FINANCIAL
SERVICES
REVIEW

Financial Services Review 12 (2003) 179–199

Valuing Defined-Benefit Plans

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Accepted 18 July 2003

Abstract

We examine issues surrounding the valuation of defined-benefit pension plans including benefit formulas, integration with Social Security, postretirement benefit increases, and default risk. We obtain a reasonable valuation with three key estimates—the level of retirement benefits, the growth rate of postretirement benefits, and the discount rate. We consider the PBGC guarantee afforded many DB pensions. Usually, benefits are essentially default-risk-free, and the discount rate can be based on Treasury yields. We also offer methodological advances over current approaches. DB valuation is crucially relevant to asset allocation decisions and has litigation implications. © 2003 Academy of Financial Services. All rights reserved.

JEL classification: G11; J26; J17; K13

Keywords: Portfolio choice; Defined-benefit pensions; Valuation

1. Introduction

Bodie, Merton, and Samuelson (1992), Scott (1995), Fraser, Jennings, and King (2000), Jennings and Reichenstein (2001), and Campbell and Viceira (2002) contend that the present value of income streams like retirement benefits should be included when making asset mix decisions. Similarly, we argue that it is inconsistent to include pension benefits when evaluating retirement income needs but exclude pension benefits when designing a portfolio

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to meet those needs. That is, it is inconsistent to include them on the forecast income statement but not the balance sheet. This article addresses this inconsistency for defined-benefit (DB) plans by detailing a valuation approach.

A complete treatment of this paper's goal requires discussion of several topics: (1) retirement-income formulas, (2) Social Security integration, (3) postretirement benefit increases, and (4) assessments of default risk. Further, the value of a DB plan to a particular participant depends upon individual-specific estimates such as life expectancy.

Once we know the individual-specific estimates, we can obtain a reasonable value estimate with three key inputs. The first is the current level of retirement benefits (or initial level of benefits if not yet retired). The second key input is the growth rate in postretirement benefits. The third is the discount rate; in most cases, benefits are essentially risk free.

Although classical approaches to asset allocation ignore pension value, new research has encouraged its valuation and inclusion in the family portfolio. Scott (1995) includes assets that generate spending money or that can be sold for spending money in the family investment portfolio; her first-generation approach to pension valuation uses Treasury bonds for the discount rate (with an essentially ad hoc inflation adjustment). The second-generation approach of Fraser, Jennings, and King (2000) uses inflation-indexed Treasuries to obtain a market-based inflation adjustment in valuing Social Security pensions. Jennings and Reichenstein (2001, 2002b) and Goodman (2002) advance the Fraser et al. technology by switching from simple life expectancies—point estimates of mortality—to using the probability distribution of mortality. In this paper, we advance the literature in both technical and general ways. We add precision by using midyear cash flows, semiannual bond pricing, and the full yield curve, and we apply the techniques in the realm of defined-benefit plans.

Section 2 provides background of DB plans. Section 3 discusses estimation issues associated with (1) benefit formulas, (2) integration with Social Security, (3) payout options, (4) potential adjustments in postretirement benefits, and (5) determining the discount rate. Section 4 applies the approach to asset allocation, while Section 5 covers an advanced variation. Section 6 offers a research agenda, and Section 7 concludes.

2. Defined benefit plans

Despite the proliferation of defined-contribution plans, DB plans remain popular. From Table 1, typical government-sponsored public plans have a benefits formula based on earnings, do not pay into Social Security, and the benefits formula is not integrated with Social Security. About half of public plans provide automatic increases in postretirement payments, with half of these tied to inflation. In contrast, typical business-established private plans are governed by the Employment Retirement Income Security Act (ERISA), have a benefits formula based on earnings, do pay into Social Security, and the benefits formula is integrated with Social Security. Private plans rarely provide automatic increases in postretirement benefits.

Table 1
Public and private defined benefit pension plan design features

Benefit formulas	Public	Private
Dollar amount basis	13%	22%
Earnings basis	82%	75%
Other basis	5%	3%
Benefits integrated with SS	32%	63%
Postretirement increase		
Automatic	50%	Rare
Full CPI	27%	Rare

Source: adopted from Mitchell and Carr (1996), "State and Local Pension Plans." In Jerry S. Rosenbloom, ed., *The Handbook of Employee Benefits*, 4th Edition, Burr Ridge, IL: Irwin Professional Publishing.

2.1. Benefit formulas

Benefit formulas are usually based on earnings. From Table 1, 82% of public plans are earnings-based; among private plans, 75% are earnings-based. The most popular structure of earnings-based formula promises payments equal to a "percent" × years of service × salary. For example, the initial annual benefits level may equal 1.1% of average annual salary over the highest-earning three years multiplied by years of service.

The DB plans "percent" tends to be smaller in plans whose participants also contribute to Social Security. In these plans, employees pay Social Security taxes and are entitled to Social Security benefits. Because they receive retirement income from both the DB plan and Social Security, the DB plan needs to replace a smaller portion of preretirement income. As a separate point, their benefit formula is likely to be integrated with Social Security payments (see below). In public plans that are not part of the Social Security system, the "percent" is usually larger since it needs to be larger to replace more preretirement income. Typically, the public plan benefit formula will not be integrated with Social Security.

The less typical dollar-amount benefit formula may specify a dollar amount per month × months of service. For example, the initial annual benefits level may equal $35 per month times months of service. The payment does not vary with the participant's income. These plans often occur in unionized industries.

2.2. Integration with Social Security

Integration with Social Security is a separate issue from participation in the Social Security system. When integrated, DB benefits are reduced. Table 1 shows only 32% of public plans are integrated with Social Security benefits, while 63% of private plans are integrated.

There are two types of integration—offset and step-rate. The offset approach may reduce DB benefits by up to half of initial Social Security benefits. For example, a DB plan might specify a benefit formula that sets initial benefits using an earnings-based formula but reduces benefits by 50% of initial Social Security benefits. Although Social Security benefits subsequently increase with inflation, the reduction in DB benefits remains constant.

