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A Comparative Analysis of Retirement Portfolio Success Rates: Simulation Versus Overlapping Periods

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Abstract

One of the risks faced by retirees is the possibility of outliving money saved for the retirement years. Knowing the sustainability of withdrawal rates from a portfolio, or at least the risks associated with them, would greatly help retirees deal with this problem. Two procedures proposed to analyze the problem are Monte Carlo simulation and the overlapping periods methodology. This study compares and contrasts the implications of these two procedures for sustainable withdrawal rates from a retirement portfolio. Under some conditions, the procedures produce similar results, but in others the differences are quite large. © 2003 Academy of Financial Services. All rights reserved.

JEL classification: G1; G2

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1. Introduction

One of the many changes that the Internet has brought to retirement planning is the use of mathematical simulations to assist in retirement investment planning. Investors nearing retirement age become especially interested in how much income their portfolios will provide at reasonable risk-return tradeoffs. In other words, what would be a reasonable withdrawal rate from a retirement portfolio? Withdraw too much and the retiree dies broke, but withdraw too little and the retiree unnecessarily sacrifices a higher standard of living. A portfolio is successful only if it lasts as long as required by the retiree.

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Money Tree, U.S. News, Fidelity, Vanguard, Financial Engines, and NETirement are examples of companies that offer net-based simulations to help retirees. The typical simulation program provides Monte Carlo simulations that require input assumptions regarding personal characteristics and expected investment performance. The program then outputs estimates of the probability of retirement portfolio success assuming various initial withdrawal rates, portfolio compositions, and payout periods.

For retirees who are unfamiliar with mathematical simulation, the process may well be regarded as a "black box" out of which flows investment advice. The alternative approach to simulation relies on actual historical security returns and inflation rates. Referred to as the overlapping (or rolling) periods method, it makes intensive use of historical market returns to calculate periodic portfolio returns and end-of-period values net of planned retirement withdrawals. The sample size in the overlapping periods approach is limited to the available historical returns data, whereas simulation programs can simulate very large numbers of market returns from which a large number of hypothetical portfolios and payout periods may be evaluated. Special care, however, must be taken in simulating security returns so that the underlying time series properties (such as mean reversion) are preserved.

An obvious concern for retirees is which methodology produces more reliable results. If one approach reports significantly different portfolio success rates for the same range of withdrawal rates, portfolio compositions, and payout periods, retirees must make a methodological choice. Theoretically, if the simulation faithfully incorporates all meaningful market properties, the simulation methodology should produce highly robust results, looking at the effects of different portfolio withdrawal rates over potentially thousands of simulated holding periods. However, if actual security returns are generated from unstable distributions or different distributions over time, it might not be possible to build a realistic simulation. In such a situation, the overlapping periods methodology would be a more accurate predictor of sustainable withdrawal rates.

Fortunately, in many cases we find that the alternative methods produce similar results and no methodological choice is required. In these cases, the simulation methodology closely matches the market conditions of the overlapping methodology. There are, however, combinations of withdrawal rates, portfolio compositions, and payout periods for which the competing methodologies (and thus their underlying assumptions) produce substantially different portfolio success rates.

This study compares portfolio success rates generated by Monte Carlo simulation and the overlapping periods method for a common range of withdrawal rates, portfolio compositions, and payout periods. Conditions are identified where results of the two methods are similar and where they differ. The Monte Carlo simulation results reported differ from some published studies because of the level of sophistication employed. We simulate portfolio returns from separately simulated stock returns, bond returns, and inflation rates rather than simply assume a portfolio mean return and standard deviation. We also include correlations among security returns and inflation rates, serial correlation of inflation rates, and mean reversion in the security markets where appropriate.

2. Literature review

Recent interest in retirement planning has resulted in a growing number of studies that examine the survivability of retirement portfolios in relationship to asset allocation, payout periods, and withdrawal rates. Ho, Milevsky, and Robinson (1994) develop a model to estimate the optimal portfolio allocation between a risky security and a risk-free asset that minimizes the likelihood of withdrawals prematurely exhausting a retirement portfolio. They conclude that retirement portfolios should have larger allocations of equities than suggested by conventional wisdom on portfolio allocations by age of retiree. In a more recent study, Milevsky, Ho, and Robinson (1997) extend their earlier research by simulating financial market returns and life expectancies. The more recent analysis employing Monte Carlo simulations provides additional empirical support for larger equity allocations in retirement portfolios.

