

Dynamic retirement withdrawal planning

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Abstract

This paper develops a dynamic model of retirement withdrawal planning that allows retirees and financial planners to improve the probability of retirement portfolio success while simultaneously increasing the average withdrawal rate. The key elements of the model are periodic adjustments of retirement withdrawal rates based on both portfolio performance and remaining life expectancy, and Monte Carlo simulation of both investment returns and mortality. The inclusion of mortality in fixed planning horizon models reduces the probability of retirement-portfolio ruin by almost 50%. When compared to fixed withdrawal rate models, dynamic withdrawal management incorporating mortality reduces the probability of ruin by another 35–40% while increasing average lifetime withdrawal rates by nearly 50%. © 2006 Academy of Financial Services. All rights reserved.

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1. Introduction

As the baby boomers enter retirement, the development of retirement withdrawal planning models has become, and will continue to become, increasingly more important. Although the search for simplicity will continue, the pursuit of accuracy will cause planning and control of retirement withdrawals to become increasingly complex. The withdrawal phase of retirement planning may well require more professional guidance and expertise than the accumulation phase.

To date, much of the emphasis of retirement planning models has been on *sustainable fixed withdrawal rates* from a retirement portfolio subject to uncertain security returns over

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a *fixed planning horizon*. These models are static inasmuch as the withdrawal rate and the planning horizon are fixed.

The volatility of market returns causes fixed withdrawal rate plans with reasonable probabilities of running out of money to generate significant increases in average portfolio values (Ameriks, Veres & Warshawsky, 2001; Pye, 2001). These increases in average portfolio values suggest that excess portfolio accumulations are a cost of reducing the probability of ruin.

Furthermore, *financial* ruin should be defined as the probability of running out of money in the retirement life span, whether that span is shorter or longer than a predetermined number of years. Thus, a first step in improving the accuracy of retirement withdrawal planning is the recognition of the uncertainty of the remaining life span.

This research develops a dynamic retirement withdrawal planning model for controlling retirement withdrawal rates in the context of uncertainty in portfolio performance and remaining life span, as well. The withdrawal management process reduces *both* the probability of running out of money in retirement and excess wealth accumulation.

2. Literature review

Previous research has explored maximum safe withdrawal rates from retirement portfolios. Bengen (1994) recommends a 4% real withdrawal rate from a portfolio made up of 50% stock and 50% intermediate-term treasuries for retirees age 60–65 and finds that rate sustainable for a minimum of 33 years. He advises that initial retirement portfolios should contain at least 75% equities. Bengen (1997), Tezel (2004), and Cooley, Hubbard and Walz (1998) confirm the importance of aggressive portfolio allocations in promoting portfolio sustainability over long periods, although Ervin, Filer and Smolira (2005) find that internationally diversified portfolios would have been less successful than U.S. portfolios in providing real fixed monthly withdrawals over the same, 1930 to 2001, time period. While the conclusions of the previous researchers are based upon studying actual sequences of historic security returns, Milevsky (2001) and Ameriks et al. (2001) also find support for substantial equity allocations using simulated market returns.

Subsequently, Cooley, Hubbard and Walz (2003) compare the sustainability of withdrawals over fixed time periods under two methods for determining portfolio returns: simulation and overlapping historic periods. Differences in the probability of financial ruin from the two methods are attributable, in part, to the overweighting of midsample returns in the overlapping periods methodology. The overweighting might lead to especially poor estimates of ruin if the planning horizon is long relative to the length of the data sequence, so these conditions would favor the simulation method of analysis. Their research also emphasizes the importance of stock-dominated portfolios for sustainability of withdrawals over longer periods under either methodology. Even in stock-dominated portfolios, they find that fixed withdrawal rates must be below 4% to achieve a probability of ruin of as low as 10% over longer (25–30 year fixed) time periods.

Weiss (2001) employs randomly generated returns from probability distributions of actual returns adjusted for correlations. He finds that dynamic rebalancing of a model comprised of

50% stocks with a mean real return of 8% and 50% bonds with a mean real return of 2% results in a 10% probability of ruin if the real withdrawal rate is 4.2% and a 20% probability of ruin if the real withdrawal rate is 4.8%.

Trainor (2005) investigates the validity of assuming that market returns are normally distributed. He finds that projections of market returns and terminal wealth estimates beyond 10 years should be corrected for the lognormal nature of compound returns. Consistent with Trainor's finding, Milevsky and Robinson (2005) assume lognormally distributed real returns with a hypothetical 7% mean and 20% standard deviation. They combine the lognormal distribution of returns with the probability distribution of mortality to compute the stochastic present value necessary to support a constant real amount of retirement-lifetime spending. Milevsky and Robinson find that a 60-year-old retiree with a 4% real withdrawal rate would experience a 13.7% chance of ruin over his expected life span, but increasing the real withdrawal rate to 5% increases the probability of ruin to 22.9%.

Adjusting withdrawals has been explored as a means of reducing the chance of ruin. Bengen (2001) adopts a 63% stock portfolio but added floor (–10% of first year withdrawal) and ceiling (+25%) limits on the withdrawal rate. A 4.15% maximum safe (never failing) withdrawal rate was thus increased to 4.58% when withdrawals were allowed to rise or fall, based on portfolio performance, within the prescribed limits.