Step-rate integration is sometimes called an excess-earnings approach. This approach specifies two percentages. For example, the base rate may be 1.25% for income up to an integration level, and the excess rate may be 2% for income above that level. The maximum allowable integration level is Social Security's maximum taxable earnings base ($84,900 in 2002); plans frequently adopt this maximum. The difference between the base rate and excess rate is called the "disparity." If the base rate is at least 0.75%, then the maximum permitted disparity is 0.75%; otherwise, the maximum permitted disparity is the base rate.

For example, assume that the earnings base is $84,900, and a DB plan specifies an earnings-based formula that sets initial benefits at 1% of final pay times years of service up to the earnings base, plus 1.5% of final pay for income above that amount. If an employee had final pay of $100,000 and 24 years of service, she would receive $25,812 in DB benefits during her first year of retirement, 0.01 × (24) × $84,900 + 0.015 × (24) × $15,100.

Table 1 statistics can be somewhat misleading in that it appears that integrated plans have reduced benefits compared to nonintegrated plans. Clearly, participants in integrated plans receive reduced benefits from their DB plans; however, participants in nonintegrated DB plans often receive reduced benefits from Social Security because of the Windfall Elimination Provision (see Jennings and Reichenstein, 2002a). Thus, in practice, the combined retirement income from DB plans and Social Security are often *effectively* integrated.

2.3. Automatic and discretionary increases

About half of public plans provide automatic increases in postretirement benefits. A little over half of the automatic increases fully reflect inflation. Other plans adjust benefits by a constant percentage per year. Private plans rarely provide automatic increases in postretirement benefits; however, some have discretionary increases when the firm's financial health permits. To be conservative, we recommend assuming benefits will not be subject to discretionary increases.

3. Estimation issues

3.1. Level of retirement income

A current retiree knows her benefits. A current worker must estimate the initial level of benefits. This estimate depends upon the benefits formula, retirement age, and the payout option. While the plan's benefits formula is known, the latter two factors are individual-choice variables.

The plan specifies the benefits formula. We recommend that the estimate of initial level of retirement income reflect current average salary and current years of service but subsume the fact that the worker may not have yet attained full retirement age. For example, assume Mary's plan promises annual retirement benefits of 2% times years of service times average income during the highest four years if she retires after attaining full retirement age, defined in her plan as age plus years of service of 80 or more. Suppose Mary is 60 years old with 18 years of service, an average salary of $40,000, and expects to retire in six years. We recommend that she estimate her initial annual benefits based on 18 years of service and $40,000 but ignore the reduction in benefits that would occur if she retired today before attaining full retirement age. Although if she retires in six years, she will have 24 years of service and will likely have a higher average income, it is generally inappropriate to consider the rewards of future work when calculating the family's current asset mix. It is notionally more conservative to exclude the extra value.[1]

There are at least two potential objections to our recommended approach. First, it seems to suggest that Mary's pension is more valuable if she retires in say four years instead of six years. In fact, by continuing to work, the level of benefits grows by the compound growth rate in years of service and salary. Because of this compound effect, DB plans are especially valuable to workers who have long careers at the same firm. Woerheide and Fortner (1994) analyze the severe penalty associated with changing from one DB plan to another.

The second potential objection is that we assume the payout associated with normal retirement age even though she is younger. This represents a middle course between ignoring prospective retirement benefits in strategic asset allocation—the traditional financial planning approach—and crediting the worker with probable (but unearned) salary and years of service increases. Clearly, individual valuation should reflect planned retirement age including any early retirement penalties.

In making the distinction between accrued benefits and projected benefits, we are somewhat conservative. Our approach is akin to the Generally Accepted Accounting Principles' (GAAP) treatment of corporate pension liabilities known as the accrued benefit obligation, which is reflected on the balance sheet. The accrued benefit obligation relies on current wages and current years of service; it is a smaller estimate of a pension's liability than the GAAP projected benefit obligation, which includes projected salary growth and projected years of service at retirement.

Initial annual benefits also depend on the choice of payout options. Most plans have several different payout options including:

- Lifetime income with no payment guarantee.
- Lifetime income with payments guaranteed for 10 years. This payout would pay her a lifetime income and, should she die before receiving 120 monthly payments, her beneficiary would receive the remainder of the 120 payments. There also may be lifetime income with 5-, 15-, or 20-year guarantees. These options are called "years certain" or "term certain."
- Lifetime income with 50% to primary beneficiary—usually the spouse. This promises a lifetime income and would pay the primary beneficiary 50% of that amount. There also may be other guarantees at different percentages.
- Hybrids that combine one or more joint beneficiary options with a "term certain" guarantee.

Married participants usually select a joint payout option that promises payments for as long as either the participant or the spouse is alive. Unless the spouse waives the right in writing, the participant must select a joint payout option in many plans.

Our estimates of the value of DB benefits focus on the value of retirement benefits available for retirement. That is, we ignore term-certain guarantees that potentially extend benefits beyond an individual's or couple's lifetime. For example, if a single man chooses a lifetime income with 10-year certain guarantee, our model considers the value of lifetime payments but ignores potential payments to heirs if he dies before receiving 120 monthly payments.

3.2. *Discount rate*

Lastly, we must estimate the discount rate used to reduce future income streams to a present value. An appropriate discount rate is risk-adjusted and reflects the promised growth in postretirement benefits. Our model of the appropriate discount rate is the classic one:

$$k = Rf - g + RP \tag{1}$$

where k is the discount rate, Rf denotes the risk-free rate, g is the annual growth rate in postretirement income, and RP is the default risk premium. We discuss each of these components below.

3.2.1. *Core rate*

Like most economists, we recommend that the risk-free rate be measured by a Treasury yield. Specifically, we recommend using the adjusted Treasury yield on the bond with maturity closest to the participant's life expectancy. To be precise, the yield should be adjusted for bond's semiannual pricing convention. Section 5 presents an extension that uses the full yield curve.

