

# Time, wealth, and human capital as determinants of asset allocation

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*“Dost thou love life? Then do not squander time, for that is the stuff life is made of”. . .*  
Benjamin Franklin

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## Abstract

This paper provides a theoretical model for individuals to make asset allocation decisions based on their value of leisure time, risk tolerance, and a retirement goal. As individuals go through life cycles financial wealth increases while human capital and time decrease. A critical point is identified where individuals maximize leisure time and financial wealth. The model suggests basing asset allocation decisions on the level of wealth in relationship to the retirement goal and time, rather than the age of the individual typically used in investment planning. © 2004 Academy of Financial Services. All rights reserved.

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## 1. Introduction

This paper presents an individual life cycle model for use in the asset allocation decision process. The model assumes that individuals reach a critical point in their life where they desire to maximize leisure time as well as wealth. The model presented in this paper extends the theoretical model of Bodie et al., (1992) and empirical work of Van Eaton and Conover (2002) by illustrating how time impacts portfolio and consumption choices over an individ-

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ual's life cycle. An individual's total wealth is comprised of human capital and financial wealth. Total wealth at the start of an individual's life cycle is comprised almost entirely of human capital, which is defined as the present value of all future earnings. With the passing of time, individuals' total remaining human capital decreases while their financial wealth increases. As individuals approach retirement they rely more on financial wealth and less on human capital to achieve a retirement goal necessary to maintain a desired standard of living.

Time is an important determinate of an individual's total wealth. Increasing the amount of time devoted to labor hours increases human capital that in turn increases total wealth. In addition, increasing the amount of time devoted to labor hours, delays consumption from savings, thus, allowing savings to continue to compound and accumulate. A major constraining factor for individuals is their limited amount of time allotted to them. Do individuals live for the enjoyment of accumulating wealth or do individuals accumulate wealth so that they can enjoy leisure time? If individuals live to accumulate wealth, why do they retire? A plausible reason for retirement is related to the fact that individuals value leisure time. Individuals must choose how to allocate their time between labor hours and leisure time. Individuals, who value leisure time, maximize leisure time as well as wealth.

Traditional financial planning models suggest an asset allocation strategy that focuses primarily on the age of the individual. This strategy emphasizes the maximization of retirement wealth and ignores the importance of leisure time for individuals. An appropriate asset allocation strategy should consider all relevant factors for the individual, including the value of leisure time.

In this paper, a critical point is identified where the individual's wealth comprised of financial investments is equal to total remaining human capital. This is a very important turning point in the individual's life because the expected return from financial wealth begins to contribute more to current and future consumption needs than human capital. Section 2 provides a discussion of the issues of time horizon, time diversification, shortfall risk, and the life cycle model as a determinate for asset allocation. A life cycle model is defined in Section 3 illustrating the tradeoffs between human capital, time, and financial wealth. An important implication of the model suggests that individuals who desire to maximize both leisure time and wealth should focus on critical wealth levels as well as time horizons. The model is expanded to illustrate the impact of adding an annuity payment in Section 4. In Section 5, there is a discussion of the implementation of the model to the asset allocation decision process.

## **2. Time horizon and equity allocation**

The issues of time diversification and asset allocation are crucial for individual investors. Conventional financial planning models support the notion of time diversification and individual investors are encouraged to reduce the amount held in equities as they approach retirement. Malkiel (1996) describes individuals' tolerance towards risk as a function of their attitude as well as their capacity for risk. Malkiel suggests that risk attitude is subjective, but their risk capacity can be measured by their position in the life cycle. In practice the life cycle model of individual investment planning is widely followed. However, time horizons

identified by the model are often subjective. A related concern extending from the issue of time diversification is the inability to define a critical time horizon in applying the life cycle model for investment. Therefore, as individuals get closer to retirement, arbitrary time horizons are suggested for investors to reduce their portion in equities. Financial planners suggest smaller allocations of equities beginning 10 to 15 years before retirement. However, the appropriate time horizon for this shift out of equities to occur is somewhat ambiguous. The practice of reducing equity allocation is examined in light of time diversification, investor life cycles and shortfall risk. Time diversification suggests that above-average returns tend to offset below-average returns over long horizons. Under the notion of time diversification, investors should invest in riskier equities with higher expected returns. However, Samuelson (1963) presents a strong argument against the notion of time diversification. Samuelson suggests individuals should hold the same amount in equities regardless of time horizon if equity returns follow a random walk and individuals' utility curves resemble the logarithm of wealth as presented by Bernoulli. This mathematical proof suggesting time diversification does not exist is puzzling given that common individual investment advice and practice suggests shifting to less risky portfolios in later stages of life.

To resolve the mathematical puzzle presented by Samuelson, several cases illustrate that time horizon is important and time diversification is plausible.<sup>1</sup> Samuelson (1989) presents a case for a greater tolerance for equities in an individual's portfolio with a longer time horizon. If individual utility functions are more risk averse than the log function implied by Bernoulli, then a shift out of equities is rationale. Economists suggest power functions [i.e.,  $U(W) = -W$  or  $-1/W^2$ ] represent a more realistic utility of wealth function for individuals than the log function first assumed by Bernoulli. Samuelson states, "older people will put less into risky stocks when they have fewer years to go before the terminal date of retiring or bequeathing." He further explains, "the present discounted value of the terminal substance wealth, which is  $S/(1+i)^t$ , must in effect be put into escrow in the form of safe cash or money market funds. As  $T$ , the time left in the horizon shrinks,  $S/(1+i)^t$  will rise, thereby displacing from one's equity portfolio a larger and larger fractional share."

