An Investigation of Adjustable-rate Mortgage Pricing Features

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Abstract

Mortgage borrowers face the difficult prospect of evaluating the costs and risks associated with the choice of terms for adjustable-rate mortgages. This study uses a simulation approach to model the choices. We represent the risk of the adjustable-rate mortages with distributions of present value-cost differentials for a variety of mortgage life periods. We provide insight on the financial planning aspect by modeling the impact of mortgage-rate changes on the size of payments for adjustable-rate mortgages. Simulation can yield nonintuitive results that may lead to better decision making by borrowers. © 2003 Academy of Financial Services. All rights reserved.

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1. Introduction

Borrowers in the market for a mortgage today face a bewildering array of choices. One can choose between fixed-rate mortgages (FRMs) and adjustable-rate mortgages (ARMs). ARMs generally have the advantage of lower initial interest rates and payments. Although these rates and payments increase in subsequent years, ARMs are often advantageous to borrowers who anticipate a relatively short holding period.

Among ARMs, one can select from a wide variety of terms. Typically, they offer an interest rate and payment that remain the same for a fixed period of time. After that time, the

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lender adjusts the interest rate by adding a margin to a published interest-rate index. Both the amount of the margin and the selection of the index are indicated in the loan contract. For some ARMs, the interest rate and payment adjusts annually after the first year. For others, annual adjustment begins after 3, 5, or even 10 years. The common notation describes an ARM by indicating, first, the years until the first rate adjustment and, second, how frequently the rate adjusts. Thus, a 3/1 ARM has a three-year fixed period and adjusts annually after that time. The size of the annual adjustment is always limited—both the maximum adjustment in any given year and the maximum adjustment over the life of the loan. These limits are known as "caps" and they, too, vary from loan to loan. Templeton, Main and Orris (1996) showed that Monte Carlo simulations can shed light on the choice between an annually adjusting ARM and a FRM. This paper shows how Monte Carlo simulations can help a hypothetical borrower choose among ARMs.

The first simulation examines the choice of the length of the fixed period before the first interest-rate adjustment occurs. Borrowers can expect to pay a higher initial interest rate for the less risky ARMs that delay the date of first adjustment than for ARMs that begin adjustment sooner. We provide a simulation model that yields information on this cost and risk tradeoff. The simulation output allows a borrower to view the mean present value cost of an ARM at any date of termination, as well as probability distributions of present value-cost differentials between ARMs with different initial fixed periods. It also provides a distribution of the *breakeven period*—the number of years after initiation of the loan for which the ARM with the shorter fixed period maintains its present value-cost advantage over the ARM with a longer fixed period. Finally, the simulation permits insight into the financial planning aspect of the choice by modeling the impact of mortgage-rate changes on the size of payments for ARMs with various initial fixed periods.

The second simulation examines the choice of annual and lifetime caps for a standard one-year ARM. Borrowers can expect to pay a higher initial mortgage rate for less risky ARMs with lower caps. The same kind of output discussed above can help borrowers determine the circumstances under which they ought to be willing to pay a higher rate in exchange for the lower caps.

1. Literature review

There has been a limited academic literature so far dealing with mortgage choices, and much of that has dealt with the choice between FRMs and ARMs. Most authors have focused on discovering which variables significantly influence actual borrower choice.[1] Two papers have used Monte Carlo simulations to shed light on that choice. Tucker (1991) showed that simulation can reveal important information related to the expected costs of mortgages to borrowers. He simulated interest-rate changes to demonstrate the present value-cost differential between ARMs and FRMs assuming a variety of opportunity discount rates.[2] His analysis showed that ARMs were often the lower cost alternative for borrowers who anticipated a shorter life for their mortgage and had higher opportunity cost discount rates.

Templeton, Main and Orris (1996) extended the simulation approach to making the ARM vs. FRM choice by including additional results important to borrowers. Their simulation

results provided information on the present value-cost differentials, the breakeven period (the holding period at which the present value cost of the ARM begins to exceed that of the FRM), and the payment size, using actual loan interest-rate data from both low and high interest-rate environments. They provided both expected values and distributions of these values, allowing a hypothetical borrower to consider both cost and risk in order to make a more informed choice.

This paper applies the methodology of simulation to the choice among ARMs with various terms. Specifically, we examine ARMs with different fixed periods before the adjustments in the mortgage interest rate begin and different caps. The benefit of the approach is to illuminate the borrower's options. In most cases, the ultimate choice still depends on the borrower's willingness to accept a higher expected present value mortgage cost in return for a reduced variability of cost or payments.

2. Methodology

The present value cost of a mortgage is the sum of the discount points, the present value of the payments, and the present value of the payoff balance. The discount points and the interest portion of the payment are taken on an after-tax basis. Mathematically, the present value cost can be expressed as

$$\text{PVCOST} = M \cdot (p) \cdot (1 - tx_j) + \left[\sum_{t=1}^{T} \frac{[pmt_t - I_t \cdot (tx_j)]}{(1 + r_j)^t} \right] + \frac{B_T}{(1 + r_j)^T} \quad (1)$$

where M = the mortgage amount; p = the discount points rate; tx_j = the marginal personal tax rate of borrower j; T = the assumed life of the mortgage in years; pmt_t = the mortgage payment at year t; I_t = the interest portion of the payment in dollars at year t; r_j = the opportunity cost discount rate for borrower j; and B_T = the payoff balance at year T.

