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Within-horizon exposure to loss for dollar cost averaging and lump sum investing

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Abstract

Based on a statistic known as "first-passage time probability" that accounts for exposure to loss during the entire investment horizon, it is shown that dollar cost averaging relative to lump sum investing can significantly reduce the probability, magnitude, and duration of enduring a large loss. This is especially relevant to investors with minimum loss thresholds, possible interim withdrawal needs, changing asset allocations, and/or an uncertain retirement date. For investing in stocks with a 5-year horizon, the probability of enduring a loss can be reduced from over 90% to less than 50%, the dollar amount of the conditional expected shortfall can be reduced by 65%, and the expected time one may have to endure a loss is reduced from 1.5 years to 4 months. © 2005 Academy of Financial Services. All rights reserved.

JEL classifications: C15; G11

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1. Introduction

It is not often investors are saddled with the enviable problem of investing large lump sums (LS), but with an aging population that has more wealth than ever before, large inheritances will become increasingly more common for the next generation of investors. Many of these heirs will decide to invest these sums for their own future retirement plans and deciding whether to invest it all at once or employ dollar cost averaging (DCA) will be a major conundrum.

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The major advantage of DCA over LS investing touted by many financial planners is that it allows investors to gradually invest in the stock market or some other risky asset while reducing the risk of buying completely in at a market high. One need only do a quick search on Google to find a variety of sites expounding this advantage, including such notables as Wells Fargo, Met Life, and CNN Money. Even Burton Malkiel's classic text "A Random Walk Down Wall Street" discusses this advantage (Malkiel, 2003).

However, most studies suggest implementing DCA reduces returns without a large enough reduction in risk to justify it relative to lump sum (LS) investing. Constantinides (1979) first demonstrated the inferiority of DCA relative to LS investing when the decision is based solely on expected return. On a risk-return basis, the answer is not as obvious, but a substantial amount of empirical research agrees that DCA is suboptimal relative to LS investing, (Knight & Mandell, 1993; Williams & Bacon, 1993; Rozeff, 1994; Leggio & Lien, 2003).

In DCA's defense, some empirical studies have suggested that DCA has merit in some narrowly defined circumstances, (Israelsen, 1999; Abeysekera & Rosenbloom, 2000; Milevsky & Posner, 2003). More recently, using an option theory framework, Dubil (2004) finds DCA can reduce the expected dollar amount of shortfall upon liquidation of the investment, although the probability of shortfall is found to be approximately the same for both strategies, and the difference in shortfall amounts tends to disappear as the investment horizon is extended.

Thus, even the support for DCA had not been overwhelming. However, previous studies have never directly addressed the risk of loss during the investment horizon that DCA is tailored made to mitigate. This study remedies this oversight and addresses the question as to whether DCA can reduce the probability of loss throughout the investment horizon substantially enough to justify its use.

Using a statistic known as "first-passage time probability," the results of this study show that DCA can significantly reduce the probability, magnitude, and duration of enduring a large loss anytime within the investment horizon. Inputting returns and standard deviations based loosely on the U.S. stock market, it is found that, for a 10-year time horizon, a 5-year DCA averaging strategy can reduce the probability of ever experiencing a loss from over 90% to less than 60%, reduce the conditional mean expected shortfall by 45%, and reduce the expected time one has to weather a loss during the investment horizon from over 22% to less than 10%. Other investment horizons have qualitatively similar results.

2. Why within-horizon risk is important

Kritzman and Rich (2001) poignantly point out that within-horizon risk is far greater by a magnitude than end-of-horizon risk. This risk is especially relevant to investors with (1) minimum loss thresholds, (2) possible interim withdrawal needs, (3) changing asset allocations and/or, (4) an uncertain time horizon or retirement date.

