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Wealth and Risk from Leveraged Stock Portfolios

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Abstract

A modest amount of leverage enhances the performance of stock portfolios in the long run. However, higher amounts of leverage produce dramatic declines in long-run wealth. Using probability distributions constructed from value-weighted stock index returns and a borrowing rate two percentage points higher than Treasury bills, the maximum median ending wealth is achieved with an asset allocation of 170% stock. We use Value at Risk (VaR) to measure downside risk over a range of asset allocations and holding periods.

JEL classification: G10; G11

Keywords: Asset allocation; Leverage; Value at Risk

1. Introduction

An investor is said to buy stock "on margin" when a portion of the purchase price is financed by a loan from his or her brokerage firm. This leveraged strategy magnifies investment profits when stock prices rise, but also deepens losses when the market declines. As a recent example, many companies in the technology sector experienced spectacular increases in their share prices during late 1999 and early 2000. Some investors attempted to exploit these gains with margin purchases. But when the market suddenly turned downward in April 2000, these same investors were facing huge losses. Further losses were experienced in technology downturns during 2001.

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These recent declines draw attention to the potential risk in leveraged portfolios. But are the losses so large that they wipe out the bull market gains? We examine this question from a long-run perspective, by constructing probability distributions of returns on leveraged portfolios over various holding periods. Our findings show that a modest amount of leverage enhances the growth of wealth in the long run.

An interesting result from our research is that high amounts of leverage produce dramatic declines in long-run wealth. This result is consistent with an investment rule developed in the 1950s. According to the maximum expected log (MEL) rule, maximum growth of wealth over the indefinitely long run is achieved by investing each period so as to maximize the expected value of the logarithm of (1 + single-period return). In our empirical results the highest median ending wealth is achieved at an amount of leverage equal to the MEL optimum.

Section 2 reviews previous research on leveraged portfolios, long-run returns, and the MEL rule. Section 3 describes the data used in the current study. Section 4 outlines two empirical techniques that can be applied to this research area. Our findings are presented in Section 5. Section 6 describes some implications for investors, and Section 7 concludes the paper.

2. Literature review

Little attention has been given in the academic literature to investment strategies using leveraged portfolios. Among the few published studies, Grauer and Hakansson (1985, 1986) apply multi-period portfolio theory to the construction and rebalancing of portfolios. The Ibbotson SBBI monthly return series for common stocks, long-term corporate bonds, long-term government bonds, and Treasury bills are used in the 1985 study, with small stocks also included in the 1986 study. Investors are allowed to borrow at the call money rate plus one percent. However, Grauer and Hakansson do not explore the impact of using leverage every period.

Ferguson (1994) demonstrates that long-run returns decline as leverage increases. A key factor is that although the underlying securities have limited liability, leverage magnifies their volatility and leads to a positive probability of bankruptcy. He also shows that long-term returns can be disappointing even when bankruptcy is not possible. Ferguson's diagrams are based on binomial examples with only two equally likely stock returns. It is, therefore, difficult to apply his results to real-world investment portfolios.

One method of introducing realistic portfolio characteristics is to use observed capital market history. Although this has not been done for leveraged portfolios, we can apply the resampling technique used by Butler and Domian (1991) for unlevered portfolios. This technique is described below in Section 4. The same technique is used by Butler and Domian (1993) for retirement planning, and by Hickman et al. (2001) for life-cycle investing. Thaler and Williamson (1994) note that the results of Butler and Domian (1991) have implications for university endowment funds.

Some of the earliest research on long-run investing includes Kelly (1956), Latané (1957, 1959), and Markowitz (1959). These studies are all "concerned with a hypothetical investor who neither consumes nor deposits new cash into his portfolio, but reinvests his portfolio each period to achieve maximum growth of wealth over the indefinitely long run" (Markowitz, 1976, p. 1273). According to these authors, such an individual should invest each period to maximize $E \log(1 + r)$, where E is the expectation operator and r is the single-period return. Markowitz (1976) shows how this MEL rule can be derived by using the weak and strong laws of large numbers.

Samuelson (1963, 1969) disputes the MEL solution, arguing that expected utility maximization is the appropriate goal. According to Samuelson, each period the investor should maximize single-period expected utility, and in the long run this provides a higher expected utility than the MEL strategy. However, Markowitz (1976) refutes Samuelson's objections.

3. Data

Our data include monthly returns on a common stock index, Treasury bills, and inflation. Each series contains 888 values over the period January 1926 through December 1999. The stock returns are from the CRSP value-weighted index, including dividends and other distributions. Ibbotson SBBI series are used for Treasury bills and inflation.

We focus on real returns because inflation-adjusted return distributions are more stable over time than their nominal counterparts. The real stock returns have a mean of 0.007599 and a standard deviation of 0.055172, whereas the real T-bill returns have a 0.000586 mean and 0.005362 standard deviation. These correspond to real annualized percentage rates of return of 9.51% and 0.71% on stocks and T-bills, respectively. The difference between these annual returns, 8.80%, is a very high equity risk premium, in part due to superior stock appreciation during the 1990s.

4. Methodology

Our goal is to examine and interpret probability distributions of ending wealth from leveraged portfolios over various holding periods. Because there is a one-to-one correspondence between ending wealth and annualized rate of return, the entire wealth distribution can be obtained from the probability distribution of returns. But it is difficult to empirically assess long holding period returns, regardless of leverage, because there are so few observations. For example, returns could be computed from some strategy over January 1971 through December 1995, and again for January 1972 through December 1996. However, these would not be independent observations because of the substantial overlap between the two periods. Even with 100 years of data, there are just four independent 25-year periods.