We constructed a model in Excel spreadsheet software to compare the present value costs of ARMs with different contract terms over the first 15 years of a 30-year mortgage using Eq. (1). The model was simulated by using the Monte Carlo technique in @RISK simulation software. Table 1 shows contract terms for the ARMs included in the two simulations of this study. A Midwestern mortgage broker provided the loan contract terms data for the week of January 24, 1997; thus, the simulations in this study were performed with actual quoted loan terms as parameters. For all mortgages we examined, the term was 30 years, there were no points charged, and the index used for setting adjusted mortgage rates was the one-year constant maturity yield of U.S. Treasury securities. We assume the initial amount of all mortgages is $100,000. Table 1 shows the margins over the index rate for each ARM and the annual and lifetime caps applicable to each mortgage. It also shows the initial mortgage rate, which is usually below the index plus margin, reflecting what is known as a "teaser discount." This teaser discounts applies only to the initial mortgage rate. After the mortgage rate begins to adjust, it will be equal to just the index rate plus the margin subject to any caps.

Modeling interest-rate changes over the life of the mortgage is the engine that drives these simulations. Though the loan parameters can be taken from actual contracts, the modeler

Table 1
Contract terms of ARMs

Mortgage	Caps			Margin	Teaser discount	Initial mortgage rate
	First adjustment	Subsequent adjustment	Lifetime adjustment			
A: ARMs from Simulation 1						
1/1 ARM	2.00%	2.00%	6.00%	3.00%	2.735	5.875%
3/1 ARM	2.00%	2.00%	6.00%	2.75%	1.235%	7.125%
5/1 ARM	2.00%	2.00%	6.00%	2.75%	0.860%	7.500%
7/1 ARM	3.00%	2.00%	5.00%	2.50%	0.235%	7.875%
10/1 ARM	3.00%	2.00%	5.00%	2.75%	0.360%	8.00%
B: ARMs from Simulation 2						
1/1 ARM	2.00%	2.00%	6.00%	3.00%	2.735%	5.875%
1/1 ARM	1.00%	1.00%	5.00%	2.875%	1.860%	6.625%

Note: All ARM adjustments are tied to the 1-year constant maturity yield on U.S. Treasury Securities, which was 5.61% for the week of January 24, 1997. Also, recall that the notation for ARMs indicates the number of years before the first adjustment and then the frequency of adjustment. Thus, with a 3/1 ARM, 3 years pass before the first rate adjustment. Thereafter, the rate adjusts annually.

must select parameters related to the rate changes. All the ARMs listed in Table 1 have adjustable rates tied to the one-year constant maturity yield of U.S. Treasury securities. First, we collected weekly data on this index, as reported by the Federal Reserve, from December 1987 through November 1996. We calculated annual changes in this index by taking each observed value and subtracting the index value from 52 weeks prior. The annual changes appear to be approximately normally distributed around zero. See Fig. 1 for a histogram of annual changes over the period 1987–1996. We calculated a standard deviation of annual index-rate changes of 1.52 percentage points. To run the simulation, we simply observe the first-year index rate. The index rate for the second year is the index-rate value for the first year plus the product of a randomly selected z-score from a normal distribution and the
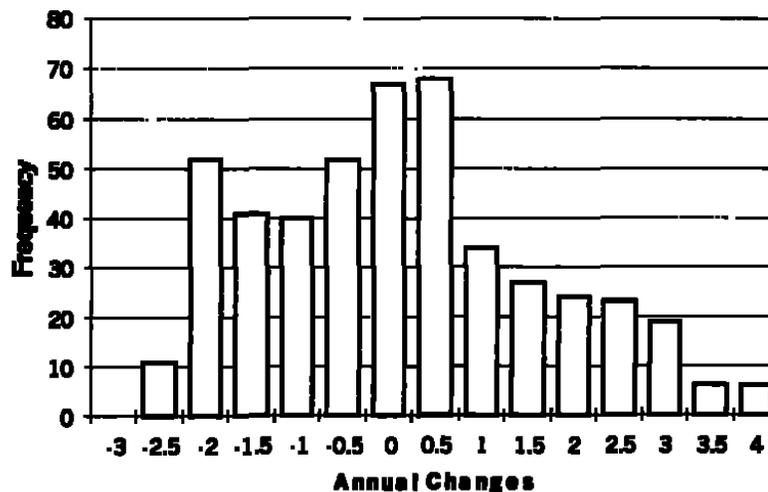

Fig. 1. Distribution of interest rate changes. December 1987 through November 1996.

standard deviation of annual changes. Subsequent index rates in the same simulation are modeled similarly. An index-rate floor of 3% (i_{min}) and a ceiling of 10% (i_{max}) were selected to keep the simulated index rate within a reasonable range. The effect of simulating interest rates in this way is to make the expected index value for any period approximately equal to the previous period's index value.[3]

A composite rate for a given year is determined by adding the appropriate margin to the simulated value of the index rate. The composite rate is subjected to annual and lifetime caps specified in the ARM contract to arrive at the simulated mortgage interest rate for each period. For simplicity, the payments of each ARM are treated as annual rather than monthly. The payment in any year is the constant annuity amount that would completely amortize the remaining balance over the remaining life of the loan, given that year's mortgage interest rate. For an ARM that adjusts annually, the payment is recalculated each year, based on the simulated mortgage interest rate for that year. For an ARM that adjusts annually after n years, the payment for the first n years is the constant annuity amount that would completely amortize the loan over 30 years at the initial interest rate. After n years, the payment is calculated in the same manner as the one-year ARM's payment.