2.1. Minimum loss thresholds

Conventional wisdom has held that short-run fluctuations should not matter as long as the investment horizon is relatively long. However, Chhabra, (2004) makes a strong argument for

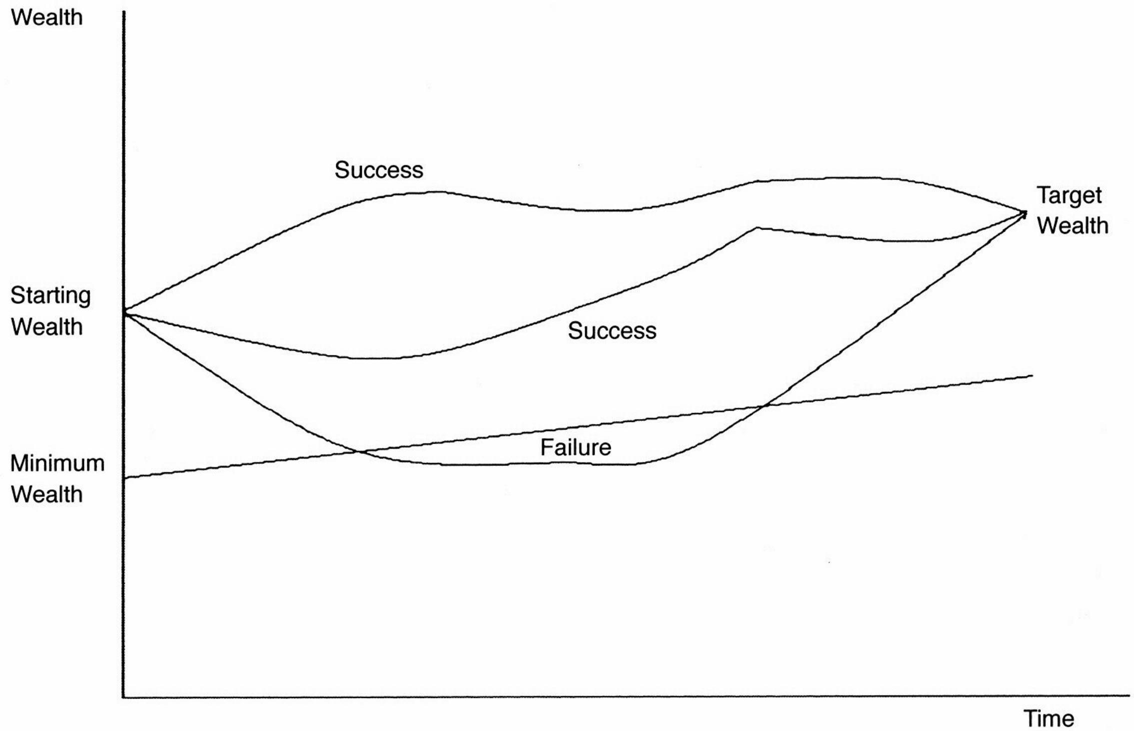

Fig. 1. The return sequence matters. All three paths lead to the same target wealth, but an investor who experiences the bottom path and crosses the minimum wealth threshold may be unable, or unwilling to continue with the investment to actually attain the target wealth.

why this line of reasoning is not necessarily valid. After breaking risk into three subcategories, Chhabra shows that the "ideal portfolio" takes into account personal, market, and aspirational risk. Most relevant here is the personal risk component which involves protecting yourself from anxiety regarding a dramatic decrease in your lifestyle. The essence of Fig. 1, which is taken from Chhabra's study, shows that all return states lead to success, but the one that falls below the minimum acceptable wealth may not be tenable to the investor. As Chhabra so elegantly puts it, "A stream may have an average depth of five feet, but a traveler wading through it will not make it to the other side if its midpoint is 10 feet deep."

The basis premise here is simply that there is a class of investors who do not wish or cannot accept a certain decline in their wealth level at any time. Casual observation suggests that many investors fall in this category. After the 2000 stock market crash, many investors postponed their retirement simply because the boom of the 1990s changed their aspirations. Before this boom, many of them likely would have been happy to retire with the wealth accumulated even after the crash, but it is the drop in wealth level at any time during the investment horizon that is often not palatable to the investor, not the absolute amount attained.

2.2. Possible interim withdrawal needs

Within-horizon risk is also relevant for those investors who may require access to the funds before the end of the investment horizon. Unexpected expenses may be even more relevant today, as an aging population will likely have more "health" emergencies. If this causes the investing process to stop, or worse, funds need to be withdrawn while the market is down, end-of-horizon probabilities will be of little use. Knowing the probability, magnitude, and duration of losses that may have to be endured during the entire investment horizon would help investors make a much more informed decision about not only where, but also how they should be investing to account for possible emergency withdrawals.