Despite this difficulty, there are two techniques that can be used to determine long-run probability distributions from the available data. First, simulations can be performed, with parameters calibrated to the data. The simulation approach is very flexible since one can examine the effect of changing parameter values. For example, Leibowitz and Langetieg (1989) develop a model assuming a 4% common stock risk premium over Treasury Bonds. They conclude that there is a 24% chance that common stocks will underperform Treasury

Bonds over a 20-year investment horizon. If these simulations are repeated with a higher risk premium, a lower probability of underperformance is obtained.

A second technique is the resampling approach presented by Butler and Domian (1991, 1993). The resampling technique is used in the current paper; simulations will be explored in later research. The procedure is implemented as follows:

1. Randomly select one of the 888 months. Record the observed real stock and T-bill returns for this month.
2. Compute the portfolio returns for asset allocations ranging from 0% stock up to 300% stock. For stock proportions below 100%, the remainder of the allocation is in Treasury bills. Stock proportions exceeding 100% are achieved with borrowed funds at rates described below.
3. Repeat the previous steps $n \times 12$ times with replacement and compound the monthly returns to construct one representative n-year ($n = 1, 5, 10, 20, 30, 40$) holding-period return for each asset allocation.
4. Perform the entire procedure 1,000,000 times to generate n-year holding-period return distributions from the observed history of real monthly returns.

The resulting distributions are presented and interpreted in the next section.

Two different borrowing rates are used for the second step of the above procedure. We initially assume equal lending and borrowing rates using the real Treasury bill returns. Later in the paper we explore the impact of borrowing at two percentage points above the T-bill rate. This higher rate is a reasonable approximation to actual margin loan rates.

Another practical constraint is the 50% initial margin requirement faced by U.S. investors. The maximum stock proportion consistent with this requirement is 200%. Nevertheless, it is useful to study higher stock proportions to determine the extent to which the margin requirement is a binding constraint. Margin requirements are lower in some countries, e.g., Canada, which requires 30% for large stocks. Also, investors could use derivatives to increase their effective leverage.

5. Results

We begin by considering distributions in the form of annualized real returns over the various holding periods. For stock proportions above 100%, borrowing is assumed to be at the historical real T-bill rate. The 5-year and 20-year distributions are presented as histograms in Figs. 1 and 2. Histograms for other holding periods are available from the authors upon request.

Each histogram displays returns in one-percentage-point intervals. For example, the top left panel in Fig. 1 shows returns from a 50% stock, 50% T-bill allocation over a 5-year holding period. The tallest bar in this diagram is centered at a 4% real return with a height of 0.08959. This indicates that among the 1,000,000 5-year returns for the 50–50 asset allocation, 89,590 were between 3.5% and 4.5%.

Two features are readily apparent in these histograms. First, for a given holding period, the distribution of returns spreads out as the stock proportion is increased. The spreads are