There are potential criticisms to any Rf estimates. First, interest on Treasury debt is tax-exempt at the state and local level, potentially biasing the observed value. Second, the discount rate used to value DB postretirement benefits could be higher because postretirement benefits are illiquid. However, we are not interested in the liquidity-adjusted "fair market value" when valuing pensions for retirement planning, but rather in the individual-specific valuation.

The growth rate, g, depends upon increases in postretirement income. If postretirement income increases with inflation, then the plan promises a constant real income, and we can discount the constant *real* income by an appropriate *real* interest rate; in this case, the value, $Rf - g$, can be estimated by the yield on the maturity-appropriate Treasury Inflation Protection Security (TIPS). Both postretirement income and TIPS cash flows increase with inflation. Note that this approach relies on a market-based inflation expectation, not an idiosyncratic forecast. If the retirement benefit increases by a fixed percentage each year, then we can deduct the fixed percentage, g, from the appropriate nominal risk-free rate.

3.2.2. *Default risk*

The last estimate is the size of the default risk premium, RP. There is negligible default risk on the promises of most DB plans.

To assess a plan's default risk, one should examine the layers of protection backing the plan's promises. For private plans, there are three. The first is pension assets. ERISA provides incentives for private plans to be, or quickly become, fully funded. Plans must report their funding ratio (ratio of pension asset to accrued pension liabilities). The second layer of protection is the firm's assets and ability to contribute funds to the pension. The federal Pension Benefit Guaranty Corporation provides the third layer. The PBGC guarantees most private-plan benefits of up to \$40,705 a year (in 2001) for each participant.[2] Some private plans are excluded; for example, plans offered by small professional service firms and those offered by church groups are not usually insured. Some benefits are excluded; for example, the PBGC does not insure cost-of-living increases. Overall, however, the PBGC guarantee is broad and strong. In addition to having a pool of funds accumulated from its insurance premiums to back this guarantee, the PBGC also has a priority claim in bankruptcy. Retirement income from private plans appears secure, especially retirement income up to the PBGG guarantee limit.

There are two layers of protection backing public plans. The first layer is pension assets. The second layer is the ability of the state or local government to contribute funds. Most participants in public plans are in large state plans. They are usually well funded with funding ratios exceeding 90%, although exceptions exist. In addition, they offer the second layer of protection insofar as the state would likely make good on any projected shortfall. The major difficulty comes in assessing the default risk of underfunded plans, especially underfunded plans of local governments. If pension assets are insufficient then the ability of the local government to raise funds through debt or taxes becomes important. The capability to meet unfunded pension obligations can be roughly assessed from the state or local government's credit rating. If the government's credit rating is Baa2, then $(Rf + RP)$ can be estimated at the maturity-appropriate yield on Baa2 *corporate* bonds. The risk premium, RP, is the difference between this Baa2 corporate yield and the Treasury yields with the same maturity. We use the corporate yield because municipal debt is federally tax-exempt. Since pension distributions are subject to taxation, the tax-exempt municipal yield is inappropriate.[3]

McLeod, Moody, and Phillips (1993) present a taxonomy of pension risks that include investment risks like inflation, liquidity, marketability, portfolio, reinvestment, and default as well as pension-specific risks like plan type, funding level, asset mix, regulatory noncompliance, and plan modifications. We subsume all of these risks in the risk premium.

Note that default risk valuation scenarios are not always clean-cut. If the PBGC guarantee applied to a private plan with default risk and the expected benefits exceeded the guarantee level, valuation would be more complex. The pension promise could be split into the excess portion and the guaranteed portion valued with and without a default risk premium.

3.3. Applying the model

Mary is a single participant in a plan with no automatic increases in retirement benefits and no integration with Social Security benefits. (This example corresponds to Table 2.) She is 60 years old with 18 years of service and an average income during the highest four years of \$40,000. She expects to retire and begin receiving benefits at age 66 and to select the lifetime income option with no guaranteed payment period. Although she has not attained normal retirement age—here, any combination of age plus years of service totaling 80—she will have attained it by her expected retirement date. The benefit formula is 2% × years of service × average salary in the highest four years.

We project retirement income of \$14,400 or 0.02 (18) \$40,000 beginning in six years. The

Table 2
Calculations of Present Values of Expected Cash Flows and Cash Flows through Life Expectancy for a 60-Year Old Single Female Beginning Benefits at Age 66

Age	Probability of Being Alive	Expected Cash Flows	Present Value of Expected Cash Flows	Present Value of Cash Flows through Life Expectancy
66	0.957274	$13,784.75	$\$\ 13{,}784.75/(1.0496)^{6.5}$	$\$\ 14{,}400/(1.0496)^{6.5}$
67	0.946788	$13,633.75	$\$\ 13{,}633.75/(1.0496)^{7.5}$	$\$\ 14{,}400/(1.0496)^{7.5}$
⋮				
83	0.586627	$ 8,447.43	$\$\ 8{,}447.43/(1.0496)^{23.5}$	$\$\ 14{,}400/(1.0496)^{23.5}$
84	0.549959	$ 7,919.42	$\$\ 7{,}919.42/(1.0496)^{24.5}$	$\$\ 5{,}760/(1.0496)^{24.5}$
85	0.511728	$ 7,368.88	$\$\ 7{,}368.88/(1.0496)^{25.5}$	$ 0
⋮				
120	0.000009	$ 0.13	$\$\ 0.13/(1.0496)^{60.5}$	$ 0
			$123,552	**$131,184**

The discount interest rate is 4.967%. All payments are assumed to occur at mid-year. In this example, the present value of cash flows through life expectancy has a 6.2% upward bias compared to the present value of expected cash flows.

payments will not increase since the plan does not provide increases in postretirement income. The present value of retirement income is the value at age 60 of benefits of $14,400 a year (or $1,200 a month) to begin at midyear at age 66 and continuing for the rest of her life.

This state-sponsored DB plan is well funded and, although a shortfall is not anticipated, it is strongly expected that the state would fulfill the pension obligation. Benefits, once begun, will remain constant. We use the 25-year Treasury bond yield of 4.9% to estimate the risk-free rate. The 4.9% Treasury yields are based on the semiannual bond convention. The effective annual yield is 4.96% or $(1.0245)^2 - 1$, where 0.0245 is 4.9%/2.

