FINANCIAL
SERVICES
REVIEW

Financial Services Review 12 (2003) 319–337

The mortgage refinance decision: an equation-based model

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Accepted 29 October 2003

Abstract

Given recent historically low interest rates, homeowners have been heavily involved in the process of refinancing their home mortgages. This paper examines current refinance analysis methods, and finds most advocate various forms of break-even analysis. In essence, these methods primarily differ with respect the variables considered, complexity of analysis, and solution accuracy. This paper develops an equation-based model that considers the generally relevant variables, while decreasing the complexity of analysis to the solution of an equation. In addition, the model permits an effective comparison of alternative loan terms, determines the expected refinance savings, and solves for the opportunity gain or loss of a refinance deferral.

JEL classification: D14; D81; C63

Keywords: Refinancing; Mortgage; Loans; Prepayment

1. Introduction

Given recent historically low interest rates, homeowners have been heavily involved in the process of refinancing their home mortgages. This paper examines current refinance analysis methods, and finds most advocate various forms of break-even analysis. In essence, these methods primarily differ with respect the variables considered, complexity of analysis, and solution accuracy. This paper develops an equation-based model that considers the generally relevant variables, while decreasing the complexity of analysis to the solution of an equation. In addition, the model permits an effective comparison of alternative loan terms, determines

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1057-0810/03/$ – see front matter

the expected refinance savings, and solves for the opportunity gain or loss of a refinance deferral.

Section 2 of the paper examines a number of variations on payback analysis, beginning with the most fundamental payback approach. This basic model is then extended to include consideration of additional variables, with discussions emphasizing the relevance of each variable to the analysis. Section 3 examines the mathematical relationships inherent in the refinance decision, and develops an equation-based model that calculates the required break-even period and expected refinance savings. A refinance analysis example follows in Section 4, and Section 5 includes a summary and concluding remarks.

2. Payback analysis methods

The below discussions begin with a presentation of the basic payback model, and follow with extensions that include the effect of additional variables. While the addition of each variable improves the accuracy of the payback solution, the obvious tradeoff is an increase in the complexity of the analysis. Accordingly, the intent of this section is to present alternative payback methods that differ with respect to the variables included, and examine the estimation bias associated with the exclusion of each variable.

2.1. Payback

The payback approach is definitely the easiest to use and understand of all the refinance analysis methods. Given a before-tax rate on a new loan ($r_{NL,BT}$) that is lower than the before-tax rate on the original loan ($r_{OL,BT}$), the borrower will realize periodic savings by refinancing. In its most fundamental form, the borrower simply divides the total cost of refinancing (TCR) by the periodic payment savings to arrive at the number of periods to break-even (n). If the borrower expects to stay in the new loan beyond the break-even point, the refinance transaction is expected to yield savings and the borrower should consider refinancing.

To solve for the payback or break-even period, the borrower needs to determine the total cost of refinancing and periodic payment savings. The total cost of refinancing is equal to the sum of all *incremental* costs realized with acceptance of the refinance transaction, and the periodic payment savings is equal to the difference in the before-tax payment of the original loan (BTP_{OL}) and that of the new loan (BTP_{NL}). It is important to note that this payback method requires determination of the original and new loan payments using the same loan amount and number of payment periods. The applicable loan amount is that remaining on the original loan ($OLB_{t=0}$), and the number of payment periods (m_{OL}) is based on the borrower's planned payment schedule. Note that although the new loan balance ($NLB_{t=0}$) may be greater than that remaining on the original loan, inclusion of this additional amount in both loans results in an erroneous increase in the periodic refinance savings. In addition, use of different payment periods will generally result in a greater payoff differential, which is not factored into the break-even solution of this model. After determination of the total cost of refinancing and periodic payments, an estimate of the required break-even period as advocated by Boone et al. (1996) can be solved using the following equation:

$$n = \frac{TCR}{BTP_{OL} - BTP_{NL}} \tag{1}$$

[For calculation of the before-tax payments, see Eq. (1A) in Section 1 of the Appendix.]

Although this model yields a relatively straightforward and easily solved payback estimate, the model does not include a number of variables generally considered relevant to refinance analysis. As is evident in an examination of Eq. (1), this model does not consider the tax deductibility of interest expense, time value of money, or tax effect associated with a rate buy-down on the original and/or new loan. In addition, the model does not account for the difference between the new and original loan balance throughout the payment period. Since the original and new loan payments must be determined using the same loan life, the near-term principal payments on the new loan will be greater than those of the original loan. This is because payments toward principal grow in value at their respective before-tax rates, thus requiring a higher initial principal payment on the new lower-rate loan to realize loan payoff within the same payment period. Therefore, the new loan balance will be lower than that of the original loan throughout the payment schedule, and the difference at break-even represents accumulated savings in the form of a lower payoff balance on the new loan. Consequently, excluding this loan balance differential has the effect of understating the periodic savings and overstating the estimated payback period. With respect to the deductibility of interest expense and time value of money effects, their exclusion results in an overstatement of the periodic savings and understatement of the payback period. A more detailed discussion of these effects will be addressed below, as will issues related to the amortization of rate buy-down points.