2.3. Changing asset allocations

Over the last 20 years, the stock market has averaged almost 12%, but according to Dalbar's Quantitative Analysis of Investor Behavior (QAIB) study, individual equity investors are only averaging 4%. How is this possible? Imprudent individual investor behavior causes many investors to sell after market declines. The 2004 data showing inflows and outflows to equity mutual funds confirm this behavior still continues. For these investors, the within-horizon risk actually takes precedent because large losses within the investment horizon causes them to change their asset allocation. End-of-horizon goals have little chance of being met because within-horizon risk is not known or is not adequately being accounted for.

2.4. Uncertain time-horizon

Yet another disturbing statistic by Dalbar's QAIB study shows that over the last 20 years, the average time an equity mutual fund holder stayed invested is only 2.5 years. This empirical reality also helps explain why the average equity investor does so poorly relative to the average equity mutual fund. Equity investors who should have long-term horizons are apparently moving their assets long before they are expected to do so. The range of values at the end of a 10- or 20-year investment horizon has little meaning for investors if they "get out" early. Investors may start out with a long horizon in mind, and most financial planners would recommend a minimum 15-year time-horizon before investing substantially in equities. However, the reality is that the actual time horizon an average equity investor stays invested is much shorter. Couple this with the fact that the time horizon is often cut short because of losses, accounting for interim risk becomes critically important. With an uncertain time horizon, risk throughout the time horizon needs to be addressed.

Finally, even if an investor is not subject to the above four issues, it is likely they are still concerned about within-horizon risk for the same reasons they worry about the results at the end of the horizon. The simple fact is that we will always be uncertain about the actual outcome of any investment over any time horizon even if the expected return and variance is known with certainty. Unfortunately, even these two factors are not known and early losses can cause a great deal of anxiety that often leads to poor long-term investment decisions.

3. Within-horizon risk

This study formally defines within-horizon risk as the probability one will lose a particular amount at anytime within the investment horizon. To capture this risk, a statistic known as "first-passage time probability" first described by Karlin and Taylor (1975) can be used. This statistic measures the probability of a first occurrence within a finite horizon. This statistic measures not only the probability that an investment will lose a particular amount at the end of the time horizon, but also at any time within the time horizon (Kritzman & Rich, 2002). This statistic is written as:

$$\Pr_w = N\frac{\ln(1 + L) - \mu_c T\}}{\sigma_c \sqrt{T}} + N\frac{\ln(1 + L) + \mu_c T}{\sigma_c \sqrt{T}}(1 + L)^{2\mu_c/\sigma_c^2} \quad (1)$$

where: $N[]$ = cumulative normal distribution function,
L = cumulative percentage loss in periodic units,
μ_c = annualized expected return in continuous units,
T = number of years in horizon,
σ_c = annualized standard deviation of continuous returns.

Because compounding causes discrete returns to be lognormally distributed, inputs are in continuous units that are normally distributed which can then be used within the normal distribution function.

To convert a discrete return and standard deviation into their continuous counterparts, note the following:

$$\mu_c = \ln(1 + \mu_d) - \sigma^2 c/2 \quad (2)$$

$$\mu_c = \sqrt{\ln[\sigma^2{}_d/(1 + \mu_d) + 1]} \quad (3)$$

where μ_c = continuous expected return
μ_d = discrete expected return
σ_c = continuous standard deviation
$\sigma^2{}_d$ = discrete variance.

Also note that the continuous mean is a function of the variance. This is because the lognormal distribution is not symmetric and the value of the variance affects the spread of the distribution that in turn affects the mean.

Returning to Eq. (1), the first part is simply the end-of period probability that a return will decrease to some level. The second part of Eq. (1) calculates the probability of a particular loss anytime within the investment horizon. Thus, the probability of loss at anytime within the time horizon will always be at least as great as the probability of a loss at the end of the horizon. In addition, this risk increases with the time horizon as opposed to end-of-horizon risk that decreases with time. This type of risk measurement is especially applicable for comparing DCA to LS investing because a major touted advantage of DCA is to mitigate the probability of getting into the market and suffering early large losses.

4. Methodology

To determine the within-horizon risk for both DCA and LS investing, theoretical values are inputted into Eq. (1) based loosely on approximate empirical values observed in the U.S.

markets. To represent a 100% stock portfolio, 50/50 stock/bond portfolio, and 100% bond portfolio, the expected returns are set at 10%, 8%, and 6%, respectively, while the standard deviations are set at 20%, 10%, and 5%, respectively. The Treasury bill rate is fixed at 4% resulting in a 6% expected equity premium.[1] Time horizons of 1, 5, 10, and 20 years are examined while loss limits are set from 10% to 25%.