The opportunity discount rate is the assumed after-tax return the borrower could earn on invested funds. We use an after-tax opportunity discount rate of 4% for both simulations described in this study. An alternative way to think about the opportunity discount rate is that it is the borrower's best alternative borrowing rate. Under that view, the rate would be higher for most people. The marginal tax rate used in these simulations is 28%, a rate typical of higher income borrowers.

3. Results

3.1. Simulation comparing periods before initial adjustment

We ran a 1,000-iteration simulation by using the data for the five ARMs shown in Panel A of Table 1. Table 2 shows the mean present value (PV) costs for all five ARMs for each of the hypothetical termination dates, from 1 to 15 years. In Table 2, the ARM with the lowest mean cost as of any year is shown in boldface.

Table 2 allows us to get a general feel for the way the PV costs of the ARMs are likely to behave over different holding periods. The difference between the mean PV cost of the lowest and the highest cost ARM, for any year of termination, ranges from about $1,500 (10/1 ARM − 1/1 ARM) at Year 1 to about $2,100 (1/1 ARM − 7/1 ARM) at Year 15. The lower initial rate of the 1/1 ARM is cost advantageous to borrowers with very short holding periods. Once the rate adjustments begin and the teaser discount disappears, this mortgage quickly becomes a higher expected cost alternative. Each mortgage appears to be the lowest expected cost alternative for holding periods approximating its fixed period. The one exception to this observation is the 7/1 ARM. Curiously, it has the lowest margin, which might be attractive to borrowers. On the other hand, it also has the lowest teaser discount, which in its case would be in effect for the first seven years of the mortgage holding period.

Table 2
Mean PV costs for the five ARMs for different termination years 1–15

Year	PV 1 Yr ARM	PV 3/1 ARM	PV 5/1 ARM	PV 7/1 ARM	PV 10/1 ARM
1	100,221	101,087	101,346	101,606	101,692
2	101,564	102,121	102,628	103,136	103,305
3	103,317	103,104	103,848	104,593	104,841
4	105,207	104,516	105,007	105,978	106,302
5	107,047	106,165	106,108	107,295	107,692
6	108,840	107,869	107,484	108,545	109,010
7	110,583	109,537	109,072	109,731	110,261
8	112,256	111,158	110,682	111,103	111,446
9	113,848	112,714	112,231	112,532	112,566
10	115,378	114,206	113,717	113,905	113,624
11	116,846	115,633	115,140	115,223	114,914
12	118,244	116,997	116,500	116,484	116,255
13	119,566	118,290	117,790	117,680	117,532
14	120,811	119,508	119,005	118,806	118,736
15	121,984	120,660	120,155	119,873	119,878

Note: Boldface indicates the ARM with the lowest mean PV cost as of any year.

The low teaser discount makes this mortgage a poor choice for risk-neutral borrowers with any holding period up to 15 years, even though it has the lowest margin over index.

Fig. 2 shows the mean annual before-tax payments for the five mortgages for different years of the mortgage holding period. The maximum difference between payments ranges from about $1,700 in Year 1 to about $800 in Year 10, and about $300 in Year 15. All of the ARMs have payments based on the same index. Thus, after the initial fixed period for each mortgage, its payment begins to converge with the other adjusting payments. The pattern of initial mortgage rates, teaser discounts, and margins means that, as each ARM goes from its initial fixed rate to the adjustable rate, it goes from having the lowest expected payment to being somewhere up in the pack. For example, the 7/1 ARM has the lowest expected payment in Year 6 and Year 7 (just after the 5/1 ARM begins to adjust, but before the 7/1 ARM begins its adjustments). Once the 7/1 ARM begins to adjust, it has higher expected payments than the 10/1 ARM, the last remaining ARM still operating with a teaser discount.

First, let us consider a borrower contemplating a short holding period for his mortgage. The mean PV cost would be the criterion by which a risk-neutral borrower would judge mortgages. If the borrower expected to hold the mortgage for four years, for example, he would choose the 3/1 ARM, because that mortgage has the lowest expected PV cost for that holding period. The cost of the 5/1 ARM is only $491 higher, however. A risk-averse borrower might find it useful to compare the 3/1 and the 5/1 ARMs directly for holding periods in the neighborhood of four years.

