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After-Tax Valuation of Tax-Sheltered Assets

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Abstract

Valuing tax-sheltered assets on an after-tax basis has many applications. This paper develops models that accommodate annuitized withdrawals from tax-sheltered accounts, an after-tax mutual fund cost of capital, and some variation in the tax rate over time. Annuitized withdrawals significantly decrease the after-tax value of tax-sheltered accounts compared to a single withdrawal over the same time period. Also, after-tax mutual fund discount rates significantly increase the after-tax cost of capital thereby decreasing after-tax valuations. Examples illustrate that using after-tax values can change investors' effective equity exposure by as much as ten percentage points. © 2002 Academy of Financial Services. All rights reserved.

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1. Introduction

Since the proliferation of tax preferred savings vehicles (such as pension funds, traditional IRAs, 401ks, and Roth IRAs) tax-sheltered assets have become a significant part of investors' portfolios. Consequently, it has become increasingly important to value these assets on an after-tax basis. Reichenstein (1998) argues that, if these assets are to be used for income needs during retirement, using after-tax valuations to calculate a family's asset mix is appropriate since goods and services are purchased with after-tax dollars. This idea, however,

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is not consistent with current practice, which tends to rely on pre-tax market values and can greatly distort a family's asset mix calculation.

Calculating the after-tax value of assets in tax-sheltered accounts can be important for other reasons as well. It can serve as basis for determining an individual's credit limit if retirement assets are to be used as collateral for a loan. It may also be useful in calculating the realizable value for estates involved in litigation, like probate or divorce proceedings. Valuing retirement assets on an after-tax basis can also be an important factor in an individual's optimal withdrawal policy or the timing of one's retirement. Therefore, developing a methodology for valuing tax-sheltered assets on an after-tax basis can be important for individual investors.

As a first approximation to valuing tax-sheltered assets on an after-tax basis, Reichenstein (1998) multiplies the pre-tax value of pension assets by one minus the tax rate. This approach, however, assumes that the funds would be immediately withdrawn in their entirety and therefore subject to taxation. Sibley (2002) proposes a more sophisticated framework by deriving a taxable equivalent; that is, the amount of taxable assets that would produce the same after-tax cash flow as a withdrawal from the tax-sheltered account at some future date. This approach is useful because it accounts for the value of assets earning tax-deferred or a tax-free returns before they are withdrawn. His approach is limited, however, because it focuses on single withdrawals and assumes that returns on the taxable equivalent are fully taxed annually at a single, constant tax rate.

The purpose of this paper is to extend the prior literature in three ways. First, the methodology developed in this paper for the after-tax valuation tax-sheltered assets accommodates an annuitized withdrawal policy, which more closely approximates the usual withdrawal patterns from retirement assets, in a more direct fashion. To be sure, one could repeatedly apply a model for single withdrawals to each annuity payment and sum the results to arrive at the same answer (see Sibley, 2002). But having models that specifically accommodate annuitized withdrawals provides heuristics and quicker results that may be useful to investors and those advising them.

Second, the methodology developed in this paper assumes a more typical tax structure for returns earned on a taxable equivalent. Rather than restricting the analysis to assets having returns that are fully taxed annually as ordinary income, the approach introduced here incorporates more ubiquitous investments with lighter tax burdens, namely mutual funds. The proliferation of mutual funds as investment vehicles for individual investors increases the importance of understanding their impact on the after-tax valuation of tax-sheltered assets, and therefore, motivates the following analysis.

Mutual funds have inherent tax deferral characteristics in that their returns have three different components for tax purposes. A portion of a mutual fund's return is attributable to the current income of the underlying investments and is subject to annual taxation as ordinary income. Another portion is attributable to capital gains realized when fund managers sell securities and is taxed annually at the capital gains tax rate. The remainder of the return, unrealized capital gains, is taxed only when the investor sells the mutual fund. This more complex tax structure can significantly affect an analysis of after-tax valuation that relies only on simple annual taxation. Finally, this paper extends prior work by allowing for some variation in tax rates over time. This modification is motivated by Horan, Peterson, and

McLeod (1997) who show that changing tax brackets can significantly affect an investor's optimal choice between a traditional IRA and a Roth IRA.

The tables developed in this paper are analogous to present value interest factor tables found in most finance texts. The difference is that the underlying models represent valuations of a dollar invested in various tax-sheltered accounts. In addition, the tables make important distinctions regarding the taxation scheme associated with the appropriate opportunity cost of capital. As such, investors and those advising them may use these tables to approximate the after-tax value of retirement assets. Alternatively, one could use the models introduced in this paper directly, making appropriate modifications to the parameters to reflect their unique situation.

The remainder of this paper is organized as follows. Section 2 reviews the literature and derives a more straightforward formulation of an after-tax mutual fund valuation from the literature for use in the remainder of the paper. Section 3 introduces the after-tax valuation of single withdrawals from tax-sheltered accounts using two different opportunity costs of capital. The first discount factor is based on an investment that is fully taxed annually as ordinary income as introduced by Sibley (2002). The second discount rate is based on the less oppressive tax structure of a mutual fund investment. Section 4 derives after-tax valuation models assuming withdrawals from a tax-sheltered account are annuitized. Again, two distinct discount factors are used, each representing different taxation schemes. Section 5 introduces some illustrative examples that demonstrate how the tables in this paper might be used. Section 6 concludes and introduces avenues for future research.

2. Literature review

2.1. Prior research

Prior research has recognized the importance of valuing tax-sheltered assets on an after-tax basis and has made contributions in developing models for that purpose. Reichenstein (1998) proposes multiplying the value of a tax-sheltered account by one minus the tax rate, but this adjustment is overly aggressive because it assumes assets would be immediately withdrawn and hence subject to tax. Sibley (2002) offers a different approach by deriving a taxable equivalent, or amount of taxable assets that produces the same after-tax cash flow as an after-tax withdrawal from a retirement account. Interestingly, he shows that the after-tax value of tax-sheltered accounts can actually be greater than their pre-tax values for long time horizons and high returns. This result obtains because the value of the tax deferral grows for long time horizons and high returns, eventually outweighing the tax disadvantage to withdrawing the funds. This paper extends this line research by developing models that investors and practitioners can more easily use to accommodate annuitized withdrawals and a more complex tax structure to determine the taxable equivalent.

Any analysis based on the notion of finding a taxable equivalent that produces the same after-tax cash flows as some withdrawal at a future date must begin with frameworks developed for valuing future accumulations (or future values) of different tax-sheltered savings accounts to determine the size of future withdrawals. Crain and Austin (1997) develop a mathematical model to analyze the choice between taxable investments, deductible IRAs, non-deductible IRAs and Roth IRAs. They recognize that mutual funds (even as taxable investments) have inherent tax deferral characteristics since a significant portion of the return is taxed at a lower capital gains tax rate when the fund is sold. In addition, a portion of the return that is taxed annually is considered capital gains and given preferential tax treatment. Making a distinction between ordinary income tax rates and capital gain tax rates, Crain and Austin (1997) build on the work of Randolf (1994) who examines similar issues with mutual funds that make periodic taxable distributions. Although he makes no distinction between ordinary and capital gains tax rates, Randolf (1994) demonstrates that mutual funds with high turnover and distributions (such as some aggressive growth funds) are best held in tax-sheltered accounts, while mutual funds with low turnover and distributions (such as index funds) should be placed in taxable accounts when both tax deferred and taxable savings accounts are used.