Pye (2001) employs a withdrawal management technique that permits the retiree to withdraw the lower of the previous amount (in real terms) or the amortized current portfolio value using the plan length and expected portfolio rate of return. Assuming that security returns are distributed normally with a hypothetical 9% mean real rate of return and an 18% standard deviation, Pye reports that a 4.5% real withdrawal rate would require that 17% of retirees reduce real withdrawals within 20 years. Furthermore, the “worst off” 5% of retirees will need to reduce their withdrawals by at least 47%, even as the median portfolio value has approximately doubled. Guyton (2004) imposes systematic decision rules (restrictions) on withdrawal rate increases and subsequent make-ups for foregone withdrawal rate increases. For retirees willing to maintain the previous withdrawal rate in years after investment losses, and also cap inflationary increases in the withdrawal rate at 6% (without subsequent make-ups for either), the 40-year always safe initial withdrawal rate for an 80% equity portfolio is increased to 6.2%.

3. Research design and data

This research presents several important modifications to the fixed planning horizon, fixed withdrawal rate, model. First, the model assumes that retirees plan for their retirement lifetime rather than a fixed number of years. Retirees will, therefore, adjust their withdrawal rates as they see their withdrawal sustainability threatened by *either* reduced returns or increased expected longevity (life span). And second, the model presents a modification of the withdrawal control limits proposed by Bengen (2001), Pye (2001), and Guyton (2004). The control limits signal withdrawal rate decreases necessary to reduce or delay impending financial ruin and affordable withdrawal rate increases to avoid excess accumulation.

The research methodology model employs Monte Carlo simulation with Latin Hypercube

sampling and 10,000 realizations (iterations).¹ The simulations are performed on a single portfolio allocation which consists of 65% large-cap stocks and 35% intermediate-term U.S. government bonds. The 65/35 allocation is based, in part, on the conclusions of Bengen (1994) and Milevsky (2001) that the portfolio should contain 50–75% common stocks to promote sustainability. In addition, Ervin et al. (2005), Tezel (2004), Cooley et al. (2003), and Ibbotson Associates (2005) show that less aggressive allocations would, predictably, generate higher probabilities of ruin (particularly over long planning horizons) and lower terminal portfolio values. Finally, a stock allocation in the 60–75% range allows comparison of research results to prior literature.

All simulation returns results are based on annual data from Ibbotson Associates (2005), and following Ibbotson Associates (2005), we assume that the asset-class return relatives (1 + return) are lognormally distributed. Over the 1926 through 2004 data period, the arithmetic mean real return to large-cap stocks is 9.17%, and the standard deviation of real large-cap returns is 20.27%. The arithmetic mean real return to intermediate-term government bonds is 2.48%, and the standard deviation of real intermediate-term government bond returns is 6.86%. The correlation between real large-cap stock returns and real intermediate-term government bonds is 0.14. There is no serial correlation in historic large-cap stock returns, but intermediate-term government bond returns show a first-order autocorrelation coefficient of 0.23. While the cross-correlation between the asset classes and the serial correlation within intermediate-term bond returns are not statistically significant, they are maintained in the Monte Carlo simulations. This procedure generates random asset-class returns exhibiting the distributional properties of the historic asset-class returns and the historic correlations, as well. The Kolmogorov-Smirnov goodness of fit test concludes that the distributions of simulated asset-class returns are not different from the distribution of actual historic asset-class returns. The simulated returns, descriptive of historic asset-class returns, are assumed to represent future asset-class returns, as well.

Finally, all returns are assumed to be from tax-deferred accounts (before-tax), and asset management fees or costs are ignored. Withdrawals from the portfolio are made at the beginning of the year with annual rebalancing. Ending portfolio amounts are presented as multiples of the initial portfolio amount.

The simulation model is developed and presented in a step-wise fashion beginning with a fixed planning horizon, then adding mortality, and finally, adding alternative withdrawal rate adjustment mechanisms. The step-wise development and presentation allows the reader to compare results to earlier research and to earlier stages of the model development.

3.1. Model 1: the investment model for a fixed withdrawal rate over a fixed planning horizon

Model 1 assumes that the retiree withdraws a fixed fraction of the initial portfolio at the beginning of the first year of retirement. The remainder of the portfolio is reinvested at simulated real rates of return for the specified portfolio until the next withdrawal. Subsequent withdrawals maintain the fixed real withdrawal amount over the fixed planning horizon. Financial ruin is indicated when the fixed real withdrawal causes the portfolio balance to

Table 1

Model 1 simulation results of the probability of ruin and the distribution of ending portfolio values, as a function of the real fixed withdrawal rate over 30 years

Real withdrawal rate	Probability of ruin	Average ending portfolio*	Percentiles of the distribution of ending portfolio values†				
			95%	75%	50%	25%	5%
3.5%	3.43%	3.65	11.32	4.72	2.43	1.08	.09
4.0%	7.53%	3.20	10.45	4.17	2.03	.77	-.05
4.5%	13.44%	2.76	9.54	3.61	1.62	.46	-.07
5.0%	20.99%	2.34	8.75	3.07	1.23	.01	-.09
5.5%	29.66%	1.95	7.90	2.55	.83	-.06	-.10
6.0%	38.80%	1.60	7.10	2.04	.45	-.08	-.11
6.5%	48.30%	1.28	6.33	1.55	.07	-.10	-.13
7.0%	57.41%	1.00	5.93	1.05	-.08	-.11	-.14

* Ending portfolios values are multiples of the initial portfolio value.