2.2. Payback with taxes

As mentioned above, another variable that is usually relevant to refinance analysis is the tax effect associated with the deductibility of interest expense. Because refinancing at a lower rate reduces the interest tax shield, inclusion of this tax effect decreases the periodic savings and increases the break-even estimate. Therefore, the effect of this lost tax shield is more material the greater the borrower's marginal tax rate (T). Consistent with the basic payback method just discussed, this model also requires determination of the original and new loan payments using the same loan amount and number of payment periods. Using a payback model advocated by Gitman and Joehnk (1996) that includes an approximation of this tax effect, the required break-even period can be estimated with the following equation:

$$n = \frac{TCR}{(BTP_{OL} - BTP_{NL})(1 - T)} \tag{2}$$

For reasons previously discussed, this model's use of the before-tax *payment* differential also understates the refinance opportunity's periodic savings. Even though the understated savings estimate results in an understated tax effect, the offset is only partial and the net effect of using an after-tax *payment* differential is an overstatement of the estimated payback period.

2.3. Payback with taxes and amortization of discount points

Assuming a borrower expects to stay in the new loan for an extended period, he or she may want to consider a rate buy-down. In this instance, the borrower pays a lump sum of money to the lender in return for obtaining a lower-than-market rate. As per existing tax law, the cost of buying down the rate (or points) on a new loan (P_{NL}) can be amortized over the contract life of the new loan (m_{NL}). In addition, if the original loan has un-amortized points (P_{OL}) and the new loan is from a different lender, this un-amortized balance is immediately deductible. [For tax treatment details regarding rate buy-downs see Appendix Section 6, Rev. Proc. 92–12 and 92–12A, Federal Tax Handbook Sections 1745 and K-5183, and Bird and McCraw (1993).]

Given that the after-tax rate on the new loan ($r_{NL,AT}$) is lower than the after-tax rate on the original loan ($r_{OL,AT}$), the borrower will realize periodic interest savings and the required break-even period can be estimated using the following equation:

$$n = \frac{TCR - (P_{OL})T}{OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\frac{P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}}\right)T} \quad (3)$$

This model is similar to that advocated by Timmons and Winfield (1997), with the exception that the above model includes the current year tax savings corresponding to un-amortized points on the original loan. Note also that use of an estimated after-tax *interest* differential in place of the periodic *payment* savings eliminates the necessity to determine the original and new loan payments using an equal number of payment periods. This is because the payments toward principal are not included in this model, thus the number of payment periods do not need to be aligned to minimize the estimation bias resulting from a loan balance difference at break-even. In sum, even though this model incorporates the somewhat restrictive assumption of an interest-only loan, it has the advantage of eliminating the bias associated with a loan balance difference at break-even and corresponding requirement of using an equal number of payment periods.

2.4. Discounted payback with taxes

An additional factor that generally has a material effect on refinance analysis is the time value of money. Because the cost of refinancing is incurred today whereas the savings are realized over time, the periodic savings should be discounted at the borrower's opportunity cost of capital corresponding to an equivalent risk investment. As discussed in detail in the following section, the discount rate advocated in this paper is the after-tax rate on the new loan. The greater the after-tax rate and the smaller the periodic savings as a percentage of the total refinance costs, the greater the importance of discounting the refinance savings.

By including in the model the time value of money effect, the borrower realizes break-even when the present value of the periodic savings is equal to the total cost of refinancing. Expressed in equation form, the discounted payback is realized when

$$TCR = \sum_{t=1}^{n} \frac{OLB_{t=0}(r_{OL,AT} - r_{NL,AT})}{(1 + r_{NL,AT})^t} \tag{4}$$

or equivalently when

$$TCR = OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) \left[\frac{1}{r_{NL,AT}} - \frac{1}{r_{NL,AT}(1 + r_{NL,AT})^n} \right] \tag{5}$$

Note that the above term in brackets represents the present value interest factor of an annuity for (n) periods and rate ($r_{NL,AT}$). Solving for (n), the discounted payback can be estimated using the following equation:

$$n = \frac{\ln\left[\dfrac{OLB_{t=0}(r_{OL,AT} - r_{NL,AT})}{OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) - TCR(r_{NL,AT})} \right]}{\ln(1 + r_{NL,AT})} \tag{6}$$

Consistent with the prior model, this break-even approximation method assumes an interest-only loan. Because principal payments on the original loan result in interest savings that exceed those corresponding to equivalent principal payments on the new loan, the interest-only assumption results in a somewhat overstated periodic savings estimate and understated break-even solution. [This effect is discussed in detail in Section 4 of the Appendix. In addition, details regarding the derivation of Eqs. (4) and (5) are covered in Section 2 of the Appendix.]

2.5. Discounted payback with taxes and amortization of discount points

Adding into the discounted payback model a rate buy-down tax effect, the refinance opportunity's discounted payback can be expressed as:

$$TCR - (P_{OL})T = \left[OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\frac{P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}} \right) T \right] \cdot \left[\frac{1}{r_{NL,AT}} - \frac{1}{r_{NL,AT}(1 + r_{NL,AT})^n} \right] \tag{7}$$

Solving for (n), the discounted payback can be estimated using the following equation:

$$n = \frac{\ln\left[\dfrac{OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\dfrac{P_{NL}}{m_{NL}} - \dfrac{P_{OL}}{m_{OL}} \right) T}{OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\dfrac{P_{NL}}{m_{NL}} - \dfrac{P_{OL}}{m_{OL}} \right) T - r_{NL,AT}[TCR - (P_{OL})T]} \right]}{\ln(1 + r_{NL,AT})} \tag{8}$$

2.6. Discounted spreadsheet models

With recent software developments and increased use of spreadsheet analysis, a number of discounted payback spreadsheet models have been proposed. Randle and Johnson (1996) advocate an approach that uses a refinance opportunity's before-tax savings and a discount rate equal to the before-tax rate on the new loan. Rose (1992) includes the effect of taxes, and proposes use of a discount rate equal to the before-tax rate on the new loan. This model, however, does not account for the refinance savings resulting from a difference in the original and new loan balance at break-even. Chen (1997) advocates use of the refinance opportunity's before-tax savings, and suggests that the applicable discount rate depends on the type of investments the borrower typically includes in his or her portfolio. G-Yohannes (1988) includes the applicable tax effects, and advocates use of the borrower's opportunity cost of capital on a long-term investment. Finally, Marquardt and Woerheide (1988) propose a rule of thumb model based on the coefficients derived from a simulation analysis. Their findings generate a set of coefficients that can be used on the borrower's marginal tax rate, expected loan life, and rate spread to determine the break-even level of refinance cost as a percentage of the loan balance. Consistent with this paper, the authors advocate using the after-tax rate on the new loan as the applicable discount rate. In summarizing, each of these spreadsheet models differ with respect to the variables included, complexity of analysis, and discount rate advocated.