The exact implementation of DCA follows other studies in that DCA is premised on being fully invested by the end of either one, three, or five years. Only monthly periodic investing is analyzed as it is easily employed and is a likely scenario for most investors. As an example, for the 1-year DCA strategy, in the first month, 1/12 of wealth is invested in the risky asset and the remaining is invested at the risk-free rate. In the second month, 1/11 of the T-bill portfolio is moved into the risky asset, and so on until the entire portfolio is invested in the risky asset at the end of the year.

Although the use of Eq. (1) for determining the probability of shortfall for LS investing is relatively straightforward, this is not the case for DCA. The problem is that the expected return and standard deviation inputs are not constant throughout the implementation of the DCA strategy as the investor moves wealth from the risk-free to the risky asset. To get around this issue, the probability of shortfall can be measured for each incremental investment in the risky asset and multiplied by the percentage of the portfolio invested at that time. These values are then summed.

In addition, the percentage of the portfolio that remains invested in the risk-free asset must also be accounted for as it earns the risk-free rate. This is accomplished by increasing the within-horizon loss limit associated with any probability of loss to account for the return attained while funds were still invested in the risk-free asset. For example, the probability of a 15% loss may actually be based on numbers for the probability of a 16.5% loss because 1.5% would have been earned in the risk-free asset during the implementation of the DCA strategy. The values added to the loss limits depend on the risk-free rate and the length of the implementation period. In this study, they ranged from 1.8% for the 1-year DCA strategy to 9.8% for the 5-year DCA implementation strategy.[2]

To further determine the relative merits between DCA and LS investing, mean terminal values of each strategy are given along with the end-of-horizon probability of shortfall, and the end-of-horizon conditional expected shortfall. This is done so the results of this study can be directly compared to Dubil's (2004) results. Note that the conditional expected shortfall is defined as the mean shortfall conditioned on whether a shortfall actually occurs. Unlike Dubil, this study is able to analytically derive these results based on Eq. (1) without resorting to Monte Carlo methods. The shortfall probability is just the first part of Eq. (1), and the conditional expected shortfall is just the sum of the incremental probabilities for every loss amount from 0% to 100% divided by the total probability of shortfall multiplied by the initial investment.

For example, the procedure in the case of a $100,000 investment would be to first find the probability of a 100% loss and divide by the probability of a loss greater than 0%. This is then multiplied by $100,000. The next step would entail finding the incremental probability between a 100% loss and a 99.75% loss, dividing by the probability of a loss greater than 0%, and then multiplying by $99,875, which is the average loss amount within this interval. This process is simply continued all the way to a 0% loss and the amounts are then summed.

Table 1
Theoretical probabilities for losses of 10 to 25% or more for LS and DCA investing in simulated 100% stock, 100% stock/bond, and 100% bond portfolios assuming a 1-year investment horizon

	Stock		Stock/bond		Bond	
	End-of-horizon	Within-horizon	End-of-horizon	Within-horizon	End-of-horizon	Within-horizon
LS						
10% or more loss	15.3%	41.8%	2.7%	8.7%	0.0%	0.1%
15% or more loss	9.0%	23.6%	0.5%	1.6%	0.0%	0.0%
20% or more loss	4.7%	11.9%	0.1%	0.2%	0.0%	0.0%
25% or more loss	2.1%	5.2%	0.0%	0.0%	0.0%	0.0%
DCA						
10% or more loss	8.8%	21.8%	0.7%	2.1%	0.0%	0.0%
15% or more loss	3.9%	9.6%	0.1%	0.2%	0.0%	0.0%
20% or more loss	1.6%	3.7%	0.0%	0.0%	0.0%	0.0%
25% or more loss	0.5%	1.2%	0.0%	0.0%	0.0%	0.0%

Note: Expected returns are set at 10%, 8%, and 6%, while standard deviations are set at 20%, 10%, and 5%, respectively. The risk-free rate is set at 4%.

Using the same process and logic behind the end-of-horizon conditional expected shortfall, this study also calculates the within-horizon conditional expected shortfall. In addition, it is informative to not only know the probability that a loss may be experienced during the investment horizon, but also know how long it may have to be endured. To that end, the expected percentage of time that the asset is below its initial value relative to the entire investment time horizon is calculated. Because there is no obvious analytical solution to this last question, Monte Carlo simulation is employed and the average time that the risky asset is below the initial value is calculated based on 10,000 simulated return paths.