In addition, empirical investigations of shortfall risk question shifting out of equities and into bonds. Vora and McGinnis (2000) find that bond-heavy portfolios can result in consumption shortfalls and stock-heavy portfolios can result in large windfalls. Ameriks et al. (2001) examine the performance of portfolios with various weights in equities in relationship to sustainable withdrawal rates for rolling 20, 30, and 40 year retirement periods. They find that the most conservative portfolios, with the lowest percentage in equities, result in significantly less maximum withdrawal rates for the 30 and 40-year retirement periods. They found the results for the 20-year retirement period were about the same regardless of the percentage in equities. These findings question the conventional advice of reducing the percentage in equities as retirement approaches.

The life cycle model presents a very compelling argument for the reduction of allocation to equities as individuals approach retirement. Bodie (2003) suggests, "A person's welfare depends not only on her end-of-period wealth but also on the consumption of goods and leisure over her entire lifetime." Bodie et al. (1992) present a life cycle model that suggests individuals choose the rate of consumption, asset allocation of investments, and the amount

of leisure and labor time. An investor's total retirement portfolio, or total wealth, is made up of both financial assets and human capital.

Bodie et al. (1992) suggest that investors can allocate more of their investments to equities when they are young, because they have more flexibility to adjust their human capital at a young age. In other words, individuals can work more hours to make up for losses when they are young. Furthermore, they assume that human capital is less risky, and therefore, an individual's investment portfolio can take on more risk when their total wealth is comprised more of less risky human capital. As an individual goes through his or her life cycle human capital is depleted and financial investments increase in value. Therefore, they suggest reducing the amount invested in equities as a greater portion of the individual's wealth is comprised of financial investments instead of human capital.

While the model presented by Bodie et al. (1992) is very interesting in theory, it is very difficult to implement in practice because of the parameter estimates required in the model. More recently, Gollier (2002) suggests that an individual with liquidity constraints has a shorter time horizon for consumption, and therefore, an increase in risk aversion. Van Eaton and Conover (2002) empirically investigate the life cycle model and find that individual financial circumstances and portfolio horizons play an important role in asset allocation decisions.

Van Eaton and Conover (2002) provide a framework for testing the empirical life cycle model of Bodie et al. (1992) by simplifying two assumptions of their model. First, they assume there is no labor supply flexibility. Therefore, individuals cannot adjust their labor hours later in life to make up for low returns on investments. Second, they assume that an individual's total wealth is comprised of financial investments and human capital. Human capital is defined as the present value of future cash contributions. While the empirical investigation of Van Eaton and Conover (2002) are interesting their model does not allow for flexibility in the labor supply. This limits the model and does not allow for the tradeoff of leisure time and labor hours.

### **3. A life cycle model based on time and wealth**

The model developed in this paper extends the Bodie et al. (1992) life cycle model by quantifying the tradeoff between time and wealth. Furthermore, the model presents a dynamic process that allows flexibility for individuals' unique needs and preferences. Specifically individuals are not assumed to always solely maximize wealth. In addition to maximizing wealth, individuals make decisions to maximize their standard of living and leisure time. Social security and tax impacts are not considered explicitly in the examples or model. However, an individual could easily adjust their retirement goal appropriately to account for these variables.<sup>2</sup>

Total wealth for an individual is comprised of human capital and financial wealth. At the start of an individual's life cycle total wealth is comprised almost entirely of human capital (*HC*) which is defined as the present value of future earnings power. Human capital decreases at an increasing rate over time. Eventually human capital is totally depleted for all individuals and total wealth is comprised entirely of financial wealth.

The model summarizes choices throughout an individual's life regarding the allocation of time, investment, consumption, and savings that are consistent with an individual's desired standard of living and allocation of leisure time. All individuals are provided a finite amount of time in life ( $T$ ). Individuals make choices throughout their life regarding the allocation of their time towards labor hours,  $l$ , and leisure time,  $t$ . The total amount of time allocated to labor hours and leisure time is constrained by the following function,  $T = l + t$ . Increasing allocation of time towards labor hours, results in increasing the individuals earnings power.

In addition to making choices regarding time, choices are made throughout an individual's life regarding consumption,  $C$ , and savings,  $A$ . An individual's current standard of living is directly related to current consumption,  $C_o$ . An individual's desired standard of living after retirement is equivalent to future consumption,  $C_r$ . Individuals are assumed to desire the same standard of living after retirement that they have before retirement,  $C_o = C_r$ . The individuals earnings power,  $EP$ , is used for both consumption and savings,  $EP = C_o + A$ .