3. Equation derivations

This section begins with a discussion of the discount rate appropriate to refinance analysis, and then develops an equation-based model that includes consideration of all the previously discussed variables. The derived equations allow the borrower to accurately determine a refinance opportunity's discounted payback, net present value, and potential opportunity gain or loss associated with a refinance deferral.

3.1. Discount rate

Because the cost of refinancing is incurred today whereas the associated savings are realized over time, these savings should be discounted at the borrower's opportunity cost of capital corresponding to an equivalent-risk investment. Assuming the investor holds a well-diversified portfolio, *modern portfolio theory* suggests that the applicable risk depends on the relationship of the refinance savings with the returns of the market, as measured by the refinance opportunity's beta. Given that the refinance savings through the realized loan life are relatively certain, the risk inherent in these savings is primarily driven by uncertainty regarding the new loan life. Consequently, the refinance opportunity's beta hinges on the relationship of the new loan life with the returns of the market. While any number of scenarios could be argued where market returns and the life of the new loan may be related, the expected general relationship will not likely result in a material beta. However, even accepting the possibility of a small beta, the borrower should still not be able to find a higher equivalent-risk return in the financial marketplace than the borrower's currently available cost of capital on the new loan. Because the new loan rate includes the lender spread, the after-tax rate on new loan principal payments is materially greater than that of an equivalent-risk instrument trading in the financial marketplace. In sum, assuming even a moderate level of risk, the borrower's highest alternate rate of return on an equivalent-risk instrument will arguably be the after-tax rate on the new loan.

3.2. Discounted payback (break-even) derivation

The discussions will begin with an evaluation of the mathematics behind a break-even analysis. Stated in its most simplistic form, a break-even analysis involves a determination of the required period of time the borrower must remain with a new lower-rate loan to save an amount equal to the cost of refinancing. In an examination of this statement, it is important to note that the savings realized with acceptance of the refinance transaction occur over a period of time. Therefore, given a nontrivial cost of capital, it can be argued that an accurate break-even solution requires the use of discounted cash flow analysis. Furthermore, because the borrower's current cost of capital applicable to the investment in question is the after-tax rate on the new loan, all future expected cash flows should be discounted at this rate. Given these basic principles, the borrower will achieve break-even when the present value of the after-tax payments on the new loan ($\mathrm{ATP}_{\mathrm{NL},t}$) and remaining balance ($\mathrm{NLB}_{t=n}$) is less than the present-value of the after-tax payments on the original loan ($\mathrm{ATP}_{\mathrm{OL},t}$) and remaining balance ($\mathrm{OLB}_{t=n}$) by the total cost of refinancing. Stated in equation form, a refinance opportunity's break-even is the point at which

$$\sum_{t=1}^{n} \frac{ATP_{OL,t}}{(1+r_{NL,AT})^t} + \frac{OLB_{t=n}}{(1+r_{NL,AT})^n} = \sum_{t=1}^{n} \frac{ATP_{NL,t}}{(1+r_{NL,AT})^t} + \frac{NLB_{t=n}}{(1+r_{NL,AT})^n} + TCR \tag{9}$$

In now defining the components of Eq. (9), note that the first term on the LHS represents the present value of the after-tax payments on the original loan through break-even, and the second term is equal to the present value of the remaining original loan balance. The first two terms on the RHS represent the new loan equivalents of those just described, and the last term represents the total cost of refinancing.

Toward the effort of simplifying Eq. (9) into a form that can be easily solved without the use of amortization tables, the following analysis will break down the individual terms of Eq. (9) into their respective components. Beginning with the LHS of Eq. (9), the analysis will first examine the present value of the after-tax payments on the original loan. As expressed in Eq. (10) below, this term is comprised of the present value of the principal payments, the present value of the after-tax interest expense on an interest-only loan, and the present value of the after-tax interest expense not realized because of payments on principal. (For derivation details see Appendix Sections 2 through 4.)

$$\sum_{t=1}^{n} \frac{ATP_{OL,t}}{(1+r_{NL,AT})^t} = \sum_{t=1}^{n} \frac{a_{OL}(1+r_{OL,BT})^{t-1}}{(1+r_{NL,AT})^t} + \sum_{t=1}^{n} \frac{OLB_{t=0}(r_{OL,AT})}{(1+r_{NL,AT})^t}$$

$$- a_{OL}(r_{OL,AT}) \sum_{v=2}^{n} \frac{\sum_{t=2}^{v} (1 + r_{OL,BT})^{v-t}}{(1 + r_{NL,AT})^{v}} \tag{10}$$

Alternatively expressed, the first term on the RHS of Eq. (10) represents the present value of the principal payments, and the combined value of the second and third term represents the present value of the after-tax interest expense.