Fig. 3 shows the difference in PV cost (cost of the 5/1 ARM − cost of the 3/1 ARM) for holding periods up to 15 years for the 1,000-iteration simulation. The heavy black line shows the mean difference for each termination year. The two dotted lines include 90% of the observations between them. Notice that there is no variation around the mean for the first three years of termination. This is because both the 3/1 and 5/1 ARMs have fixed interest
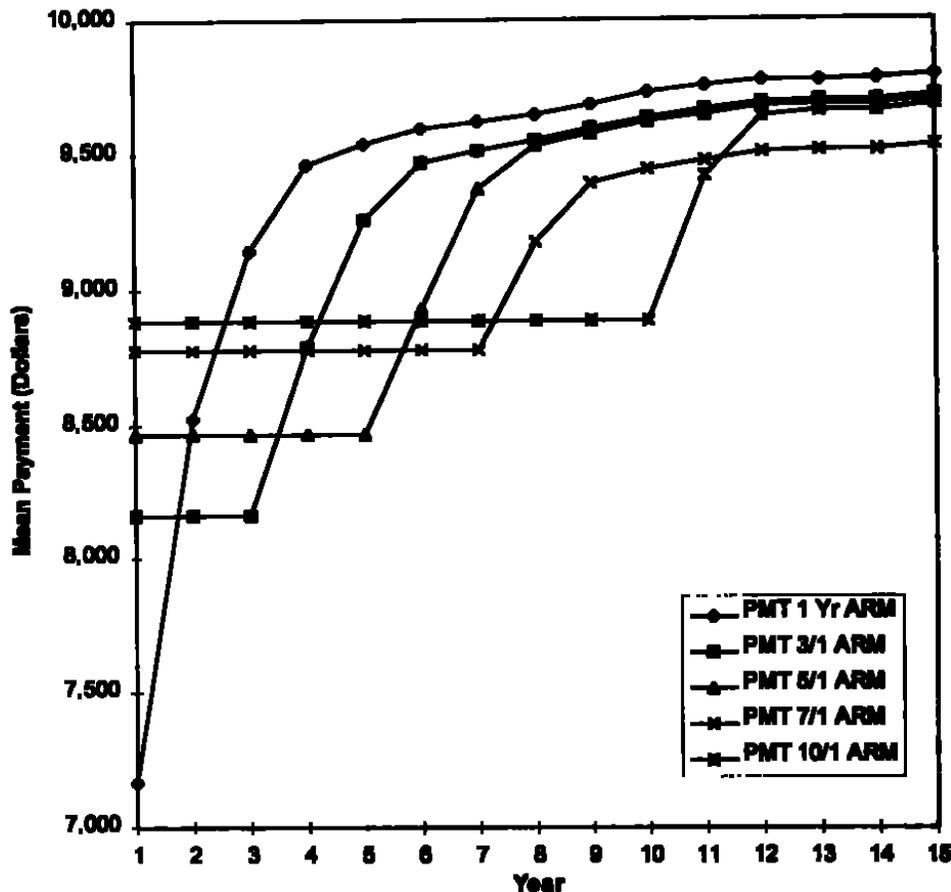
Fig. 2. Mean annual payments for all five loans over time.

rates and payments for the first three years. Starting in the fourth year, the 3/1 ARM's interest rate begins to adjust. In Year 4 and Year 5, the 3/1 ARM's interest rate adjusts, whereas the 5/1 ARM's does not. Thus, the 3/1 ARM is riskier than the 5/1 ARM for someone contemplating a mortgage of length greater than three years. Note, however, that the riskiness of the 3/1 ARM compared to the 5/1 ARM does not increase after the sixth year. This is because, starting in the sixth year, the two ARMs will have the same interest rate (except for differences caused by cap limitations on the 5/1 ARM as it begins to adjust). Any variation in the PV cost of these two ARMs is thus caused entirely by variation in the 3/1 ARM's rate in Year 4 and Year 5.

From this chart, we can see that a potential borrower contemplating holding an ARM for four years would almost certainly be better off with the 3/1 than with the 5/1 ARM. There is a very small probability that the PV cost of the 5/1 ARM would be lower than that for the 3/1. By contrast, there is a sizable probability that the 3/1 ARM could have an even larger PV cost advantage over the 5/1 ARM. However, the picture is not so favorable for the 3/1 ARM if the holding period is expected to be five years. In that case, their expected PV costs are virtually the same, but it is equally likely that the PV cost will be higher for the 3/1 as lower. As noted above, this variation in the PV cost difference between these two ARMs is
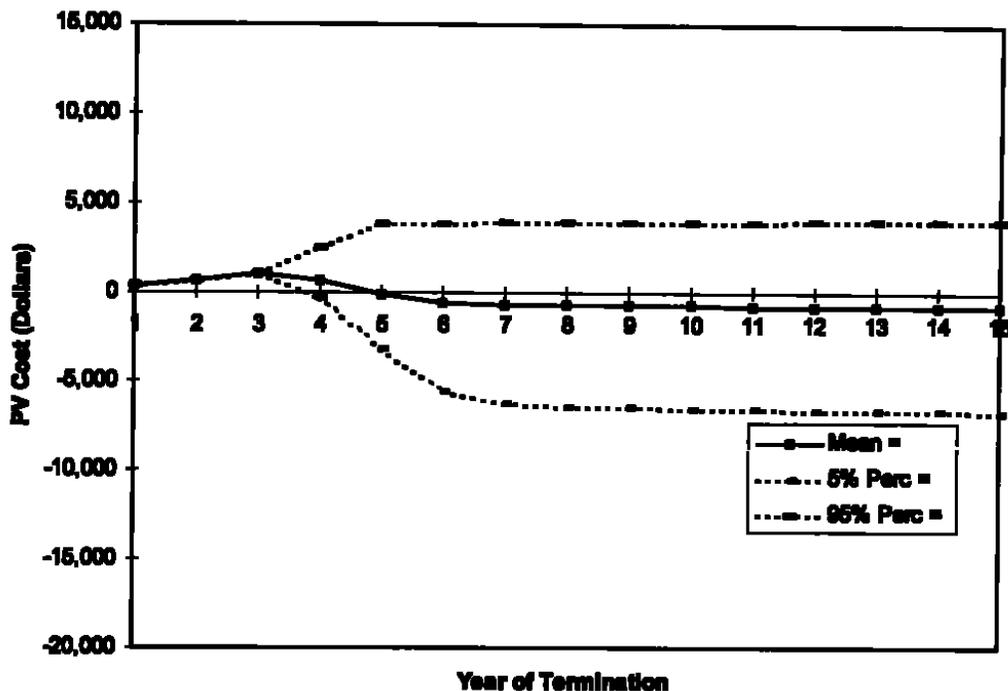
Fig. 3. Difference in PV cost between 5/1 ARM and 3/1 ARM. The heavy black line is the mean difference in PV cost (5/1 ARM − 3/1 ARM). Ninety percent of the observations lie between the dotted lines.