The expected wealth at retirement,  $FV$ , is a function of current savings,  $PV$ , future savings payments,  $A$ , and an expected rate of return on investments,  $r$ . Individuals choose a retirement goal,  $FV$ , consistent with their desired standard of living and risk tolerance,  $r$ . While human capital decreases at an increasing rate over time, financial wealth increases at an increasing rate. Therefore, all individuals reach a point in life where their current financial wealth,  $PV$ , is equivalent to remaining human capital,  $HC$ . At this point of intersection a critical time horizon,  $t^*$ , and critical wealth level,  $PV^*$ , are defined. The critical point is defined by each individual's unique preferences for leisure time and standard of living. Before reaching the critical point individuals have more time than wealth. Therefore, individuals focus on maximizing wealth to increase both current and future standards of living. However, beyond the critical point, individuals desire to maximize both leisure time and wealth.

The asset allocation process is a dynamic process in which individuals change their preferences for desired standard of living or leisure time even after an original critical wealth level is achieved. Thus, individuals who desire to maximize their standard of living will maximize wealth and increase future retirement goals if a critical point is reached earlier in life than originally anticipated. Because individuals are allocated a finite amount of time in life, they eventually reach a point in their life where they are either voluntarily or involuntarily forced to adjust their consumption and retirement goals. At this point in life individuals are concerned with maximizing leisure time in addition to wealth.

Whether an individual reaches retirement voluntarily or involuntarily they are eventually forced to adjust consumption based on their inability to convert human capital into financial wealth. Individuals may voluntarily decide to retire before depleting human capital if they value leisure time more in later stages of life. Therefore, they determine retirement goals that are consistent with a desired standard of living, before and after retirement, and a desired amount of leisure time. Some individuals are forced to retire early because of unexpected human capital depletion. Unemployment, physical or mental limitations force individuals to lower current and future standards of living consistent with a lower anticipated retirement goal.

### 3.1. Defining human capital

Consistent with Bodie et al. (1992) and Van Eaton and Conover (2002) the amount of remaining human capital,  $HC$ , throughout an individual's life is equal to the present value of all future earnings, illustrated by Eq. (1).

$$HC = \sum_{i=1}^t \frac{EP_i}{(1+r)^i} = \frac{EP}{r} \left( 1 - \frac{1}{(1+r)^t} \right) \quad (1)$$

Bodie et al. (1992) and Van Eaton and Conover (2002) assume that human capital is less risky than equity and discount future earnings at the risk-free rate.

An appropriate discount rate to determine the present value of an individual's remaining human capital is unobservable and unknown. It is possible that different occupations have varying degrees of uncertainty regarding future earnings. For example, assume a tenured professor has more certain future earnings than a plant manager without a college degree who is employed in a cyclical industry. Therefore, allowing for different discount rates to measure human capital provides more flexibility in applying the model. The model presented in this paper does not require that human capital be discounted at the risk-free rate of return. However, examples in this paper future earnings are discounted at the risk-free rate of return, consistent with Bodie et al. (1992). Also consistent with Bodie et al. (1992), it is assumed that an individual's labor supply is flexible throughout his or her life. Therefore, it is possible for an individual to adjust the amount of remaining human capital in the event that the expected return on the investment portfolio is lower than originally expected. Other cases where an unexpected event occurs may also influence an individual to make adjustments to labor hours.

Proposition 1: Human capital for an individual is defined as the discounted future earnings until retirement.

The decreasing function in Fig. 1 illustrates the depletion of human capital over an individual's life cycle. The shape of human capital is concave, indicating that an individual's human capital decreases at an increasing rate over time. On the  $x$ -axis, the time to retirement,  $t$ , approaches 0, as an individual ages. The individual's remaining human capital is shown at every point in time before retirement.

### 3.2. Standard of living as a function of earnings power, consumption, and savings

All individuals convert human capital into financial wealth to meet savings and consumption needs. Throughout individuals' lives, individuals make choices regarding savings, consumption, and standards of living based on their perceived value of time, and expectations of future earnings and rates of returns associated with their savings. Individuals may increase the amount of savings and reduce the amount of current consumption if they desire to increase their retirement wealth and in turn their future standard of living and consumption. Casual observation suggests that individuals' consumption patterns tend to gravitate towards their desired standard of living. For example, individuals often purchase homes with