Evaluating now the second term on the LHS of Eq. (9), it is readily apparent that the present value of the remaining loan balance at break-even can be equivalently expressed as the present value of the original loan balance, less the present value of the reduction in principal at break-even. (For derivation details of this latter term, see Section 5 of the Appendix.) Accordingly, the second term on the LHS of Eq. (9) can be broken into its two components and expressed as

$$\frac{OLB_{t=n}}{(1 + r_{NL,AT})^{n}} = \frac{OLB_{t=0}}{(1 + r_{NL,AT})^{n}} - \frac{a_{OL} \sum_{t=1}^{n} (1 + r_{OL,BT})^{n-t}}{(1 + r_{NL,AT})^{n}} \tag{11}$$

Before moving on to the RHS of Eq. (9), it is important to note that the break-even solution is not impacted by the payment structure of the new loan or whether the refinance costs are included in the new loan balance. Because the discount rate is equal to the after-tax rate on the new loan, any payments toward principal realize after-tax interest savings equal to the after-tax rate on the new loan. Therefore, toward the effort of simplifying the break-even equation without any affect on the break-even solution, the new loan balance and principal payments will be set equal to those of the original loan.

Restating the new loan components in a form consistent with Eqs. (10) and (11), the present value of the after-tax payments on the new loan can be expressed as

$$\sum_{t=1}^{n} \frac{ATP_{NL,t}}{(1 + r_{NL,AT})^{t}} = \sum_{t=1}^{n} \frac{a_{OL}(1 + r_{OL,BT})^{t-1}}{(1 + r_{NL,AT})^{t}} + \sum_{t=1}^{n} \frac{OLB_{t=0}(r_{NL,AT})}{(1 + r_{NL,AT})^{t}} \tag{12}$$

$$- a_{OL}(r_{NL,AT}) \sum_{v=2}^{n} \frac{\sum_{t=2}^{v} (1 + r_{OL,BT})^{v-t}}{(1 + r_{NL,AT})^{v}}$$

Likewise, the present value of the new loan's remaining balance at break-even can be stated as

$$\frac{NLB_{t=n}}{(1 + r_{NL,AT})^{n}} = \frac{OLB_{t=0}}{(1 + r_{NL,AT})^{n}} - \frac{a_{OL} \sum_{t=1}^{n} (1 + r_{OL,BT})^{n-t}}{(1 + r_{NL,AT})^{n}} \tag{13}$$

By now substituting Eqs. (10) through (13) into Eq. (9), note that the first terms of Eqs. and (10) and (12) cancel one another out. The same is true for both terms in Eqs. (11) and (13). Consequently, Eq. (9) can be simplified and restated as

$$TCR = \sum_{t=1}^{n} \frac{OLB_{t=0}(r_{OL,AT} - r_{NL,AT})}{(1 + r_{NL,AT})^t} - a_{OL}(r_{OL,AT} - r_{NL,AT}) \sum_{v=2}^{n} \frac{\sum_{t=2}^{v} (1 + r_{OL,BT})^{v-t}}{(1 + r_{NL,AT})^v} \quad (14)$$

In now evaluating the components of Eq. (14), the first term on the RHS can be interpreted as the present value of the after-tax interest savings assuming interest-only loans, and the second term as the present value of the after-tax interest savings not realized because of scheduled principal payments. Combined, these two terms represent the present value of the after-tax interest savings through break-even. Therefore, the break-even period can be interpreted as the number of periods (n) required for the present value of the after-tax interest savings to equal the total cost of refinancing.

Recognizing that the first term on the RHS of Eq. (14) is an annuity and the second term a geometric series within a series, the break-even equation can be simplified and the payback period solved using the following restated break-even equation

$$TCR = OLB_{t=0}(r_{OL,AT} - r_{NL,AT})\left[\frac{1}{r_{NL,AT}} - \frac{1}{r_{NL,AT}(1 + r_{NL,AT})^n}\right] - \frac{a_{OL}(r_{OL,AT} - r_{NL,AT})\{r_{NL,AT}[(1 + r_{OL,BT})^n - 1] - r_{OL,BT}[(1 + r_{NL,AT})^n - 1]\}}{r_{NL,AT}(r_{OL,BT})(r_{OL,BT} - r_{NL,AT})(1 + r_{NL,AT})^n} \quad (15)$$

[For derivation details regarding the above two RHS terms, see Appendix Sections 2 and 4, respectively.]

Including in the analysis the existence of an original and/or new loan rate buy-down effect, Eq. (14) becomes

$$TCR - (P_{OL})T = \sum_{t=1}^{n} \frac{OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\frac{P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}}\right)T}{(1 + r_{NL,AT})^t} - a_{OL}(r_{OL,AT} - r_{NL,AT}) \sum_{v=2}^{n} \frac{\sum_{t=2}^{v} (1 + r_{OL,BT})^{v-t}}{(1 + r_{NL,AT})^v} \quad (16)$$

Restating Eq. (16) in its simplified form, the refinance opportunity's break-even period can be solved using

$$TCR - (P_{OL})T = \left[OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\frac{P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}} \right) T \right] \cdot \left[\frac{1}{r_{NL,AT}} - \frac{1}{r_{NL,AT}(1 + r_{NL,AT})^n} \right] - \frac{a_{OL}(r_{OL,AT} - r_{NL,AT})\{r_{NL,AT}[(1 + r_{OL,BT})^n - 1] - r_{OL,BT}[(1 + r_{NL,AT})^n - 1]\}}{r_{NL,AT}(r_{OL,BT})(r_{OL,BT} - r_{NL,AT})(1 + r_{NL,AT})^n} \tag{17}$$

[Note that the equation for determining the initial principal payment on the original loan (a_{OL}) can be found in Section 1 of the Appendix as Eq. (2A). For derivation details regarding the original and new loan rate buy-down tax effect, see Appendix Sections 6.1. and 6.2.]