Pye (1999, 2000) provides additional examples of simulation studies in retirement planning and provides a useful explanation of Monte Carlo simulation as applied in retirement and endowment fund planning. He reports the results of simulations that examine sustainability of withdrawals from portfolios with mean real returns of 6% to 8% and standard deviations of returns of 15.9% to 18.0% in log-normal distributions. Pye concludes that conservative withdrawal rates are sustainable over long payout periods if real portfolio returns are expected to be 8% with a standard deviation of about 18%. He also notes that lower initial withdrawals reduce the chance of a shortfall in the future. In the case of fixed pension income from a defined benefit retirement plan, the retiree's real income can be sustained if 26% to 40% of the fixed payment is reinvested.

Cooley, Hubbard, and Walz (1998, 1999, 2001) employ overlapping periods of historical stock returns and corporate bond returns to investigate the sustainability of a wide set of withdrawal rates (3% to 12%) over multiple payout periods (15, 20, 25, and 30 years) within different portfolio asset allocations. Cooley et al., conclude that a withdrawal rate of 7% from portfolios of 50% or more equities should be considered sustainable for planning purposes. For shorter payout periods (15 years or less) they find that withdrawal rates of 8% to 9% can generally be sustained. However, the adjustment of periodic withdrawals to maintain purchasing power of retirement income greatly lowers the sustainability of withdrawal rates to the 4% to 5% range.

Three studies by Bengen (1994, 1996, 1997) are additional examples of applications of the overlapping periods methodology. Using security returns data reported by *Ibbotson Associates*, Bengen (1994) concludes that if the market behaves in the future the way it has in the past, the typical retirement portfolio should have a 50% to 75% equity allocation, which allows for a 4% inflation-adjusted withdrawal rate for 35 years. Bengen's (1996) second study reports essentially no effect on the sustainability of a 4% annual withdrawal rate when the percentage of stocks in a retirement portfolio is gradually reduced over time. In the third study, Bengen (1997) found that using quarterly security returns instead of annual returns did not alter his previous results. Bengen (1997) also suggests that small-cap equities in the place of large-cap equities in retirement portfolios can significantly increase sustainability of withdrawal rates over 30-year payout periods.

Like previous research on this topic we ignore the transaction costs associated with monthly portfolio rebalancing, and there are practical reasons for doing so. Individual portfolios may be institutionally managed or personally managed, and management fees and brokerage fees differ by firm and account type. Investors who pay a fixed asset management fee might not pay any additional fees for monthly rebalancing. Monthly rebalancing in an asset allocation mutual fund is done by the management of the investment company. Thus, specifying a transaction cost that would be generally applicable to most investors is next to impossible. Also, the integrity of the comparative analysis that we present in this paper is not impaired by omitting transaction costs in the two methodologies that we compare.

3. Methodology

To compare results from the two methodologies—simulation and overlapping periods—both are used to calculate portfolio success rates from month-end portfolio values net of monthly withdrawals for payout periods of 15, 20, 25, and 30 years. A portfolio success rate is the percentage of past payout periods during which the retirement portfolio provided planned withdrawals and finished the period with a positive value. Portfolio success rates are computed for nominal and for inflation-adjusted annual withdrawal rates of 3% through 12%. Sample portfolios in both methodologies consist of monthly rebalanced asset allocations of 100% common stocks, 75% stocks/25% bonds, 50% stocks/50% bonds, 25% stocks/75% bonds, and 100% bonds, where bonds are long-term, high-grade corporate bonds. Stocks are represented by Standard & Poor's 500 Index, and bonds are represented by the corporate bond index as constructed by Ibbotson Associates (2002).

3.1. Monte Carlo simulation

Monte Carlo simulation as applied here proceeds in the following manner. A withdrawal rate and a portfolio asset allocation are specified for the simulation of a payout period. Consistent with the given historical means and standard deviations of monthly security returns and correlations among security returns, the simulation draws random values for the first month's equity and bond returns. The total return on the portfolio is calculated for the period and added to the arbitrary beginning balance of $1,000. A monthly withdrawal from the portfolio is made, and the ending value of the portfolio net of that period's withdrawal is the beginning value for the next period, unless the portfolio value is zero or negative, in which case the portfolio fails. In contrast to the fixed monthly withdrawals, in the second year of the inflation-adjusted analyses, an inflation rate is used to adjust that year's monthly withdrawals.

For subsequent months, equity returns and bond returns (and inflation rates where appropriate) are randomly drawn based on assumed distributional characteristics and correlations. Month-end retirement portfolio values net of withdrawals are calculated to complete the payout period of 180, 240, 300, or 360 simulated months (15, 20, 25, or 30 years). Completion of an entire payout period concludes the first simulated iteration in the Monte Carlo simulation. The previous steps are then repeated an additional 999 times for a total of 1,000 iterations for each combination of withdrawal rate, asset allocation, and payout period.