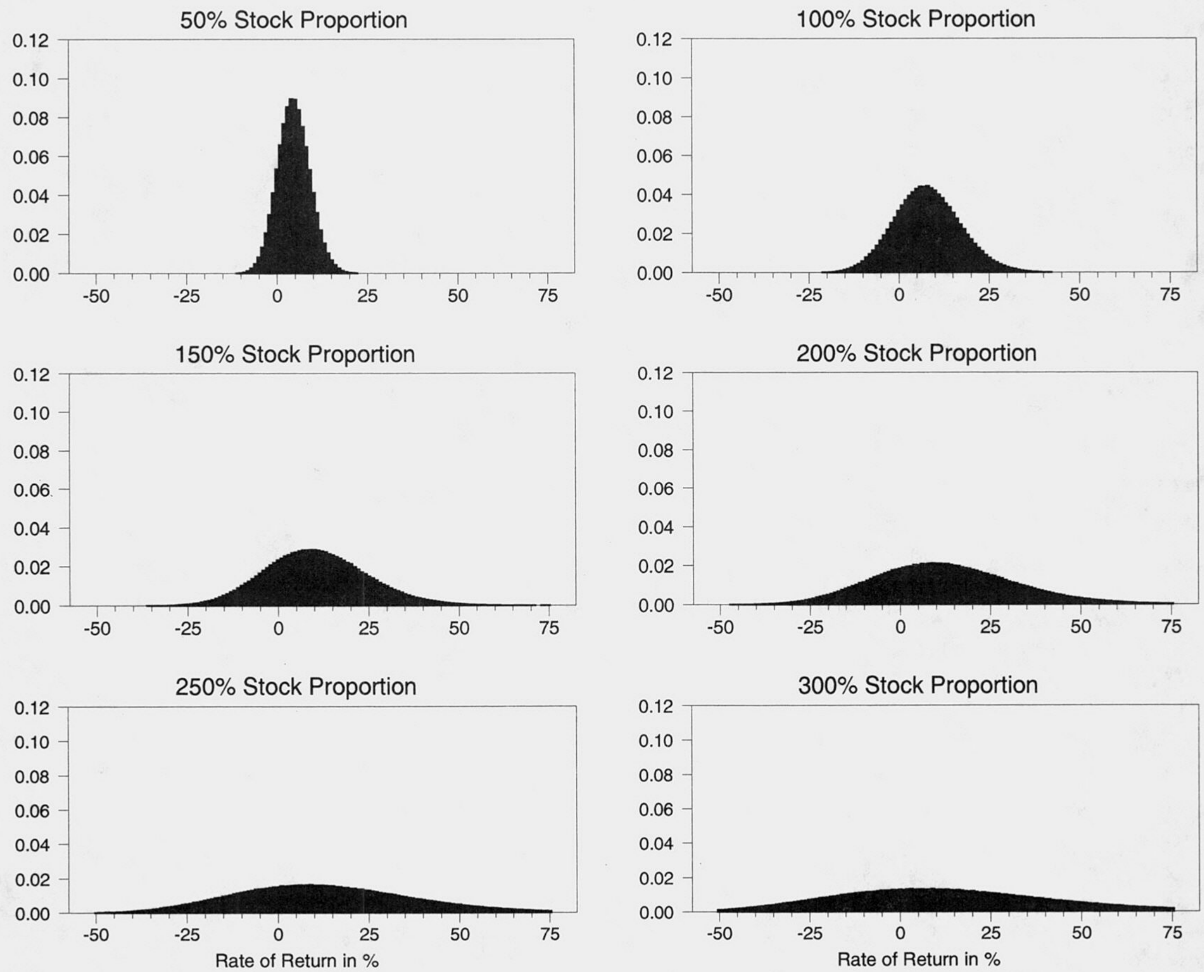

Fig. 1. Histograms of annualized real returns, 5-year holding period. The probability distributions are obtained by resampling from real monthly stock and Treasury bill returns over 1926 to 1999. Compounding 60 monthly returns produces a 5-year holding period return, and the procedure is repeated 1,000,000 times to generate the distributions.

so wide for the highly leveraged 5-year distributions that the tails extend beyond the range depicted in the diagrams. With a 300% stock proportion, almost 1% of the 5-year outcomes have annual returns lower than −50%, and ~4% have annual returns greater than 75%.

The second feature is that for a given asset allocation, the distribution becomes more concentrated as the holding period is increased. Consider allocations with a 200% stock proportion. The return distribution has a standard deviation of 0.195 over the 5-year holding period, but just 0.097 for the 20-year holding period. Similar reductions in standard deviation are apparent for the other asset allocations.

This second feature is sometimes cited as evidence of a "time diversification" effect for 100% stock portfolios. Because of the more concentrated return distributions, time diversification proponents argue that stock portfolios are less risky over long holding periods. However, as McEnally (1985) observes, even though return distributions get narrower, the distributions of ending wealth become more spread out. Investors should indeed view wealth,

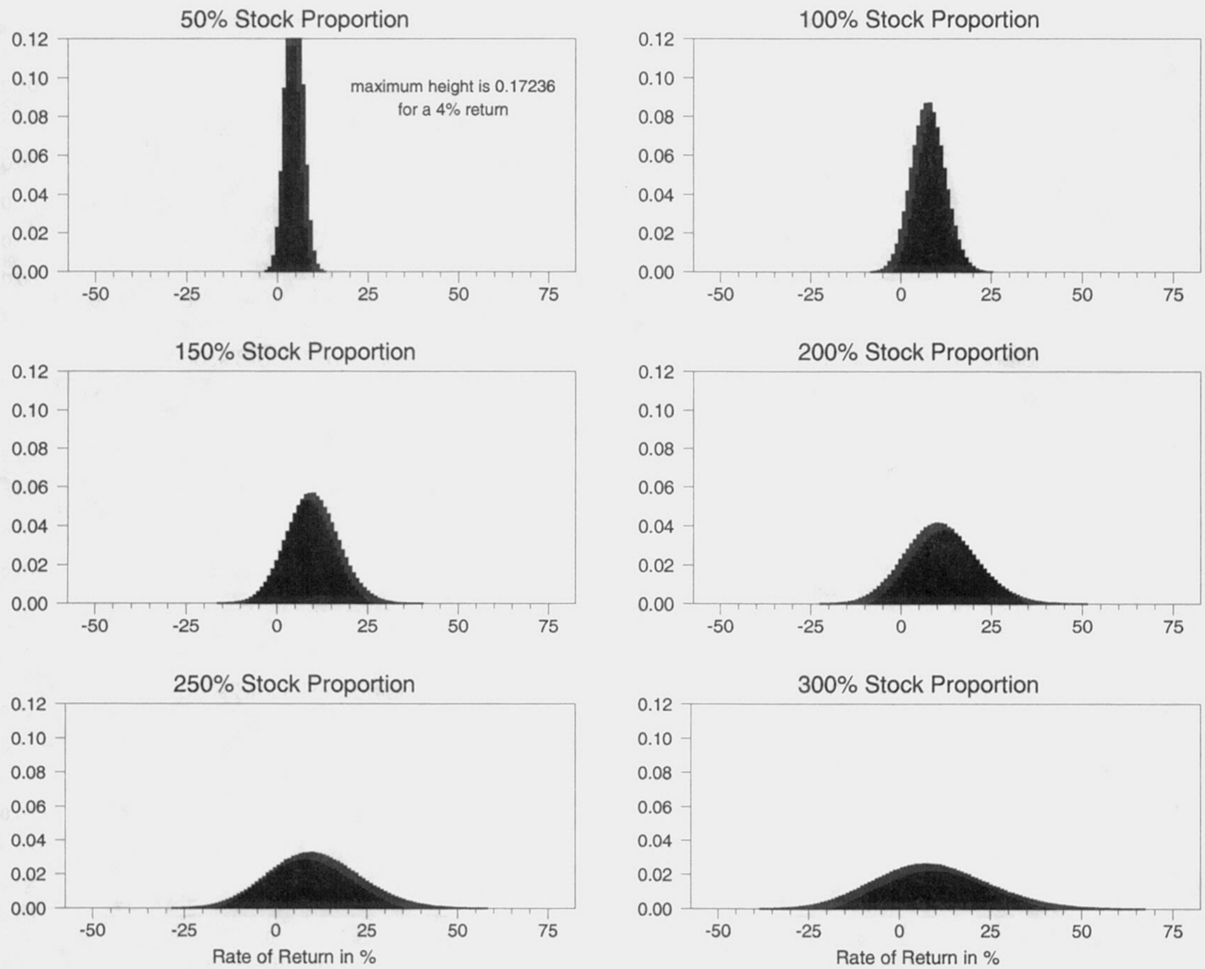

Fig. 2. Histograms of annualized real returns, 20-year holding period. The probability distributions are obtained by resampling from real monthly stock and Treasury bill returns over 1926 to 1999. Compounding 240 monthly returns produces a 20-year holding period return, and the procedure is repeated 1,000,000 times to generate the distributions.

rather than return, as their ultimate goal. Thus we now turn to examining ending wealth from leveraged portfolios.

