolio? Are the bequest motives reasonable in the light of lifetime consumption targets? The best investment/insurance options depend on several factors some of which (e.g., health status) are subject to change throughout the

Table 7 Preferred option summary

Case	Preferred insurance option	Preferred allocation option
1	Hold	20–80
2	Sell	20–80
3	Hold	20–80
4	Sell	20–80
5	Lapse	20–80
6	Hold	40–60

applicable planning horizon. Given the stochastic nature of the process, simulating shortfall probabilities allows financial planners, from time to time, to re-evaluate the decision options investors face as their financial asset portfolio evolves. Thus, initial decisions can be periodically revisited as part of a dynamic, integrated financial and actuarial asset management approach.

Notes

1. The recommendation assumes that the insurance premium loading is sensible, that the initial wealth endowment is modest, that there are beneficiary stakeholders holding a claim to future wages, and other reasonable conditions.
2. This study does not consider households with members over age 64.
3. A further distinction can be made between maximizing utility in low consumption states (state preference utility) and maximizing utility overall economic states–expected utility (Sharpe, 2007).
4. For example, Yaari (1965) assumes that markets are complete. This is equivalent to asserting that all risks to economic security are spanned by payoffs from tradable financial assets or by actuarial instruments. Many lifecycle models assume constant relative risk aversion. This is equivalent to asserting that the investor does not revise risk tolerance as total wealth increases or decreases. Many models assume equivalent risk aversion for utility of consumption as well as bequest functions. This is equivalent to asserting that the insurance demand function is homogeneous in wealth and across all states of nature—alive or dead.
5. HJB equations determine the best path for a possibly non-linear process. Dynamic programming sets the terminal objective and works recursively to find the optimal decision sequence most likely to attain the end goal. When the sequence incorporates stochastic elements, dynamic programming can take the form of a free-boundary problem. The best decision in a current state is independent of past decisions—the concern is the best path (gradient) given the current realizations of the relevant variables. See Ho, Milevsky, and Robinson (1997) for an example of optimizing asset allocation dynamically with both financial assets and actuarial instruments—that is, annuities.
6. For a summary of past literature, see Zietz (2003), and Liebenberg, Carson, and Drumm (2010). The latter study provides a comprehensive survey of results from

previous studies regarding the variables of interest in determining life insurance demand as well as their level of statistical significance and, when significant, their sign.

7. In a low interest rate environment the option to annuitize the cash value—at guaranteed annuity pricing factors offered in contracts issued in higher-rate periods—may be attractive for investors willing to trade liquidity for lifetime income enhanced by annuity mortality credits. This paper does not explore this dimension of insurance-related retirement income planning. However, a non-forfeiture provision offering constant dollar annuity benefits does not exist in the U.S. insurance market.
8. Brown (1999) tests four hypotheses purporting to explain the demand to hold life insurance during retirement: (1) to insure against the loss of Social Security benefits on the death of a spouse—insurance is purchased to re-allocate life-contingent income; (2) inertia—policies are a residue from attempts made earlier in life to insure human capital; (3) estate tax planning; and, (4) payment of funeral expenses. Brown concludes that evidence for hypothesis one is mixed, for hypothesis two the evidence is not convincing, that hypothesis three applies only to a small fraction of the elderly population, and that hypothesis four is the most likely explanation for the widespread ownership of insurance among the elderly.
9. The traditional utility maximization approach solves for the maximum expected utility of consumption plus bequests over the investor's lifetime. For a discussion regarding differences and similarities between utility and shortfall approaches, see Collins (2011). Milevsky (1998) argues that the probability of consumption shortfall is the relevant measure of risk for retirees lacking strong bequest motives. In a response to Milevsky–published as a reply to Milevsky and Robinson's article entitled "Self-Annuitization and Ruin in Retirement" (Milevsky and Robinson, 2000)–J.R. Brown criticizes use of a shortfall risk metric. From an economic theory perspective, the probability of ruin assumes, once the investor selects a distribution amount, he will myopically continue to spend this precise amount irrespective of an increase or decrease in the probability of ruin because of investment results, inflation paths, and so forth. According to Brown, the risk of ruin measures the probability of an event that no rational investor would permit. Albrecht, Maurer, and Ruckpaul (2001) contribute to the debate over the merits of shortfall probability as a risk metric by pointing out that although the shortfall risk approach seems to be a "preference free" metric, whenever a shortfall risk implies a target benchmark or dollar-value portfolio floor, it is not preference free. In this respect it is similar to the traditional utility-based risk evaluations which explicitly define the investor's risk-aversion function. Stutzer (2003) further develops the Albrecht, Maurer, and Ruckpaul observations and argues that shortfall probability is an allowable risk metric. Dus, Maurer, and Mitchell (2005) compare shortfall analysis to the traditional maximization of discounted expected utility of consumption and bequest. Additionally, they apply shortfall analysis to various distribution policy elections for portfolios with and without annuity contracts. Stout (2008) argues that the optimal portfolio is the minimum probability of ruin portfolio—a safety first preferencing criteria.
10. The empirical evidence is from Brown's research of the Asset and Health Dynamics of the Oldest Old (AHEAD) survey (Brown, 1999).