3.3. Net present value (NPV)

With a minor modification to Eq. (16), the net present value of the refinance opportunity can now be determined. By replacing the break-even variable (n) with one representing the projected life of the new loan (z), the net present value of the refinance opportunity can be expressed as

$$NPV = \sum_{t=1}^{z} \frac{OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\frac{P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}} \right) T}{(1 + r_{NL,AT})^t} - a_{OL}(r_{OL,AT} - r_{NL,AT}) \sum_{v=2}^{z} \frac{\sum_{t=2}^{v} (1 + r_{OL,BT})^{v-t}}{(1 + r_{NL,AT})^v} + (P_{OL})T - TCR \tag{18}$$

and solved using

$$NPV = \left[OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\frac{P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}} \right) T \right] \cdot \left[\frac{1}{r_{NL,AT}} - \frac{1}{r_{NL,AT}(1 + r_{NL,AT})^z} \right] - \frac{a_{OL}(r_{OL,AT} - r_{NL,AT})\{r_{NL,AT}[(1 + r_{OL,BT})^z - 1] - r_{OL,BT}[(1 + r_{NL,AT})^z - 1]\}}{r_{NL,AT}(r_{OL,BT})(r_{OL,BT} - r_{NL,AT})(1 + r_{NL,AT})^z} + (P_{OL})T - TCR \tag{19}$$

Therefore, the NPV of the refinance opportunity is equal to the present value of after-tax interest savings, net of the total cost of refinancing.

Note that Eqs. (16) through (19) assume the new loan is from a different lender than that of the original loan. If the new loan is from the same lender, the un-amortized buy-down balance on the original loan must be amortized over the life of the new loan. Given this scenario, Eqs. (16) through (19) change from that stated above. [See Appendix Section 6.3. for a discussion of this tax effect, along with the restated final form break-even and net present value equations.]

4. A refinance analysis example

Toward the objective of providing an example and check figures for the previously developed model, three loan alternatives will be examined as noted below. Specifics regarding the original loan include an outstanding balance of \$250,000, a before-tax rate of 8.0%, a remaining contract life of 20 years, and an un-amortized buy-down balance of \$1,400. Lastly, the borrower's marginal tax rate is 38.6%.

Additional Input Information		Output Information	
Loan A:		Break-even:	19.2 months
Lender:	New	$NPV_{3\ YRS}$:	\$2,760
Term:	30 YRS	$NPV_{7\ YRS}$:	\$9,282
Annual Before-Tax Rate:	6.50%	$NPV_{11\ YRS}$:	\$13,930
Refinancing Costs:	\$4,000	$NPV_{16\ YRS}$:	\$17,279
Rate Buy-Down Cost:	\$0	$NPV_{20\ YRS}$:	\$18,073
Total Cost of Refinancing:	\$4,000		
Loan B:		Break-even:	28.5 months
Lender:	New	$NPV_{3\ YRS}$:	\$1,417
Term:	30 YRS	$NPV_{7\ YRS}$:	\$9,205
Annual Before-Tax Rate:	6.25%	$NPV_{11\ YRS}$:	\$14,805
Refinancing Costs:	\$4,000	$NPV_{16\ YRS}$:	\$18,902
Rate Buy-Down Cost:	\$2,500	$NPV_{20\ YRS}$:	\$19,927
Total Cost of Refinancing:	\$6,500		
Loan C:		Break-even:	44.9 months
Lender:	Original	$NPV_{3\ YRS}$:	(\$1,892)
Term:	30 YRS	$NPV_{7\ YRS}$:	\$7,262
Annual Before-Tax Rate:	6.00%	$NPV_{11\ YRS}$:	\$13,911
Refinancing Costs:	\$5,000	$NPV_{16\ YRS}$:	\$18,858
Rate Buy-Down Cost:	\$5,500	$NPV_{20\ YRS}$:	\$20,177
Total Cost of Refinancing:	\$10,500		

In now evaluating the above output, it is readily apparent that the borrower will not realize expected savings from refinancing unless he or she expects to remain in the new loan beyond 19.2 months. Moreover, it is also clear that the borrower's preferred loan option depends in large part on his or her expectations and uncertainty regarding the life of the new loan. For example, if the borrower is fairly certain as to the expected new loan life, Loan A yields the greatest savings for an expected life of approximately 7 years or less. Alternatively, Loan B generates the greatest savings for expected lives extending somewhat beyond 7 years and through a little more than 16 years. Finally, for periods extending beyond 16 years, the borrower realizes the greatest expected savings by refinancing with Loan C. However, keep in mind that these loan selections represent the borrower's preferred options as based in part on the premise that the borrower's expected new loan life is fairly certain. If the borrower expects to stay in the new loan for another 8 years but could move in as few as three, the borrower may decide to accept the lower expected savings of Loan A in lieu of the higher expected savings and risk of Loan B. Another consideration is that the life of the new loan does not wholly depend on the moving plans of the borrower. For example, the new loan life could be terminated sooner than expected because of a subsequent refinance decision.

Note that the net present value calculations are only computed through 20 years. This is because savings associated with the refinance opportunities cannot accrue beyond the remaining life of the original loan. Therefore, this model will not always reveal the preferred loan when the expected new loan life extends beyond the remaining original loan life. Note, however, that the previously illustrated alternative loan scenario properly identifies loan C as the preferred loan for periods extending somewhat beyond 16 years and through the loan's 30-year life. This is because Loan C is the lowest rate loan and revealed itself as the highest NPV alternative within the remaining life of the original loan. Finally, a comparison of loans with different lives is only appropriate using this model when the expected life of the new loan falls within the shortest contract life of the alternative new loans.