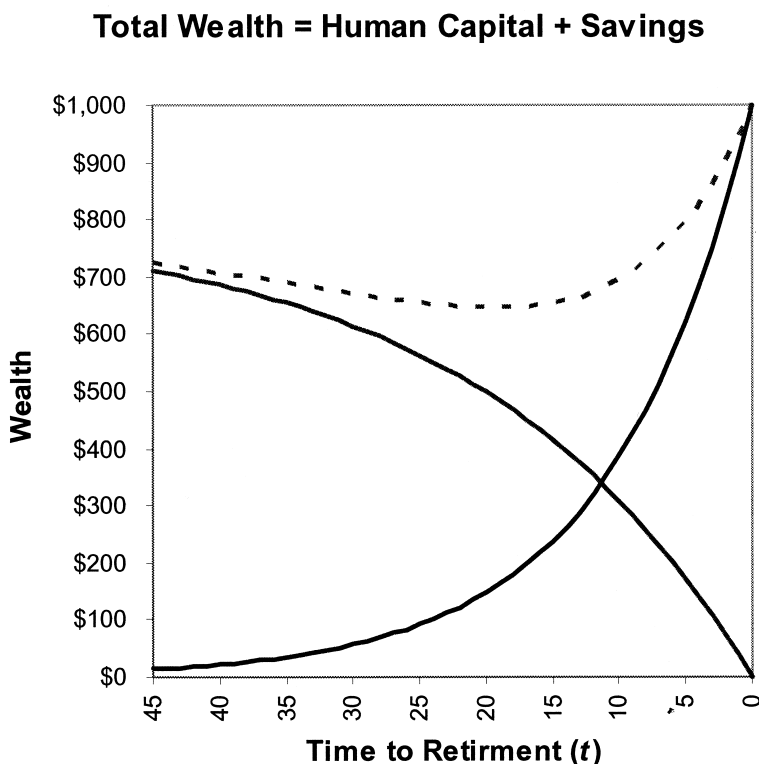


Fig. 1. illustrates an individual's total wealth (top dashed line) is comprised of human capital and financial wealth over an anticipated working life of 45 years. Total wealth is initially comprised almost entirely of human capital that is eventually depleted as financial wealth increases. Human capital is defined as the present value of future earnings (constant at \$40,000/year in this example) with a 5% risk-free discount rate. The increasing function represents current savings as the present value of a retirement goal of \$1,000,000 with a 10% discount rate.

a price that is consistent with their expected future earnings. House values are observable measures of individuals' standards of living, consistent with their future expected earnings at specific points in the individuals' lives. In this paper, individuals are assumed to desire a standard of living after retirement equivalent to their standard of living before retirement.

The standards of living for individuals over their life cycles are a function of earnings power, consumption and savings. Earnings power is defined as the annual wages less taxes and all work related expenses. For example consider an individual who earns a gross annual salary of \$50,000. The total \$50,000 is not available for consumption and savings because of federal, state and city taxes. In addition, other work-related expenses, such as childcare, clothing, and transportation, may be necessary for certain individuals. Work related expenses are defined as those that are no longer required after the individual stops working.<sup>3</sup> Human capital is overestimated if work related consumption expenses are not considered.

Each individual makes choices regarding how to allocate time between labor hours and leisure hours. Increasing the amount of time allocated to labor hours increases the amount of wages and thus earnings power for the individual. If all individuals desire only to maximize wealth, then they will limit the amount of leisure time to a level that is required for their

optimal health for their occupation. Conversely, it is assumed that individuals desire to maximize both wealth and leisure time. Therefore, they make choices throughout their lives based on their value of time as well as wealth. The earnings for each individual reflect an individual's intrinsic value of time. Individuals are only willing to give up more leisure time if their wages per unit of time increases.

Proposition 2: Individuals value leisure time and wealth. Therefore, they make choices that will maximize both leisure time and wealth.

Thus, the individual's flexibility in labor supply is determined in part by the individual's perception of time versus wealth that is reflected by wages per unit of time. For example consider two individuals, one with a salary of \$100 per hour and the second with a salary of \$10 per hour. Assume that both individuals enjoy spending as much time as possible on the golf course. The individual able to make \$100 per hour may decide to hire someone to mow his lawn for \$15 per hour. This individual is able to buy time, because his value of time is higher than the individual who mows the lawn. The individual who makes \$10 per hour is not willing to hire someone to mow his lawn because he would require 1.5 hours of labor for the gain in one hour of leisure time. All individuals make choices regarding the allocation of time consistent with the value of their leisure time relative to the value of labor time.

The process of making investment and time allocation choices requires a dynamic model that changes over time with the individual's unique circumstances, choices, and desires. In examples using the dynamic model, individuals are assumed to create a budget at the start of every year and use this information in making labor supply and consumption choices. Because of deviations from the mean expected return, an individual's actual savings for retirement deviates from the required amount necessary to achieve a retirement goal, implied by the time value of money formula. The individual may decide to adjust his or her retirement goal, standard of living, earnings power, risk tolerance or time horizon. Individuals, who recognize realized returns fluctuate over time, make few, if any, adjustments based on market fluctuations in realized returns when they have a relatively long time horizon. The realized returns over longer time periods are more likely to reflect expected returns. Eventually individuals are forced to accept standards of living consistent with realistic retirement goals the closer they are to retirement.

The model suggests individuals make consumption choices that maximize their desired terminal wealth, standard of living and leisure time. Young individuals are expected to maximize their retirement goals over time. Individuals who act consistent with the assumption of wealth maximization increase their terminal wealth level or retirement goal with unexpected increases in wealth. Under the assumption of wealth maximization an individual should never retire until he or she is physically unable to work any longer. In the real world, many individuals retire before depleting all human capital. Individuals who voluntarily retire make choices based on their desire to maximize wealth and leisure time.

The model is first constructed under the assumption that no savings are added to financial wealth. Thus, earnings power is used entirely for current consumption. This assumption is later relaxed in Section 4, where the earnings power is used for both savings and consumption as an annuity is introduced to the model. Even if an investor chooses to use all of their remaining human capital for consumption, human capital still plays an important role in

determining the investor's ability to achieve a retirement goal. If an individual chooses to use human capital to meet consumption needs, savings continue to accumulate without withdrawals being used for consumption.