The simulated portfolios that complete the payout periods with values greater than zero are deemed successful, and those that are depleted before the end of the payout period are failures. From the 1,000 iterations the number of successful portfolios as a percentage of 1,000 is recorded as the portfolio success rate for that particular combination of withdrawal rate, asset allocation, and payout period. The simulation process is then repeated for another combination until the entire study is completed for all combinations analyzed.

Like other recent studies (see Campbell, Lo, and MacKinlay, 1997, and Pye 1999, 2000), this study assumes that monthly continuously compounded security returns are normally distributed, which is equivalent to simple returns being lognormally distributed. Thus, all security returns are assumed to be generated from a lognormal distribution. Recent empirical studies, starting with Fama and French (1988) and Poterba and Summers (1988), also indicate that equity returns may be mean reverting. Specifically, these studies show that if equity prices contain a stationary "noise" component, equity returns should follow an autoregressive moving average process of order 1 [ARMA(1,1)]. We estimated an ARMA(1,1) model using the S&P 500 equity returns as calculated by Ibbotson Associates for the 1946 to 2001 period. We find the autoregressive coefficient for this sample period to equal 0.7278, and the coefficient is significant at the 0.0001 level. The moving average component equals 0.7727 and also is significant at the 0.0001 level. Thus, like previous researchers, we find evidence of mean reversion in equity returns. The intercept or mean monthly return for the ARMA model is 0.0103 and the standard error is 0.0414. These values are incorporated in the generation of the simulated equity returns.

When analyzing subperiods for mean reversion of stock returns, we found that the autoregressive coefficient is not significantly different from zero after 1969. Thus the inclusion of a mean reversion assumption in Monte Carlo simulation of future stock returns depends on the analyst's opinion of the distribution of those future returns. Because we are comparing simulation results with overlapping periods results for the full 1946 to 2001 sample, we chose to incorporate the mean reversion assumption described above in the simulated portfolio returns and success rates.

Monthly corporate bond returns were generated from a lognormal distribution with a mean monthly return of 0.0050 and a standard deviation of 0.0219. As with monthly equity returns, monthly corporate bond returns constructed by Ibbotson Associates (2002) over the 1946 to 2001 period were used to fit the distribution. The correlation between equity and bond returns was assumed to be 0.39, the actual correlation between monthly equity returns and monthly bond returns during the 1946 to 2001 period.

Monthly inflation rates were generated from a normal distribution with a mean of 0.0034 and standard deviation of 0.0046. Additionally, a first order auto-correlation of 0.46 was assumed for monthly inflation rates. Unlike equity returns, inflation rates and bond returns were not found to be mean reverting. Correlations of -0.16 and -0.11 were assumed between monthly inflation and equity and bond returns, respectively. These values are based on the actual first order auto-correlation of inflation rates and the actual correlations between equity and bond returns and inflation during the 1946 to 2001 period.

3.2. Overlapping periods

The overlapping periods methodology, sometimes referred to as rolling periods, calculates end-of-period portfolio values from historical stock returns and bond returns. Bierwirth (1994) and Cooley et al. (1998, 1999, 2001) are example applications of the overlapping methodology to the problem of sustainable withdrawal rates. At the completion of the analysis of a payout period, the sample payout period is "rolled" forward by one year, and another analysis is conducted. Thus from a total of 56 years of monthly returns, for example, there are 42 overlapping 15-year payout periods (56 − 15 + 1).

In this methodology as reported in Cooley et al. (2001) the first month's ending portfolio value is the arbitrary beginning value of $1,000 plus January 1946 returns to S&P 500 stocks and to long-term, high-grade corporate bonds less that month's withdrawal. The second month's ending portfolio value is the previous month's ending value plus the monthly portfolio returns less another withdrawal. That process of calculating month-end rebalanced portfolio values gross of returns and net of withdrawals continues through a payout period. At the completion of the first analysis of an n-year payout period, the first year's returns, say 1946, are dropped and the n-year sample is moved forward by one year. The process of analyzing payout periods and dropping the earliest year and adding the next year's returns continues until the available returns data are exhausted. In the inflation-adjusted analysis, the second year's monthly withdrawals are increased by the previous year's rate of inflation (or decreased for periods of deflation).

The overlapping periods methodology represents an attempt to maximize the number of observed payout periods from a limited number of historical periods. By overlapping the payout periods, this methodology produces success rates that rely heavily on the returns from the middle years of the overall study period. For example, consider the 27 30-year payout periods during 1946 to 2001: 1946 is included in only one of the 30-year periods; 1947 is included in two periods and so on until 1972, which is included in all 27 30-year periods, as are 1973 to 1975; then, 1976 is included in 26 of the periods, 1977 in 25 of the periods and so on until 2001, which is included in only one 30-year period. Because the sample payout periods overlap, the end-of-period portfolio values are not serially independent. Although there are 27 observed 30-year periods, they do not represent 27 independent observations. Shorter payout periods mitigate this problem as illustrated with the 15-year payout period. In this case, the years 1960 through 1987 are each used in 17 15-year payout periods, with years before 1960 and after 1987 being used fewer times. The larger number of shorter periods lessens the heavy reliance on a few middle years and provides a more reliable sample of payout periods.