Horan, Peterson, and McLeod (1997) also recognize the importance of a mutual fund tax structure in choosing between different types of retirement accounts. They extend the Crain and Austin analysis by developing a more robust model that allows investors to drop into lower tax brackets upon withdrawal of retirement assets. This approach is motivated by a Panel Study of Income Dynamics (PSID) as interpreted by Bernheim, Skinner, and Wienberg (1997) who show that, on average, retirement income is about 64% of pre-retirement income, suggesting that marginal tax rates for retirees are likely to fall over their investment horizon. Horan, Peterson, and McLeod (1997) find that declining withdrawal tax rates significantly increase the attractiveness of the tax-deductible IRAs.

Krishnan and Lawrence (2001) and Horan and Peterson (2001) offer similar analyses that make different assumptions about how tax savings from tax-deductible options are invested. For example, Krishnan and Lawrence (2001) show that when tax savings are invested in a fully taxable investment rather than another tax-sheltered account the relative attractiveness of the Roth IRA increases significantly. Horan and Peterson (2001) show that the Roth IRAs attractiveness is significantly compromised if tax savings are invested in a taxable mutual fund, which has a much lighter tax burden. Although the analysis provided in this paper requires no such assumptions, the models used in these studies are related to those developed here. Most of the aforementioned studies use simulations to augment the analysis and provide heuristics for different scenarios. This paper follows a similar protocol and presents after-tax present value interest factors tables that can be used to determine after-tax values for a dollar in a tax-sheltered account under different withdrawal patterns and cost of capital tax structures.

2.2. A reformulation for expositional ease

Crain and Austin (1997) introduce a model subsequently used by Horan, Peterson, and McLeod (1997) and Horan and Peterson (2001) for finding the future value of a taxable investment, like a mutual fund, that has returns with different taxable components. The purpose of this section is to provide a more simple and intuitive exposition of that expression to be used for the remainder of the paper. Mutual funds are required to distribute interest income, dividend income, and short-term capital gains to fund shareholders on a prorata basis as dividends, which are taxable to the investor as ordinary income. Long-term capital gains realized by selling appreciated stock are distributed to shareholders on a pro rata basis as well and are taxable to investors at the long-term capital gains rate. Unrealized capital gains accrue on a tax-deferred basis until the investor sells the fund. Crain and Austin show that for a taxable mutual fund investment, a dollar invested for n years has a before-withdrawal tax future value of

$$FV_{TXb} = (1 + r - rp_{oi}t_{oi} - rp_{cg}t_{cg})^n \tag{1}$$

where

r = the expected pre-tax rate of return on the investment;
n = the number of years until the investment is sold for withdrawal;
t_{oi} = the marginal tax rate on ordinary income over the term of the investment;
t_{cg} = the marginal tax rate on capital gains over the term of the investment;
p_{oi} = the percent of annual return distributed to shareholders as ordinary income; and
p_{cg} = the percent of annual return distributed to shareholders as capital gains.

The pre-tax return r is reduced by the taxes attributable to distributions of ordinary income and realized capital gains paid each year, resulting in an annual after-tax return, $(r - rp_{oi}t_{oi} - rp_{cg}t_{cg})$. A capital gain is also recognized when mutual fund shares are sold. The capital gains tax is based on the before-withdrawal tax future value in Eq. (1) less the adjusted basis, which is composed of the initial investment plus dividend and capital gain distributions (on which tax has already been paid) less the income tax on those distributions. The after-withdrawal tax future value of a dollar invested in a taxable mutual fund investment is

$$FV_{TX} = (1 + r - rp_{oi}t_{oi} - rp_{cg}t_{cg})^n - t_{cg}\begin{bmatrix} (1 + r - rp_{oi}t_{oi} - rp_{cg}t_{cg})^n - 1 \\ - rp_{oi}(1 - t_{oi})\dfrac{(1 + r - rp_{oi}t_{oi} - rp_{cg}t_{cg})^n - 1}{(r - rp_{oi}t_{oi} - rp_{cg}t_{cg})} \\ - rp_{cg}(1 - t_{cg})\dfrac{(1 + r - rp_{oi}t_{oi} - rp_{cg}t_{cg})^n - 1}{(r - rp_{oi}t_{oi} - rp_{cg}t_{cg})} \end{bmatrix} \tag{2}$$

Crain and Austin (1997) present a more thorough development of Eq. (2). The second term reduces the before-withdrawal-tax future value by the capital gains tax paid when mutual fund shares are sold at the time of withdrawal. The capital gain is the future market value, $(1 + r - rp_{oi}t_{oi} - rp_{cg}t_{cg})^n$, less the adjusted basis, which is increased by the amount of taxes that have been previously paid annually.

Fortunately, this unwieldy equation can be simplified. Rearranging terms on the right hand side of Eq. (2) yields:

$$FV_{TX} = (1 + r^*)^n(1 - T^*) + T^* \tag{3}$$

where

$r^* = r - rp_{oi}t_{oi} - rp_{cg}t_{cg}$, or the annual after-tax return; and

$T^* = t_{cg}(1 - p_{oi} - p_{cg})/(1 - p_{oi}t_{oi} - p_{cg}t_{cg})$.

Eq. (3) is more simple and intuitive than Eq. (2) and will be used to compute discount factors for the opportunity cost of capital associated with a mutual fund investment in the following analysis.

3. Single withdrawals from tax-sheltered accounts

Sibley (2002) introduces the idea of valuing tax-sheltered accounts by discounting future after-tax withdrawals from the account at an after-tax cost of capital, thereby deriving a taxable equivalent. The after-tax rate of return in his analysis assumes that the entire annual return is taxed each year as ordinary income. This section follows a similar approach but introduces models using discount factors based on a typical mutual fund taxing scheme outlined in the previous section whereby a portion of the return is taxed each year as distributed ordinary income, a portion as distributed capital gain, and the remainder as capital gain when the investment is sold. The result is an opportunity cost of capital associated with a typical mutual fund investment, which is appropriate to use if an investor were to use a mutual fund to invest the taxable equivalent in lieu of the tax-sheltered vehicle.

3.1. Single withdrawal from a fully deductible IRA

The future after-tax withdrawal after n years of investment return from a fully deductible traditional IRA as introduced by Crain and Austin (1997) is

$$FV_{FDed} = (1 + r)^n(1 - T_n) \tag{4}$$

where T_n is the applicable tax rate upon withdrawal. T_n and t_{oi} are similar in that they are both ordinary income tax rates. They are different in that t_{oi} applies over the term of the investment, and T_n is the prevailing tax rate upon withdrawal. As such, these models allow for some variation in the tax rate over time. Although this paper uses the parlance of fully deductible traditional IRAs, Eq. (4) applies to other fully deductible, tax deferred accounts such as 401(k)s, 403(b)s, and pension assets, as well. Sibley (2002) arrives at the after-tax valuation by discounting the value in Eq. (4) at an after-tax rate of return that is fully taxed annually as ordinary income, yielding

$$PV_{FDedOrd} = \frac{(1 + r)^n(1 - T_n)}{[1 + r(1 - t_{oi})]^n}. \tag{5}$$

The numerator is simply the future value from Eq. (4), and the denominator discounts this value back to the present using an after-tax rate of return.