### 3.3. Time versus wealth

The basic time value of money (TVM) formula illustrates the relationship of time and wealth. Assume an individual is unable to contribute any additional payments for savings and must rely only on current savings to meet their retirement goals. Eq. (2) illustrates the time value of money formula that defines current wealth,  $PV$ , with respect to a retirement goal or future lump sum,  $FV$ , discounted by an expected rate of return,  $r$ .

$$PV = \frac{FV}{(1 + r)^t} \quad (2)$$

where:  $FV$  = retirement goal.

$PV$  = current wealth today.

$t$  = time remaining to retirement.

$r$  = the expected rate of return on investments.

Individuals with no ability to add to savings or who voluntarily choose to consume all current earnings must rely on current wealth to grow over time to meet future consumption needs after human capital is depleted. The increasing function in Fig. 1, illustrates the accumulation of wealth for an individual with an anticipated working life of 45 years. The present or current level of wealth on the x-axis is determined by Eq. (2) given the time remaining to retirement, an expected return of 10%, and a retirement goal of \$1,000,000. At every point on the curve, this risk-averse individual desires to have a current wealth level that is greater than or equal to the amount on the x-axis given the time remaining to retirement. For example, if the individual desires to retire in 20 years, the individual who is not planning on making any additional payments to savings should have at least \$148,643.63.

### 3.4. Maximizing time and wealth

Traditional financial planning models suggest individuals reduce the proportion invested in equities according to the age of the individual. Under traditional financial planning models, the amount of current wealth is seldom considered in the asset allocation decision and the reduction in equities is typically a function of the individual's age. If an individual focuses entirely on time horizon without being cognizant of the present level of wealth, the individual is acting in a manner that maximizes terminal wealth without the consideration that time is limited for individuals. If an individual disregards the current level of wealth, the individual is more likely to experience larger fluctuations in the end amount of wealth available for retirement. Van Eaton and Conover (2002) illustrate this with empirical examples. If leisure time is valued by the individual, the asset allocation decision should not be made solely on the assumption of wealth maximization. Rather the individual desires to maximize wealth relative to their value of leisure time.

Fig. 1 illustrates the composition and level of total wealth over time. Because human capital is decreasing at an increasing rate and financial wealth is increasing at an increasing rate, the lines must intersect. It is interesting to note that total wealth is fairly level before reaching the critical wealth and critical time horizon. Beyond the critical point of intersection total wealth increases at an increasing rate consistent with the accumulation of financial wealth. *The critical point represents a time in the individual's life where the individual begins to value leisure time more than an increased standard of living or wealth.*

**Proposition 3:** A critical wealth level for asset allocation exists where the value of an individual's remaining human capital is equal to financial wealth.

At a critical level of wealth, the individual's total human capital remaining is equal to the amount of the individual's financial wealth. The individual is indifferent between his or her value of time and wealth at this critical point. Beyond this point wealth continues to build at an increasing rate to the desired retirement goal while human capital continues to decrease at an increasing rate and is eventually depleted. Time also eventually expires for all individuals. To the left of the critical wealth level financial wealth is less than the individual's remaining human capital. Thus, individuals have more flexibility in labor supply as they are willing to increase labor hours for increased earnings. Before reaching a critical wealth level, an individual relies more on human capital to meet current and future consumption needs than financial wealth. Young individuals are more concerned with increasing wealth to meet their financial goals and forego leisure time in exchange for labor hours to increase earnings power. In addition to being a potential source for savings, earnings power allows wealth to accumulate while consumption needs are met with human capital. In short, *to the left of the critical point the individual has more time and human capital than money.*

Conversely, to the right of the critical wealth level, individuals become less and less flexible with respect to labor supply. As time passes the gap between financial wealth and human capital broadens. To the right of the critical point the individual has more money than human capital and time. Beyond the critical point the expected return from financial wealth begins to contribute more towards terminal wealth than human capital.

Individuals go through two phases in life with respect to a critical point. Before the critical point individuals are only interested in maximizing wealth. At the critical point individuals become cognizant of the increased value of leisure time. They begin to make choices regarding investments and leisure time that are consistent with a desired current and future standard of living. Because human capital and time are limited they make choices that are consistent with maximizing leisure time and wealth. Eventually, as financial wealth compounds, the amount of earnings generated from labor hours is not as valuable relative to the return from financial wealth. At some point individuals begin to allocate more of their remaining total time,  $T$ , to leisure time rather than labor hours. To the right of the critical point, leisure time is becoming more valuable as time depletes for the individual. A decrease in an individual's ability and/or desire to work causes a decrease in earnings power. The combined decreases in time and earnings power per unit of time result in the reduction of the individual's total human capital.

The model requires individuals to set a lump sum retirement goal that is consistent with their desired standard of living and desired amount of leisure time. As long as labor hours

are flexible, individuals are free to adjust their desired terminal wealth and standard of living. In some cases, individuals are able to adjust their earnings power and/or time to retirement that together influence their total remaining human capital.