3.3. Calculation steps

Calculations of monthly portfolio returns in both methodologies assume monthly rebalancing of portfolios to the desired allocation of stocks and bonds. Month-end portfolio values that determine portfolio success rates after nominal withdrawals are calculated as follows in both methodologies:

$$V_t = V_{t-1}(1 + R_t) - W_t \tag{1}$$

in which V_t is the remaining value of the portfolio at the end of month t, V_{t-1} is the value of the portfolio at the beginning of the month net of the previous month's withdrawal, R_t is the rate of return on the portfolio for month t, and W_t is the amount of money withdrawn from the portfolio at the end of the month. Where monthly withdrawals are fixed throughout the payout period, W_t is the initial value of the portfolio ($1,000) multiplied by annual withdrawal rate. For example, 6% of $1,000 equals $60, which divided by 12 produces $5 per month.

Month-end portfolio values that determine portfolio success rates after inflation-adjusted withdrawals are calculated as follows:

$$V_t = V_{t-1}(1 + R_t) - W_t(CPI_{Y-1}/CPI_0) \tag{2}$$

in which the variables are defined as in Eq. (1) except (CPI_{Y-1}/CPI_0) is the inflation adjustment [based on the Consumer Price Index (CPI) for urban consumers] for each year's monthly withdrawals. That is, a current year's monthly withdrawals are calculated by adjusting the previous year's total withdrawals up or down by the previous year's CPI value relative to the CPI value at the beginning of the n-year payout period. The adjusted monthly withdrawals are then fixed for the year. By using this procedure, the retiree maintains the purchasing power of each year's withdrawals with a one-year lag throughout the payout period. Lagging the inflation adjustment would enable a retiree to adjust withdrawals by a known rate of inflation.

In either methodology one could assume beginning-of-month withdrawals instead of end-of-month withdrawals. The initial withdrawal of a payout period would simply scale back the initial investment capital by that first month's withdrawal. In that case the withdrawal rates analyzed would apply to the reduced initial amount of investment capital. Thus the portfolio success rates reported here are unaffected by the end-of-month versus beginning-of-the-month timing decision.

The portfolio success rates reported below in Table 1 are developed using Eq. (1) to calculate terminal values of portfolios and are the result of constant withdrawal amounts through the payout periods without adjustment for inflation. The portfolio success rates in Table 2 assume that monthly withdrawals are inflation-adjusted annually and are the result of portfolio terminal values calculated with Eq. (2). That is, the investor is assumed to initiate withdrawals at a specific withdrawal rate and then adjust each subsequent year's, thus month's, withdrawal amount by the previous year's percentage change in the Consumer Price Index. The objective of inflation adjustment in Eq. (2) is to maintain the purchasing power of the monthly withdrawal amount.

4. Results

Tables 1 and 2 compare the simulation findings to the updated results from a previous study (Cooley et al., 2001), which uses the overlapping periods methodology. In the bodies of Tables 1 and 2 the portfolio success rate comparisons are reported as x/y in which x is the

Table 1
Portfolio success rates with fixed monthly withdrawals: simulated versus overlapping periods (percent of all past payout periods supported by the portfolio)