It is quite likely that, if an investor were to have the after-tax equivalent in lieu of the tax-sheltered account, the proceeds might be invested in a mutual fund with a different taxation scheme thereby making the after-tax mutual fund return the appropriate cost of capital for the discount rate. If so, the numerator in Eq. (5) should be discounted using the

Table 1
After-tax value of a dollar in a fully deductible IRA withdrawn as a single cash flow using either an after-tax mutual fund discount rate or a fully taxable discount rate.

r	Investment horizon in years (n) 5	10	15	20	25	30	35	40
	Panel A: after-tax mutual fund discount rate							
5%	0.755	0.789	0.821	0.853	0.883	0.913	0.943	0.973
6%	0.762	0.801	0.839	0.876	0.912	0.947	**0.983**	**1.018**
7%	0.768	0.813	0.857	0.899	0.940	0.981	1.021	1.062
8%	0.774	0.825	0.874	0.921	0.967	**1.013**	1.059	1.106
9%	0.780	0.837	0.890	0.942	**0.994**	1.045	1.097	1.150
10%	0.786	0.848	0.907	0.963	1.020	1.077	1.135	1.195
11%	0.792	0.859	0.922	0.984	1.046	1.108	1.173	1.240
12%	0.798	0.870	0.938	**1.004**	1.071	1.140	1.211	1.285
13%	0.804	0.880	0.953	1.024	1.096	1.171	1.249	1.331
14%	0.809	0.890	0.967	1.044	1.122	1.203	1.288	1.378
15%	0.814	0.900	**0.982**	1.063	1.147	1.234	1.326	1.425
r	*Panel B: fully taxable discount rate*							
5%	0.770	0.823	0.881	0.942	**1.007**	1.077	1.152	1.232
6%	0.780	0.845	0.915	**0.991**	1.073	1.163	1.259	1.364
7%	0.790	0.866	0.950	1.042	1.143	1.254	1.375	1.508
8%	0.800	0.888	**0.986**	1.095	1.216	1.350	1.499	1.665
9%	0.809	0.910	1.023	1.149	1.292	1.452	1.633	1.835
10%	0.819	0.932	1.060	1.206	1.372	1.561	1.775	2.020
11%	0.829	0.954	1.098	1.264	1.455	1.675	1.928	2.219
12%	0.838	0.976	1.137	1.324	1.542	1.795	2.091	2.435
13%	0.848	**0.999**	1.177	1.386	1.632	1.923	2.265	2.668
14%	0.858	1.022	1.217	1.450	1.727	2.057	2.450	2.919
15%	0.867	1.045	1.258	1.515	1.825	2.198	2.648	3.189

Bold figures indicate approximate break-even points between pre-tax and after-tax values.

Assumptions: a) $p_{oi} = 0.0699$ and $p_{cg} = 0.4423$, b) the investor's tax rate on ordinary income is 28% for the life of the investment, and c) capital gains are tax at 20%.

after-tax relationship in Eq. (3). In other words, if the alternative to holding assets in a tax-sheltered account is investing in a mutual fund, the after-tax valuation of each dollar in the tax-sheltered account is

$$PV_{FDedMut} = \frac{(1 + r)^n(1 - T_n)}{(1 + r^*)^n(1 - T^*) + T^*}. \tag{6}$$

The numerators in Eqs. (5) and (6) are the identical future values. The denominator in Eq. (6), however, will generally be significantly greater than that in Eq. (5) using average distribution rates of growth funds reported by Crain and Austin (1997) (i.e., $p_{oi} = 0.0699$ and $p_{cg} = 0.4423$) because the tax burden associated with mutual funds is lighter. Therefore, Eq. (6) will generally result in a lower after-tax valuation. This relationship is illustrated in Table 1, which displays the after-tax valuation of a dollar in a fully deductible IRA under the two different scenarios using the average annual distribution rates of growth funds reported by Crain and Austin (1997).

As predicted, the after-tax values in Panel A that are calculated using Eq. (6) are less than those in Panel B, which are calculated using Eq. (5). The intuition for this result is that tax-sheltered accounts become more (less) valuable in high (low) tax rate environments. In the extreme case, they have no value in the absence of taxes. Therefore, after-tax values discounted using the tax structure of a mutual fund investment (which has significant tax deferral characteristics and low tax obligation in the form of capital gains) are lower than values derived from fully taxable discount rates, suggesting the discount rate is an important factor in finding the after-tax value of tax-sheltered assets. Not surprisingly, the difference grows and becomes quite substantial for long time horizons and large expected returns, approaching a factor of two.

A quick example of how an investor or financial planner would use Table 1 might be helpful. Consider a retired investor with \$200,000 invested in equity mutual funds in a fully deductible IRA account. If the expected return for equity mutual funds is 12% (the approximate average annual return on equities since 1926) and if retirement assets were expected to be withdrawn as a lump sum in 10 years, the after-tax present value interest factor is 0.870 (Panel A). Therefore, the after-tax value is 0.870 × \$200,000, or \$174,000. Using a fully taxable discount rate (Panel B) the after-tax value is 0.976 × \$200,000, or \$195,200.

An important inference to draw from Table 1 is that after-tax present values (not future values) are directly related to the time horizon and investment return because the value of the tax deferral characteristics associated with the tax-sheltered account is amplified for long time horizons and high returns. In fact, the after-tax value for single withdrawals can be greater than the pre-tax value for long time horizons and high returns because the value of the account's tax deferral characteristics eventually outweighs the impact of the withdrawal tax under these conditions. Although not reported here, the results are substantially the same for investors in the 31% tax bracket.

The results change significantly if the tax rate upon withdrawal T_n drops from 28% to 15% and are displayed in Table 2. Predictably, the after-tax value increases dramatically under these circumstances since the tax burden upon withdrawal decreases. In fact, after-tax values are greater than pre-tax values for relatively short time horizons, as much as 20 years shorter than scenarios in which the tax rate is constant (see Table 1). For example, continuing with the previous example, the after-tax value of the \$200,000 IRA invested in stock mutual funds and withdrawn as a lump sum in 10 years is \$205,400, or 1.027 × \$200,000, compared to only \$174,000 when tax rates remain constant. The values using the after-tax mutual fund rate continue to be less than those associated with an investment taxed annually at the ordinary income tax rate because the value of the tax shelter decreases in less oppressive tax environments. Similar models and tables can be developed for partially deductible IRAs, in which only a portion of the contribution is tax deductible. The reader can find these derivations in the Appendix, and the associated tables are available from the author.

2.2. *Single withdrawal from a Roth IRA*

A similar process can be followed to find the after-tax value of a Roth IRA withdrawn as a single cash flow. Using a fully taxed discount rate the after-tax value of single withdrawal from a Roth IRA is

Table 2
After-tax value of a dollar in a fully deductible IRA withdrawn as a single cash flow using either an after-tax mutual fund discount rate or a fully taxable discount rate.

	Investment horizon in years (n)							
	5	10	15	20	25	30	35	40
r	*Panel A: after-tax mutual fund discount rate*							
5%	0.891	0.931	0.969	**1.006**	1.043	1.078	1.114	1.148
6%	0.899	0.946	**0.991**	1.034	1.077	1.118	1.160	1.201
7%	0.907	0.960	1.011	1.061	1.110	1.158	1.206	1.254
8%	0.914	0.974	1.032	1.087	1.142	1.196	1.251	1.306
9%	0.921	0.988	1.051	1.113	1.173	1.234	1.295	1.358
10%	0.928	**1.001**	1.070	1.137	1.204	1.271	1.340	1.411
11%	0.935	1.014	1.089	1.162	1.234	1.308	1.385	1.464
12%	0.942	1.027	1.107	1.186	1.265	1.346	1.429	1.517
13%	0.949	1.039	1.125	1.209	1.294	1.383	1.475	1.572
14%	0.955	1.051	1.142	1.232	1.324	1.420	1.520	1.627
15%	0.962	1.063	1.159	1.255	1.354	1.457	1.566	1.682
r	*Panel B: fully taxable discount rate*							
5%	0.909	0.972	1.040	1.112	1.189	1.271	1.360	1.454
6%	0.921	**0.997**	1.080	1.170	1.267	1.373	1.487	1.610
7%	0.932	1.023	1.122	1.230	1.349	1.480	1.623	1.781
8%	0.944	1.048	1.164	1.293	1.435	1.594	1.770	1.966
9%	0.955	1.074	1.207	1.357	1.525	1.715	1.927	2.166
10%	0.967	1.100	1.251	1.424	1.619	1.842	2.096	2.384
11%	0.978	1.126	1.296	1.492	1.718	1.977	2.276	2.620
12%	0.990	1.153	1.342	1.563	1.820	2.120	2.468	2.874
13%	**1.001**	1.179	1.389	1.636	1.927	2.270	2.674	3.149
14%	1.013	1.206	1.437	1.711	2.039	2.428	2.893	3.446
15%	1.024	1.233	1.485	1.789	2.155	2.595	3.126	3.765