### 3.5. Defining a critical wealth level and time horizon

At some point in every individual's life there is a critical wealth level,  $PV^*$ , where the amount of financial wealth for the individual is equal to the total remaining human capital,  $HC^*$ . Eq. (3) represents the point of intersection in Fig. 1 where the right hand side of the equation is equal to the remaining human capital,  $HC^*$ , and the left hand side of the equation is equal to the critical wealth level,  $PV^*$ .

$$\frac{FV}{(1 + r_1)^{t^*}} = \frac{EP}{r_2} \left( 1 - \frac{1}{(1 + r_2)^{t^*}} \right) \quad (3)$$

It is not possible to mathematically solve for a closed form solution for the critical time horizon, assuming two different discount rates in Eq. (3). Assuming the discount rate for human capital,  $r_2$ , is equivalent to the discount rate for financial wealth,  $r_1$ , and solving for the critical time horizon,  $t^*$ , in Eq. (3) yields Eq. (4) shown below. The derivation for Eq. (4) is found in Appendix A.

$$t^* = \ln \left( \frac{FVr + EP}{EP} \right) / \ln(1 + r) \quad (4)$$

Eq. (4) presents a more conservative estimate that could be considered in cases where the individual has greater uncertainty regarding future earnings. Greater uncertainty of future earnings could potentially be a reflection of the industry an individual is employed or physical or mental limitations of the individual. While Eq. (4) is convenient for approximating a critical time horizon, Eq. (3) is not limited to specific discount rates for human capital or specific expected rates of return for financial wealth. Bodie et al. (1992) requires that human capital is discounted at the risk-free rate to justify the shift in allocations out of equities. Therefore, in applying the model, the discount rate for human capital is assumed to be equal to the risk-free rate of return. Eq. (3) is used as the model for cases with no savings annuity. The critical time horizon,  $t^*$ , is determined by individuals' unique preferences and values based on their desired future standards of living (as a function of  $FV$  and  $r_1$ ) and remaining human capital,  $HC$ . Increasing time to retirement allows financial wealth more time to compound and increases terminal wealth or the retirement goal. However, eventually all individuals voluntarily or involuntarily retire. Therefore, time remaining to retirement is constrained by the limited amount of remaining human capital.

## 4. Including an annuity payment into the model

Individuals with the ability to add to their retirement savings have more flexibility in the choices they make. To add to retirement savings, the individual's current earnings power

must be sufficient to generate income in excess of the individual's current standard of living or consumption needs. When a constant annuity payment is introduced to the model, time to retirement,  $t$ , is defined as a function of the retirement goal,  $FV$ , current financial wealth,  $PV$ , and the annuity savings payment,  $A$ , based on time value of money equations. Earnings power is used for consumption or savings. The level of earnings power determines a reasonable standard of living and retirement goal for the individual. Greater earnings power provide more flexibility in choices for individuals to contribute more towards savings and wealth to increase their standard of living in the future or consume more today and increase their current standard of living.

The more an individual relies on an annuity payment to assist in meeting the retirement goal, the lower the critical wealth level is for that individual. Consistent with Bodie et al. (1992) and Van Eaton and Conover (2002), individuals with greater earnings power have more flexibility in saving for retirement. Individuals with greater earnings power have more flexibility in savings and consumption choices. Individuals with the ability to add more savings to their wealth portfolio every year are able to take on more risk. They can make up for losses more easily and therefore, have a lower critical wealth level. As financial wealth exponentially compounds for the individual over time, the savings annuity plays a decreasing role in contributing to the retirement goal. In addition, as individuals age they have less flexibility in their labor supply.

Eq. (5) defines the critical time horizon,  $t^*$ , when an annuity savings payment until retirement is included into the model. Notice that if there are no annuity payments,  $A = 0$ , Eq. (5) is the equivalent of Eq. (3).

$$\frac{FV - A \frac{(1 + r_1)^t - 1}{r_1}}{(1 + r_1)^t} = \frac{EP}{r_f} \left( 1 - \frac{1}{(1 + r_f)^t} \right) \quad (5)$$

Similar to the example with no annuity, the critical time horizon is approximated by Eq. (6). The derivation for Eq. (6) is provided in Appendix B. Consistent with Eq. (4), financial wealth is invested at the risky rate of return and human capital is assumed to be discounted at the same risky rate of return.

$$t^* = \ln \left( \frac{FVr + EP + A}{EP + A} \right) / \ln(1 + r) \quad (6)$$

## 5. Application of the model to the asset allocation decision process

Individuals allocate assets consistent with their desired expected return and risk tolerance levels. In addition, individuals' choices regarding savings and consumption are related to a desired current and future standard of living consistent with expected future retirement goals and life styles. While their desired standards of living may change over time, individuals

make investment and time allocation decisions based on their retirement goals that are consistent with desired standards of living. Before reaching a critical wealth level an individual has more time than money and maximizes wealth and the retirement goal. Therefore, the individual is 100% invested in equities before accumulating a critical amount of wealth. At some point in life the individual begins to value leisure time more and is not willing to maximize wealth without recognizing the opportunity cost of lost leisure time. The asset allocation decision should focus on a critical wealth level and not simply time remaining to retirement, if the individual desires a specific retirement goal and values leisure time.

### *5.1. Example with no annuity savings payment and a constant earnings power*

An individual's remaining human capital is defined by a constant earnings power of \$40,000 per year discounted at the risk free rate of return of 5%. Assume the individual is currently 20 years old with a savings of \$13,719 and desires to retire at age 65 with a retirement goal of \$1,000,000. To simplify the example assume the individual desires to maintain a constant standard of living that requires the consumption of total earnings power of \$40,000 every year. Therefore, no payments are added to savings.<sup>4</sup> Assume that annual realized returns fluctuate around an expected return of 10% such that the individual's financial wealth grows over time from \$13,719 at age 20 to \$318,631 at age 53, as illustrated by Table 1.