11. Appendix II provides details of the risk model used in this paper. Although most portfolio risk models assume a lognormal return distribution as a matter of convenience either for algebraic calculations or for programming Monte Carlo simulation algorithms, the econometric evidence clearly indicates, over relevant planning horizons, that such a distribution is ill fitted to financial asset returns. There are a number of credible alternatives to the two-parameter, symmetric family of Gaussian distributions. Alternative models include normal distribution mixtures, GARCH models, and Vector Autoregressive models. The risk model used in this paper uses a two-state, Markov Regime Switching model. A good discussion of the advantages of using this model in the context of an uncertain life span, risk of ruin, and financial asset evolutions can be found in Hardy (2003). For a broad overview of risk modeling in a more general context, the reader is directed to Alexander (Alexander, 2008). Ang and Bekaert (2004) provide a helpful discussion in the context of portfolio design and asset management.
12. However, rerunning the risk model periodically provides a practical way to achieve a dynamic investment program. Model inputs are, of course, revised to reflect the current state of nature including age, health, and so forth. Assuming that personal circumstances, goals, and investment performance are reviewed periodically, the client can determine when it is beneficial, if ever, to exercise the option for policy surrender. As stated earlier, this approach is akin to an optimal 'free-boundary' type problem where an irrevocable decision to lapse or surrender insurance is made at the time when the option is first "in the money"—that is, the optimal stopping time.
13. The risk model described in this paper can, however, readily accommodate these factors.
14. The risk model can accommodate differences in consumption and bequest utility by assigning an adjusted value for each dollar of terminal wealth and periodic consumption. For example, a couple may assign a $1 value for each dollar available for lifetime consumption and a $0.50 value for each dollar available for bequests. Adjusting the weighting of utility of consumption relative to the utility of bequests—by assigning different subjective discount factors—is common in closed-form models where the form of the functions is the same. In this article, when the investor has no bequest motives, they assign a value of $1 to periodic consumption and $0 to terminal wealth. When they do have bequest motives, we assume they value a dollar of periodic consumption and terminal wealth equally.
15. We ran an alternative set of simulations for Case 1 through Case 4 under the assumption that the death benefit is a nominal amount (not inflation adjusted by virtue of dividends funding paid-up additions). In these cases the inflation-adjusted benefit plays a key role in the demand to hold the insurance policy. Policies providing only nominal dollar payoffs are significantly less attractive, and, in most cases, the decision rule is to surrender the policy. Interested readers may obtain details at *www.schultzcollins.com/insurance_demand*. If the consumer seeks insurance during working years with the secondary goal of enhancing portfolio cash flow sustainability in retirement, it is helpful for the agent to explain the tradeoffs

between participating and non-participating whole life, or between fixed benefit and a fixed benefit + cash value insurance rider for universal life policies.