Payout period	Withdrawal rate as % of initial portfolio value									
	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100% Stocks										
15 years	100/100	100/100	100/100	99/100	97/100	94/100	87/90	80/83	69/74	57/64
20 years	100/100	100/100	99/100	97/100	93/100	86/95	77/81	66/65	54/51	45/51
25 years	100/100	99/100	98/100	95/100	89/100	81/84	70/63	58/47	48/44	36/31
30 years	100/100	99/100	97/100	93/100	86/100	77/81	66/56	53/41	43/37	32/30
75% Stocks/25% bonds										
15 years	100/100	100/100	100/100	100/100	98/100	95/100	89/95	78/81	65/64	51/55
20 years	100/100	100/100	100/100	98/100	95/100	87/95	74/76	60/54	45/46	34/41
25 years	100/100	100/100	99/100	96/100	90/100	78/81	63/53	50/44	36/34	25/22
30 years	100/100	100/100	98/100	94/100	86/100	71/67	57/44	44/30	31/22	20/11
50% Stocks/50% bonds										
15 years	100/100	100/100	100/100	100/100	99/100	96/100	89/95	75/74	59/52	39/43
20 years	100/100	100/100	100/100	99/100	94/100	84/89	70/59	49/43	30/24	18/16
25 years	100/100	100/100	100/100	96/100	88/100	74/59	53/41	34/16	20/6	10/3
30 years	100/100	100/100	99/100	93/100	81/85	66/41	42/11	27/0	15/0	7/0
25% Stocks/75% bonds										
15 years	100/100	100/100	100/100	100/100	99/100	95/100	83/81	64/45	41/29	22/24
20 years	100/100	100/100	100/100	98/100	92/100	75/57	53/30	30/19	13/16	5/8
25 years	100/100	100/100	99/100	95/100	80/56	57/25	32/16	14/6	5/3	1/0
30 years	100/100	100/100	98/100	89/96	69/22	44/11	22/4	8/0	2/0	1/0
100% Bonds										
15 years	100/100	100/100	100/100	100/100	98/100	90/64	69/48	43/33	22/24	8/17
20 years	100/100	100/100	100/100	96/86	81/46	55/41	28/27	12/14	4/8	1/8
25 years	100/100	100/100	97/94	85/53	57/31	30/25	13/9	4/0	1/0	0/0
30 years	100/100	99/99	93/59	71/26	41/19	19/7	7/0	2/0	0/0	0/0

Note: Each cell of the table reports portfolio success rates derived by Monte Carlo simulation and the Overlapping Periods methodology. Portfolio success rates are rounded to the nearest whole percentage. Stock returns are monthly returns to Standard & Poor's 500 Index; bond returns are monthly returns to high-grade, long-term corporate bonds. The returns data are those reported by Ibbotson Associates.

portfolio success rate from Monte Carlo simulation and y is the portfolio success rate using the overlapping periods methodology. Table 1 reports comparative portfolio success rates based on fixed monthly withdrawals where the withdrawals are calculated from a range of annual withdrawal rates. If a minimum success rate of 75% is used to indicate sustainability of a withdrawal rate, then Table 1 shows that withdrawal rates as high as 7%, and in many cases 8%, can be sustained by stock-dominated portfolios. The success rates from both methodologies illustrate the importance of a substantial equity allocation in retirement portfolios. This finding is consistent with Ho et al. (1994) and Milevsky et al. (1997), who report similar results for Canadian securities.

More risk-averse users of Table 1 might object to using 75% as the required minimum success rate. After all, a withdrawal rate that historically succeeded three out of four times also failed one out of four. Such failures in the future would require the retiree to reduce withdrawals below those planned and to live on a lower income. To anticipate and to lower the probability of these midcourse corrections, the retiree might wish to require a higher

Table 2
Portfolio success rates with inflation-adjusted monthly withdrawals: simulated versus overlapping periods (percent of all past payout periods supported by the portfolio)

Payout period	Withdrawal rate as % of initial portfolio value									
	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100% Stocks										
15 years	100/100	100/100	98/100	93/95	85/83	73/76	59/71	47/62	34/52	23/43
20 years	100/100	97/100	91/92	80/76	66/68	52/59	37/46	24/41	16/32	10/22
25 years	98/100	93/100	82/83	67/63	51/47	36/40	23/33	16/27	9/27	6/20
30 years	97/100	88/92	75/68	59/52	42/44	28/40	19/32	11/32	7/20	4/12
75% Stocks/25% bonds										
15 years	100/100	100/100	99/100	94/95	83/81	68/74	52/64	37/52	24/43	13/31
20 years	100/100	98/100	90/92	75/73	60/65	41/51	26/43	15/27	7/22	3/11
25 years	99/100	91/100	77/77	61/50	40/40	25/30	15/23	7/13	3/7	1/0
30 years	96/100	84/92	67/68	48/44	30/32	17/32	9/12	4/8	2/0	1/0
50% Stocks/50% bonds										
15 years	100/100	100/100	99/100	94/98	82/76	61/64	40/55	21/40	10/29	5/14
20 years	100/100	99/100	89/89	70/68	45/49	24/35	12/22	4/11	2/3	1/3
25 years	99/100	91/100	71/67	44/37	23/27	10/10	4/0	1/0	0/0	0/0
30 years	98/100	81/84	53/44	30/32	12/12	5/0	2/0	0/0	0/0	0/0
25% Stocks/75% bonds										
15 years	100/100	100/100	99/100	93/98	75/74	44/55	21/31	8/19	2/14	1/12
20 years	100/100	98/100	85/84	53/54	24/30	8/8	3/8	0/5	0/3	0/3
25 years	99/100	86/93	55/40	21/7	7/0	2/0	0/0	0/0	0/0	0/0
30 years	94/100	66/60	29/8	9/0	3/0	1/0	0/0	0/0	0/0	0/0
100% Bonds										
15 years	100/100	100/100	97/100	84/83	54/45	23/19	9/19	2/14	0/12	0/12
20 years	100/100	93/100	64/62	27/19	10/8	2/8	0/5	0/5	0/3	0/3
25 years	95/100	65/57	25/7	8/0	1/0	0/0	0/0	0/0	0/0	0/0
30 years	79/92	35/0	10/0	2/0	0/0	0/0	0/0	0/0	0/0	0/0