The ending wealth distributions can be easily calculated from the percentage returns depicted in the previous histograms. Along with the 5-year and 20-year returns depicted in Figs. 1 and 2, we also use returns over 1, 10, 30, and 40 years. These wealth distributions have a large skewness over the longer holding periods. We initially focus on the medians of the wealth distributions, and later consider the left tails.

Fig. 3 displays the median ending wealth per dollar of initial investment, over holding periods ranging from 1 to 40 years. Because these are calculated from the real return distributions, the amounts are in real dollars. While the previous figures included results for just six asset allocations, we now use 31 allocations in 10-percentage-point increments: 0% stock, 10% stock, 20% stock, . . . , 300% stock.

For the 1-year holding period, the graph in the top left panel of Fig. 3 is almost flat.

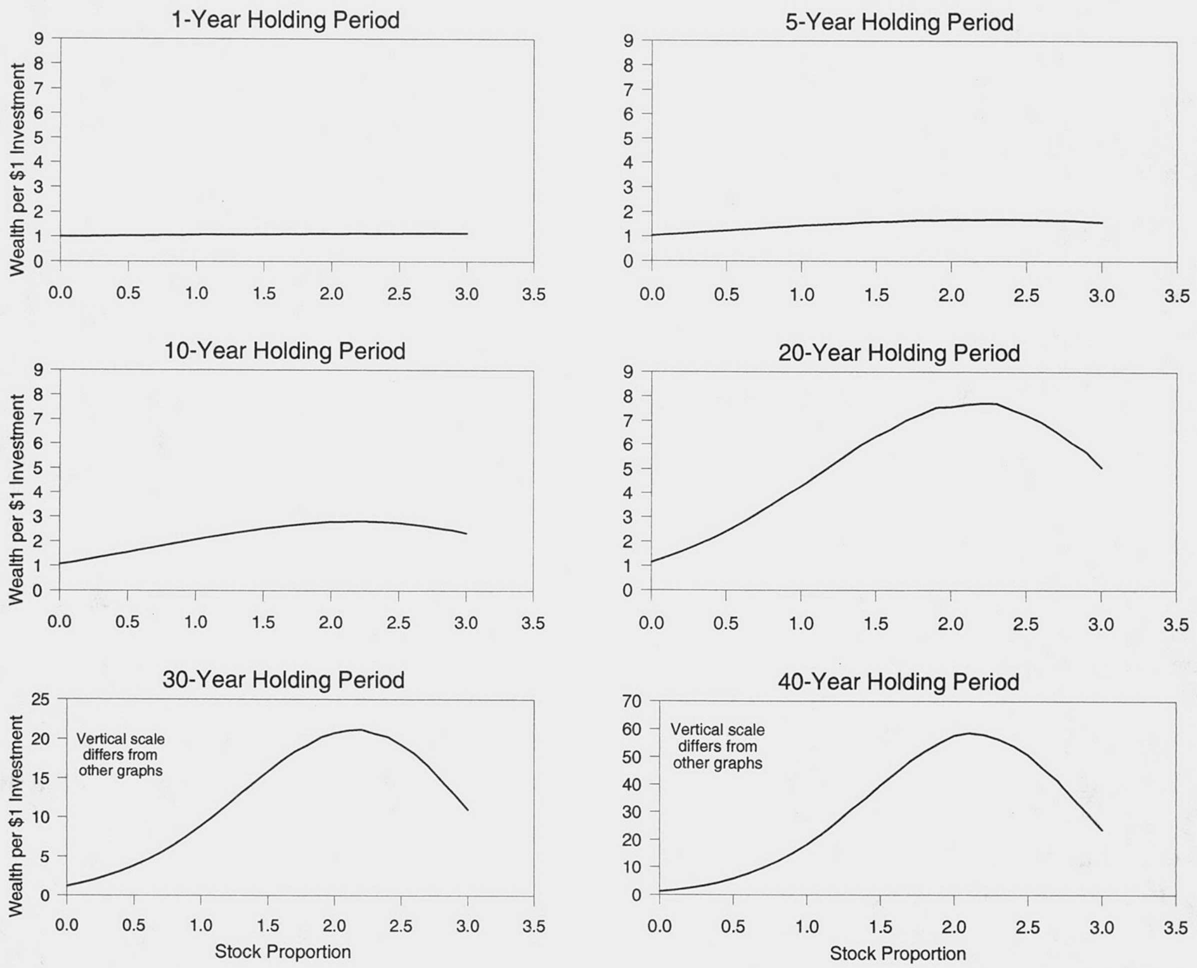

Fig. 3. Median ending real wealth per dollar of initial investment; borrowing rate equals the T-bill rate. Ending wealth distributions can be computed from the return distributions for any asset allocation and any holding period. These figures plot the medians of ending wealth.

Rounded to the nearest penny, a dollar invested in an all-T-bill portfolio (i.e., the 0.0 stock proportion) produces median ending wealth of $1.01. This rises to $1.08 with 100% stock, $1.13 with 200% stock, and $1.15 with 300% stock. The 5-year holding period graph appears to be nearly linear with a slightly greater upward slope, reaching its $1.71 maximum at a 230% stock proportion.