There are two methods of estimating the value of pension benefits. The first method estimates the present value of *expected future cash flows*. Each year's expected cash flow is $14,400 times the probability of being alive (Society of Actuaries, 2000). The expected cash flow at age 66 is $13,784.75, and it decreases each subsequent year through age 120, the end of the mortality tables. The present value of lifetime benefits is the product of $14,400 and the multiple, where the multiple is the present value if benefits were $1 per year. Based on the model with a 4.96% discount rate, the exact multiple is 8.58. The present value of expected cash flows is $123,552. Alternatively, this multiple can be estimated from Table 4. The multiples for a 60-year-old female when the discount rates are 5% and 6% are, respectively, 8.53 and 7.41. Using extrapolation, the estimated multiple is 8.57, or $8.53 + 0.04(8.53 - 7.41)$, and the estimated value is $123,408.

The second method estimates the present value of *cash flows through expected life*. At age 60, Mary will live 24.4 years (assuming average life expectancy). The present value of an ordinary annuity of $14,400 for 18.4 years is $171,187. After adjusting for the midyear convention, the value at age 66 is $175,381 or $171,187 × (1.0245). Discounting this lump sum at 4.967% for six years gives a present value at age 60 of $131,171.

In finance, an asset's value is traditionally estimated with expected cash flows. Experimentation suggests that the present value of cash flows through life expectancy is an upward biased estimate of the preferred estimate based on expected cash flows. However, the cash-flows-through-life-expectancy approach is much easier to estimate, easier to explain, and accommodates atypical life expectancies. For singles with average life expectancy, the two estimates are close. So most people will prefer to calculate the present value of cash flows through life expectancy and adjust for the 3% to 7% bias. If we use the midpoint, 5%, and the \$131,171 estimate, the bias-adjusted estimated value is \$124,925.

The pretax value of the DB plan varies with the estimation method, but within a relatively narrow range. Present value estimates are:

PV Estimates	Method
\$123,552	Expected cash flows (Multiple from model; Table 2)
\$123,408	Expected cash flows (Multiple from extrapolation; Table 4)
\$131,171	Cash flows through life expectancy (no adjustment for bias)
\$124,925	Cash flows through life expectancy (adjusted for estimated 5% bias)

Because the financial profession typically values DB plans at zero when calculating a family's asset allocation, any of these methods is a substantial improvement over current use.

As noted above, there is a distinction between the present value of expected cash flows and the present value of cash flows through life expectancy. For singles with average life expectancy, it is much easier to estimate the present value of cash flows through life expectancy and adjust for the bias. In general, we cannot use the present-value-of-cash-flows-through-life-expectancy approach for couples. For example, suppose a spouse receives pension benefits of \$2,000 a month and his spouse, if she outlives him, will receive \$1,500 a month for the rest of her life. Even if we know the couple's joint life expectancy is 20 years, we cannot calculate the pension's value because payments depend upon who is alive. The exception is if the survivor receives 100% benefits; in this case, we could estimate the pension's value as the present values of benefits through their joint life expectancy (and then adjusts for the bias).

Tables 3 and 4 present multiples for single males and females, respectively. Table 5 presents couple's multiples when there are no automatic benefit increases. Table 6 presents couple's multiples when the postretirement discount rate is 3% and is designed to accommodate increases in postretirement benefits. Tables 5 and 6 include survivor's annuities of both 50% and 100%. Scenarios not covered by the tables—fractional discount rates, 75% survivor benefits, and so forth—can be interpolated. Alternatively, see Reichenstein and Jennings (2003) for additional tables with more detail including tables for couples with different ages.

3.4. Other retirement ages

Our present-value-of-expected-cash-flows tables assume retirement at age 66. Necessarily, different tables exist for different retirement ages. We set the retirement age at 66 because it is the Social Security Full Retirement Age for a large portion of the Baby Boom generation. The tables can be used for people who will retire around age 66. If retirement is at age 65, one year earlier, the multiple can be approximated by adding $1/(1 + k_1)^{65.5-n}$ to the age-66 multiple, where n is the participant's age today; in essence, the multiple increases

Table 3
Multiples for single males

Age	$k_1=5\%$			$k_1=6\%$		
	$k_2=5\%$	$k_2=4\%$	$k_2=3\%$	$k_2=6\%$	$k_2=4\%$	$k_2=3\%$
35	2.16	2.34	2.55	1.49	1.75	1.90
45	3.56	3.86	4.20	2.70	3.16	3.44
55	5.93	6.43	7.00	4.95	5.79	6.31
60	7.75	8.40	9.15	6.79	7.94	8.64
65	10.34	11.21	12.21	9.50	11.10	12.09
70	9.70	10.41	11.21	9.07	10.41	11.21
75	8.02	8.51	9.04	7.59	8.51	9.04
85	4.92	5.10	5.30	4.75	5.10	5.30
Age	$k_1=7\%$			$k_1=9\%$		
	$k_2=7\%$	$k_2=4\%$	$k_2=3\%$	$k_2=9\%$	$k_2=4\%$	$k_2=3\%$
35	1.04	1.31	1.42	0.51	0.74	0.80
45	2.07	2.60	2.83	1.23	1.76	1.92
55	4.16	5.22	5.69	2.97	4.26	4.64
60	5.97	7.50	8.17	4.68	6.71	7.31
65	8.75	11.00	11.98	7.53	10.80	11.76
70	8.51	10.41	11.21	7.56	10.41	11.21
75	7.19	8.51	9.04	6.51	8.51	9.04
85	4.59	5.10	5.30	4.30	5.10	5.30

Note: Multiples represent the present value of \$1 starting at retirement at age 66. The variable k_1 is the pre-retirement discount rate, and k_2 is the postretirement discount rate. If a pension does not include post-retirement increases then k_1 equals k_2. If a default-risk-free pension receives inflation-indexed post-retirement increases then k_1 is the nominal Treasury bond rate and k_2 is the real Treasury bond (TIPS) rate.

by approximately the present value of \$1 to be received at midyear of age 65. If retirement is at age 67, the multiple can be approximated by subtracting $1/(1 + k_1)^{66.5-n}$ from the age-66 multiple, where n is the participant's age today; in essence, the multiple decreases by approximately the present value of \$1 to be received at midyear of age 66. If retirement is at age 68, the multiple can be approximated by subtracting $1/(1 + k_1)^{66.5-n} + 1/(1 + k_1)^{67.5-n}$ from the age-66 multiple; in essence, the multiple decreases by approximately the present value of \$1 to be received at midyears of ages 66 and 67 (and so on). These adjustments produce multiples that are only approximate; actual multiples are affected by mortality risk.