† The distributions of ending portfolio values show significant kurtosis and skewness at all withdrawal rates. At a 4.5% fixed withdrawal rate, the standard deviation of the ending portfolio value is 3.77, kurtosis is 33.28, and skewness is 3.96.

become negative. The initial portfolio value is V_0 , say \$1,000,000, and subsequent beginning-of-year portfolio values are given by Eq. (1), below:

$$V_{t,i} = [V_{t-1,i} - (W) V_0] (1 + R_{t-1,i}) \quad (1)$$

where $V_{t,i}$ is the prewithdrawal value of the portfolio at the beginning of year t for iteration number i , W is the constant real withdrawal percentage of the initial portfolio, and $R_{t,i}$ is the simulated portfolio rate of return in year t for iteration number i . In simulation terminology, RUIN_t is a conditional variable taking the value TRUE to indicate financial ruin if $[V_{t,i} - (W) V_0] < 0$. The subscript t increases to the number of years in the fixed planning horizon, and the model is simulated for 10,000 iterations.

Simulated results of portfolio performance for Model 1 are presented in Table 1. A fixed 4.5% withdrawal rate over a 30-year horizon, for example, will fail 13.44% of the time. Lower and higher withdrawal rates significantly affect both the probability of ruin and the average ending portfolio value. The average ending portfolio may be viewed as buffer against ruin, but it also indicates excess portfolio accumulation and foregone retirement consumption. Most portfolios have grown significantly during retirement while the retiree was constrained to a fixed real withdrawal of 4.5% of the initial portfolio. Hence, the fixed withdrawal rate assumption is reducing the potential withdrawals of retirees to protect the portfolio from ruin. This suggests that performance-based withdrawal rate adjustments would be productive in reducing the probability of ruin while increasing the average withdrawal rate, as well.

3.2. Model 2: the investment model for a fixed withdrawal rate adjusted for mortality

Assuming that the retiree seeks sustainable withdrawals over the remainder of the retiree's lifetime, the probability of financial ruin must rely upon a forecast of retiree longevity. Model

2 maintains the fixed real withdrawal rate assumption but adjusts the investment model, year-by-year, for the probability of death. By incorporating the uncertain retirement life span, Model 2 generates a more meaningful probability of financial ruin.

The mortality table for the total population from the Centers for Disease Control, National Center for Health Statistics for the total U.S. population (Centers for Disease Control, 2002) is used to determine the year-by-year probability of death in a Boolean distribution. For a given year in retirement, the first simulated activity in Model 2 is to determine the value of $DEATH_t$. $DEATH_t$ is a conditional (true or false) variable simulated from the conditional probability of death in the t^{th} year of retirement, given that the retiree was alive the previous year. A withdrawal is attempted only if $DEATH_t$ is false. As before, $RUIN_t$ remains a conditional variable indicating financial ruin if $[V_{t,i} - (W) V_0] < 0$. The model, then, is described by Eqs. (2) and (3) below:

$$V_{t,i} = [V_{t-1,i} - (W) V_0] (1 + R_{t-1,i}) \text{ if } DEATH_t = \text{false and } RUIN_t = \text{false}, \quad (2)$$

otherwise

$$V_{t,i} = V_{t-1,i} \quad (3)$$

The time subscript now increments to 101 minus the initial retirement age, at which time all retirees are assigned $DEATH_t = \text{true}$ as the mortality table is truncated at age 101. The ending portfolio value is $V_{t,i}$ at the earlier of financial ruin or death.

Incorporating mortality provides retirees a more accurate probability of sustaining a real income level during their lifetime. A 4.5% fixed real withdrawal rate will ruin 13.44% of the time in 30 years; but if the retirement age is 60, the probability that the portfolio will ruin before death is only 7.16%.

Careful examination of Table 2 also reveals that as withdrawal rates increase, the costs of the higher withdrawal rates, in terms of increased probabilities of ruin, also increase. However, the increases in the probability of ruin from higher withdrawal rates and the variability of ending portfolios are less for later retirement ages, thereby confirming the increased risk of early retirement. Later retirement reduces the probability of ruin by reducing the time that the portfolio is exposed to market uncertainty, and reducing the uncertainty of longevity, as well.