16. The portfolios are assumed to invest in diversified baskets of United States and foreign equities with a tilt towards small capitalization stocks and value stocks. Other equity positions include diversified US REITS (real estate investment trusts) and emerging market stocks. On the fixed income side, the portfolios invest in diversified baskets of United States and foreign bonds with short and intermediate maturities. Global fixed income investments are hedged to minimize translation risk between currencies. See Table 8 in Appendix I for allocation details.
17. Vanduffel, Dhaene, Goovaerts, and Kaas, (2003) provide a helpful review of the mathematics of determining the ability of initial wealth (the "provision") to fund a series of liability payments when wealth is invested in a stochastic return process. In the case of retirement income, the liability is deterministic but its present value is subject to changes in the discount rate. The threshold requirement for feasibility is that the stochastic present value of the provision is equal to or greater than the stochastic present value of the required payments.
18. Alternatively, terminal wealth can be thought of as a fund for payment of end-of-life medical and long-term care expenses. See, for example, Freedman (2008).
19. In terms of financial economics, this represents a smoothed consumption path despite the periodic change in the distribution target's dollar value. A smoothed consumption path is one in which the marginal utility of consumption remains equal over all time periods.
20. We reiterate that unassailable, bright line financial decision rules are rare, and that one should not rely exclusively on any single risk model or preferencing criterion. If termination of insurance coverage is an irreversible decision, it may generate substantial regret in the event of a near-term change of health. This article does not address the real-option value to postpone surrender in the face of uncertain future mortality prospects.
21. This conclusion differs from Lin and Grace (2005), who "... see a negative relationship between age difference between spouses and life insurance demand. This suggests that when the age difference between spouses increases, the household tends to use other methods instead of life insurance to manage their risks ..." However, the data set (Survey of Consumer Finances) underlying their study does not include households in which the primary wage earner is retired.

Appendix I: Portfolio allocations and data sources

Asset allocations are presented in the table below. The investments in each asset class are diversified baskets of securities, and the portfolios are rebalanced to the target allocations every 12 months. The input data consists of the time series of monthly returns described in Table 9. If the benchmark index did not exist for the entire period of 1973 through 2009, the covariance matrix is calculated based on the partial data with shrinkage of off-diagonal elements if necessary to insure matrix invertibility.

Table 8 Asset allocation

Portfolio	100% equity	80–20	60–40	40–60	20–80	100% bonds
Large blend	20%	15%	12%	8%	4%	0%
Large value	20%	15%	12%	8%	4%	0%
Small blend	6%	5%	3%	2%	1%	0%
Small value	8%	6%	5%	3%	2%	0%
Real estate	6%	5%	4%	3%	1%	0%
Int'l large blend	8%	7%	5%	3%	1%	0%
Int'l large value	8%	7%	5%	4%	2%	0%
Int'l small blend	6%	5%	3%	2%	1%	0%
Int'l small value	8%	7%	5%	3%	2%	0%
Emerging markets	10%	8%	6%	4%	2%	0%
Total equity	100%	80%	60%	40%	20%	0%
Short term bonds	0%	10%	20%	30%	40%	50%
Intermediate term bonds	0%	5%	10%	15%	20%	25%
Global bonds	0%	5%	10%	15%	20%	25%
Total fixed income	0%	20%	40%	60%	80%	100%

Table 9 Benchmark indexes

Asset Class	Proxy Benchmark Index	Start Date	End Date
Large Blend	S&P 500	1/73	12/09
Large Value	Fama-French Large Value	1/73	12/09
Small Blend	CRSP NY/AM/NM 6–8	1/73	12/09
Small Value	Fama-French Small Value	1/73	12/09
Real Estate	FTSE NAREIT Equity REITS	1/73	12/09
Int'l Large Blend	MSCI EAFE	1/73	12/09
Int'l Large Value	MSCI EAFE Value	1/75	12/09
Int'l Small Blend	DFA Int'l Small Cap	1/73	12/09
Int'l Small Value	S&P EPAC Small Value	1/89	12/09
Emerging Markets	S&P/IFCI Emerging Composite	1/89	12/09
Short Term Bonds	US 1 Yr Const Mat	1/73	12/09
Intermediate Term Bonds	BarCap US Gov't/Credit Intermediate	1/73	12/09
Global Bonds	Citi WGBI	1/85	12/09

Appendix II: Risk model

The risk model simulated in this paper incorporates several 'moving parts' which are best characterized as random variables. These include:

The Planning Horizon—the applicable planning horizon can either be fixed (e.g., a university endowment will need to finance a new building in exactly seven years) or variable (an indeterminate length). When the planning horizon is measured by life span, the application simulates sample lifetimes using a Society of Actuaries annuity table based on "white collar" retirees from Defined Benefit Pension Plans. This table is conservative (i.e., exhibits a force of mortality lower than general population tables used by Social Security); and, therefore suggests a higher likelihood of a long life. Unless otherwise indicated, the simulation reflects longevity expectations that assume good health.