5. Summary and concluding remarks

The current methods used to evaluate a mortgage refinance opportunity employ various forms of break-even analysis. These methods differ considerably with respect to the variables included, complexity of analysis, and discount rate advocated. This paper examines these alternate break-even models, with discussions emphasizing the relevance of each variable to the analysis. In addition, an equation-based model is developed that considers each of these variables, while decreasing the complexity of analysis to the solution of an equation. Through use of this model, the borrower can easily and accurately determine a refinance opportunity's required break-even period and expected savings. Finally, the model also allows the borrower to quantify the potential opportunity gain or loss associated with a refinance deferral.

Acknowledgment

The author would like to thank Conrad Ciccotello and an anonymous referee for their considerable assistance and helpful comments.

Appendix: Formula derivation details

1. Original loan before-tax payment (BTP_{OL}) and initial principal payment (a_{OL})

The before-tax payment (principal and interest-only) on the original loan (BTP_{OL}) can be determined as follows:

$$BTP_{OL} = \frac{OLB_{t=0}(r_{OL,BT})(1 + r_{OL,BT})^{m_{OL}}}{(1 + r_{OL,BT})^{m_{OL}} - 1} \tag{1A}$$

[The solution of this equation is used as an input variable to break-even Eqs. (1) and (2). Note that the new loan equivalent of this equation only changes with respect to the loan rate. These models require that the new loan balance and number of payment periods must equal those of the original loan.]

In addition, the initial period principal payment of the original loan (a_{OL}) can be determined using

$$a_{OL} = \frac{OLB_{t=0}(r_{OL,BT})}{(1 + r_{OL,BT})^{m_{OL}-1}} \tag{2A}$$

[The solution of this equation is used as an input variable to break-even Eqs. (17) and (26A), as well as net present value Eqs. (19) and (27A).]

2. Present value of the after-tax interest savings through break-even on an interest-only loan ($PVISIO_n$)

The present value of the original loan after-tax interest *expense* assuming an interest-only loan ($PVIEIO_n$) through period (n) can be expressed as

$$PVIEIO_n = \sum_{t=1}^{n} \frac{OLB_{t=0}(r_{OL,AT})}{(1 + r_{NL,AT})^t} \tag{3A}$$

Recognizing that the payment stream is an ordinary annuity, (3A) can be equivalently stated as

$$PVIEIO_n = OLB_{t=0}(r_{OL,AT})\left[\frac{1}{r_{NL,AT}} - \frac{1}{r_{NL,AT}(1 + r_{NL,AT})^n}\right] \tag{4A}$$

Therefore, the present value of the after-tax interest *savings* from refinancing assuming interest-only loans ($PVISIO_n$) through period (n) can be expressed as

$$PVISIO_n = \sum_{t=1}^{n} \frac{OLB_{t=0}(r_{OL,AT} - r_{NL,AT})}{(1 + r_{NL,AT})^t} \tag{5A}$$

or equivalently as

$$PVISIO_n = OLB_{t=0}(r_{OL,AT} - r_{NL,AT})\left[\frac{1}{r_{NL,AT}} - \frac{1}{r_{NL,AT}(1 + r_{NL,AT})^n}\right] \tag{6A}$$

[Eqs. (5A) and (6A) are first used in break-even Eqs. (4) and (5), respectively.]

3. Present value of the principal payments through break-even ($PVPP_n$)

Each payment on a conventional fixed rate mortgage includes both a principal and interest expense component. Because payments on principal reduce the interest expense component of subsequent payments, each principal payment can be regarded as growing in value by the before-tax rate on the loan. Therefore, the present value of the principal payments on the original loan through break-even ($PVPP_{OL,n}$) can be expressed as

$$PVPP_{OL,n} = \frac{a_{OL}}{(1 + r_{NL,AT})^1} + \frac{a_{OL}(1 + r_{OL,BT})^1}{(1 + r_{NL,AT})^2} + \frac{a_{OL}(1 + r_{OL,BT})^2}{(1 + r_{NL,AT})^3} + \cdots + \frac{a_{OL}(1 + r_{OL,BT})^{n-1}}{(1 + r_{NL,AT})^n} \tag{7A}$$

Expressing (7A) in a mathematically concise manner yields

$$PVPP_{OL,n} = \sum_{t=1}^{n} \frac{a_{OL}(1 + r_{OL,BT})^{t-1}}{(1 + r_{NL,AT})^t} \tag{8A}$$

[Note that this term is first introduced in the body of the paper in Eq. (10) and occurs again in Eq. (12).]

4. Present value of the after-tax interest savings not realized because of principal payments ($PVISNR_n$)

In addition to the previously discussed fact that principal payments reduce the loan balance, these same principal payments also reduce the amount of interest expense. Given that Section 2 of the Appendix determined the present value of the interest savings from refinancing assuming interest-only loans, the below discussions develop the present value of the interest savings not realized because of principal payments. In other words, the present value of the interest savings from refinancing can be interpreted as equal to the present value of the interest savings assuming interest-only loans, less the present value of the interest savings not realized because of principal payments. This principal payment effect is because payments toward principal on the original loan realize a higher after-tax rate than those of the new loan.