3.5. After-tax valuation

People are concerned about the amount of goods and services they can consume in retirement. Accordingly, we recommend that the calculation of the asset allocation be based on asset's after-tax values. For example, 401(k) accounts contain before-tax funds, while taxable accounts generally contain after-tax funds. Equating pretax funds from a 401(k) and after-tax funds from a taxable account in an asset allocation is an apples-to-oranges comparison—because of taxes due on the 401(k). Making the proper after-tax adjustment can also have a major impact on the asset mix decision. See Reichenstein (1998) for details; he

Table 4
Multiples for single females

Age	$k_1=5\%$			$k_1=6\%$		
	$k_2=5\%$	$k_2=4\%$	$k_2=3\%$	$k_2=6\%$	$k_2=4\%$	$k_2=3\%$
35	2.42	2.64	2.90	1.66	1.97	2.16
45	3.96	4.33	4.76	2.99	3.55	3.90
55	6.56	7.17	7.89	5.44	6.46	7.10
60	8.53	9.32	10.24	7.41	8.80	9.68
65	11.26	12.30	13.52	10.26	12.19	13.40
70	10.71	11.59	12.60	9.94	11.59	12.60
75	9.12	9.75	10.46	8.55	9.75	10.46
85	5.95	6.22	6.52	5.70	6.22	6.52
Age	$k_1=7\%$			$k_1=9\%$		
	$k_2=7\%$	$k_2=4\%$	$k_2=3\%$	$k_2=9\%$	$k_2=4\%$	$k_2=3\%$
35	1.14	1.47	1.62	0.56	0.83	0.91
45	2.27	2.91	3.20	1.33	1.97	2.17
55	4.54	5.83	6.41	3.21	4.75	5.23
60	6.48	8.32	9.15	5.02	7.45	8.18
65	9.40	12.07	13.27	7.99	11.85	13.03
70	9.26	11.59	12.60	8.13	11.59	12.60
75	8.05	9.75	10.46	7.19	9.75	10.46
85	5.47	6.22	6.52	5.06	6.22	6.52

See explanation in Table 3.

advocates adjusting pretax funds to after-tax funds by multiplying their pretax value by (1 − the expected tax rate in retirement).

The proper after-tax adjustment is not as easy as it initially appears. First, the state tax component of the adjustment should reflect the anticipated retirement state's taxes. For example, popular retirement destinations like Arizona and Florida might have lower taxes than a worker's current state. Second, the state income tax provisions for pension income are complex and varied—often with massive exemptions for pension income. The tax adjustment, thus, might be different for income from DB plans versus other taxable income. Baer (2001) details the idiosyncrasies of state pension taxation. Third, while after-tax valuation work by Sibley (2002) and Horan (2002) is interesting with respect to tax-deferred retirement accounts, the Reichenstein (1998) approach is appropriate for DB plans.

4. Asset allocation implications

In this section, we revisit Mary, the example from Section 3, to demonstrate the difference between her asset allocation as traditionally defined (based on her financial portfolio) and her asset allocation based on her expanded portfolio. We make some additional assumptions: She has $150,000 in a 401(k) and $50,000 of bonds in a taxable account. The book value and market value of the bonds are $50,000. She is entitled to Social Security benefits with a pretax present value of about $211,000.[4]

Table 5
Joint-survivor multiples for couples same age: no automatic increase in postretirement income

Age	**5%**			**6%**			**7%**			**9%**		
	50% to widow	50% to widower	100% to survivor	50% to widow	50% to widower	100% to survivor	50% to widow	50% to widower	100% to survivor	50% to widow	50% to widower	100% to survivor
38	2.93	3.08	3.35	2.07	2.16	2.35	1.47	1.53	1.66	0.76	0.79	0.85
47	4.58	4.80	5.22	3.52	3.68	3.99	2.72	2.83	3.07	1.66	1.72	1.85
56	7.20	7.53	8.15	6.02	6.28	6.78	5.07	5.27	5.68	3.65	3.77	4.05
62	9.86	10.28	11.05	8.73	9.07	9.72	7.77	8.05	8.61	6.24	6.44	6.85
65	11.62	12.08	12.90	10.59	10.97	11.68	9.69	10.01	10.63	8.23	8.46	8.93
68	11.97	12.47	13.32	11.11	11.53	12.29	10.35	10.72	11.39	9.10	9.38	9.91
71	10.98	11.50	12.36	10.26	10.72	11.48	9.62	10.02	10.70	8.55	8.86	9.42
77	8.90	9.47	10.26	8.44	8.95	9.67	8.01	8.47	9.14	7.28	7.66	8.23
83	6.85	7.40	8.08	6.57	7.08	7.72	6.32	6.79	7.38	5.87	6.27	6.80
89	5.07	5.55	6.10	4.92	5.37	5.90	4.78	5.20	5.70	4.52	4.90	5.36

Note: Multiples represent the present value of $1 starting retirement at age 66. Pre-retirement and postretirement discount rates are the same; there are no automatic post-retirement benefit increases. 50% and 100% reflect the relative level of the survivor's annuity.