3.3. Model 3: the dynamic model: variable withdrawal rates adjusted for mortality

The dynamic model allows modifications in the retiree's withdrawal rate similar to those proposed by Pye (2001). The simulation model assumes that the retiree, informed of the average tradeoffs between higher withdrawal rates and the probability of financial ruin in his remaining lifetime, selects an initial withdrawal rate. However, before each subsequent withdrawal, the retiree compares the prior withdrawal rate to the withdrawal rate which would be supported (amortized) by the retirement portfolio balance over the retiree's expected remaining lifetime² at the average real portfolio return across overlapping historic time periods of the retiree's expected life and portfolio allocation. The retiree, or possibly his financial planner, thereby monitors the progress of the retirement portfolio year-by-year to

Table 2

Model 2 simulation results of the probability of ruin and the distribution of ending portfolio values, at the earlier of death or ruin, as a function of the real withdrawal rate

Real withdrawal rate	Retirement age	Probability of ruin	Average ending portfolio*	Percentiles of the distribution of ending portfolio values†				
				95%	75%	50%	25%	5%
3.5%	55	3.38%	3.70	13.06	4.17	1.92	.99	.16
	60	1.99%	2.87	9.17	3.28	1.65	.97	.28
	65	1.11%	2.29	6.74	2.62	1.44	.94	.37
	70	0.46%	1.86	4.97	2.14	1.30	.93	.44
4.0%	55	6.35%	3.26	11.88	3.61	1.63	.82	−.04
	60	4.18%	2.56	8.39	2.92	1.44	.83	.08
	65	2.47%	2.07	6.22	2.37	1.30	.84	.21
4.5%	70	1.19%	1.70	4.62	1.97	1.20	.85	.33
	55	10.64%	2.82	10.66	3.10	1.36	.60	−.07
	60	7.16%	2.25	7.62	2.55	1.25	.67	−.05
	65	4.53%	1.85	5.66	2.12	1.17	.72	.04
5.0%	70	2.48%	1.55	4.22	1.80	1.11	.76	.20
	55	15.93%	2.42	9.51	2.63	1.11	.37	−.08
	60	11.46%	1.95	6.74	2.20	1.08	.50	−.08
	65	7.65%	1.64	5.09	1.90	1.05	.60	−.06
5.5%	70	4.21%	1.40	3.88	1.64	1.01	.67	.06
	55	22.66%	2.04	8.32	2.18	.92	.10	−.10
	60	16.97%	1.68	6.01	1.88	.94	.30	−.09
	65	11.66%	1.44	4.56	1.66	.95	.46	−.08
6.0%	70	6.64%	1.26	3.56	1.49	.95	.56	−.06
	55	29.20%	1.69	7.17	1.76	.71	−.07	−.11
	60	22.41%	1.42	5.35	1.60	.79	.10	−.11
	65	15.98%	1.25	4.06	1.45	.85	.30	−.10
6.5%	70	9.93%	1.12	3.23	1.34	.88	.45	−.09
	55	36.69%	1.38	6.08	1.40	.47	−.08	−.12
	60	28.74%	1.18	4.66	1.33	.62	−.07	−.12
	65	20.71%	1.07	3.60	1.27	.72	.14	−.11
7.0%	70	13.71%	.99	2.91	1.21	.79	.33	−.10
	55	44.04%	1.10	5.11	1.11	.23	−.10	−.14
	60	35.55%	.97	4.00	1.11	.44	−.09	−.13
	65	26.36%	.90	3.20	1.11	.60	−.04	−.13
	70	18.11%	.86	2.62	1.09	.70	.20	−.12

* Ending portfolios are multiples of the initial portfolio value.

† The distributions of ending portfolio values show extreme kurtosis and significant skewness at all withdrawal rates. At a 4.5% fixed withdrawal rate, the standard deviation of the ending portfolio value is 3.42, kurtosis is 143.7, and skewness is 7.47.

receive signals that the withdrawal rate could, or should, be changed. However, following the lead of Bengen (2001), the model allows several types of controls in the withdrawal rate adjustment process. The controls are necessary to avoid overreactions to unanticipated market gains, which may subsequently threaten the survivability of the portfolio. A well-designed set of withdrawal change controls, adjusted for the retiree's risk tolerance in pursuit of higher real withdrawal rates, would reduce or delay financial ruin and would allow affordable increases in the withdrawal rate, as well.

The dynamic model employs three types of controls on withdrawal rate changes: (1)

portfolio deviation thresholds, (2) withdrawal adjustment rates, and (3) absolute withdrawal rate limits. Portfolio deviation thresholds act as preconditions to changing the real withdrawal rate. An upward threshold of deviation establishes a minimum portfolio balance necessary for withdrawal rate increases, a downward threshold of deviation establishes the maximum portfolio balance necessary for withdrawal rate decreases. The thresholds of deviation delay changes in the withdrawal rate until the portfolio reveals signs of significant over or under performance. The thresholds, then, limit responses to short-term fluctuations in portfolio performance while allowing the retiree to adjust to market trends. Withdrawal adjustment rates limit the extent of withdrawal rate changes in response to a signal that the portfolio value has broken a threshold of deviation. The upward adjustment rate prescribes the rate of adjustment towards a higher withdrawal rate, while the downward adjustment rate sets the rate of adjustment towards a lower withdrawal rate. Withdrawal adjustment rates reduce the chance of overreacting to improved portfolio performance that is subsequently reversed, and they might also be used to smooth the income effects of poor portfolio performance. Absolute withdrawal rate limits, an upward limit and a downward limit, constrain the range of permissible withdrawal rates to prevent unsustainable increases and/or intolerable decreases in the withdrawal rate.