The amount of the after-tax interest expense on the original loan not realized because of principal payments for a single period (v) is equal to the accumulated reduction in principal through period (v) times the after-tax rate on the loan. Accordingly, the present value of the after-tax interest expense on the original loan not realized for a single period (v) resulting from cumulative principal payments ($PVIENR_{OL,v}$) can be expressed as

$$PVIENR_{OL,v} = [a_{OL}(1 + r_{OL,BT})^{v-2} + a_{OL}(1 + r_{OL,BT})^{v-3} + \cdots + a_{OL}(1 + r_{OL,BT})^{v-v}]\frac{r_{OL,AT}}{(1 + r_{NL,AT})^{v}} \quad (9A)$$

For example, the first payment toward principal (a_{OL}) occurs one period from today, and thus results in a corresponding period two interest reduction equal to $(a_{OL})(r_{OL,AT})$, with a present value equal to $(a_{OL})(r_{OL,AT})/(1 + r_{NL,AT})^2$. Likewise, the present value of the period three interest reduction resulting from period one and two principal payments is equal to the combined value of $(a_{OL})(1 + r_{OL,BT})(r_{OL,AT})/(1 + r_{NL,AT})^3$ and $(a_{OL})(r_{OL,AT})/(1 + r_{NL,AT})^3$. Note that this process is consistent with the principal reduction discussions covered in the immediately preceding section of this Appendix. Expressing (9A) in a mathematically concise manner yields

$$PVIENR_{OL,v} = \frac{a_{OL}(r_{OL,AT}) \sum_{t=2}^{v} (1 + r_{OL,BT})^{v-t}}{(1 + r_{NL,AT})^{v}} \quad (10A)$$

Because this process occurs for each period (v) through break-even, the present value of the original loan interest expense not realized because of principal payments across (n) periods can be expressed as

$$PVIENR_{OL,n} = a_{OL}(r_{OL,AT}) \sum_{v=2}^{n} \frac{\sum_{t=2}^{v} (1 + r_{OL,BT})^{v-t}}{(1 + r_{NL,AT})^{v}} \quad (11A)$$

Solving for the sum of this geometric series within a series, the present value of the original loan interest expense not realized because of principal payments through period (n) can be restated as

$$PVIENR_{OL,n} = \frac{a_{OL}(r_{OL,AT})\{r_{NL,AT}[(1 + r_{OL,BT})^{n} - 1] - r_{OL,BT}[(1 + r_{NL,AT})^{n} - 1]\}}{r_{NL,AT}(r_{OL,BT})(r_{OL,BT} - r_{NL,AT})(1 + r_{NL,AT})^{n}} \quad (12A)$$

Toward the objective of determining the difference in this value for the original and new loan, the new loan equivalents of (11A) and (12A) will now be determined. Given that the discount rate is equal to the after-tax rate on the new loan, the payment structure of the new loan does not impact the break-even solution. This is because any payments toward principal realize interest savings equal to the after-tax rate on the new loan. Therefore, the new loan principal payments will be set equal to those of the original loan. Using similar logic to that used in deriving Eq. (11A), the present value of the new loan interest expense not realized because of principal payments across (n) periods can be expressed as

$$PVIENR_{NL,n} = a_{OL}(r_{NL,AT}) \sum_{v=2}^{n} \frac{\sum_{t=2}^{v} (1 + r_{OL,BT})^{v-t}}{(1 + r_{NL,AT})^{v}} \tag{13A}$$

and the simplified equivalent as

$$PVIENR_{NL,n} = \frac{a_{OL}(r_{NL,AT})\{r_{NL,AT}[(1 + r_{OL,BT})^{n} - 1] - r_{OL,BT}[(1 + r_{NL,AT})^{n} - 1]\}}{r_{NL,AT}(r_{OL,BT})(r_{OL,BT} - r_{NL,AT})(1 + r_{NL,AT})^{n}} \tag{14A}$$

Therefore, the present value of the interest *savings* not realized with refinancing because of principal payments is equal to the difference between (11A) and (13A), which can be expressed as

$$PVISNR_{n} = a_{OL}(r_{OL,AT} - r_{NL,AT}) \sum_{v=2}^{n} \frac{\sum_{t=2}^{v} (1 + r_{OL,BT})^{v-t}}{(1 + r_{NL,AT})^{v}} \tag{15A}$$

and equivalently stated as the difference between (12A) and (14A), or

$$PVISNR_{n} = \frac{a_{OL}(r_{OL,AT} - r_{NL,AT})\{r_{NL,AT}[(1 + r_{OL,BT})^{n} - 1] - r_{OL,BT}[(1 + r_{NL,AT})^{n} - 1]\}}{r_{NL,AT}(r_{OL,BT})(r_{OL,BT} - r_{NL,AT})(1 + r_{NL,AT})^{n}} \tag{16A}$$

Note that this term is interpreted as interest *savings* not realized because the interest-only savings is overstated by this amount. [As stated above in Eqs. (15A) and (16A), these terms are first introduced in the body of the paper as components of Eqs. (14) and (15).]

5. Present value of the cumulative principal reduction at break-even ($PVCPR_n$)

As previously discussed in Section 3 of the Appendix, each principal payment can be regarded as growing in value by the before-tax rate on the loan. Therefore, the initial principal payment on the original loan (a_{OL}) will result in a principal balance reduction at break-even (PR_n) equal to

$$PR_n = a_{OL}(1 + r_{OL,BT})^{n-1} \tag{17A}$$

Because each subsequent payment toward principal (a_{OL}) will have a similar effect, the cumulative principal balance reduction at break-even (CPR_n) can be expressed as

$$CPR_n = a_{OL}(1 + r_{OL,BT})^{n-1} + a_{OL}(1 + r_{OL,BT})^{n-2} + \cdot \cdot \cdot + a_{OL}(1 + r_{OL,BT})^{n-n} \tag{18A}$$

Therefore, given that the after-tax rate on the new loan ($r_{NL,AT}$) is the applicable discount rate, the present value of the cumulative principal reduction at break-even ($PVCPR_n$) can be expressed as

$$PVCPR_n = \frac{a_{OL}\sum_{t=1}^{n}(1 + r_{OL,BT})^{n-t}}{(1 + r_{NL,AT})^{n}} \tag{19A}$$

[For a discussion of the applicable discount rate, see Section 3.1 of the body of the paper. Note that this equation is first introduced as an individual term in Eq. (11).]