Bold figures indicate approximate break-even points between pre-tax and after-tax values.
Assumptions: a) $p_{oi} = 0.0699$ and $p_{cg} = 0.4423$, b) the investor's tax rate on ordinary income is 28% and drops to 15% when the funds are withdrawn, and c) capital gains are tax at 20%.

$$PV_{RothOrd} = \frac{(1 + r)^n}{[1 + r(1 - t_{oi})]^n}, \tag{7}$$

which is introduced by Sibley (2002). The numerator is the after-tax future value of the Roth IRA when the funds are withdrawn with no tax consequence and the denominator represents the discount factor associated with a fully taxable cost of capital. Using the cost of capital associated with a mutual fund tax structure, the after-tax value of Roth IRA withdrawn as a single cash flow is

$$PV_{RothMut} = \frac{(1 + r)^n}{(1 + r^*)^n(1 - T^*) + T^*}. \tag{8}$$

Table 3 shows the after-tax value of a dollar in a Roth IRA withdrawn as a single cash flow for various returns and time horizons for an investor in the 28% tax bracket. All of the values are greater than one and greater than those for traditional IRAs because withdrawing funds

Table 3
After-tax value of a dollar in a Roth IRA withdrawn as a single cash flow using either an after-tax mutual fund discount rate or a fully taxable discount rate.

	Investment horizon in years (n)							
	5	10	15	20	25	30	35	40
r	*Panel A: after-tax mutual fund discount rate*							
5%	1.049	1.095	1.140	1.184	1.227	1.269	1.310	1.351
6%	1.058	1.113	1.166	1.217	1.267	1.316	1.365	1.413
7%	1.067	1.130	1.190	1.248	1.305	1.362	1.418	1.475
8%	1.075	1.146	1.214	1.279	1.343	1.407	1.471	1.536
9%	1.084	1.162	1.237	1.309	1.380	1.452	1.524	1.598
10%	1.092	1.178	1.259	1.338	1.416	1.496	1.576	1.660
11%	1.100	1.193	1.281	1.367	1.452	1.539	1.629	1.722
12%	1.108	1.208	1.302	1.395	1.488	1.583	1.682	1.785
13%	1.116	1.222	1.323	1.422	1.523	1.627	1.735	1.849
14%	1.124	1.236	1.344	1.450	1.558	1.670	1.788	1.914
15%	1.131	1.250	1.364	1.477	1.593	1.714	1.842	1.979
r	*Panel B: fully taxable discount rate*							
5%	1.069	1.144	1.223	1.308	1.399	1.496	1.600	1.711
6%	1.083	1.173	1.271	1.376	1.491	1.615	1.749	1.895
7%	1.097	1.203	1.320	1.447	1.588	1.741	1.910	2.095
8%	1.110	1.233	1.369	1.521	1.689	1.875	2.082	2.313
9%	1.124	1.264	1.420	1.597	1.795	2.017	2.267	2.549
10%	1.138	1.294	1.472	1.675	1.905	2.167	2.466	2.805
11%	1.151	1.325	1.525	1.756	2.021	2.326	2.678	3.082
12%	1.165	1.356	1.579	1.839	2.141	2.494	2.904	3.382
13%	1.178	1.387	1.634	1.925	2.267	2.671	3.146	3.705
14%	1.191	1.419	1.690	2.013	2.398	2.857	3.403	4.054
15%	1.204	1.451	1.747	2.105	2.535	3.053	3.677	4.429

Assumptions: a) $p_{oi} = 0.0699$ and $p_{cg} = 0.4423$, b) the investor's tax rate on ordinary income is 28% for the life of the investment, and c) capital gains are tax at 20%.

from a Roth IRA has no tax consequence. Dropping to a lower tax bracket upon withdrawal has no impact for the same reason. Consider an investor with a $200,000 Roth IRA invested in equity funds (assuming a 12% annual expected rate of return). If the funds are withdrawn as a lump sum in 10 years, the Roth IRA has an after-tax value of $241,600, or 1.208 × $200,000, which is much greater than the $174,000 after-tax value for the fully deductible IRA. Results for an investor in the 31% tax bracket are virtually identical though not reported here in the interest of parsimony.

4. Annuitized withdrawals from tax-sheltered accounts

The preceding section derives after-tax values of tax-sheltered accounts for a single withdrawal. A more typical withdrawal pattern is for investors to draw on their assets more evenly over time. The purpose of this section to derive after-tax models assuming funds are withdrawn as an annuity rather than as a single cash flow and to compare the results to those in the previous section. It is possible, of course, to simply value an annuity as a series of single payments. But models that specifically accommodate annuitized withdrawals provide heuristics and quicker results that may be useful to investors or those advising them.

One way to view withdrawing funds as an annuity instead of a single cash flow is to consider the duration of the cash flow streams. In the case of the single withdrawal, the duration of the cash flow stream is simply the timing of the withdrawal. For an annuity having the same time horizon, the duration is significantly less than half (particularly for long time horizons) of that for the single withdrawal since duration is the weighted average timing of the payments using present values as weights and since the weights will be larger for nearer term payments. Therefore, after-tax values of tax-sheltered accounts assuming annuitized withdrawals should be much less than values for single cash flows.

4.1. Annuitized withdrawals from a fully deductible IRA

The first step to valuing an annuitized pattern of withdrawals from a tax-sheltered account is to find the annuity payment that can be sustained by the current value of the account over n years while earning a pre-tax return of r. These annuity payments can then be adjusted for withdrawal taxes and discounted using an after-tax cost of capital. The before-tax annuity payment supported by a dollar in a fully deductible IRA over n years generated by the account is

$$PMT_{FDed} = \frac{1}{1/r - 1/r(1+r)^n}. \tag{9}$$

where the denominator is the well-known present value interest factor for an annuity. This annuity withdrawal is fully taxable at the ordinary rate, and the after-withdrawal-tax annuity payment is determined by multiplying the value in Eq. (9) by $(1 - t_{oi})$. This after-withdrawal tax payment can then be discounted using the after-tax discount rate associated with a fully taxable investment to find the after-tax value, yielding

$$PV_{FDedAnnOrd} = \frac{(1 - t_{oi})}{1/r - 1/r(1+r)^n}\left\{\frac{1}{r(1 - t_{oi})} - \frac{1}{r(1 - t_{oi})[1 + r(1 - t_{oi})]^n}\right\}. \tag{10}$$

The term to the left of the curly brackets is the annual payment after-withdrawal taxes that can be generated from each dollar in a tax-sheltered account. The term inside the curly brackets is the present value interest factor of an annuity using the after-tax return, $r(1 - t_{oi})$, as the discount rate when the cost of capital is fully taxed annually as ordinary income.

A similar process may be used to determine the after-tax value of a fully deductible IRA withdrawn as an annuity using a discount factor associated with a mutual fund. Using the derivation found in the Appendix, the after-tax value of a dollar in a fully deductible IRA having annuitized withdrawals discounted at the after-tax cost of capital for a mutual fund is

Table 4
After-tax value of a dollar in a fully deductible IRA withdrawn as an annuity using either an after-tax mutual fund discount rate or a fully taxable discount rate.