The individual determines the critical time horizon and wealth level by inputting his or her unique expectations and preferences into Eq. (3). Table 1 illustrates that in this example the critical time horizon,  $t^*$ , is 11.33 years using Eq. (3). At 11.33 years, the individual has \$339,642 of financial wealth that is approximately equal to his or her expected remaining human capital. The individual remains 100% invested in equities until \$339,642 of wealth is acquired. At this critical level of wealth, the individual anticipates retiring in 11.33 years with \$1,000,000 to meet future desired standard of living needs after retirement. Before this point, his or her remaining human capital exceeded financial wealth. Therefore, consistent with Bodie et al. (1992) the individual invests 100% of financial wealth in risky assets. When the individual's financial wealth exceeds the remaining human capital, the individual gradually shifts out of equities consistent with the decrease in human capital as a percentage of total wealth. Thus, the individual reduces the allocation to equities after a critical wealth level is obtained, to maintain an optimal percentage of equities to total wealth.

#### *5.1.a. Equity allocation when realized returns are equivalent to expected returns*

If the realized return is exactly equal to the expected return, over the next 12 years, then the individual reduces the amount of money invested in equities to maintain the optimal maximization of wealth and leisure time as illustrated by Table 1. The reduction in equities reflects the individual's desire to obtain the goal of \$1,000,000 to meet future standard of living needs and to retire at an optimal point in life. Assume that at age 54, the individual's wealth has grown exactly 10% over the past year to \$350,494. Assuming the individual has not changed his or her value of leisure time and desired standard of living, Table 1 reflects the reduction in equities for financial wealth. The reduction in equities causes the expected

Table 1

Illustrates the amount of human capital and present value of financial wealth for an individual with a retirement goal of \$1,000,000.

Years to retirement	Financial wealth	Human capital	Total wealth	Equities to financial wealth	Equities to total wealth	Expected return
45	13,719	710,963	724,682	100.0%	1.9%	10.0%
40	22,095	686,363	708,458	100.0%	3.1%	10.0%
35	35,584	654,968	690,552	100.0%	5.2%	10.0%
30	57,309	614,898	672,207	100.0%	8.5%	10.0%
25	92,296	563,758	656,054	100.0%	14.1%	10.0%
20	148,644	498,488	647,132	100.0%	23.0%	10.0%
15	239,392	415,186	654,578	100.0%	36.6%	10.0%
14	263,331	395,946	659,277	100.0%	39.9%	10.0%
13	289,664	375,743	665,407	100.0%	43.5%	10.0%
12	318,631	354,530	673,161	100.0%	47.3%	10.0%
11.33	339,642	339,727	679,369	100.0%	50.0%	10.0%
11	356,686	332,257	688,943	96.5%	50.0%	9.8%
10	407,218	308,869	716,087	88.0%	50.0%	9.4%
9	458,531	284,313	742,844	81.0%	50.0%	9.1%
8	510,419	258,529	768,948	75.4%	50.0%	8.8%
7	564,016	231,455	795,471	70.5%	50.0%	8.5%
6	619,082	203,028	822,110	66.4%	50.0%	8.3%
5	676,189	173,179	849,368	62.8%	50.0%	8.1%
4	735,438	141,838	877,276	59.7%	50.0%	8.0%
3	797,371	108,930	906,301	56.8%	50.0%	7.8%
2	861,882	74,376	936,258	54.3%	50.0%	7.7%
1	929,325	38,095	967,420	52.1%	50.0%	7.6%
0	1,000,000	—	1,000,000	50.0%	50.0%	7.5%

The present value of financial wealth is equal to  $\$1,000,000/(1.10)^t$ . The remaining human capital is equal to the present value of an annuity with a 5% risk-free discount rate. Earnings power is assumed to be constant at \$40,000 for this example.

return to decrease. Therefore, the next critical wealth level for this individual is determined by a smaller expected return. Using this model the individual adjusts the percentage of investment in equities based on the current wealth level and desired time to retirement.

#### 5.1.b. *Equity allocation when realized returns are less than expected returns*

Using the same illustration in Table 1, assume that instead of realizing a 10% return on the wealth of \$318,631 at age 52, the individual's actual return on risky investments is -10%. The individual now has wealth of \$286,768. Now the individual must make choices related to consumption, savings, and leisure time given the new current wealth level. If the individual still desires a \$1,000,000 retirement goal to maintain a constant standard of living after retirement, she expects to now work approximately 13 years instead of 11 years to obtain the goal. If the individual values wealth more than time at this level of wealth, the individual will continue to be 100% invested in equities. If the individual decides to lower the retirement goal, his or her value of leisure time is greater than the desired standard of living and the model is rerun with the new parameters.

### *5.1.c. Equity allocation when realized returns are greater than expected returns*

Assume that instead of realizing an expected 10% return the individual's actual return is 30% on the initial wealth of \$318,631 at age 52. Now at age 53, the current financial wealth level is \$414,220 and the individual anticipates retiring in 9.25 years rather than 11 years. Assuming the individual desires the same standard of living, equities are reduced to 88.0% to reflect the desired optimal mix to maximize both leisure time and wealth. The individual will maintain the 88.0% investment in equities, until the financial wealth either grows or shrinks to the next level. If financial wealth grows to \$458,531 then equities are reduced to 81.0% of financial wealth. If financial wealth decreases to \$356,686, then equities are increased back to 96.5%.