The Economy—the risk model divides economies into two regimes: A Bear Market regime (defined as a 20% or greater peak to trough price decline for the Capital Appreciation S&P 500 stock index); and, a Bull Market regime. Using historical data from January 1973 through the end of 2009, the historical lengths of bull and bear markets are determined. The simulation uses a Markov-switching regime model (with a random selection for the initial economic regime) to determine the sequence of market conditions that the investor will face. The probability (p) that the initial economy is in a bull market regime or a bear market regime (1 – p) is based on historical frequencies. For all future periods, the model determines the probability of remaining in a bear market given that the last month was a bear; or, of switching from a bear to a bull market given the total duration of the bear market to date. Similar calculations are made for the probability of remaining in or leaving a bull market regime.

Inflation—the model proxies inflation by the Consumer Price Index (CPI). Changes in the inflation rate are primary determinates of the likelihood that periodic investment returns are either positive or negative. The econometric model specifies the inflation generating process as a serially correlated random variable with a "smoothed" reversionary factor. Specifically, the algorithm regresses the average value of the previous 12 month's inflation against the average value of the next 12 months.

When the application has not yet produced 12 monthly simulated values, the algorithm recursively calculates the average of the preceding 12 months by using the initial value to replace any missing terms. Therefore, the value for average prior 12 month inflation in the second month is 11/12 * the initial value + 1/12 * the value in the first month. The persistence coefficient determines the speed of CPI mean reversion. The coefficient's value is calculated via a regression of the rolling 12-month CPI against the rolling forward 12-month CPI. Thus, the model assumes that inflation is an Ornstein-Uhlenbeck process that includes a term for autocorrelation as well as a term for mean-reversion.

Investment Returns—the model generates investment returns utilizing common matrix algebra techniques. Utilizing separate variance/covariance matrices from historical bull and bear market regimes, the application executes a Cholesky decomposition. It may also adjust dependence relationships by shrinking extreme off-diagonal elements to assure matrix invertibility. The Cholesky matrix algebra operation "divides" a variance/covariance matrix into upper and lower triangle matrices which make them equivalent to the square root of a variance matrix. If there exists a lower triangle matrix **C** such that the historical matrix $\mathbf{V} = \mathbf{CC^t}$, then **C** is a Cholesky matrix. The application simulates combinations of return series where each historical return series ($\vec{x}$) is transformed (by subtracting the mean and dividing by the standard deviation) into an independent standard normal variable ($\vec{z}$). The computer's random number generation function can readily simulate future evolutions for each independent return vector by drawing values for uncorrelated zero-mean variables. Pre-multiplying the vectors of simulated independent returns by **C** $C(\vec{z})$ restores their equivalency to each original return series $(\vec{x}) = C(\vec{z})$. The variance of the independent vectors is easily determined; and, pre and post multiplication of the variance of ($\vec{z}$) by the appropriate lower triangle decomposition matrix **C** and its inverse restores the correlation structure by generating the required variance/covariance matrix. $[\mathbf{V} = \mathbf{C}V(\vec{z})\mathbf{C}^t]$

Financial asset return series usually cannot be characterized as normal (bell-curve) distributions. Portfolio investment risk defined by the first two moments of multivariate symmetric distributions (i.e., Gaussian, Student's *t*, etc..) is often misleading. Monte Carlo simulations based on a normal distribution cannot realistically capture the frequency and magnitude of tail-risk events (leptokurtosis). To avoid this deficiency, the application utilizes two normal distributions (bull and bear) with separately calculated means and variances for each regime. The distributions, according to the Markov transition probabilities described above, enable the model to capture the risk of outlier results that mirror real world frequencies rather than risks that are largely pre-determined by theoretical parameter inputs.

Additionally, a regime switching approach captures dynamic correlation and time-varying risk premia over different market conditions. Thus, instead of using average unconditional correlation values determined by the historical data, the application applies the historical correlation values conditioned on bull and bear market data. For example, over the entire sample period, an asset class may exhibit a mean of 10% and a standard deviation of 20%. However, during bull markets, the parameter values may be +18% mean and 15% standard deviation; while, in bear markets, the parameter values may be −23% and 25%, respectively. Thus, simply using the unconditional mean, standard deviation and correlation values for the aggregate historical period cannot capture realistic asset price behavior.

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