caused entirely by the variation in the 3/1 ARM's mortgage interest rate in Year 4 and Year 5. The 3/1 ARM's higher risk, combined with its slightly higher cost, makes it a less attractive option than the 5/1 ARM for a risk-averse borrower contemplating a five-year holding period. This sort of conclusion is evident with the simulation approach we have developed. It might be less obvious from a simple examination of the initial rates and other terms of the various ARMs.

Examining other outputs from the simulation can give the borrower additional insights. Fig. 4 shows the distribution of "breakeven" years for the 3/1 vs. the 5/1 ARM. The shorter fixed period ARM will have the PV cost advantage for short holding periods. Fig. 4 shows how long that initial cost advantage lasts in the 1,000-iterations of the simulation. The Figure shows that the cost advantage lasted just three years in 43% of the iterations. It lasted four years in 11% of the iterations. Cumulatively, there is about a 56% chance that the cost advantage of the shorter mortgage will last five years or less. From the sixth year on, these two ARMs will have virtually the same mortgage interest rate; thus, 44% of the time the cost advantage lasts the entire 15 years.

A comparison of likely payments for various ARMs would be useful for a borrower in planning his or her budget. Fig. 2 shows mean before-tax payments for the first 15 years for the five ARMs from Panel A of Table 1. Comparing the mean payments for the 3/1 and 5/1 ARMs, we see that the 3/1 ARM is likely to have a higher payment, starting in the fourth year, because its interest rate begins to adjust, and the teaser discount is removed. By the eighth year, the mean payments of the two ARMs are virtually identical, because their mortgage interest rates are based on the same index and margins. So, from the standpoint of
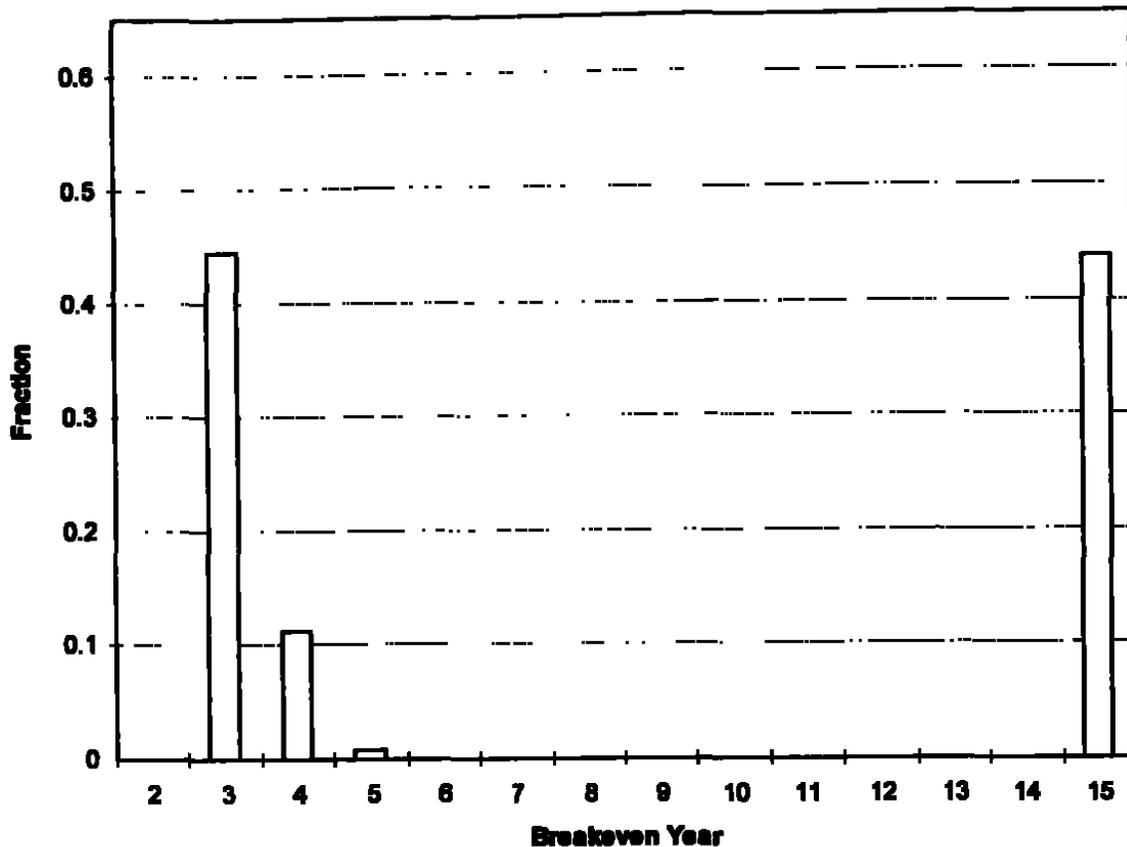
Fig. 4. Proportion of iterations in which the 5/1 ARM's PV cost exceeded the PV cost of the 3/1 ARM until the year in question. For example, in about 44% of the iterations, the PV cost of the 3/1 ARM began to exceed the PV cost of the 5/1 ARM in the fourth year. In about the same number of cases, the PV cost of the 5/1 ARM exceeded the 3/1 ARM's PV cost for at least 15 years.

budget planning, the two mortgages are likely to differ over a period of about eight years, with the 3/1 ARM having the (certain) advantage during the first three years and the 5/1 ARM having the probable advantage during the next five years.