	Investment horizon in years (n) 5	10	15	20	25	30	35	40
r	*Panel A: after-tax mutual fund discount rate*							
5%	0.740	0.753	0.765	0.774	0.781	0.787	0.792	0.796
6%	0.743	0.759	0.771	0.780	0.787	0.793	0.797	0.800
7%	0.747	0.763	0.776	0.785	0.792	0.797	0.801	0.803
8%	0.750	0.768	0.781	0.790	0.796	0.800	0.803	0.805
9%	0.753	0.772	0.785	0.793	0.799	0.803	0.805	0.806
10%	0.756	0.775	0.788	0.796	0.801	0.804	0.806	0.807
11%	0.759	0.779	0.791	0.799	0.803	0.806	0.807	0.807
12%	0.761	0.782	0.794	0.801	0.805	0.806	0.807	0.808
13%	0.764	0.785	0.796	0.802	0.806	0.807	0.808	0.808
14%	0.766	0.787	0.798	0.804	0.806	0.807	0.808	0.808
15%	0.769	0.789	0.800	0.805	0.807	0.808	0.808	0.808
r	*Panel B: fully taxable discount rate*							
5%	0.749	0.772	0.793	0.814	0.833	0.851	0.867	0.882
6%	0.754	0.781	0.806	0.829	0.851	0.870	0.888	0.904
7%	0.759	0.790	0.818	0.844	0.867	0.888	0.906	0.922
8%	0.765	0.799	0.830	0.858	0.882	0.903	0.921	0.937
9%	0.770	0.807	0.841	0.870	0.896	0.917	0.935	0.949
10%	0.775	0.815	0.851	0.882	0.908	0.929	0.946	0.959
11%	0.779	0.823	0.861	0.893	0.919	0.939	0.955	0.967
12%	0.784	0.831	0.871	0.903	0.929	0.948	0.963	0.974
13%	0.789	0.838	0.879	0.912	0.937	0.956	0.970	0.979
14%	0.793	0.845	0.888	0.920	0.945	0.963	0.975	0.984
15%	0.798	0.852	0.895	0.928	0.952	0.969	0.980	0.987

Assumptions: a) $p_{oi} = 0.0699$ and $p_{cg} = 0.4423$, b) the investor's tax rate on ordinary income is 28% for the life of the investment, and c) capital gains are tax at 20%.

$$PV_{FDedAnnMut} = \frac{(1 - t_{oi})}{1/r - 1/r(1 + r)^n} \left\{ \frac{\frac{(1 + r^*)^n - 1}{r^*}(1 - T^*) + nT^*}{(1 + r^*)^n(1 - T^*) + T^*} \right\}. \tag{11}$$

The term to the left of the curly brackets is the same annual payment after-withdrawal taxes found in Eq. (10). The term inside the curly brackets represents the present value interest factor of an annuity using an after-tax cost of capital associated with a mutual fund. See Appendix for a derivation.

Table 4 presents after-tax values of a dollar in a fully deductible IRA using Eqs. (10) and (11). The after-tax value continues to be greater for the fully taxable discount rate, long investment horizons, and high pre-tax return since the value of the tax shelter increases in these situations. The most noticeable result, however, is that after-tax values are much lower than those reported in Table 1 for single withdrawals, especially for long time horizons. In fact, there are no longer scenarios for which after-tax values are greater than pre-tax values. The reason for the divergent results is that annuitized withdrawal

Table 5
After-tax value of a dollar in a Roth IRA withdrawn as an annuity using either an after-tax mutual fund discount rate or a fully taxable discount rate.

r	Investment horizon in years (n) 5	10	15	20	25	30	35	40
	Panel A: after-tax mutual fund discount rate							
5%	1.028	1.046	1.062	1.075	1.085	1.093	1.100	1.105
6%	1.033	1.054	1.070	1.083	1.093	1.101	1.107	1.111
7%	1.037	1.060	1.078	1.091	1.100	1.107	1.112	1.115
8%	1.042	1.066	1.084	1.097	1.106	1.112	1.115	1.118
9%	1.046	1.072	1.090	1.102	1.110	1.115	1.118	1.120
10%	1.050	1.077	1.095	1.106	1.113	1.117	1.120	1.121
11%	1.054	1.082	1.099	1.109	1.116	1.119	1.121	1.121
12%	1.057	1.086	1.103	1.112	1.117	1.120	1.121	1.122
13%	1.061	1.090	1.106	1.115	1.119	1.121	1.122	1.122
14%	1.064	1.093	1.109	1.116	1.120	1.121	1.122	1.122
15%	1.067	1.096	1.111	1.118	1.121	1.122	1.122	1.122
r	*Panel B: fully taxable discount rate*							
5%	1.040	1.072	1.102	1.130	1.157	1.182	1.204	1.225
6%	1.047	1.085	1.120	1.152	1.182	1.209	1.233	1.255
7%	1.055	1.097	1.137	1.172	1.205	1.233	1.258	1.280
8%	1.062	1.109	1.153	1.191	1.225	1.255	1.280	1.301
9%	1.069	1.121	1.168	1.209	1.244	1.274	1.298	1.318
10%	1.076	1.133	1.182	1.225	1.261	1.290	1.314	1.332
11%	1.083	1.144	1.196	1.240	1.276	1.305	1.327	1.344
12%	1.089	1.154	1.209	1.254	1.290	1.317	1.338	1.353
13%	1.096	1.164	1.221	1.267	1.302	1.328	1.347	1.360
14%	1.102	1.174	1.233	1.278	1.313	1.337	1.355	1.366
15%	1.108	1.183	1.243	1.289	1.322	1.345	1.361	1.371

Assumptions: a) $p_{oi} = 0.0699$ and $p_{cg} = 0.4423$, b) the investor's tax rate on ordinary income is 28% for the life of the investment, and c) capital gains are tax at 20%.

schemes have much shorter durations than single withdrawal schemes thereby decreasing the value of the tax shelter. Therefore, using single withdrawal models can dramatically bias valuations upward if withdrawal patterns are more evenly distributed over time.

For example, if one withdraws funds as an annuity over ten years from an IRA invested in equity mutual funds, the after-tax value is $156,400, or 0.782 × $200,000, compared to $174,000 assuming a single withdrawal in ten years. The difference becomes much greater for long time horizons and demonstrates that the assumed withdrawal pattern significantly affects the after-tax value of tax-sheltered accounts. Because all the values in Table 4 fall between 0.74 and 0.81, one could develop a rule of thumb that after-tax values of fully deductible IRAs are approximately 75% to 80% of their pre-tax value regardless of return and time horizon. Similar models and tables can be developed for partially deductible IRAs, in which only a portion of the contribution is tax deductible. The reader can find these derivations in the Appendix, and the associated tables are available from the author.