The sequence of operations embedded in the simulation model represents the sequence of decisions involved in changing the real withdrawal rate. In considering a withdrawal rate increase, the retiree or his financial planner first computes the portfolio amount required to sustain the existing withdrawal amount, $[W_{t-1} (V_0)]$, over the retiree's expected remaining life, L . The required portfolio amount is the present value of an annuity due, $PVIFADue_{L, Avg r}$, computed at the average portfolio rate of return that has been historically realized, $Avg r$, in overlapping periods of time equal to the retiree's expected remaining life. The upward threshold required portfolio amount is

$$UPTHR\$ = (1 + UPTHR) [(W_{t-1}) (V_0) (PVIFADue_{L, Avg r})], \quad (4)$$

where $UPTHR$ is the upward threshold of deviation, stated as a multiple of the required portfolio, by which the current portfolio exceeds the required amount.

No change in the real withdrawal rate is permitted unless the portfolio balance exceeds $UPTHR\$$. An upward threshold of 1.0, for example, would require that the portfolio exceed twice the amount necessary to sustain the prior withdrawal rate. An upward threshold of 1.0 might be considered a portfolio "cushion" of 100%. If the retiree's portfolio balance exceeds the upward threshold amount, $UPTHR\$$, the retiree's expected remaining life, L , and the average portfolio rate of return, $Avg r$, are used to calculate a new withdrawal rate that amortizes the remaining portfolio. Therefore, the upward threshold constrains changes in the withdrawal rate as in Eqs. (5) and (6), below:

$$W_t = (V_t / PVIFADue_{L, Avg r}) / V_0, \text{ if } V_t > UPTHR\$ \quad (5)$$

otherwise,

$$W_t = W_{t-1}. \quad (6)$$

Higher upward thresholds would be expected to reduce the probability of ruin as increased withdrawals can only occur after significant portfolio growth.³ The cost of the reduced probability of ruin would be lower average withdrawal rates.

Downward thresholds impact the timing of withdrawal rate decreases with higher downward thresholds causing withdrawal rates to be reduced at earlier signals of portfolio deficiency or underperformance. Downward thresholds greater than zero maintain a pre-established cushion against inadequate future portfolio performance, while downward thresholds below zero delay the recognition of portfolio underperformance and increase the probability of subsequent ruin.

The second control, the rate of withdrawal adjustment, is the fractional rate of adjustment from the prior withdrawal rate towards the new rate. The retiree can choose, depending upon his risk tolerance, to completely or partially adjust to the new withdrawal rate. An upward adjustment rate of 50% would adjust the prior withdrawal rate only 50% toward the new, higher withdrawal rate. Provided that the constraint of Eq. (5) is met, the upward adjustment rate (UPRATE) would alter Eq. (6) to Eq. (7) below:

$$W_t = W_{t-1} + (\text{UPRATE}) \{[(V_t / \text{PVIFADue}_{L, \text{Avg } T}) / V_0] - W_{t-1}\} \quad (7)$$

A higher (approaching 1.0) upward adjustment rate would be associated with higher probabilities of ruin as retirees adjust more completely to abnormally high market returns, and therefore, reduce the portfolio buffer against subsequent market declines or underperformance. A lower (approaching zero) upward adjustment would be more conservative and have the opposite impact on the probability of ruin. The effects of adjustment rates on average withdrawal rates are not clear a priori. Incomplete upward adjustment, for example, reduces withdrawal rates in the short-term but may increase the average withdrawal rates in the future by delaying financial ruin beyond periods of superior market performance.

One could also enforce a downward adjustment rate. Higher downward adjustment rates would be more conservative and reduce the probability of ruin by forcing earlier recognition of market declines to preserve any available portfolio buffer against yet further market declines.

The third control is an absolute limit on the maximum and minimum withdrawal rates. In the event of continuing poor market performance, the minimum withdrawal rate is necessary to prohibit the remaining portfolio from being amortized over the remaining expected life by a withdrawal rate below a minimally acceptable lifestyle. Retirees would be advised to set their personal minimum withdrawal rate at the lowest minimum they can tolerate as the alternative is ruin.

The three types of controls interact in their impact on the probability of financial ruin and the average withdrawal rate. For example, to the extent that overreactions are limited by partial adjustment rates, there would be less need to constrain withdrawal rate changes by absolute withdrawal rate limits and/or portfolio deviation thresholds. The controls have the common purpose of reducing short-term reactions to market performance to preserve financial viability over the longer term, while also allowing affordable increases in the withdrawal rate. The following section explores the interaction of the upward threshold of deviation and the upward adjustment rate, and their combined effect on the probability of ruin, average withdrawal rate, and ending portfolio value.