With holding periods of 10 years or more, there are pronounced declines in ending wealth at higher amounts of leverage. In each of these longer holding periods the maximum median ending wealth is achieved at ~220% stock. The longer the holding period, the greater is the decline in wealth per dollar as leverage approaches 300%. The median ending wealth for the 10-year holding period is $2.83 for a 220% stock proportion and $2.36 for the 300% stock proportion. This represents a 16.6% decline in ending wealth due to the higher leverage. The same leverage increase produces a 34.4% wealth decline over 20 years, 48.0% over 30 years, and a 59.1% decline for the 40-year holding period.

Given the huge 8.80% equity risk premium in the historical data, it may seem surprising

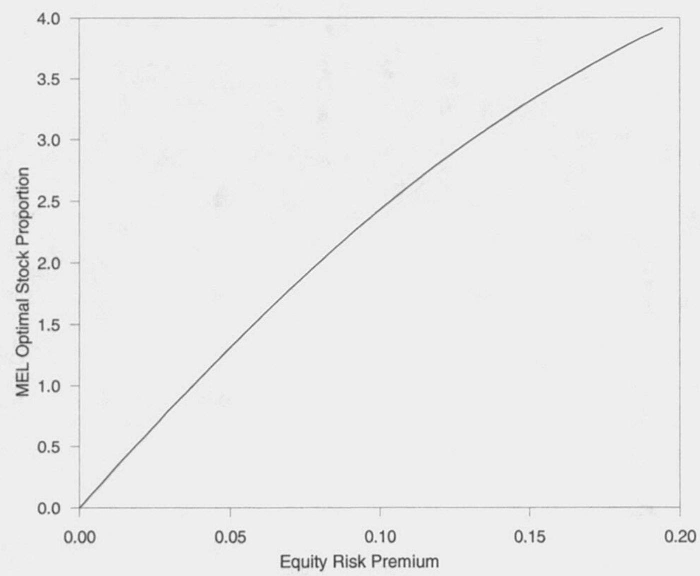

Fig. 4. Relation between equity risk premium and MEL optimal stock proportion. The MEL rule is presented by Markowitz (1976) and earlier authors. This figure plots the Young and Trent (1969) MEL approximation for a range of equity risk premia.

that a 300% stock allocation would be less profitable than 220%. Though the wealth declines shown in Fig. 3 may be perplexing at first glance, they are due to the well-known result that multi-period compounded returns decline as volatility increases. Higher leverage magnifies the volatility in the underlying stock returns. Our wealth declines follow directly from the return declines described by Ferguson (1994).

Ferguson also notes the impact of potential bankruptcy. This is not a problem for the leverage amounts considered in our study. The worst month in our data is September 1931 with a 29% nominal decline in the stock market, a tiny 0.03% nominal monthly T-bill return, and a monthly inflation rate of −0.44% (i.e., deflation). An investor with at least a 350% stock proportion at the beginning of this month would have been bankrupt at the end of the month. However, the investor would still be solvent with 300% stock, the maximum leverage we consider.

The MEL rule, as presented by Markowitz (1976) and earlier authors, was briefly outlined in Section 2 above. We now examine what the MEL rule recommends for a range of equity risk premia. For simplicity we assume a zero real monthly borrowing/lending rate. Note that this zero rate is just slightly below the historical real T-bill average. Recall that the MEL rule is to maximize $E \log(1 + r)$. According to Young and Trent (1969), $E \log(1 + r)$ can be approximated by $\log(1 + E(r)) - \frac{1}{2}(Var(r)/(1 + E(r))^2$ for a wide class of distributions. We use this approximation with the variance determined from the 1926–1999 real monthly stock series, and with monthly stock returns ranging from 0 to 0.01 in increments of 0.0001. For each assumed value of the monthly stock return, the optimal stock proportion is chosen to maximize the Young and Trent MEL approximation. Results are plotted in Fig. 4, with the stock returns annualized for ease of interpretation.

Fig. 4 shows that if the annual equity risk premium is 3.75%, then the MEL-optimal asset allocation is exactly 100% stock. In order for the MEL rule to select a 200% stock allocation,