4.2. Annuitized withdrawals from a Roth IRA

Finding the after-tax annuity payment supported by each dollar in a Roth IRA requires only a straightforward application of the present value of an annuity relationship since there are no tax implications. Therefore, the after-tax annuity payment for each dollar in a Roth IRA is simply

$$PMT_{Roth} = \frac{1}{1/r - 1/r(1+r)^n}. \tag{12}$$

The after-tax value is determined by multiplying this payment by the after-tax present value interest factor of annuity for either a fully taxable investment or a mutual fund investment, yielding

$$PV_{RothAnnOrd} = \frac{1}{1/r - 1/r(1+r)^n}\left\{\frac{1}{r(1-t_{oi})} - \frac{1}{r(1-t_{oi})[1+r(1-t_{oi})]^n}\right\} \tag{13}$$

or

$$PV_{RothAnnMut} = \frac{1}{1/r - 1/r(1+r)^n}\left\{\frac{\frac{(1+r^*)^n - 1}{r^*}(1-T^*) + nT^*}{(1+r^*)^n(1-T^*) + T^*}\right\}, \tag{14}$$

respectively. These values differ from those in Eqs. (10) and (11) by a factor of $(1 - t_{oi})$, which reflects the fact that withdrawals from Roth IRAs are tax-free whereas fully deductible IRA withdrawals are taxed at the ordinary rate. These values are reported in Table 5 for a typical mutual fund, and various time horizons and returns. All of the after-tax values are greater than one, and increase with the time horizon and return. In addition, after-tax values using the after-tax mutual fund cost of capital are smaller than those using the fully taxable cost of capital at the ordinary rate. The most dramatic result, however, is that these after-tax values assuming an annuitized withdrawal pattern are dramatically less than those assuming a lump sum withdrawal in Table 3, indicating that the withdrawal pattern is a significant component in the after-tax valuation.

Obviously, many other scenarios are possible. The purpose of this paper, however, is not to report simulations that exhaust all possibilities. Instead, we offer expository simulations and, more importantly, models that investors and financial planners can use and modify for their special circumstances. The next section reviews how these models might be put into practice.

5. Practical applications

The purpose of this section is to provide some illustrative examples of how these models and tables might be used by individual investors or financial planners. Two different situations are investigated—calculating an individual's or family's asset mix and determining one's credit limit for borrowing against a 401(k) or 403(b) account. Other applications,

Table 6
Personal balance sheet based on pre-tax and after-tax market values assuming a fully taxable cost of capital.

	Pre-tax			After-tax (Reichenstein) n = 0 years			After-tax single withdrawal n = 30 years			After-tax annuity withdrawal n = 30 years		
	Market value	% of assets	% of Fin. assets	Market value	% of assets	% of Fin. assets	Market value	% of assets	% of Fin. assets	Market value	% of assets	% of Fin. assets
Assets												
Cash	$15,000	1.7		$15,000	1.9		$15,000	1.1		$15,000	1.6	
Financial assets												
Taxable mutual funds (stock)	$100,000	11.6	16.7	$100,000	12.4	18.4	$100,000	7.4	9.3	$100,000	10.7	14.9
Roth IRA (stock)	$300,000	34.7	50.0	$300,000	37.1	55.1	$748,200	55.6	69.2	$395,100	42.3	59.0
Fully-deductible IRA (bonds)	$200,000	23.1	33.3	$144,000	17.8	26.5	$232,600	17.3	21.5	$174,000	18.6	26.0
Total financial assets	$600,000	69.4	100%	$544,000	67.2	100%	$1,080,800	80.3	100%	$669,100	71.6	100%
Home	$250,000	28.9		$250,000	30.9		$250,000	18.6		$250,000	26.8	
Total assets	$865,000	100%		$809,000	100%		$1,345,800	100%		$934,100	100%	
Liabilities & equity												
Credit card balances	$10,000	1.2		$10,000	1.2		$10,000	0.7		$10,000	1.1	
Mortgage	$190,000	22.0		$190,000	23.5		$171,821	12.8		$171,821	18.4	
Equity	$665,000	76.9		$609,000	75.3		$1,163,979	86.5		$752,279	80.5	
Total liabilities & equity	$865,000	100%		$809,000	100%		$1,345,800	100%		$934,100	100%	

The investor is retired, 65 years old, in the 28% tax bracket, and planning withdrawals over 30 years. Stocks and bonds are expected to return 12% and 6%, respectively. The taxable mutual fund was just purchased, and the new, 30-year, fixed-rate mortgage carries an 8% interest rate.

such as determining estate value in divorce or probate proceedings, are straightforward extensions of these examples.

5.1. An individual's asset mix

Table 6 presents a balance sheet for a hypothetical investor who is retired, 65 years old, in the 28% tax bracket, and planning withdrawals over 30 years. Stocks and bonds are expected to return 12% and 6%, respectively. The taxable mutual fund was just purchased, and the mortgage is a new 30-year, fixed rate loan carrying an 8% interest rate. Assumed pre-tax values are presented in the first set of columns. A review of the financial assets reveals $600,000 in investments, which represents 69.4% of total assets. The taxable mutual fund and Roth IRA are both invested in equities, while the fully deductible IRA is invested in bonds. On a pre-tax basis, two-thirds of the financial assets are invested in equities and one-third in bonds, representing a reasonable asset allocation. Also, the investor's personal debt ratio is about 23.2%.

The second set of columns presents after-tax market values using the approach introduced by Reichenstein (1998) whereby tax-sheltered assets are multiplied by one minus the tax rate. Although the investor's financial assets as a percent of total assets do not change dramatically under this approach, the investor's asset allocation does. Equity exposure has increased from 67% to almost 74%, and the personal debt ratio has increased slightly to 24.7%. Changes become more dramatic when after-tax values are calculated using a fully taxable opportunity cost of capital as introduced by Sibley (2002). These results are presented in the last two sets of columns in Table 6.

Assuming a lump sum withdrawal in 30 years and a fully taxable cost of capital, the after-tax value of financial assets increases markedly to 80.3% of total assets, and the investor's equity exposure increases to 78.5% of financial assets. Notice, as well, that the personal debt ratio has declined dramatically to 13.7%. The after-tax Roth IRA value is calculated by multiplying the pre-tax value of $300,000 by the appropriate after-tax present value interest factor (30 years at 12%) from Table 3, Panel B. The after-tax deductible IRA value is calculated by multiplying the $200,000 pre-tax value by the appropriate present value factor (30 years at 6%) from Table 1, Panel B. These figures provide a very different financial picture than those calculated using pre-tax values.

Some of the difference can be mitigated if withdrawals are annuitized over 30 years. The tax-deductible IRA and Roth IRA values are determined using Table 4, Panel B and Table 5, Panel B, respectively. In this case, the value of the tax-sheltered assets presented in the final columns of Table 6 decreases substantially, financial assets are 71.6% of total assets, stock comprises 73.9% of the investor's asset allocation, and the debt ratio is 18.5%. This example demonstrates that using pre-tax values for tax-sheltered assets can distort an investor's effective investment exposure, asset allocation, and use of leverage.

The astute reader may notice that the value of the mortgage has decreased in the final two sets of calculations. In keeping with the notion that assets should be valued on an after-tax basis, liabilities should also be computed on an after-tax basis. Because interest on home mortgages is typically tax-deductible, the actual after-tax obligation is less than the principal amount. The appropriate way to calculate the after-tax value is to determine the interest

Table 7
Personal balance sheet based on pre-tax and after-tax market values assuming an after-tax mutual fund cost of capital.