Let us now consider a borrower contemplating a longer holding period. Suppose a borrower expected to hold the mortgage for eight years. Table 2 shows that the 5/1 ARM has the lowest mean PV cost for that holding period. The closest competitors are the 3/1 and 7/1 ARMs. They exceed the 5/1 ARM's mean PV cost by $476 and $421, respectively. Because they all have approximately the same mean PV cost, it is likely that a borrower's choice would be made in this case on the basis of variability of PV cost and payments. The 5/1 ARM is riskier than the 7/1 ARM. The adjustment (and likely increase) in the 5/1 ARM's interest rate in Year 6 and Year 7 open up the possibility that its PV cost could exceed the 7/1 ARM's cost by Year 8. Examination of Figs. 5 and 6, showing the difference in PV costs and the distribution of breakeven holding period, respectively, suggests a probability of slightly less than 0.5 that this will occur. We get a similar story if we focus on the payments for the 5/1 ARM and the 7/1 ARM in Fig. 2: the 5/1 ARM is approximately as costly as the 7/1 ARM, but the former is riskier than the latter. A risk-averse borrower might well choose the 7/1
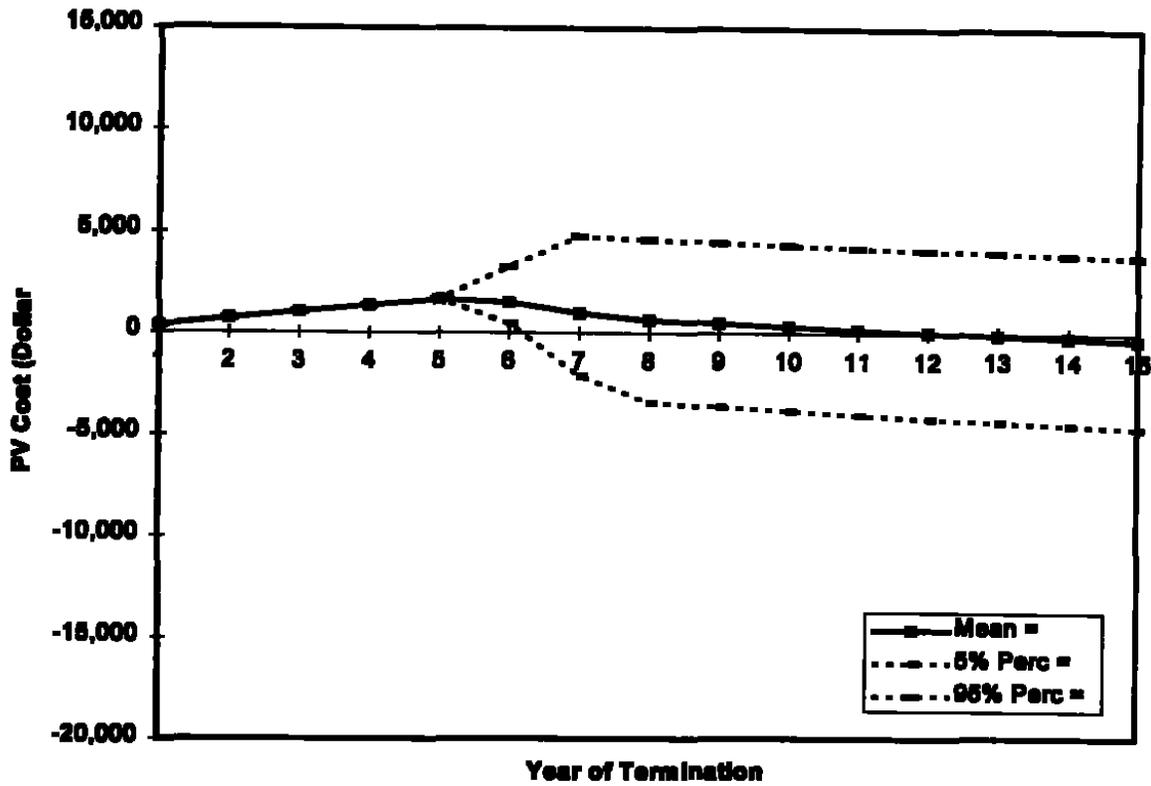
Fig. 5. Difference in PV cost between 7/1 ARM and 5/1 ARM. The heavy black line is the mean difference in PV cost (7/1 ARM − 5/1 ARM). Ninety percent of the observations lie between the dotted lines.

ARM if he or she expected to hold the mortgage for eight years. The choice will depend on the borrower's degree of risk aversion and the importance to the borrower of predictable payments. Similar analysis suggests that both a risk-neutral and a risk-averse borrower with an eight-year holding period would prefer the 5/1 ARM to the slightly costlier and significantly riskier 3/1 ARM.