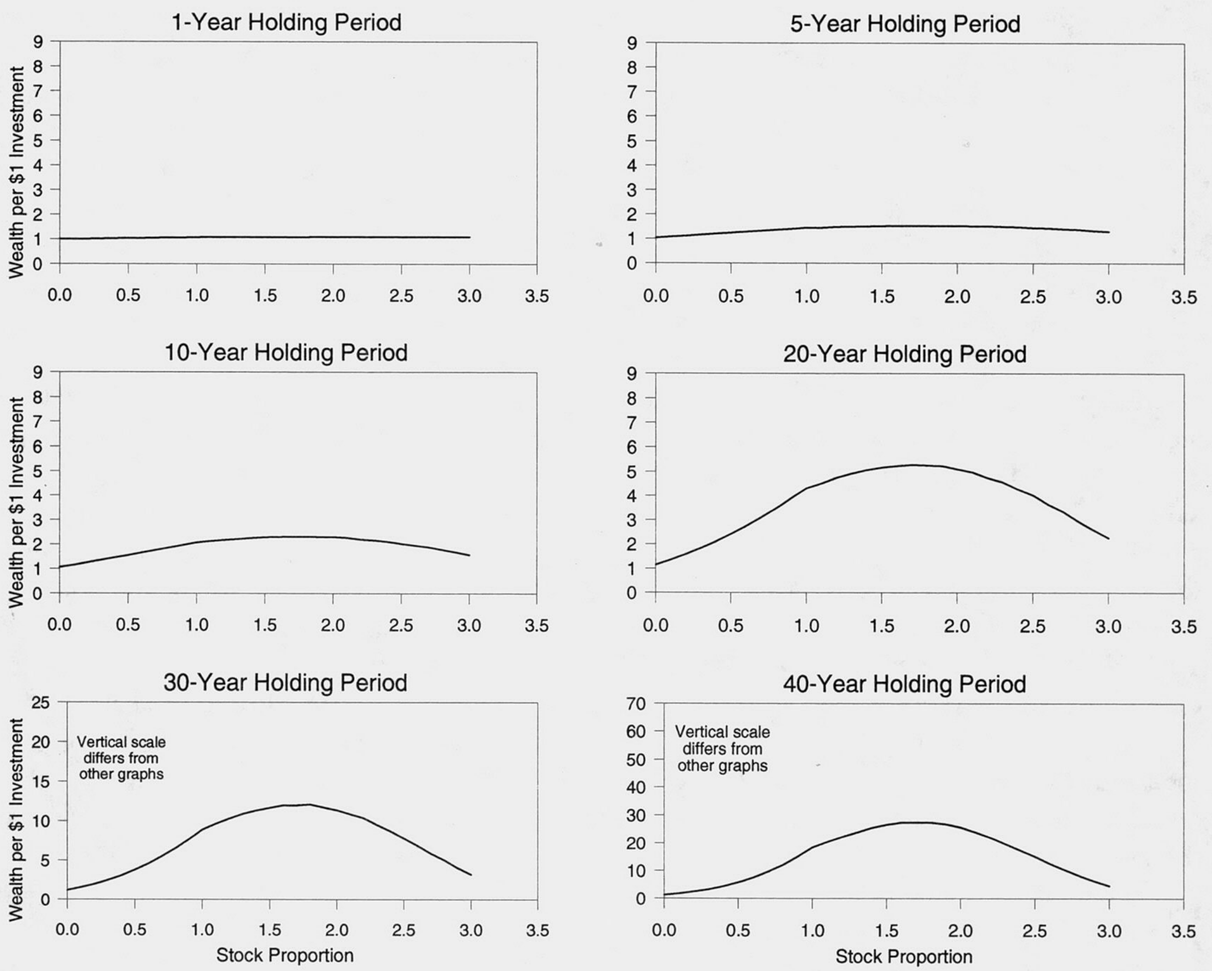

Fig. 5. Median ending real wealth per dollar of initial investment; borrowing rate is 2% over the T-bill rate. Ending wealth distributions can be computed from the return distributions for any asset allocation and any holding period. These figures plot the medians of ending wealth. In contrast to Fig. 3, a higher borrowing rate is used for each stock proportion above 1.0.

the risk premium would need to be 7.97%. A 218% stock allocation is MEL-optimal for an 8.80% annual equity premium. This last result supports our resampling findings that, to the nearest 10%, 220% stock maximizes median ending wealth over long holding periods.

The results depicted in Figs. 1 to 4 assume equal borrowing and lending rates. However, differential rates are more realistic and indeed have been utilized in many theoretical models. Margin loans to individual investors are typically a couple of percentage points higher than the T-bill rate. The benchmark rate for margin loans is the broker call rate, which was 3.75% in late November 2001. In contrast, the Treasury bill yield was ~2.25%. Most brokerage firms add an additional mark-up above the call money rate, particularly for small margin loans.

We study the impact of borrowing at 2% above the T-bill rate. We calculated new ending wealth distributions by applying the higher borrowing rate to the same resampled returns from which Fig. 3 was obtained. Fig. 5 displays the resulting ending wealth.

The higher borrowing rate will obviously reduce ending wealth, and the impact increases with the amount of leverage. Similarly, a greater wealth reduction occurs over longer holding periods. For each holding period, the maximum wealth is achieved at a lower amount of leverage than before. In the longer holding periods shown in Fig. 5, the maxima are at 170% stock proportions, compared to 220% in Fig. 3. Note that 170% is well below the maximum leverage achievable with a 50% margin requirement. As in Fig. 3, the maxima in Fig. 5 are consistent with the MEL rule. The 2% higher borrowing rate is effectively a reduction in the risk premium from 8.80% to 6.80%, and at this lower risk premium the MEL rule gives a 174% stock allocation.

Both Figs. 3 and 5 display medians instead of means. Because of positive skewness, the means are substantially higher than the medians. For the 40-year holding period and 200% stock proportion, the median ending wealth in Fig. 3 is \$57.78, whereas the mean is \$1,017.57. This mean is approximately the 85th percentile of the distribution. Due to the skewness of the distributions, medians are the more informative measure of ending wealth.

We end this section with a look at the left-hand tails of the wealth distributions. Consider the example of a 200% stock proportion over a 20-year holding period. The median ending wealth was shown in Fig. 3 as \$7.59. In contrast, the 5th percentile of the distribution is just \$0.40, and the 1st percentile is only \$0.11. These percentiles imply that there is a 5% chance of losing at least \$0.60 of the \$1.00 initial wealth, and a 1% chance of losing at least \$0.89.

Financial risk management often uses the 1st or 5th percentiles of distributions for the measure known as "Value at Risk" (VaR). As defined by Jorion (2001), VaR summarizes the worst loss over a target horizon with a given level of confidence. Note that the examples in the previous paragraph only consider the potential loss at the *end* of a very long horizon. In practice, VaR is typically used to measure losses *within* a much shorter horizon, such as the potential daily losses throughout a 1-year period. Nevertheless, we believe it is helpful to express our potential investment loss as a VaR.