Table 3
Control values for Model 3 simulations

Control	For withdrawal rate increases	For withdrawal rate decreases
Threshold of deviation*	1.4	0.0
Rate of adjustment	20%	100%
Absolute limits on withdrawal rates	40%	3%

* Thresholds of deviation are multiples of the portfolio value necessary to sustain the prior withdrawal rate over the expected remaining lifetime at historic rates of portfolio returns.

3.3.1. Example controls

To examine the productivity of the controls in reducing the probability of ruin, we continue with the same example of a 60-year old at retirement and the same portfolio allocation, but we impose what we consider to be reasonable withdrawal rate controls for portfolio thresholds of deviation, withdrawal rate adjustments, and absolute withdrawal rate limits. Then, we examine the sensitivity of the probability of financial ruin, the average withdrawal rate, and the ending portfolio value to changes in the controls. The initial set of control values are given in Table 3, below.

This choice of controls is somewhat subjective, but designed for the conservative retiree. The relatively high upward threshold forces security in maintaining the existing withdrawal rate before the rate is increased, while the lower downward threshold allows earlier reactions (reductions) in the withdrawal rate when the existing rate is not sustainable. Similarly, the relatively low rate of upward adjustment of withdrawal rates (20%) maintains security of withdrawal rate sustainability, while the higher downward adjustment rate forces more complete corrective action. The relatively high upper bound on withdrawal rates (40%) is protected by the upward threshold of deviation and the (partial) rate of adjustment to higher rates. The lower bound on withdrawal rates (3%) is to prevent possibly the worst kind of financial ruin, one that is long and drawn out.

Simulated results for the withdrawal rate controls specified in Table 3 produce a 4.43% probability of ruin, a 6.63% average withdrawal rate, and an average ending portfolio 1.07 times the beginning amount. An initial portfolio of \$1,000,000, for example, would yield an average real income of \$66,300 per year, with a 4.43% probability of ruin, and an average ending portfolio value at the earlier of death or ruin of \$1,070,000. These results are far superior to those of Model 2, which incorporated retiree mortality but maintained fixed withdrawal rates. To achieve a probability of ruin below 5% without adjusting withdrawal rates (Model 2), the fixed (and therefore the average) withdrawal rate would be below 4.1% (Table 2); and the average ending portfolio value would exceed \$2,500,000.

3.3.2. Sensitivity analysis of the impacts of the upward threshold of deviation and the upward adjustment rate on the probability of ruin, average withdrawal rate, and ending portfolio value

Fig. 1 shows the sensitivity of the probability of ruin to the upward threshold of deviation and the upward adjustment rate while holding the other controls at the values specified in Table 3. Higher upward thresholds of deviation and lower upward adjustment rates generated

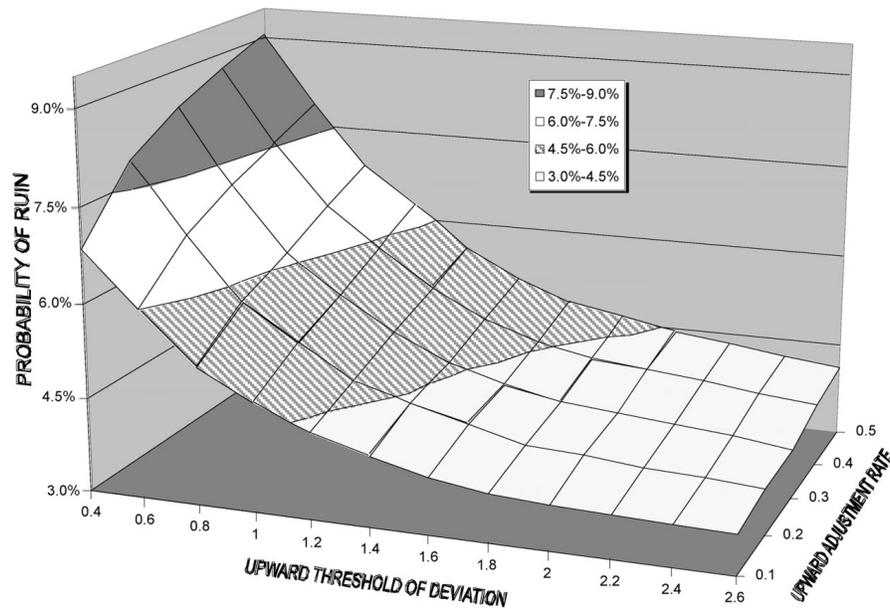


Fig. 1. Model 3 simulation results of the sensitivity of the probability of ruin to values of the upward threshold and upward adjustment rate.

lower probabilities of ruin, as expected. In isolation, the upward threshold has the greater impact on the probability of ruin, suggesting that upward thresholds would be the first control to install in reducing the probability of ruin. Upward thresholds of deviation above 1.4, however, yield only modest reductions in the probability of ruin as higher upward thresholds decrease the probability of ruin at a decreasing rate.

Fig. 1 presents four ranges of the probability of ruin separated by iso-ruin lines which depict combinations of the upward adjustment rate and the upward threshold of deviation generating a given probability of ruin. The iso-ruin lines reveal that even those portfolios already protected from ruin by a relatively large upward threshold of deviation can be further protected by relatively low upward adjustment rates.