6. Present value of the rate buy-down tax effect through break-even (PTE_n)

In the interest of brevity, note that the below derivations assume the use of discounted cash flow analysis. If not, as is the case for the model applicable to Eq. (3), the only difference is that the discounting aspects of the below equations should be eliminated.

6.1. New loan points tax effect ($PTE_{NL,n}$)

As per existing tax law, the cost of buying down the rate (or points) on a new loan (P_{NL}) can be amortized over the contract life of the new loan (m_{NL}). The applicable buy-down cost is determined by multiplying the discount points (r_p) by the new loan balance ($NLB_{t=0}$), where each discount point is equal to 0.01. Therefore, the present value of the new loan points tax effect ($PTE_{NL,n}$) through the break-even period (n) can be stated as

$$PTE_{NL,n} = \sum_{t=1}^{n}\frac{\dfrac{r_p(NLB_{t=0})T}{m_{NL}}}{(1 + r_{NL,AT})^{t}} \tag{20A}$$

or equivalently as

$$PTE_{NL,n} = \sum_{t=1}^{n}\frac{\left(\dfrac{P_{NL}}{m_{NL}}\right)T}{(1 + r_{NL,AT})^{t}} \tag{21A}$$

6.2. Combined points tax effect assuming new lender

If the original loan has an un-amortized buy-down balance and the new loan is financed with a *new lender*, there will be two tax effects. The un-amortized balance is immediately deductible and the periodic tax savings from the remaining amortization period will not be realized with refinancing. The present value of the original loan points tax effect ($PTE_{OL,n}$) through the break-even period (n) can be stated as

$$PTE_{OL,n} = (P_{OL})T - \sum_{t=1}^{n} \frac{\left(\frac{P_{OL}}{m_{OL}}\right)T}{(1 + r_{NL,AT})^t} \tag{22A}$$

Therefore, the present value of the *combined* original and new loan tax savings from amortization of the rate buy-down points can be stated as

$$PTE_n = (P_{OL})T + \sum_{t=1}^{n} \frac{\left(\frac{P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}}\right)T}{(1 + r_{NL,AT})^t} \tag{23A}$$

Therefore, to determine the break-even period of a refinance opportunity including a rate buy-down tax effect on both the original and new loan, Eq. (23A) should be added to the RHS of Eq. (14), yielding Eq. (16).

6.3. Combined points tax effect assuming original lender

Alternatively, if the original loan has an un-amortized buy-down balance and the new loan is financed with the *original lender*, the original un-amortized buy-down balance should be amortized over the life of the new loan. In this instance, the present value of the original loan points tax effect ($PTE_{OL,n}$) through the break-even period (n) becomes

$$PTE_{OL,n} = \sum_{t=1}^{n} \frac{\left(\frac{P_{OL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}}\right)T}{(1 + r_{NL,AT})^t} \tag{24A}$$

Assuming the new loan is with the original lender, the present value of the *combined* original and new loan tax savings from amortization of the rate buy-down points can be stated as

$$PTE_n = \sum_{t=1}^{n} \frac{\left(\frac{P_{OL} + P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}}\right)T}{(1 + r_{NL,AT})^t} \tag{25A}$$

Therefore, given this possible scenario, a number of the equations stated throughout this paper will require a second version to account for this different tax treatment. Specifically, these equations include (3), (7), (8), and (16) through (19). However, given that (3), (7), and

(8) represent break-even approximation equations, and (16) and (18) are not final form equations used in any calculations, only Eqs. (17) and (19) are restated below. Equation (17) is the final form break-even equation that includes consideration of all variables, and under this scenario should be stated as

$$TCR = \left[OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\frac{P_{OL} + P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}} \right) T \right] \cdot \left[\frac{1}{r_{NL,AT}} - \frac{1}{r_{NL,AT}(1 + r_{NL,AT})^n} \right] - \frac{a_{OL}(r_{OL,AT} - r_{NL,AT})\{r_{NL,AT}[(1 + r_{OL,BT})^n - 1] - r_{OL,BT}[(1 + r_{NL,AT})^n - 1]\}}{r_{NL,AT}(r_{OL,BT})(r_{OL,BT} - r_{NL,AT})(1 + r_{NL,AT})^n} \quad (26A)$$

Similarly, the final form net present value Eq. (19) becomes

$$NPV = \left[OLB_{t=0}(r_{OL,AT} - r_{NL,AT}) + \left(\frac{P_{OL} + P_{NL}}{m_{NL}} - \frac{P_{OL}}{m_{OL}} \right) T \right] \cdot \left[\frac{1}{r_{NL,AT}} - \frac{1}{r_{NL,AT}(1 + r_{NL,AT})^z} \right] - \frac{a_{OL}(r_{OL,AT} - r_{NL,AT})\{r_{NL,AT}[(1 + r_{OL,BT})^z - 1] - r_{OL,BT}[(1 + r_{NL,AT})^z - 1]\}}{r_{NL,AT}(r_{OL,BT})(r_{OL,BT} - r_{NL,AT})(1 + r_{NL,AT})^z} - TCR \quad (27A)$$

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