The graphs shown in Figs. 6 and 7 use solid lines for the 99% confidence levels (i.e., losses at the 1st percentiles of our distributions). Dashed lines for 95% confidence levels are obtained from the 5th percentiles. Because VaR is specifically for losses, it is not meaningful where ending wealth is greater than \$1.00. Those situations are displayed as 0.00 VaR values on the graphs. The borrowing and lending rates are equal in Fig. 6. Fig. 7 uses the T-bill + 2% borrowing rate.

These figures show some opposing effects on VaR as the holding period is lengthened. For the 200% stock proportion in Fig. 6, the 95% VaR is \$0.66 for 10 years, falling to \$0.60 for 20 years, \$0.43 for 30 years, and just \$0.08 for 40 years. Such a pattern seems to support the traditional view about time diversification. Yet, the 99% VaR is roughly constant at about \$0.85 over these same time horizons. Some risk-averse investors would be unwilling to accept this potential loss.

6. Tax implications for individual investors

The numerical analyses used in this paper do not examine the impact of income taxes. But while many investment strategies become less desirable when taxes are considered, the

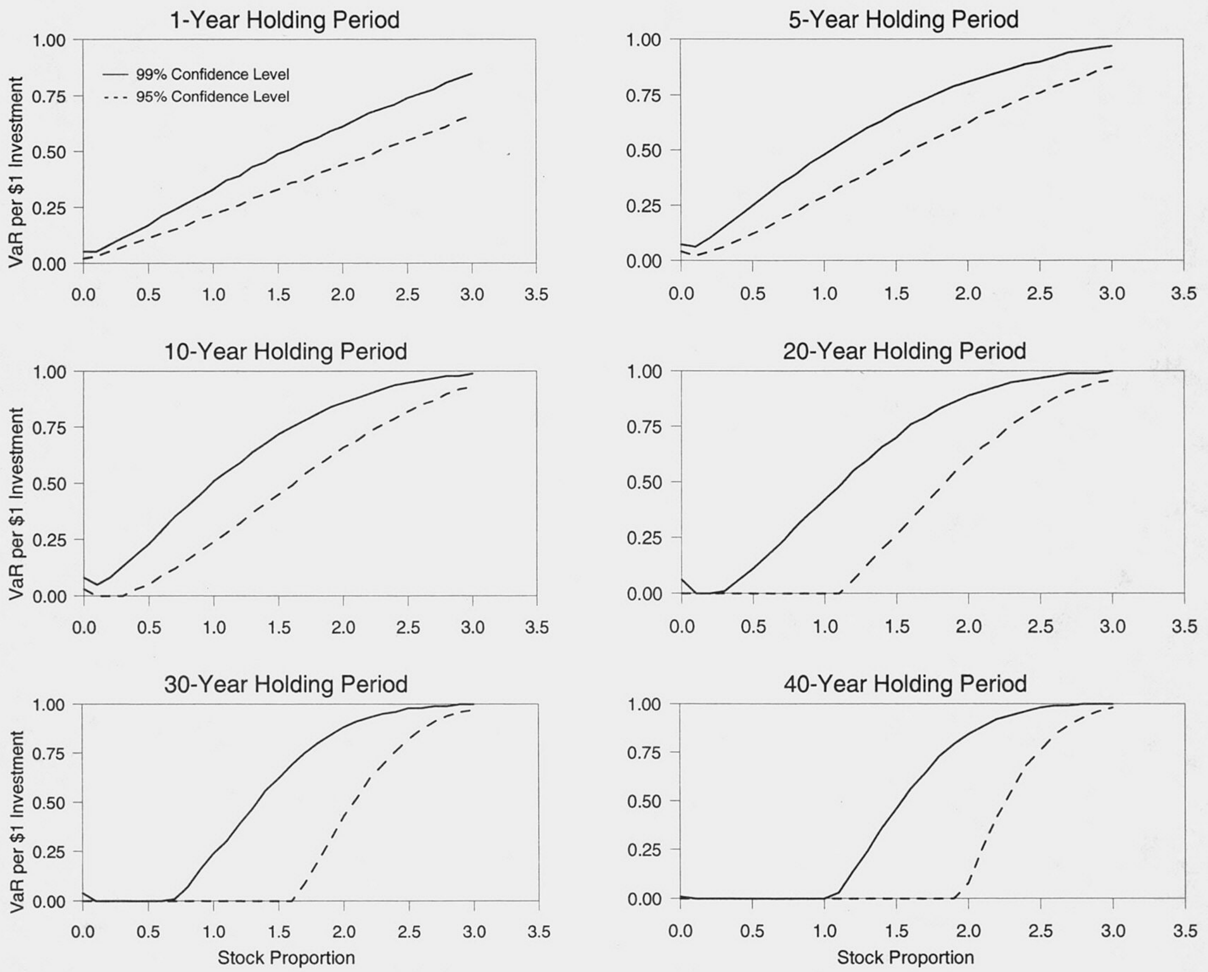

Fig. 6. VaR over various holding periods; borrowing rate equals the T-bill rate. VaR summarizes the worst losses with a given level of confidence. These figures present the potential losses at the end of various holding periods.

benefits of leverage are enhanced by some provisions of the U.S. tax code. The key feature is the deductibility of margin interest. If a U.S. taxpayer borrows money and uses it to buy some investment, the interest paid is "investment interest" which can be deducted up to the amount of net investment income. As described in IRS Publication 550 (2000), net investment income includes interest, dividends, annuities, and royalties, but generally does not include capital gains. If the margin interest rate is double the dividend yield on the stock portfolio, a 200% stock allocation produces an amount of dividends exactly equal to the margin interest paid. Thus, income tax on dividends is eliminated. Such a strategy could be used to generate long-term capital gains, which are traditionally taxed at a lower rate than ordinary income.