Finally, to empirically validate whether the "first-passage time probability" statistic is accurate for this type of analysis, Monte Carlo simulation is employed to test values calculated by Eq. (1). The 10,000 iterations are run and empirical probabilities are generated based on the different time horizons and loss limits. The empirical probabilities do validate the theoretically generated values, as all values are within plus/minus a couple of percentage points or within a few hundred dollars of the simulated values. Thus, only theoretical values are presented.

5. Results

5.1. Loss probabilities for 1-year horizons

Table 1 presents the theoretical results for end-of horizon and within-horizon risk under both DCA and LS investing for a simulated 100% stock, 50/50 stock/bond, and a 100% bond portfolio assuming a 1-year time-horizon. Although most investors will have a much longer investment horizon, it is informative to observe the difference between LS and DCA for the three asset classes and determine under what circumstances DCA should be considered.

The first notable finding in Table 1 demonstrates the remarkable contrast between end-of-horizon and within-horizon risk. As an example, with an LS strategy, the end-of-horizon probability of being down 15% or more for investing in stocks is only 9%, but the probability of being down 15% or more sometime during a 1-year horizon is over 23%. For the 1-year DCA averaging strategy, these numbers are only 3.9% and 9.6% respectively. Clearly within-horizon risk is much greater than end-of-horizon risk for either strategy and if weathering losses during the investing process is of concern to investors, standard measures of analysis are missing a large component of risk. In addition, it does appear DCA can help mitigate both types of risk.

Table 1 also makes it fairly clear that those investors who are only considering investing in a 100% bond portfolio or even a 50/50 stock/bond portfolio face little risk of any significant loss using either LS or DCA. Thus, it does not appear DCA supplies any major reduction in the probability of a loss over an LS strategy for these complete portfolio compositions. Thus, it is not a recommended strategy from a "portfolio framework" for those investors considering investing in less than 50% stocks.

However, investors may still want to consider DCA for the stock portion of their portfolio if they are concerned with suffering major losses in portions of their portfolios. Because Table 1 demonstrates that the probability of loss is fairly small for investors considering 100% bond portfolios or even 50/50 stock/bond portfolios, the rest of the results will be limited to the risk characteristics of the stock portfolio only.

5.2. Loss probabilities for extended investment horizons

Table 2 shows results for LS and DCA investing assuming investment horizons of 5, 10, and 20 years with probability of loss limits set from 10% to 25%. As can be seen, both the end-of-horizon and within-horizon probability of suffering a particular loss is smaller under the DCA strategies relative to LS investing, and the longer the averaging, the more dramatic the decline. This is particularly the case for within-horizon risk.

As an example, the within-horizon probability of a 20% or greater loss for a five- year horizon and using a 1-year DCA averaging strategy is 22.4% compared to the LS strategy that has a 28.7% probability of loss. This is obviously not a huge reduction in absolute percentage points, nor should one expect it to be because after the first year, both the LS and DCA strategies are fully invested in stocks with four years yet remaining in the time horizon. However, as one reads further down the table, extending the DCA averaging period to three years reduces the probability of losing 20% or more to 15.2%, and with a 5-year averaging period, the probability of losing 20% or more falls all the way to 7.1%.

This more substantial reduction in the probability of loss for the longer DCA strategies continues to hold even when assuming a 10- or 20-year investment horizon. For a 20-year horizon, a 3-year DCA averaging strategy reduces the probability of experiencing a 20% loss from 33.5% to 23.0%, whereas a 5-year DCA strategy can further reduce this probability to 17.4%. Thus, given the risk and return characteristics of the stock market, it appears a 1-year DCA strategy is not adequate and a 3-year or longer averaging period is needed for a significant reduction in risk.