Note: Each cell of the table reports portfolio success rates that were calculated by Monte Carlo simulation and then by the Overlapping Periods methodology. Portfolio success rates are rounded to the nearest whole percentage. Stock returns are monthly returns to Standard & Poor's 500 Index; bond returns are monthly returns to high-grade, long-term corporate bonds. The returns data are those reported by Ibbotson Associates. The Consumer Price Index values that were used to inflation adjust withdrawals are published by the U.S. Department of Labor at http://wwws.bls.gov.

success rate, say 90%, and withdraw smaller dollar amounts in the earlier years of retirement. As Table 1 shows, the requirement of 90% success reduces the sustainable withdrawal rate to 6% for stock-dominated portfolios. Other sensitivity analyses may proceed in like manner.

In general, the portfolio success rates reported in Tables 1 and 2 decline with higher withdrawal rates, longer payout periods, and higher bond allocations. Because of their limited upside potential, bonds provide limited support for the higher withdrawal rates. Bond-dominated portfolios can be expected to support a 7% withdrawal rate only for the shorter 15 and 20-year payout periods. For the longer payout periods, bond-dominated portfolios support withdrawal rates closer to 5% and perhaps 6%.

Table 2 reports portfolio success rates generated from the simulation and overlapping periods methodologies when monthly withdrawals are adjusted for inflation each year. As shown in Table 2, inflation-adjusted withdrawals greatly reduce the sustainability of the

Table 3
Difference in portfolio success rates with fixed monthly withdrawals: simulation minus overlapping periods

Payout period	Withdrawal rate as % of initial portfolio value									
	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100% Stocks										
15 years	0	0	0	−1	−3	−6	−3	−3	−5	−7
20 years	0	0	−1	−3	−7	−9	−4	1	3	−6
25 years	0	−1	−2	−5	−11	−3	7	11	4	5
30 years	0	−1	−3	−7	−14	−4	10	12	6	2
75% Stocks/25% bonds										
15 years	0	0	0	0	−2	−5	−6	−3	1	−4
20 years	0	0	0	−2	−5	−8	−2	6	−1	−7
25 years	0	0	−1	−4	−10	−3	10	6	2	3
30 years	0	0	−2	−6	−14	4	13	14	9	9
50% Stocks/50% bonds										
15 years	0	0	0	0	−1	−4	−6	1	7	−4
20 years	0	0	0	−1	−6	−5	11	6	6	2
25 years	0	0	0	−4	−12	15	12	18	14	7
30 years	0	0	−1	−7	−4	25	31	27	15	7
25% Stocks/75% bonds										
15 years	0	0	0	0	−1	−5	2	19	12	−2
20 years	0	0	0	−2	−8	18	23	11	−3	−3
25 years	0	0	−1	−5	24	32	16	8	2	1
30 years	0	0	−2	−7	47	33	18	8	2	1
100% Bonds										
15 years	0	0	0	0	−2	26	21	10	−2	−9
20 years	0	0	0	10	35	14	1	−2	−4	−7
25 years	0	0	3	32	26	5	4	4	1	0
30 years	0	0	34	45	22	12	7	2	0	0

Note: The differences above are calculated from the portfolio success rates that are reported in Table 1. Each difference is the success rate derived from Monte Carlo simulation less the success rate derived from the overlapping periods methodology.

range of withdrawal rates. At the 75% success rate standard, sustainable withdrawal rates drop two or three percentage points depending on the payout period and common stock composition of the portfolio. Once again, stock-dominated portfolios outperform the bond-dominated portfolios. Even with stock-dominated portfolios, however, the sustainable withdrawal rate for the longer payout periods is only in the 4% to 5% range. For the more demanding 90% standard, only withdrawal rates below 4% are supported.

Comparing the findings in Tables 1 and 2 reveals similarity in the reported success rates for the 7% nominal withdrawal rate and the 4% real withdrawal rate. For example, the simulated result for the 50% stocks-50% bond portfolio and a 30-year payout shows an 81% success rate for the 7% fixed withdrawal and an 81% success rate for the 4% inflation-adjusted withdrawal rate. This finding roughly implies an annual inflation rate of 3%, which approximates the actual historical inflation rate of 4.1%.