Table 6
Joint-survivor multiples for couples same age: postretirement discount rate at 3%

Age	**5%**			**6%**			**7%**			**9%**		
	50% to widow	50% to widower	100% to survivor	50% to widow	50% to widower	100% to survivor	50% to widow	50% to widower	100% to survivor	50% to widow	50% to widower	100% to survivor
38	3.52	3.08	3.35	2.70	2.85	3.12	2.07	2.19	2.40	1.23	1.31	1.43
47	5.49	4.80	5.22	4.59	4.84	5.30	3.84	4.05	4.43	2.70	2.85	3.12
56	8.65	7.53	8.15	7.87	8.29	9.02	7.16	7.54	8.21	5.95	6.27	6.82
62	11.84	10.28	11.05	11.40	11.97	12.95	10.98	11.53	12.47	10.20	10.71	11.58
65	13.97	12.08	12.90	13.83	14.49	15.57	13.71	14.35	15.43	13.45	14.09	15.15
68	14.09	12.47	13.32	14.09	14.79	15.91	14.09	14.79	15.91	14.09	14.79	15.91
71	12.72	11.50	12.36	12.72	13.43	14.52	12.72	13.43	14.52	12.72	13.43	14.52
77	10.00	9.47	10.26	10.00	10.72	11.67	10.00	10.72	11.67	10.00	10.72	11.67
83	7.47	7.40	8.08	7.47	8.13	8.91	7.47	8.13	8.91	7.47	8.13	8.91
89	5.41	5.55	6.10	5.41	2.85	3.12	5.41	5.95	6.56	5.41	1.31	1.43

Note: Multiples represent the present value of $1 starting retirement at age 66. Pre-retirement discount rate is given in the top row; postretirement discount rates is 3%; there are automatic post-retirement benefit increases equal to the difference in the rates. 50% and 100% reflect the relative level of the survivor's annuity.

Table 7

Demonstration of strategic asset allocation implications

Financial Portfolio

Stocks in 401(k)	$80.0	40% stocks
Bonds in 401(k)	$70.0	60% bonds
Bonds in taxable account	$50.0	
Total	$200.0	

Total Family Portfolio

	Pre-tax	Tax Factor	After-tax	
Stocks in 401(k)	$80.0	0.73	$58.4	14% stocks
Bonds in 401(k)	$70.0	0.73	$51.1	86% bonds
Bonds in taxable account	$50.0	1.00	$50.0	
PV of Social Security	$211.0	0.7705	$162.6	
PV of DB plan	$123.6	0.73	$90.2	
			$412..3	

Note: Based on the example in Section 4. Tax factors convert pre-tax dollars to after-tax dollars. Assuming a 27% marginal tax rate, the tax factor is 0.73 for 401(k) funds. Assuming income from the DB plan is fully taxable, and 85% of income from Social Security is taxable at 27%, these tax factors are 0.73 and 0.7705 or 1 − (0.85)0.27. Dollars in thousands.

Suppose Mary, age 60, applies a (100-age)% stock allocation rule. Following traditional financial practice when calculating asset allocation, she does not distinguish between pretax funds in a 401(k) and the generally after-tax funds in a taxable account. In addition, following common financial planning advice, she places stock in the 401(k) and bonds in taxable accounts to the degree possible. Accordingly, as detailed in Table 7, her asset allocation, as traditionally measured by the financial portfolio, is 40% stocks and 60% bonds.

The total family portfolio presents her asset allocation when we distinguish between pretax and after-tax funds and include retirement income as bonds in the expanded portfolio. When we consider the present value of her DB plan and Social Security, her asset mix is dramatically different. The asset allocation of the Total Family Portfolio is 14% stocks and 86% bonds. While she thought she had 40% stocks, she actually only has 14%.

We do not claim that anyone should arbitrarily translate a traditional asset-mix decision rule developed for financial portfolios to the after-tax total family portfolio. (In Mary's case, investing 40% of her after-tax total family portfolio would require putting all of her financial portfolio in stocks.) After all, most recommendations for asset mixes were developed for traditional portfolio thinking. For retirement planning, however, we believe investors should manage the expanded portfolio—optimal planning can only occur if they look at and manage their true portfolio. Only time will tell what norms will develop for expanded portfolios, but we suspect they will be better because they have a truer view of investors' total financial picture.

5. More precision in interest rates

Hitherto, we have relied upon using one discount rate for the entire retirement income stream. For precision, we can extend the model to reflect the yield curve. Effectively, we can value each benefit payment as a zero-coupon bond. To value benefits this way we need, not the ordinary yield curve, but the zero-coupon yield curve. The *Wall Street Journal* reports the nominal Treasury zero-coupon yields curve. When benefits are inflation-indexed, we can use the inflation-adjusted zero-coupon yield curve. The market-priced "iStrip" curve can be obtained from Bloomberg; alternatively, the zero-coupon inflation-adjusted yield curve can be extracted from the ordinary with-coupon TIPS market prices using spline fitting (McCulloch and Kochin, 2000).

This seemingly small adjustment has a nontrivial impact. Consider Jane, a single 60-years-old participant in an inflation-indexed plan whose benefit formula specifies payments of $14,400 per year (based on current years of service and salary). She expects to begin benefits at age 66 with negligible default risk over her 24.4-year life expectancy. In October 2002, the closest nominal and real Treasury bonds were the 6% February 2027s and the 3% April 2028s. McCulloch's (2002) data puts the par-coupon-bond rates at 5.044% and 3.004%, respectively. Interpolating, the multiple from Table 4 is 10.19, so the present value of Jane's pension is about $146,736. Using McCulloch's (2002) spline-fit data, the forward-indexed zero curve values Jane's pension at $156,132. This value is approximately 6% higher than using point estimates for yields. (The typical shape of the yield curve implies that using multiple rates will have the largest impact for those in, or close to, retirement.)

We have detailed several small precision enhancements over the prior literature—semi-annual pricing, cash flow timing and the yield curve. Another enhancement would be to use a targeted population's (e.g., smokers) specific mortality table. These precision enhancements are crucially relevant in litigation contexts. That said, any reasonable estimate of DB pension value is superior to the common practice of ignoring it.