Table 2
Theoretical probabilities for losses of 10 to 25% or more for LS and DCA

	5-year horizon		10-year horizon		20-year horizon	
	End-of-horizon	Within-horizon	End-of-horizon	Within-horizon	End-of-horizon	Within-horizon
LS						
10% or more loss	10.7%	56.5%	5.8%	58.9%	1.8%	59.7%
15% or more loss	8.3%	40.9%	4.7%	44.0%	1.5%	45.1%
20% or more loss	6.3%	28.7%	3.8%	32.2%	1.3%	33.5%
25% or more loss	4.5%	19.5%	2.9%	23.0%	1.0%	24.4%
DCA 1-yr average						
10% or more loss	9.4%	45.7%	5.2%	48.5%	1.6%	49.5%
15% or more loss	7.2%	32.5%	4.2%	35.9%	1.4%	37.2%
20% or more loss	5.3%	22.4%	3.3%	26.0%	1.1%	27.4%
25% or more loss	3.7%	14.7%	2.5%	18.3%	0.9%	19.8%
DCA 3-yr average						
10% or more loss	9.0%	35.7%	5.4%	41.3%	1.8%	42.9%
15% or more loss	6.4%	23.9%	4.2%	29.9%	1.4%	31.7%
20% or more loss	4.3%	15.2%	3.2%	21.0%	1.2%	23.0%
25% or more loss	2.7%	9.2%	2.4%	14.3%	0.9%	16.3%
DCA 5-yr average						
10% or more loss	5.8%	20.1%	5.0%	31.2%	1.7%	33.7%
15% or more loss	3.6%	12.3%	3.8%	21.9%	1.4%	24.5%
20% or more loss	2.2%	7.1%	2.7%	14.9%	1.1%	17.4%
25% or more loss	1.2%	3.9%	1.9%	9.7%	0.8%	12.0%

Note: 1-, 3-, and 5-year averaging periods for both end-of-horizon and within-horizon loss for 5, 10, and 20-year horizons are given. Calculations are based on a 10% expected return and a 20% standard deviation for the risky-asset, while the risk-free rate is set at 4%.

5.3. Conditional expected terminal and within-horizon shortfalls

The drawback of course from using a DCA strategy is giving up expected return and the longer the DCA averaging period, the more expected return will be sacrificed. Table 3 shows that for a 20-year investment horizon, as one goes from an LS to a 5-year DCA averaging strategy, the expected annualized return falls from 10% to 9.3% which coincides to a mean terminal value falling from $672,750 to $588,014.

However, Table 3 also demonstrates the risk advantages the DCA strategy has over LS investing. Similar to Dubil's (2004) results, although the probability of a terminal shortfall is similar between the two strategies, the conditional expected shortfall is smaller using DCA, although even this advantage begins to disappear with longer horizons.[3] To account for the different shortfall probabilities and their associated expected shortfall amounts, the shortfall probability is multiplied by the conditional expected shortfall to attain what is termed the conditional mean expected shortfall. To determine the extent DCA can reduce this amount, a conditional mean expected shortfall ratio is calculated which is simply the DCA mean expected shortfall divided by the LS mean expected shortfall.

For the end-of-horizon risk, DCA does offer some moderate risk reduction. The ratios show that for a 10-year horizon with a 5-year DCA averaging period, the mean expected

Table 3
Comparison of an LS and DCA strategy for a $100,000 investment in a risky asset with a 10% expected return and a 20% standard deviation

	5-year horizon		10-year horizon		20-year horizon	
	End-of-horizon	Within-horizon	End-of-horizon	Within-horizon	End-of-horizon	Within-horizon
LS						
Mean terminal value	$161,051	$161,051	$259,374	$259,374	$672,750	$672,750
Shortfall probability	15.2%	90.7%	7.8%	91.2%	2.4%	91.4%
Conditional expected shortfall	$18,105	$15,049	$21,211	$16,405	$23,898	$17,030
Conditional mean expected shortfall[a]	$2,760	$13,643	$1,655	$14,958	$566	$15,560
Expected time of shortfall		28.8%		22.3%		15.1%
DCA 3-yr average						
Mean terminal value	$148,576	$148,576	$239,283	$239,283	$620,638	$620,638
Shortfall probability	15.9%	70.9%	8.2%	73.7%	2.5%	74.5%
Conditional expected shortfall	$13,302	$11,823	$17,459	$14,255	$20,620	$15,264
Conditional mean expected shortfall[a]	$2,121	$8,385	$1,432	$10,508	$512	$11,373
Conditional expected shortfall ratio[b]	0.77	0.61	0.87	0.70	0.91	0.73
Expected time of shortfall		14.7%		14.1%		10.4%
DCA 5-yr average						
Mean terminal value	$140,766	$140,766	$226,705	$226,705	$588,014	$588,014
Shortfall probability	13.2%	48.3%	8.0%	58.5%	2.5%	60.2%
Conditional expected shortfall	$11,013	$9,967	$16,835	$13,944	$20,407	$15,220
Conditional mean expected shortfall[a]	$1,459	$4,811	$1,350	$8,164	$502	$9,159
Conditional expected shortfall ratio[b]	0.53	0.35	0.82	0.55	0.89	0.59
Expected time of shortfall		7.3%		9.5%		8.1%

Note: The risk-free rate is set at 4%. Shortfall threshold is set at the initial investment. The conditional expected shortfall is defined at the mean shortfall conditional on the occurrence of the shortfall, while the expected time of shortfall is defined as the mean percentage of the time horizon that the investment value is below the initial investment amount.