Table 3 reports the differences between the success rates in Table 1 by methodology. The figures in Table 3 are calculated by subtracting overlapping periods portfolio success rates from simulated portfolio success rates. On average the Monte Carlo simulation success rates are higher than the overlapping periods success rates when monthly withdrawals are fixed

Table 4
Monthly returns and inflation rates

Monthly rate	1970-1981	1946-1969 & 1982-2001	1946-2001
Equity returns	0.0066 (0.0460)	0.0114 (0.0402)	0.0104 (0.0415)
Bond returns	0.0044 (0.0313)	0.0052 (0.0186)	0.0049 (0.0219)
Inflation rates	0.0064 (0.0036)	0.0026 (0.0044)	0.0034 (0.0046)

throughout the payout periods. The larger differences occur in the longer payout periods and when bonds are included in the portfolios. The largest difference occurs for the portfolio of 25% stocks and 75% bonds, the 30-year payout period, and the 7% withdrawal rate. The 47-percentage point difference stems from the 69% success rate of simulation and 22% success rate of overlapping periods. The explanation for this dramatic difference lies in the statistical properties implicit in the two methodologies.

By construction, the overlapping periods approach does not treat all sample observations with equal weight. The middle observations are included in more overlapping periods and have a more significant impact on portfolio results. For example, the January 1946 monthly bond and equity returns are used in only one overlapping periods portfolio. However, the January 1973 returns are used in all of the 30-year payout portfolios. The middle sample observations (1970–1981) dominate the overlapping portfolio results and are very different from the remaining sample observations. These differences are illustrated in the following table of monthly returns and inflation rates, with standard deviations in parentheses (Table 4).

It is important to note that the unique characteristics of the 1970 to 1981 subperiod are not statistically significant in and of themselves. That is, the null hypothesis of coefficient stability for any of the ARMA(1,1) models estimated over the entire 1946 to 2001 period cannot be rejected. The means and mean reversion seem stable over the entire sample period, inclusive of the 1970 to 1981 subsample. The lower equity and bond returns of the 1970 to 1981 period would not significantly affect the overlapping periods results (given this period is quite short relative to the entire sample period) except for the fact that this subperiod occurs roughly in the middle of the entire sample period. Thus, the 1970 to 1981 security returns and inflation rates are included in more overlapping periods than the returns of other subsamples. In other words, the overlapping methodology magnifies the effect of this uniquely negative 1970 to 1981 period, which in turn causes significant differences in portfolio success rates between simulation and overlapping periods methodologies.

Consider the effect of length of payout on the relative success rates of the two methods. We would expect the effect of this unusual period on the simulation results to be inconsequential, since the mean security returns and inflation rates that are drawn from the total sample period returns are the same for all payout period lengths. On the other hand, in view of the 1970 to 1981 returns and inflation rates, we would expect longer payout periods in the overlapping periods approach to diminish the portfolio success rates more than in the simulation approach. A longer payout length causes more overlapping periods to contain observations from the 1970 to 1981 period, diminishing portfolio performance and leading to lower portfolio success rates.

Table 5
Difference in portfolio success rates with inflation-adjusted monthly withdrawals: simulation minus overlapping periods

Payout period	Withdrawal rate as % of initial portfolio value									
	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100% Stocks										
15 years	0	0	−2	−2	2	−3	−12	−15	−18	−20
20 years	0	−3	−1	4	−2	−7	−9	−17	−16	−12
25 years	−2	−7	−1	4	4	−4	−10	−11	−18	−14
30 years	−3	−4	7	7	−2	−12	−13	−21	−13	−8
75% Stocks/25% bonds										
15 years	0	0	−1	−1	2	−6	−12	−15	−19	−18
20 years	0	−2	−2	2	−5	−10	−17	−12	−15	−8
25 years	−1	−9	0	11	0	−5	−8	−6	−4	1
30 years	−4	−8	−1	4	−2	−15	−3	−4	2	1
50% Stocks/50% bonds										
15 years	0	0	−1	−4	6	−3	−15	−19	−19	−9
20 years	0	−1	0	2	−4	−11	−10	−7	−1	−2
25 years	−1	−9	4	7	−4	0	4	1	0	0
30 years	−2	−3	9	−2	0	5	2	0	0	0
25% Stocks/75% bonds										
15 years	0	0	−1	−5	1	−11	−10	−11	−12	−11
20 years	0	−2	1	−1	−6	0	−5	−5	−3	−3
25 years	−1	−7	15	14	7	2	0	0	0	0
30 years	−6	6	21	9	3	1	0	0	0	0
100% Bonds										
15 years	0	0	−3	1	9	4	−10	−12	−12	−12
20 years	0	−7	2	8	2	−6	−5	−5	−3	−3
25 years	−5	8	18	8	1	0	0	0	0	0
30 years	−13	35	10	2	0	0	0	0	0	0

Note: The differences above are calculated from the portfolio success rates that are reported in Table 2. Each difference is the success rate derived from Monte Carlo simulation less the success rate derived from the overlapping periods methodology.