Fig. 1 enables retirees or their financial planners to make intelligent choices regarding withdrawal rate controls consistent with the retiree's risk tolerance. The effects of the controls on risk should be examined, however, for their concurrent effects on returns.

One measure of the return to withdrawal rate controls is the increase in the lifetime average withdrawal rate. Fig. 2 shows the sensitivity of the lifetime average withdrawal rate to the upward threshold of deviation and the upward adjustment rate while holding the other controls at the values specified in Table 3.

Fig. 2 presents four ranges of the lifetime average withdrawal rate separated by iso-rate lines which depict combinations of the upward adjustment rate and the upward threshold of deviation generating a given lifetime average withdrawal rate. Combinations of high upward thresholds of deviation and low upward adjustment rates generate lower lifetime average withdrawal rates as retirees constrain their withdrawals to avoid financial ruin. Very low (approaching 0) upward thresholds of deviation and very high (approaching 1) upward adjustment rates would disarm these controls, but the impacts of the other controls of Table

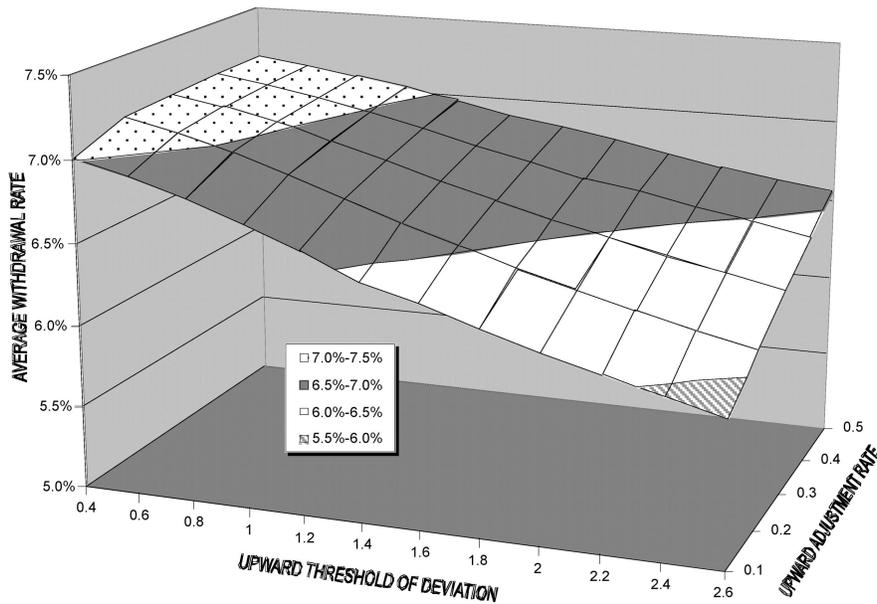


Fig. 2. Model 3 simulation results of the sensitivity of the average lifetime withdrawal rate to values of the upward threshold and upward adjustment rate.

3 would retain some advantage of withdrawal rate management as the entire average withdrawal rates surface is above 4.5%. The controls specified in Table 3 produce a lifetime average withdrawal rate of 6.63% with only a 4.43% probability of ruin.

These results are confirmed in Fig. 3, which shows the effects of the same upward

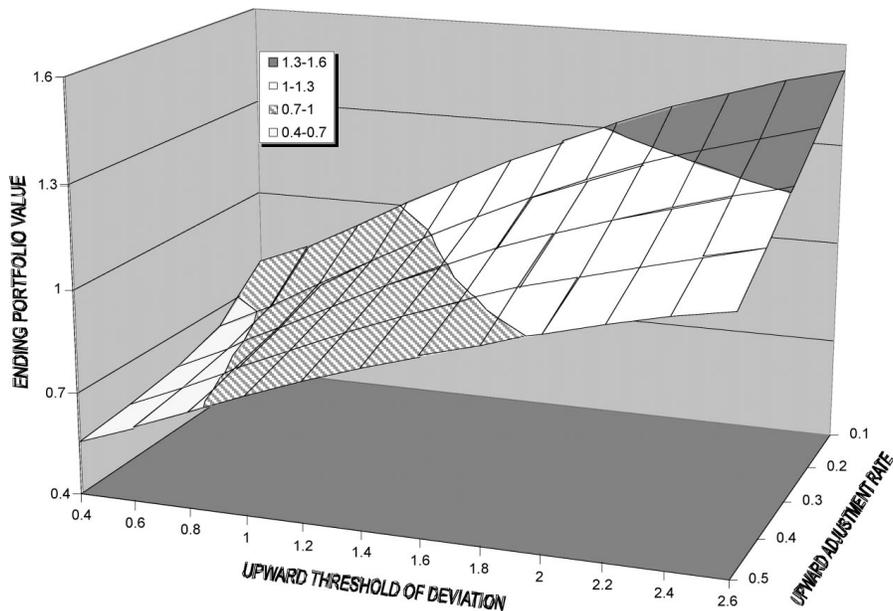


Fig. 3. Model 3 simulation results of the sensitivity of the ending portfolio value, as a multiple of the initial portfolio value, to values of the upward threshold and upward adjustment rate.

withdrawal rate controls on the ending portfolio value, another measure of the return from using the withdrawal rate controls to reduce the risk of portfolio ruin.