3.2. Simulation comparing 1/1 arms with different adjustment caps.

Both the results above and results from Templeton et al. (1996) suggest 1/1 ARMs are best suited to borrowers with short anticipated holding periods, say three years or less. It is possible for such borrowers to take some of the risk out of these ARMs by opting for lower adjustment caps. In this second simulation we compare a 1/1 ARM with the typical 2% annual and 6% lifetime caps to one with 1% annual and 5% lifetime caps.

Panel B of Table 1 shows that the initial mortgage rate is 75 basis points higher for the lower cap ARM. Because the teaser discounts are high in both cases, the first annual adjustment will probably take both mortgage rates up the maximum adjustment. This occurrence would result in a 25-basis-point advantage during the second year for the lower cap ARM. Figs. 7 and 8 show the expected payments and cost differentials over 15 years. The higher cap ARM will probably have the PV cost advantage over the first three years, but
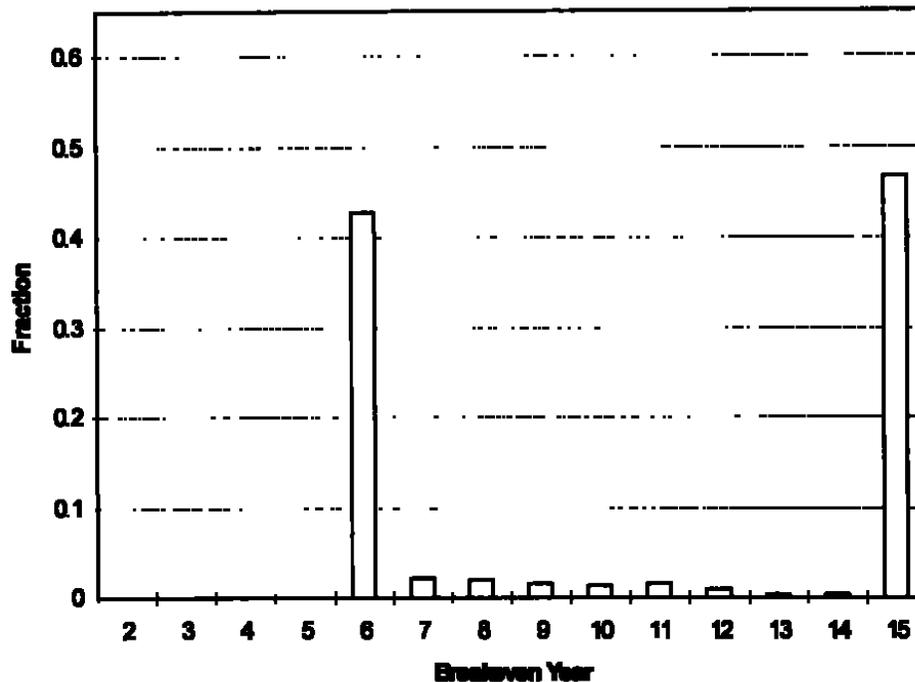
Fig. 6. Proportion of iterations in which the PV cost of the 7/1 ARM exceeds the PV cost of the 5/1 ARM until the year in question. For example, in about 42% of the iterations, the 5/1 ARM's PV cost began to exceed the PV cost of the 7/1 ARM in the seventh year. In about 47% of the cases, the 5/1 ARM retains its PV cost advantage for at least 15 years.

the advantage of lower payments for only one year. Once again, a borrower's attitude about risk and budget tolerance would determine which of these loans is preferable. For holding periods longer than three years, the lower cap loan seems much preferable, regardless of attitude toward risk, because it has both lower expected PV cost and payments. Of course, one-year ARMs are less desirable in general for borrowers with longer time horizons.

4. Conclusion

This study has applied the Monte Carlo simulation approach developed in an earlier study to the examination of contract terms of ARMs. ARMs are offered with different periods of time before their interest rates begin to adjust and different adjustment caps. Those with shorter periods of time before adjustment and higher caps typically have lower initial interest rates. Each ARM contract comes with a scheme for setting the adjusted interest rate, using an index, to which a margin is added. The initial rate quoted is invariably lower (by an amount called the teaser discount) than the sum of the index plus the margin for that ARM. The teaser is in place until the ARM's rate begins to adjust. The caps are limits to the amount by which an ARM's interest rate can adjust in a given year and over its lifetime.