Interest incurred to produce tax-exempt income is not deductible, so there is less incentive to borrow in order to fund contributions to IRA, 401(k), or 403(b) accounts. In fact, margin is simply not allowed within such tax-deferred accounts. The financial benefits of tax deferral could be less than the potential gains from leverage in fully taxable accounts. Maximizing

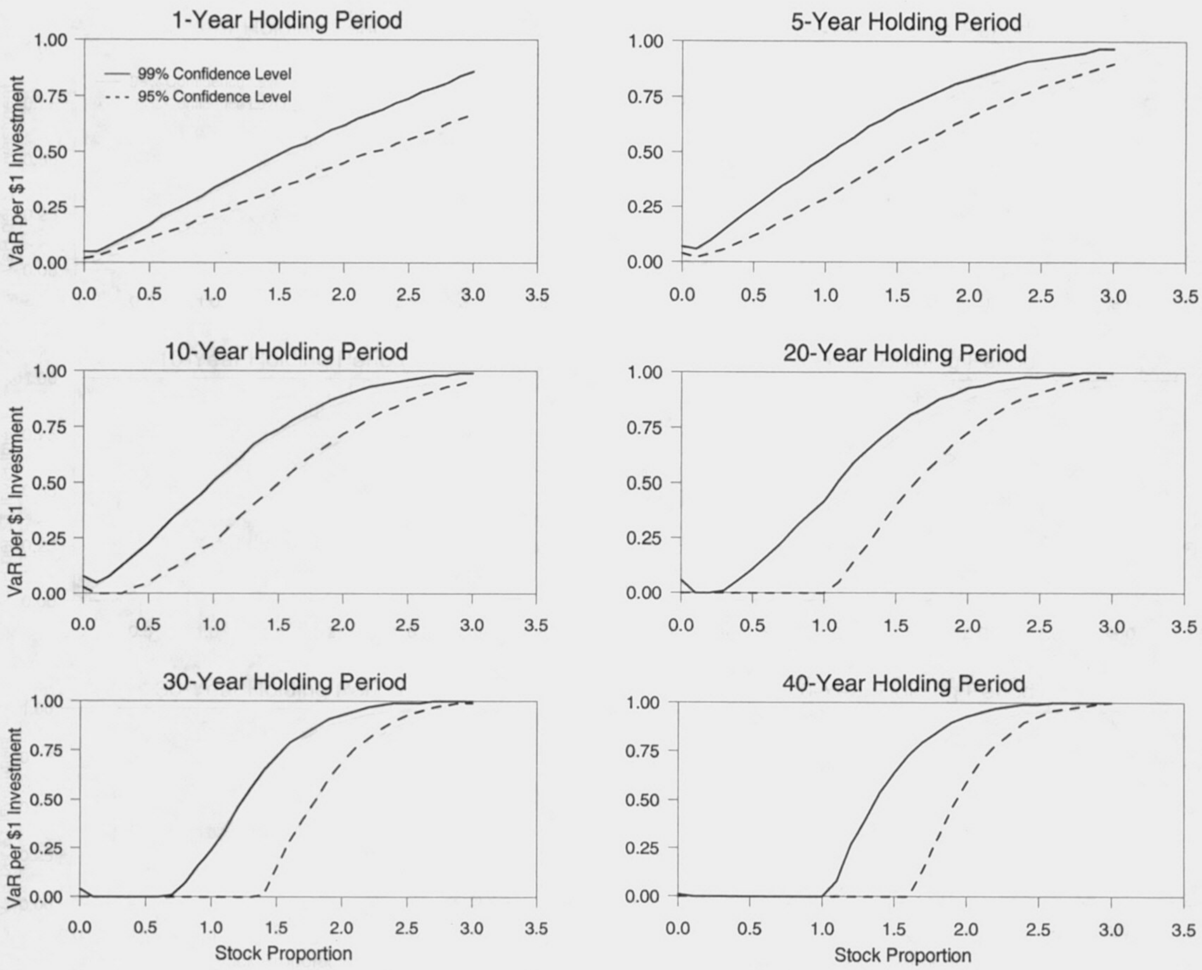

Fig. 7. VaR over various holding periods; borrowing rate is 2% over the T-bill rate. VaR summarizes the worst losses with a given level of confidence. These figures present the potential losses at the end of various holding periods. In contrast to Fig. 6, a higher borrowing rate is used for each stock proportion above 1.0.

contributions to these tax-deferred accounts may therefore be inconsistent with long-term wealth maximization.

In addition to the limitation on leverage, there are further disadvantages to tax-deferred accounts. Most have restrictions on withdrawals, such as not allowing any withdrawals before retirement, or requiring minimum annual withdrawals after retirement. Employees may find limited investment choices in the plans offered by their employers, such as 401(k) plans investing solely in the employers' common stock. On the other hand, the attractiveness of most 401(k) and 403(b) plans is enhanced by significant matching contributions from employers.

7. Conclusion

If investors could borrow at a low risk-free rate and invest the proceeds at a higher risk-free rate, they would be certain to make a profit. The greater the leverage, the larger the profit. However, this is not the case when the higher expected return is risky. The results presented in Section 5 show substantial declines in long-run wealth from large amounts of leverage. Such declines would occur with even modest amounts of leverage if the equity risk premium was lower than the 8.80% historical average.

A key finding from our research is the existence of an optimal wealth-maximizing amount of leverage for a range of holding periods. The same amount of leverage, which maximizes median 10-year wealth, is also appropriate for the 40-year period. Furthermore, this optimal amount is consistent with the MEL rule.

The wealth-maximizing allocation is 170% stock when the borrowing rate is two percentage points higher than Treasury bills. This is well below the 200% stock allowed with a 50% margin requirement. Our VaR diagrams may assist risk-averse investors in choosing lower stock proportions.

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