	After-tax single withdrawal n = 30 years			After-tax annuity withdrawal n = 30 years			After-tax single withdrawal n = 30 years			After-tax annuity withdrawal n = 30 years		
	Market value	% of assets	% of Fin. assets	Market value	% of assets	% of Fin. assets	Market value	% of assets	% of Fin. assets	Market value	% of assets	% of Fin. assets
Assets												
Cash	$15,000	1.5		$15,000	1.7		$15,000	1.2		$15,000	1.5	
Financial assets												
Taxable mutual funds (stock)	$100,000	9.7	13.1	$100,000	11.6	16.8	$100,000	8.0	10.1	$100,000	9.7	13.1
Roth IRA (stock)	$474,900	46.1	62.1	$336,000	39.1	56.5	$662,381	52.7	66.7	$468,646	45.6	61.5
Fully-deductible IRA (bonds)	$189,400	18.4	24.8	$158,600	18.5	26.7	$230,437	18.3	23.2	$192,963	18.8	25.3
Total financial assets	$764,300	74.3	100%	$594,600	69.2	100%	$992,817	78.9	100%	$761,609	74.2	100%
Home	$250,000	24.3		$250,000	29.1		$250,000	19.9		$250,000	24.4	
Total assets	$1,029,300	100%		$859,600	100%		$1,257,817	100%		$1,026,609	100%	
Liabilities & equity												
Credit card balances	$10,000	1.0		$10,000	1.2		$10,000	0.8		$10,000	1.0	
Mortgage	$171,821	16.7		$171,821	20.0		$171,821	13.7		$171,821	16.7	
Equity	$847,479	82.3		$677,779	78.8		$1,075,996	85.5		$844,788	82.3	
Total liabilities & equity	$1,029,300	100%		$859,600	100%		$1,257,817	100%		$1,026,609	100%	

In the first two scenarios, the investor is retired, 65 years old, in the 28% tax bracket, and planning withdrawals over 30 years. In the last two scenarios, the investor is 45 years old and planning to begin withdrawals over 30 years at age 65. Stocks and bonds are expected to return 12% and 6%, respectively. The taxable mutual fund was just purchased, and the new, 30-year, fixed-rate mortgage carries an 8% interest rate.

portion of each payment, subtract the value of the tax-deduction from each payment, and discount the after-tax payments at the after-tax interest rate. Because this issue falls outside the focus of this study, the after-tax value is approximated by multiplying the monthly payment by one minus the tax rate and discounting at an annual rate of 5.76%, or 8% × (1 − 0.28).

Table 7 presents results using an after-tax mutual fund cost of capital. The first two scenarios continue with the above example. For simplicity, the analysis assumes that income and capital gain distribution rates for bonds funds are similar to those for equity funds. Depending on whether one assumes a lump sum withdrawal in 30 years or an annuity-style withdrawal pattern over that time, the investor's stock allocation in the investment portfolio is either 75.2% or 73.3%, respectively. In either case, the investor's equity exposure is significantly greater than an analysis based on pre-tax values would suggest.

The last two columns of Table 7 present a slightly different scenario. The scenario is the same as above except that the investor is 45 years old and is planning for a 30-year retirement at age 65. The proper approach in this case requires us to find the future value at age 65, apply the appropriate after-tax present value factor presented in this paper, and then discount back to age 45 using the appropriate cost of capital. Taken together, the first and third steps in this process are equivalent to applying the after-tax present value interest factor for Roth IRA withdrawn in a lump sum from Table 3. These factors are calculated using the numerator to calculate the future value and the denominator to discount it at the relevant cost of capital. Under this scenario, the after-tax value of the tax-sheltered accounts increases substantially because the accounts have more time to accrue returns free from tax. As a result, the investor's after-tax equity exposure increases to 76.8% or 74.6%, depending on whether withdrawals are taken as a lump sum or annuity, respectively. The important implication from these examples is that an investor's effective after-tax asset allocation, commitment to financial assets, and even personal debt ratio can be substantially different from that implied by pre-tax values.

5.2. Determining credit limits

Some institutions, including employers, allow individual investors to borrow against their 401(k) or 403(b) accounts. This arrangement raises the question of how much credit the tax-sheltered collateral can support. Ignoring issues regarding risk differences and correlations between the loan and the collateral, one can immediately conclude that the after-tax value of the collateral is different from its pre-tax value. Again, the after-tax value will depend on the withdrawal pattern and the appropriate cost of capital. As an example, consider the 65-year old retiree, wishing to borrow for five years against a $200,000 401(k), which for tax purposes is very much like a fully deductible IRA. There are two advantages to borrowing money instead of temporarily withdrawing funds from a tax-sheltered account. First, once withdrawn, funds cannot be re-invested in a tax-sheltered vehicle save for the modest contribution limits allowed by the IRS. Second, the loan gives the investor access to retirement assets without foregoing the benefits of the tax shelter.

An after-tax value requires establishing a withdrawal time horizon. The term of the loan rather than the anticipated withdrawal horizon for the investor might be the most appropriate because a creditor will be more concerned with the withdrawal pattern for collateral purposes rather than the investor's retirement plans should the investor default. With this in mind, it is not clear whether a valuation based on a single withdrawal or an annuitized withdrawal is more appropriate. Because loan payments are in annuity form, this analysis will assume the latter. Finally, one must determine the appropriate discount rate. Since many 401(k) investors use mutual funds and since a lending institution's investment portfolio (and thus, opportunity cost of capital) is probably analogous to a mutual fund, the after-tax mutual fund discount rate is used. The proper after-tax present value interest factor is then found in Table 1, Panel A. Using a 12% expected return on equity and a five-year horizon, the factor is 0.798.

The value of the collateral is then \$159,600, or 0.798 × \$200,000. A lending institution may be willing to extend credit on, say, 80% of that amount, or \$127,680. This amount is greater than if one used Reichenstein's (1998) approach in which the after-tax value of the 401(k) is \$144,000, or \$200,000 × (1 − .28). If credit were extended on 80% of this amount, the investor could borrow only \$115,200. Determining the appropriate after-tax value for tax-sheltered assets is therefore, important in determining an investor's credit limit when borrowing against tax-sheltered assets.

6. Conclusion

Valuing tax-sheltered assets on an after-tax basis is an important factor in appropriately calculating a family's asset mix, determining credit limits for loans secured by tax-sheltered assets, and valuing assets in litigation. This paper extends the after-tax valuation methodologies introduced in previous studies. Specifically, the models accommodate valuations based on an annuitized withdrawal pattern rather than assuming all funds are withdrawn as a single cash flow. After-tax values increase with the time horizon and return because the value of the tax shelter increases as well. Valuations based on annuity-style withdrawal patterns are significantly lower than those based on single withdrawals over the same time horizon because the annuity significantly reduces the duration of the cash flow stream thereby reducing the value of the tax shelter. Therefore, it is very important to consider the withdrawal pattern in computing after-tax values for tax-sheltered accounts.

This paper also raises the question of which cost of capital is most appropriate for valuing withdrawals from tax-sheltered accounts by developing after-tax present value interest factors applicable to mutual funds, which have complex tax schemes that distinguish between distributed ordinary income, realized capital gains, and unrealized capital gains. If an investor were to use mutual funds to invest a taxable equivalent (the amount of taxable assets that would generate the same after-tax cash flows as withdrawals from the tax-sheltered account), then it is appropriate to use a cost of capital that reflects this opportunity cost. After-tax valuations based on this discount rate are significantly lower than those based on discount factors assuming returns are fully taxable as ordinary income in the year they are received. Therefore, it is important to consider the next best use of tax-sheltered assets were they to be invested outside a tax-sheltered account.

Finally, the single cash flow models developed in this paper allow for some variation in the tax rate over time. The results suggest that after-tax valuations increase significantly if investors drop into lower tax brackets when the cash is withdrawn. This situation is common for retirees whose post-retirement income is typically 64% of post-retirement income.

The tables presented in this paper are essentially present value interest factor tables designed to determine the after-tax value of a dollar in various tax-sheltered accounts. The tables make important distinctions regarding the withdrawal pattern and the taxation scheme associated with the appropriate opportunity cost of capital. As such, investors and their advisors may use them to approximate the after-tax value of retirement assets. Depending on the circumstances, an investor's calculated equity exposure can change by as much as ten percentage points according to an illustrative example. We also demonstrate how these tables might be used to determine a borrower's credit limit when borrowing against tax-sheltered assets.