We would also expect that as the equity percentage is increased, the portfolio success rates for overlapping periods for samples dominated by the 1946 to 1969 and 1982 to 2001 time periods to increase, while the success rates largely affected by the 1970 to 1981 period decrease. That expectation is observable in the success rate differences in Table 3 for the 100% equity and 75% equity portfolios and the shorter 15-year and 20-year payout periods. Thus, as the equity allocation for the retirement portfolio is increased, the portfolio success rates for the overlapping periods portfolios drawn from those pre-1970 and post-1981 periods increase relative to the success rates that are derived from simulation. Nothing analogous to that subperiod effect occurs in Monte Carlo simulation.

Table 5 presents the simulated success rates less the overlapping periods success rates from Table 2 where withdrawals are inflation-adjusted. The average value of the differences in Table 5 is negative; that is, the portfolio success rates net of inflation-adjusted withdrawals in the overlapping periods methodology are on average higher than those from the simulations. However, when viewed in sections, Table 5 tells different stories. For example, the differences in Table 5 that are related to the moderate withdrawal rates of 4% to 8% and portfolios of 25% equities to 75% equities, which is where most retirees are likely to manage their portfolios, are relatively small. Also, when the differences are examined by payout period, the large negative differences tend to occur more often in the shorter payout periods at the higher withdrawal rates.

The explanation of the payout period related differences once again lies in the different statistical assumptions implicit in the two methodologies. The shorter payout periods that occurred before and after the 1970 to 1981 subsample produced significantly higher portfolio success rates net of inflation-adjusted withdrawal. As seen in the subsample returns and inflation data reported above, the real returns to the securities are much higher in the pre-1970 and post-1981 periods than in the overall 56-year sample. The simulation study, however, makes uniform use of distributional assumptions that are derived from the entire 56-year sample, unlike the overlapping periods methodology. The resulting portfolio success rates are a function of assumed means, standard deviations, and security correlations derived from the entire 1946 to 2001 sample period. Randomness in the simulation procedure lessens the probability of catastrophic runs of high inflation and low security returns and unusual runs of low inflation and high security returns. What actually happened throughout the 1970 to 1981 period is an unlikely event in simulation!

5. Summary and implications

Despite some differences in findings, simulation and the overlapping periods methodology imply sustainability of a fixed annual 7% withdrawal rate for 30 years from portfolios with at least 50% stock. When withdrawals are adjusted for inflation, both methodologies imply sustainability of the 4% (plus) withdrawal rate for the 50% stock-50% bond portfolio over 30 years. These implications are drawn assuming that 75% is the lowest acceptable portfolio success rate. The individual's perspective (risk aversion) on the required minimum portfolio success rate will alter the interpretation of which withdrawal rates are sustainable and acceptable. For planning purposes, however, 75% seems reasonable—after all, midcourse corrections subsequently are possible and should in fact be expected. Individuals who require more assurance and demand a 90% success rate must lower their withdrawal rates by at least one percentage point.

This study shows where differences occur in portfolio success rates when using the competing methodologies of simulation and overlapping periods. Further, it provides explanations of the differences, but it does not take sides on which methodology is better. The more reliable methodology largely depends on which historical return distribution better reflects the future, which cannot be known. The overlapping periods approach overweights the return experience of the midyears of the 1946 to 2001 study period, whereas simulation assumes a stable distribution for the period based on historical means, standard deviations, and correlations among stocks, bonds, and inflation.

The impact of the overweighting of midsample period returns in the overlapping periods approach is most clearly observable when withdrawals are fixed and the payout period is long. When withdrawals are inflation-adjusted, the higher-than-average real returns in the subsamples before and after the 1970 to 1981 period resulted, on average, in higher portfolio success rates reported by the overlapping periods approach for the shorter payout periods of 15 and 20 years. Also, we note that there are only 27 overlapping 30-year payout periods during 1946 to 2001, not one of which is independent of all the others. Because the consideration of long payout periods is important in retirement planning, analysts may prefer simulation and the inherent independence for the study of longer payout periods. For short payout periods with reasonable withdrawal rates, the competing methodologies produce fairly similar results. In these cases, the overlapping periods methodology may be preferred because of its transparency in comparison to the "black box" of simulation.

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