Employing Monte Carlo simulation can shed light on the options open to a borrower. The mean and distribution of PV cost can be calculated for any ARM and compared to these
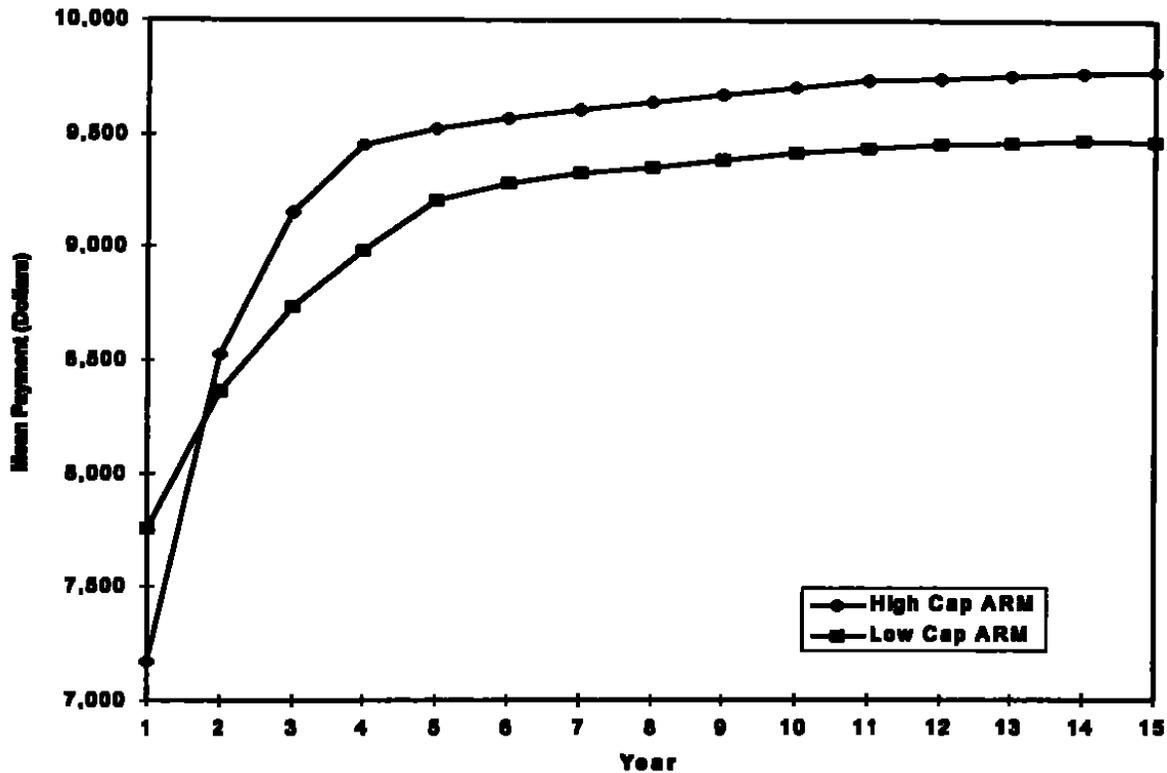
Fig. 7. Mean payments for 1/1 ARMs with high caps and low caps.

characteristics of other ARMs to illustrate the tradeoffs between cost and risk among ARMs. The frequency distribution of breakeven point for any two ARMs further helps one understand the relative advantages of different ARMs. In addition, the Monte Carlo approach allows one to compare the expected payments for different ARMs after any given number of years. Evaluating this information, in light of the borrower's attitude toward risk and expected holding period, can help the borrower make a more informed choice in this confusing market.

Notes

1. See Brueckner and Follain (1988); Dhillon et al. (1987); Goldberg and Hueson (1992); O'Brien and Wong (1990); Phillips and Vanderhoff (1991); Tucker (1989).
2. The opportunity discount rate is the after-tax rate of return a borrower could expect to earn on funds invested.
3. The procedure we use makes the expected index value rise or fall if the previous index value is not equal to the average of the floor and ceiling rates. Let i_t = the index value in period t. Then, if $i_t < (i_{max} + i_{min})/2$, then $E(i_{t+1}) > i_t$. If $i_t > (i_{max} + i_{min})/2$, then $E(i_{t+1}) < i_t$.
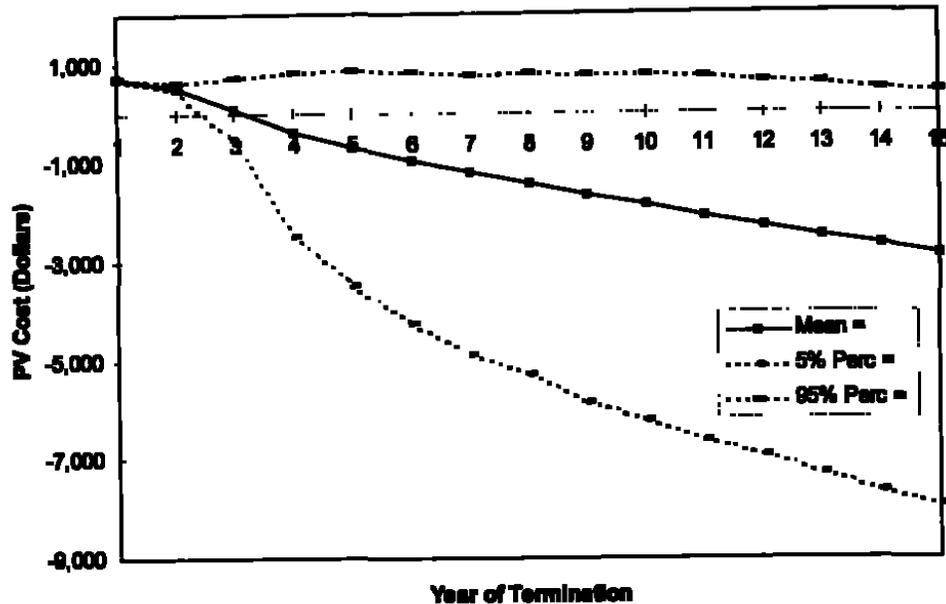
Fig. 8. Differences in PV cost between low-cap ARM and high-cap ARM. The heavy black line is the mean difference in PV cost (low-cap ARM − high-cap ARM). Ninety percent of the observations lie between the dotted lines.

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