The usual caveats apply when interpreting the results presented in this paper. First, these models assume that future tax rates, expected returns, and investors' time horizons are known and constant. Furthermore, the analysis ignores the added flexibility offered by some tax-sheltered savings vehicles. For example, Roth IRAs permit investors to withdrawal funds early without penalty under certain circumstances, which increases the attractiveness of the Roth IRA. Valuing this flexibility is difficult, but presents a fruitful avenue for future research. Nonetheless, the results presented here demonstrate that the withdrawal pattern and appropriate cost of capital are important factors in correctly valuing tax-sheltered assets.

Appendix

A.1. Single withdrawal from a partially deductible IRA

If investors make contributions to a tax-sheltered account in excess of the allowable deductible limits, the non-deductible portion of the contribution earns returns on a tax-deferred basis until the funds are withdrawn free of tax. In this sense, non-deductible contributions to traditional IRAs are equivalent to contributions to a Roth IRA that are allowed to earn returns and to be withdrawn free of tax. Sibley (2002) introduces a formula for calculating the future value of a dollar, a portion of which represents a non-deductible contribution, in an IRA and withdrawn as a single cash flow as:

$$FV_{PDed} = (1 + r)^n(1 - T_n) + aT_n \tag{A1}$$

where a is the non-deductible portion of the contribution. As before, this future value can be discounted assuming the taxable alternative is an investment having returns that are fully taxed annually or have the tax benefits associated with a mutual fund. Discounting using a fully taxed discount rate yields

$$PV_{PDedOrd} = \frac{(1 + r)^n(1 - T_n) + aT_n}{[1 + r(1 - t_{oi})]^n} \tag{A2}$$

whereas discounting with a mutual fund discount rate yields

$$PV_{FDedMut} = \frac{(1 + r)^n(1 - T_n) + aT}{(1 + r^*)^n(1 - T^*) + T^*}. \tag{A3}$$

A.2. Annuitized withdrawals from a fully deductible IRA

The annuity payment generated from the account is still represented by Eq. (9) and taxed in the same way. Using the relationship established in Eq. (3), the after-tax value of a fully deductible IRA using the after-tax opportunity costs associated with a mutual fund can be expressed as

$$PV_{FDedAnnMut} = \frac{FV_{FDedAnnMut}}{(1 + r^*)^n(1 - T^*) + T^*} \tag{A4}$$

where $FV_{FDedAnnMut}$ is the future value a fully deductible IRA having annuitized withdrawals. The numerator can be expressed as

$$FV_{FDedAnnMut} = \sum_{N=1}^{n} (1 - t_{oi}) PMT_{FDed}[(1 + r^*)^n(1 - T^*) + T^*]. \tag{A5}$$

Distributing the summation operator, factoring the constants to the front of the summation sign, and substituting the well-known future value interest factor for an annuity yields

$$FV_{FDedAnnMut} = (1 - t_{oi}) PMT_{FDed}\left[\frac{(1 + r^*)^n - 1}{r^*}(1 - T^*) + nT^*\right]. \tag{A6}$$

Substituting Eq. (9) into this expression and substituting this expression into Eq. (A4) yields the after-tax value of a dollar in a fully deductible IRA having annuitized withdrawals discounted at the after-tax cost of capital for a mutual fund, or

$$PV_{FDedAnnMut} = \frac{(1 - t_{oi})}{1/r - 1/r(1 + r)^n}\left\{\frac{\frac{(1 + r^*)^n - 1}{r^*}(1 - T^*) + nT^*}{(1 + r^*)^n(1 - T^*) + T^*}\right\}. \tag{A7}$$

The term inside the curly brackets represents the present value interest factor of an annuity using an after-tax cost of capital associated with a mutual fund. It may seem circuitous to derive a present value of an annuity by first calculating its future value and discounting back to the present. However, the mathematics associated with directly calculating the sum of the present values are prohibitive since one cannot factor multi-term denominators to the front of a summation sign.

A.3. Annuitized withdrawals from a partially deductible IRA

The approach used to find the after-tax value of a partially deductible IRA is similar to the previous section in that we find the before-withdrawal-tax payment, determine the effect of the withdrawal tax, and take the present value using our chosen after-tax discount rate.

Finding the after-tax annuity payment supported by a dollar invested in a partially deductible IRA account, however, requires a slightly different approach than that used with the fully deductible IRA because relating the payment to the present value would necessitate summing quotients with multiple terms in the denominator, making it difficult to factor terms from the denominator. Instead, one can convert the current pre-tax value of the tax-sheltered account to a future value and find the annuity payment that produces that accumulation. Using this approach, the before-withdrawal tax payment that can be generated from each dollar in a partially deductible IRA is

$$PMT_{PDed} = \frac{(1 + r)^n}{(1 + r)^n/r - 1/r} \tag{A8}$$

where the numerator is the future value of the tax-sheltered account and the denominator is the familiar future value interest factor of an annuity. These payments are taxable when withdrawn, but the portion of the withdrawal associated with the non-deductible contribution is free from taxation. Therefore, the after-withdrawal tax annuity payment is

$$PMT_{PDedAfterTax} = \frac{(1 + r)^n(1 - t_{oi}) + at_{oi}}{(1 + r)^n/r - 1/r}. \tag{A9}$$

An implicit assumption in this formulation is that the back-end tax benefits of withdrawing non-deductible contributions are taken on a prorata basis for each payment over the life of the annuity rather than applied toward the early payments. Although this assumption may or may not be realistic, it is necessary to arrive at a tractable result. The after-tax value of the partially deductible IRA can then be found by discounting this after-withdrawal tax payment using either an after-tax discount rate associated with a fully taxable investment,

$$PV_{PDedAnnOrd} = \frac{(1 + r)^n(1 - t_{oi}) + at_{oi}}{(1 + r)^n/r - 1/r}\left\{\frac{1}{r(1 - t_{oi})} - \frac{1}{r(1 - t_{oi})[1 + r(1 - t_{oi})]^n}\right\}, \tag{A10}$$

or using an after-tax discount rate associated with a mutual fund investment,

$$PV_{PDedAnnMut} = \frac{(1 + r)^n(1 - t_{oi}) + at_{oi}}{(1 + r)^n/r - 1/r}\left\{\frac{\dfrac{(1 + r^*)^n - 1}{r^*}(1 - T^*) + NT^*}{(1 + r^*)^n(1 - T^*) + T^*}\right\}. \tag{A11}$$

The terms inside the curly brackets of Eqs. (A10) and (A11) represent present value interest factors for annuities for a fully taxable investment and a mutual fund investment, respectively. They are the same factors used in Eqs. (10) and (11), respectively.

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References

Bernheim, B. D., Skinner, J. S., & Wienberg, S. (1997). What accounts for the variation in retirement wealth among U.S. households? mimeo (Stanford University).

Crain, T. L., & Austin, J. R. (1997). An analysis of the tradeoff between tax deferred earnings in IRAs and preferential capital gains. *Financial Services Review, 6* (4), 227–242.

Horan, S. M., Peterson, J. H., & McLeod, R. (1997). An analysis of non-deductible IRA contributions and Roth IRA conversions. *Financial Services Review, 6* (4), 243–256.

Horan, S. M., & Peterson, J. H. (2001). A reexamination of tax-deductible IRAs, Roth IRAs, and 401(k) investments. *Financial Services Review, 10,* 87–100.

Krishnan, S. V., & Lawrence, S. (2001). Analysis of investment choices for retirement: A new approach and perspective. *Financial Services Review, 10* (1), 75–86.

Randolf, W. L. (1994). The impact of mutual fund distributions on after-tax returns. *Financial Services Review, 3* (2), 127–141.

Reichtenstein, W. (1998). Calculating a family's asset mix. *Financial Services Review, 7* (3), 195–206.

Sibley, M. (2002). On the valuation of tax-advantaged retirement accounts. *Financial Services Review, 11* (3), 233–251.