6. A larger context and research opportunities

In a study reminiscent of Markowitz (1991), Black, Ciccotello, and Skipper (2002) propose a lifetime mean-variance optimization framework for the family—calling it "comprehensive personal financial planning." A key question in optimization is what belongs in the portfolio and the goal of optimization is estimating the efficient asset allocation. In this section, we jointly address these questions.

The answer to the question about what belongs in the portfolio depends upon the question asked. For example, if we are concerned about estate planning, last-to-die insurance belongs in the portfolio (and affects the asset allocation), but projected pension income does not. If we are concerned about retirement planning, last-to-die insurance does not belong in the portfolio but projected pension income does. In Fraser, Jennings, and King (2000), Jennings and Reichenstein (2001, 2002b) and Reichenstein and Jennings (2003), we focused on retirement planning. Does a family have sufficient resources to meet its retirement income needs? We recommend broadening the family portfolio to include the values of pension income—DB plans, military retirement, and Social Security. These assets should be viewed as bonds or real bonds in the family's expanded portfolio. Naturally, the calculation of the family's asset allocation should be based on this expanded portfolio.

Markowitz and BCS consider a broader lifetime optimization model. Therefore, they go further than we do by including in the family portfolio assets like "government benefits, insurance, expected inheritances or other family support, and human capital." In addition, they include liabilities such as mortgages and future expenditures "including but not limited to tax, housing, education and health care." To solve a lifetime optimization model, the portfolio must include most of these items.

Given our limited retirement planning scope we include the projected values of government benefits from Social Security and military retirements. In this paper, we document how to include the value of defined benefit pension plans. We would only include the value of insurance if it would provide cash flows to meet retirement needs. As such, we would exclude second-to-die life insurance (but would include first-to-die insurance) for a couple. Although we did not include the value of expected inheritances or other family support, we agree with BCS that *virtually certain* inheritances and other family support should affect the asset allocation, and thus, belong in the portfolio. For example, a prospective inheritor of an all-bond portfolio could make an ad hoc shift toward equity; more precisely, she could include the expected value of the inheritance as a bond allocation in her expanded portfolio.

Black, Ciccotello, and Skipper (2002) include human capital. Human capital provides the future cash flows to pay budget items including house payments, future retirement contributions, and more. Life insurance protects the value of human capital. The combination of human capital and life insurance is like a protective put in that it puts a floor on the value of human capital. But, should the value of human capital be included in the family portfolio? To address this issue, consider a married 40-year-old tenured professor who earns $80,000 a year. Although the present value of her future earnings may be $1,400,000, the family would need less than this amount of insurance since some of the income would go to pay taxes and for her food, clothing, housing and transportation needs. Suppose the family needs $600,000 in life insurance to cover her human capital. Now, how, if at all, should her human capital affect this family's asset allocation? The $600,000 *value* of her human capital should be protected by buying life insurance. The *risk* of her human capital should affect the family's asset allocation. Because the risk of human capital for a tenured professor is low, the rest of the family portfolio should assume more risk. In contrast, if the risk of human capital is high—as it is for entrepreneurs—the remainder of the portfolio should assume less risk. In short, we would not include the value of human capital in the expanded portfolio, but the risk of human capital would affect the portfolio's risk.[5]

Black, Ciccotello, and Skipper (2002) BCS believe several liabilities belong in the expanded portfolio including mortgages, taxes, housing, education, and health care. We agree that mortgages belong in the expanded portfolio; it is a short bond position (see Delaney and Reichenstein, 1995 and Waggle and Johnson, 2003). Just as we exclude human capital and future income, we would exclude future taxes on income. They are part of the family's budget, and budget items do not, in our view, affect the optimal asset allocation and do not belong in the expanded portfolio. As discussed in Section 3.5., we do include taxes due on current portfolio positions; because the asset values and prospective taxes are perfectly negatively correlated, taxes can be netted against pretax asset values and the net asset included in the expanded portfolio optimization. In contrast, the value of a mortgage does not move in synch with the property value, so the asset and liability are treated separately.

The next item mentioned by Black, Ciccotello, and Skipper (2002) is housing. Personal residence is an asset, but house payments are a liability. We exclude house payments as a liability. Normally, families will make such payments from the monthly budget. As before, we exclude budget items from the expanded portfolio. While the personal residence is an asset, we would not generally include it in the expanded portfolio for retirement because it does not provide cash flows to finance retirement income needs (Scott, 1995). In fact, houses require negative cash flows in the form of taxes and upkeep. However, the personal residence should affect the asset allocation in at least two circumstances. First, suppose someone plans to make a down payment on a home within the next few years. The prospective house itself is not an asset in the portfolio, but the present value of the down payment is a liability that should be included in the optimization. In consequence, the resulting asset allocation would reflect the short planning horizon. Second, when a family downsizes, it creates a positive cash flow that can be used to finance retirement needs. Suppose a family expects to downsize and free \$100,000 in cash. Before downsizing, the expanded portfolio could include this \$100,000 and view it as real estate.

Education financing is a contingent future liability. It is part of the question, "Will the family have enough to live on in retirement?" because money spent on finance education will not be available for retirement. We would include the projected cost of education in the expanded portfolio in a Black, Ciccotello, and Skipper asset-liability framework. However, quantifying the college expense liability may be quite difficult because of uncertainties about net college costs (including potential scholarships and tuition differentials) and duration (as more students extend beyond the traditional four years). We believe the timing of the liability will affect the expanded portfolio's asset allocation in an asset-liability framework. Ambiguity about education expenses should not preclude integrating this liability into the asset allocation. Including estimates of the college expense should lead to a more optimal financial strategy than ignoring this liability altogether.

Another common contingent liability is the need to finance parents' retirement. Like education, this contingent liability could affect planned savings, and the timing of these expenses should affect the portfolio's asset allocation. Thus, a couple with parents that need financial help should maintain a more conservative portfolio at least for the portion of the portfolio that is needed to finance parents' retirement. Compared to college education, the duration of this financial obligation is much less certain, but the mortality-linked techniques of Jennings and Reichenstein (2001, 2002b) and Goodman (2002) can help. If the parents live a long time, it would affect the couple's ability to finance their retirement. The couple's human capital may provide "insurance" against longevity risk for the elderly parents. If the parents live a long time, the couple can finance it by saving more each year or delaying their retirement.