## **6. Summary**

The dynamic model presented in this paper provides a framework for asset allocation decisions based on specific criteria important to unique individual investor's preferences regarding risk tolerance, time and wealth. Furthermore, the model is easily applied to practical asset allocation decisions because of its simplicity. Inputs of the model are taken directly from the individual's desired retirement goal, time to retirement, risk tolerance (expected return), earnings power and savings. The model assumes individuals desire to maximize leisure time as well as wealth. The model identifies a critical wealth level and time horizon to justify the reduction in equities in an individual's retirement portfolio.

A model is developed under the framework of time value of money functions to illustrate the tradeoff of time and wealth in the financial planning process. The following propositions develop a framework for understanding the time versus wealth life cycle model:

1. Human capital for an individual is defined as the discounted future earnings remaining until retirement.
2. Individuals value leisure time and wealth. Therefore, they make choices that will maximize both leisure time and wealth.
3. A critical wealth level for asset allocation exists where the value of an individual's remaining human capital is equal to financial wealth.

Young individuals have more time than money and their total wealth is comprised mostly of human capital in the form of potential future earnings power. Individuals use their human capital for consumption or savings. As individuals age they have more financial wealth and less time and human capital. A critical wealth level and critical time horizon are defined at the point in an individual's life where the amount of financial wealth is equivalent to the remaining amount of human capital for the individual. This is a very important turning point in the individual's life because the expected return from financial wealth begins to contribute more to consumption needs and terminal wealth (retirement goal) than human capital. The critical wealth level is a function of an individual's desired standard of living after retirement and desired leisure time.

The model is more representative of human behavior that reflects a desire to maximize both leisure time and wealth. A significant contribution of the model is the ability to

determine a critical wealth level and time horizon. Critical wealth levels identify a turning point in an individual's life where the individual relies more on financial wealth than human capital to meet current and future standard of living needs. To the left of the critical wealth level individuals remain 100% invested in equities to maximize wealth and the retirement goal. To the right of the critical wealth level financial wealth is greater than human capital and individuals begin to reduce the allocation to equities as leisure time becomes more valuable than financial wealth. Consistent with Bodie et al. (1992) with the expiration of time, the reduction of human capital justifies reductions in the amount allocated to equities.

### Notes

1. For a summary of arguments regarding time diversification see Kritzman (1994) and Van Eaton and Conover (2002).
2. For examining how social security and taxes impact retirement goals see Fraser, Jennings, and King (2000) and Reichenstein (1998).
3. There may also be benefits that may go away if the individual stops working that are not reflected in the gross wages. The most notable benefit is medical and health benefits. The individual's consumption expenditures could increase substantially in this area after retirement.
4. If the individual has an annuity savings payment Eq. 5 in Section 4 is the appropriate model.

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### Appendix A

The left-hand side of Eq. (A1) is the present value of financial wealth at the critical point. The right-hand side of Eq. (A1) is the present value of the annuity of future earnings representing remaining human capital at the critical point.

$$\frac{FV}{(1+r)^t} = \frac{EP}{r} \left( 1 - \frac{1}{(1+r)^t} \right) \quad (\text{A1})$$

$$FV = \frac{EP}{r} (1+r)^t \left( 1 - \frac{1}{(1+r)^t} \right) \quad (\text{A2})$$

$$FV = \frac{EP}{r} ((1+r)^t - 1) \quad (\text{A3})$$

$$\frac{FVr}{EP} = ((1 + r)^t - 1) \quad (\text{A4})$$

$$\frac{FVr + EP}{EP} = (1 + r)^t \quad (\text{A5})$$

$$\frac{\ln\left(\frac{FVr + EP}{EP}\right)}{\ln(1 + r)} = t \quad (\text{A6})$$

## Appendix B

The left-hand side of Eq. (B1) is the present value of financial wealth at the critical point. The right-hand side of Eq. (B1) is the present value of the annuity of future earnings representing remaining human capital at the critical point.

$$\frac{FV - A \frac{(1 + r)^t - 1}{r}}{(1 + r)^t} = \frac{EP}{r} \left(1 - \frac{1}{(1 + r)^t}\right) \quad (\text{B1})$$

$$FV - A \frac{(1 + r)^t - 1}{r} = \frac{EP}{r} (1 + r)^t \left(1 - \frac{1}{(1 + r)^t}\right) \quad (\text{B2})$$

$$rFV - A((1 + r)^t - 1) = EP((1 + r)^t - 1) \quad (\text{B3})$$

$$rFV + EP + A = (1 + r)^t(EP + A) \quad (\text{B4})$$

$$\frac{rFV + EP + A}{EP + A} = (1 + r)^t \quad (\text{B5})$$

$$\ln\left(\frac{rFV + EP + A}{EP + A}\right) = t \ln(1 + r) \quad (\text{B6})$$

$$\frac{\ln\left(\frac{rFV + EP + A}{EP + A}\right)}{\ln(1 + r)} = t \quad (\text{B7})$$

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