[a] Defined as shortfall probability multiplied by conditional expected shortfall.

[b] Ratio is defined as DCA conditional mean expected shortfall divided by LS counterpart.

shortfall is cut by 18%, and for a 20-year horizon, the value is reduced by 11%. Although these values are significant, they are not overwhelming. However, Table 3 also shows that DCA can dramatically reduce the within-horizon risk as well. For a 10-year horizon, a 5-year DCA strategy can reduce the within-horizon shortfall probability from over 90% to 58.5%, reduce the conditional mean expected shortfall by 45%, and reduce the expected time of ever experiencing a loss from 22.3% to 9.5%. For perspective, 22.3% means that one can expect to be in the red for over two years during a 10-year investment horizon. A DCA strategy can cut this time in half. Also note these last figures are not conditional on being below the initial investment and are ex ante probabilities before anything occurs. The results for the 20-year horizon are qualitatively similar. The end-of-horizon values do not even remotely reflect this substantial risk reduction that a DCA strategy can provide.

6. Conclusions

DCA continues to be recommended by financial analysts as an investment strategy despite a great deal of academic evidence suggesting that DCA is a suboptimal policy when analyzed within standard risk-return space relative to LS investing. Most academic research has found that the risk reduction of DCA does not justify the lower returns that are realized by this strategy. However, previous studies have only examined end-of-horizon risk.

This study shows that DCA clearly can be an attractive strategy when considering risk throughout the investment horizon. The reduction of this risk can be relatively dramatic. A DCA averaging period of sufficient length (at least three years), can substantially reduce the within-horizon shortfall probability, the conditional expected shortfall, and the expected time one has to endure a wealth level below the initial investment. For an investor with a 10-year horizon or more considering investing in stocks, a 5-year DCA strategy can reduce the within-horizon probability of shortfall from over 90% to less than 60%, reduce the conditional mean expected shortfall by 45%, and can reduce the expected time one may have to endure a loss by approximately 50%.

Thus, investors faced with a choice of LS or DCA investing are well advised to consider to what extent they are willing to accept losses during the investing process, and how well and how long they can deal with seeing their initial investment in the red. Based on past history that shows investors pulling out of funds after large losses, within-horizon risk is extremely relevant to an investor's decision making and DCA is an attractive strategy to help mitigate this risk.

Notes

1. Clearly a stochastic risk-free rate could be incorporated in this analysis along with a correlated risky asset, but this would not greatly affect the analysis and would only add unnecessary complexity. A fixed risk-free rate of 4% was chosen only to create the average risk-premium the market has experienced. A narrowing of the equity premium would obviously favor DCA whereas a widening would favor LS.
2. This procedure is actually only an approximation as theoretically, the loss limit for each percentage of the lump sum invested in the risky asset changes each month. As an example, consider only the first month's risky investment for a 1-year DCA averaging strategy. The loss limit is actually too low for the first six-months, too high for the second six-months, and is correct for the remainder of the time horizon. The approximation is used for simplification and does not create any significant errors as empirical values calculated from Monte Carlo methods give virtually the same probabilities.
3. Dubil also shows that the standard deviation of the terminal value is significantly smaller for DCA relative to LS. However, this is somewhat misleading as the standard deviation for the LS terminal value is based on a larger terminal value and the terminal wealth distribution is lognormally distributed. Because the LS strategy has a higher expected terminal value, extreme positive terminal values are even higher

relative to DCA. This leads directly to artificially large standard deviation estimates and cannot be used to calculate typical confidence intervals. They are especially unreliable as a downside risk measure. This fact is more or less evident by noting that the conditional expected shortfalls for both strategies are basically the same for longer horizons despite the extremely large standard deviation estimates for the up-front strategy. Thus, this study neither calculates nor reports these numbers.

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