Bequests are sometimes financial goals and could be considered a liability in the Black, Ciccotello, and Skipper "comprehensive personal financial planning" optimization. We prefer—given our narrower retirement-funding focus—to exclude bequests from the expanded portfolio and asset allocation. Testamentary bequests come after retirement, by definition, so they are not part of the question, "Will the family have enough to live on in retirement?" However, Evensky (1997), among others, recommends that retirement savers focus on the 90th percentile of mortality rather than the median life expectancy that one is 50% likely to outlive. When this retirement-funding idea is applied to a Black, Ciccotello, and Skipper asset-liability optimization, individuals build up extra "precautionary savings" against longevity and health shocks. In the median case, however, these precautionary savings are available for bequests. For more on bequests, see Ando and Modigliani (1963) and Dynan et al. (2002).

In summary, Markowitz and Black, Ciccotello, and Skipper envision a lifetime optimization model for the family, where decisions early in life affect decisions later in life. Although desirable, the profession is a long way from this goal. As an intermediate step, we encourage families to optimize an expanded asset-liability portfolio. Detailing the precise asset allocation consequences of the larger comprehensive personal financial planning context of Black, Ciccotello, and Skipper should provide fertile ground for future positive, normative, and simulation-based research.[6] Although only an intermediate step toward the ultimate goal envisioned by Markowitz and Black, Ciccotello, and Skipper, our approach concentrates on the asset allocation decision—widely considered an investor's most important decision.

7. Summary and conclusions

The goal of this paper was to estimate the present value of defined benefit retirement plans. As motivation, we argue that it is inconsistent to consider pension income streams in retirement planning, but ignore pension wealth in retirement asset allocation. We began by discussing topics that can affect the value of DB retirement income including retirement-income formulas, integration with Social Security, increases in postretirement benefits, and default risk.

We revisited the question of what counts in a family's portfolio. We build on the work of Scott (1995) and others by highlighting the importance of including pension wealth—and providing a methodology for doing so. We argue the regular payment pattern of pensions makes them bond-like and value pensions accordingly. Our approach uses three key estimates—the level of benefits, the expected growth rate in benefits and the risk-adjusted discount rate.

When planning for retirement, we conclude that after-tax pension wealth should be included in the family's expanded portfolio and included in asset-mix decisions. Optimal investment solutions are more likely to come from managing the family's true portfolio. Since the expanded portfolio distinguishes between pretax and after-tax funds and includes the value of retirement income, it provides a better picture of the family's true financial position. Decisions based on a more accurate picture of the family's position should dominate.

In the spirit of conservatism, we advocated excluding projected salary increases when computing the value of pension income; one should not count the benefits of future work as current value. This assumption has an interesting consequence under the typical earnings-based formula that bases initial retirement benefits on the product of a percentage × years of service × average salary. As time passes, the value of benefits rises because of increases in (1) years of service, (2) average salary, and (3) proximity to retirement. Consequently, the present value of a typical employee's DB plan rises sharply in the last decade before retirement. This is akin to the expected behavior of financial assets dedicated to retirement.

Further, the present value of DB wealth declines after retirement like financial assets typically do.

We acknowledge the impossibility of covering all idiosyncrasies in valuing DB benefits. However, when the financial profession ignores the value of the DB plan in calculating asset allocations, it places an implicit value of zero on DB benefits. The good news is that, even if it is impossible to obtain a precise estimate of the DB plan's value, it is easy to improve upon the implicit value of zero. This improvement is a way for financial professionals to demonstrate value to their customers.

Notes

1. Most DB plans provide statements that show projected benefits assuming work continues to full retirement age. For valuation and asset allocation purposes, we recommend that DB current value be based on current salary and current years of service.
2. The $40,705 amount applied in 2001 for single-employer plans. Those who retire before 65 receive a reduced guarantee. The amount increases with the Social Security contribution-and-benefit base. (The guarantee level increases; benefits of those under PBGC-terminated plans do not increase with inflation.) As might be expected, the limitation applies to individuals with high salaries and long tenure, especially those in generous pension plans. In a PBGC (1999) study, six in seven of those affected by the benefits maximum were airline pilots; less than 6% of plan participants were affected by the maximum, but they lost 16% of benefits. McLeod, Moody, and Phillips (1993) characterize the pension payoffs with a PBGC guarantee as an option-like collar—with a floor at the PBGC guarantee level and a cap at the promised benefit level. The PBGC (1999) study suggests that the floor guarantee is most relevant in default.
3. Admittedly, this rationale is somewhat circular since the unfunded pension liability likely hurts the municipal credit rating, but this seems to be a reasonable market-based approach to estimating the default risk premium.
4. Social Security is a special case of a DB plan and can be valued with our methods. Also, see Fraser, Jennings, and King (2000), Jennings and Reichenstein (2001) and Reichenstein and Jennings (2003, Chapter 11).
5. As a practical matter, estimating the value of an individual's human capital is complex and fraught with inaccuracies. For more on human capital, see Heaton and Lucas (2000) and Canner, Mankiw, and Weil (1997).
6. In an earlier version of this section, we hypothesized extensively about the volatility characteristics and interrelationships of the expanded portfolio assets and liabilities. While the return, risk and correlation parameters for the added liabilities and assets will be critical in deriving the asset allocation implications of a Black, Ciccotello, and Skipper optimization framework, Chopra and Ziemba (1993) show that errors in means are ten times as important as errors in variances and twenty times as important

as errors in covariances. It is likely more important to focus on what should be included in the asset-liability optimization and to model the return vector.

Acknowledgments

The authors thank the editor and referee, participants at the 2002 Academy of Financial Services conference, and Barton Waring at Barclays Global Investors. The opinions expressed are those of the authors and are not necessarily those of the USAFA, the USAF or any other federal agency. This article builds on the thinking espoused in Chapter 13 of *Integrating Investments and the Tax Code* (Reichenstein and Jennings, 2003).

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