The lines on the surface of Fig. 3 separate iso-portfolio ranges, with the middle line showing combinations of the upward adjustment rate and upward threshold of deviation that result in an ending portfolio equal to the amount at the beginning of retirement. Generally speaking, withdrawal rate management is successful as the excess accumulation surface is below 2.25 (Table 3). Furthermore, higher upward thresholds of deviation and lower upward adjustment rates (note the direction of the upward adjustment rate scale) necessary to reduce risk increase ending portfolio values measured at the earlier of death or financial ruin. The increase in ending portfolio values results from constraining withdrawal rate increases to avoid financial ruin.

Withdrawal management controls should be evaluated in their impacts on the probability of ruin and the average withdrawal rate or the distribution of ending portfolio values. In this tradeoff, the upward threshold of deviation and the upward adjustment rate show the greatest productivity on these measures of portfolio performance. The other controls not presented seem to have very little impact on portfolio performance if the upward threshold of deviation is relatively high (1.2 or above) and the upward adjustment rate is relatively low (30% or below). Decreasing the absolute maximum withdrawal rate from 40% to 8%, for example, has very little effect on the probability of ruin. It does, however, reduce the average withdrawal rate and increase the ending portfolio as very high absolute withdrawal rate limits, protected by high upward thresholds and low adjustment rates, only serve the most successful portfolios. Similarly, increasing the absolute minimum withdrawal produces modest increases in the average withdrawal rate relative to the substantial cost in terms of the higher probabilities of ruin, although retirees willing to abide by minimum withdrawal rates below 3% will be rewarded by substantially lower probabilities of ruin. Finally, downward adjustment rates below 100% increase the probability of ruin with virtually no benefit.

This research builds on the work of Guyton (2004) and Milevsky and Robinson (2005) by integrating retiree mortality and dynamic withdrawal management. The key elements of the model are periodic adjustments of retirement withdrawal rates based on both portfolio performance and remaining life expectancy, and Monte Carlo simulation of both investment returns and mortality. The results show that fixed planning horizon models significantly overstate the probability of lifetime financial ruin with the excessively long fixed planning horizons necessary to protect those who live longer than average. The results also show that fixed withdrawal rate models unnecessarily constrain affordable withdrawal rate increases from overperforming portfolios, but allow the continuation of withdrawals from underperforming portfolios. Integration of dynamic withdrawal management and mortality provides a flexible withdrawal management strategy. The integration provides higher withdrawals from portfolios with excess accumulations and forces lower withdrawals from portfolios in danger of ruin from portfolio underperformance or superannuation.

4. Conclusions

As the baby boomers enter retirement, financial planners will be pressured for more meaningful and more accurate retirement planning models, particularly withdrawal models.

Additionally, the growth of defined contribution programs will continue to transfer the responsibility for retirement management to retirees and their planners.

The purpose of retirement withdrawal planning is to arrange for withdrawals from a retirement portfolio over the retirement life span, not a fixed number of years. The probability of depleting a retirement portfolio before death is substantially lower than the probability of depleting the portfolio over a fixed 30-year time period. A fixed 4.5% real withdrawal rate has a 7.16% probability of ruin before death and a 13.44% probability of ruin before 30 years.

Furthermore, active management of withdrawal rates, based on portfolio performance and expected remaining life span, produces higher real lifetime average withdrawal rates while reducing the lifetime probability of ruin. Illustrative controls increase the real lifetime average withdrawal rate to 6.63% and reduce the probability of financial ruin before death to 4.43%. This research examines six possible withdrawal controls: maximum and minimum withdrawal rates, rates of upward and downward adjustment, and upward and downward thresholds of deviation (adjustment triggers). Results favor immediate and complete downward adjustment toward the tolerable minimum withdrawal rate. Upward adjustment, however, should be slow (cautious) and deferred until the portfolio shows significant excess accumulation.

To develop and test additional retirement portfolio management strategies, research should be directed towards the impacts of various withdrawal controls, the relationship between portfolio allocation and the withdrawal management strategy, and the risk to return relationship between the probability of financial ruin and the withdrawal rate.

Notes

1. Simulations are performed on GoldSim, Release 9.1, simulation software.
2. The expected remaining life from the mortality table is rounded up to the next whole number of years.
3. Note that the initial rate of withdrawal has a “built-in” upward threshold. Consider an initial retirement portfolio of \$1,000,000. If the retirement age is 60, the expected remaining life is rounded to 23 years, and for our portfolio, the average real return across overlapping 23-year periods is about 5.15%. At 5.15%, a 4.5% real withdrawal rate (\$45,000 annuity due) would require only \$629,318, thus the initial upward threshold is $1,000,000/629,318 = 1.59$. Therefore, depending upon the age at retirement and the withdrawal rate, retirees generally begin retirement with about 1.6 times the required portfolio. In our terminology, the initial upward threshold would be 0.6.

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