

# There's no time like the present: the cost of delaying retirement saving

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## Abstract

Many people delay joining a pension plan until well into their working lives. We use a stochastic simulation model to show the cost of this delay in terms of the higher pension contributions that must eventually be paid to ensure an adequate retirement income. We find the levels of contributions required for individuals who start saving late are so high it is questionable whether they are affordable for anyone not on a high income. We also analyze the cost in terms of reduced pension of an interrupted labor market history, such as that experienced by someone who leaves work for a period to bring up a family. © 2006 Academy of Financial Services. All rights reserved.

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## 1. Introduction

As state pensions in many countries become less generous relative to average earnings and the provision of occupational defined benefit (DB) pension plans becomes less common, defined contribution (DC) pension plans have become an increasingly important retirement saving vehicle. In most DC plans, the individual has important decisions to make in terms of whether to join, when to join, how much to pay into the plan and how to invest the contributions.

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Table 1 Participation rates in 401(k) plans by worker's age and income

Income	Age 21–29	Age 30–39	Age 40–49	Age 50–59	Average
\$25–34.9k	62.7	70.9	73.7	78.7	71.2
\$45–54.9k	84.8	86.9	86.8	87.9	86.7
\$75–99.9k	83.9	90.6	91.3	91.3	90.4
Total	62.7	75.3	78.1	80.3	74.7

*Note:* Percentage of eligible workers who have joined the plan.

Source: extracted from Table 3 of Nyce (2005).

The later an individual begins contributing to a DC pension plan, other things being equal, the larger the contributions he or she will have to pay to ensure an adequate retirement income. Interrupting contributions can also lead to a substantially reduced pension. In this paper, we use a stochastic simulation model to estimate the likely distribution of retirement incomes from a variety of alternative DC pension plan participation profiles. In particular, we show the cost in terms of higher contributions of deferring pension plan membership and of having an interrupted labor market participation history.

Our results provide serious cause for concern: they indicate that the contribution rates required to generate reasonable pension outcomes are often high, and sometimes well beyond what most individuals would be able to afford, given their other financial commitments. Our study therefore has an important role to play in educating employees about these implications and helping them develop realistic retirement saving plans.

## 2. Evidence on delayed pension plan participation

Many individuals choose not to contribute to a pension plan. For example, the National Association of Pension Funds Annual Survey of UK plans (NAPF, 2004) shows that where membership is not automatic, on average only 58% of eligible employees join their employer's DC pension plan. Some of the employees who have not joined might, of course, be contributing to their own personal pension plan, but in most cases they will have decided not to save for a pension, or to defer saving to a later stage in their career.

There is evidence that older workers are more likely to be members of their employer's pension plan. Office for National Statistics (ONS) data for the UK for 2003 show that occupational pension plan participation rates are positively related to tenure with the employer: the participation rate for employees who are eligible to join their employer's pension plan and who have less than two years' service with the employer is 61%, while the comparable figure for employees with service of five years or more is 91%.<sup>1</sup> More direct evidence of a link between age and pension plan participation comes from a study by Nyce (2005), which examined 48 US 401(k) DC pension plans covering over 300,000 employees. The results, some of which are reproduced in Table 1, show that participation rates rise with both age and income. Individuals with higher incomes are more likely to participate in the pension plan than those on lower incomes, but even after controlling for income, participation rates are higher among older workers.

### 3. Pensions in life-cycle financial planning

Standard economic theory offers an explanation as to why individuals may choose not to save at certain points in their working life. The life-cycle theory of Ando, Brumberg and Modigliani (Modigliani & Brumberg, 1954; Ando & Modigliani, 1957), and Friedman's (1957) permanent income hypothesis both imply that individuals smooth consumption over their lifetimes. In essence, in each period an individual can consume up to the annuity value of his or her expected wealth, and saving will take place only when current income exceeds this annuity value. From this perspective, a decision to defer retirement saving could simply represent a view that income will be higher in future. However, life-cycle theories have been troubled by evidence that many households fail to maintain their preretirement level of consumption in retirement, which suggests they might not have saved enough to properly smooth their lifetime spending. For instance, Banks, Blundell and Tanner (1998) find a drop in consumption at retirement that cannot be explained fully by standard consumption smoothing models. Some of the reduction in consumption expenditure is a natural consequence of withdrawal from the labor market: for example, travel costs to work are no longer incurred. However, another possible explanation for this drop in consumption is that members of these households are surprised by how low their pension is and are forced to adjust their consumption accordingly.

Behavioral economics provides an alternative view that suggests decisions to defer saving are driven by behavioral biases and thus may not represent optimal behavior. For example, Thaler (1994) argues life-cycle theory fails to consider bounded rationality, which suggests individuals cannot do the multiperiod optimization calculations that are required for life-cycle saving, and bounded self-control, which implies individuals are unable to follow through with previously identified plans to save rather than consume; "Real people have trouble both in figuring out how much to save and in implementing any given goal" (Thaler, 1994, p. 189). Laibson, Repetto and Tobacman (1998, p. 93) suggest that individuals have a "systematic tendency to err. . . in the direction of instant gratification" which they explain in terms of personal long-term discount rates being lower than short-term ones.<sup>2</sup>

Bernheim, Skinner and Weinberg (2001) found, using the Panel Study of Income Dynamics and the Consumer Expenditure Survey, that the average replacement ratio in retirement in the United States is around 64%. But there is considerable variation around this figure, even among households with similar socioeconomic characteristics. Life-cycle theory explains this variation in terms of differences in time preference rates, risk tolerance, exposure to uncertainty, and relative tastes for work and leisure at advanced ages. These factors have testable implications concerning the relation between accumulated wealth and the shape of the consumption profile. Bernheim et al. (2001) found little support for these implications. The data are instead consistent with "rule of thumb," "mental accounting," or hyperbolic discounting theories of wealth accumulation.

To the extent that decisions to defer pension saving do stem from such behavioral biases, individuals might benefit from commitment mechanisms, such as automatic enrolment of employees into the pension plan and regular saving plans, designed to mitigate the effects of the biases. (Benartzi & Thaler, 2004).

#### 4. Modeling the consequences of delayed pension plan participation

Irrespective of whether the decision to defer saving for retirement is based on rational analysis or behavioral biases, it is important to understand how much an individual needs to save over different periods to ensure an adequate income in retirement. To address this issue, we estimate the likely range of retirement incomes from a variety of different DC pension plan participation profiles. As Booth (2004) notes it is important to consider risk in pension plan accumulation and the probability of achieving a certain outcome, rather than focusing only on the most likely result. Hence, we use a stochastic model, which allows us to understand the distribution of retirement income outcomes. We use data representative of the UK to illustrate the analysis, but the broad conclusions are likely to hold across most other national markets where DC pension plans are becoming common.

We take as our benchmark the case of an average U.K. male who joins a pension plan at age 25 and retires at 65; the current state pension age for a man in the U.K. We assume that he contributes 10% of his salary each year to the pension plan, based on survey evidence that this is close to the average combined (i.e., employee plus employer) contribution rate in occupational DC schemes in the U.K. The Pensions Commission (2004) reports contribution rates of this magnitude from surveys conducted by the NAPF and the Association of Consulting Actuaries. Average United States contribution rates appear similar, with Munnell and Sunden (2006) suggesting that 9% is the typical contribution rate for a 401(k) scheme member (6% employee; 3% employer match.) While we use the 10% contribution rate in most of our analyses, we also conduct a sensitivity analysis using alternative rates.

We assume the individual's career earnings experience matches that of a typical male employee in the U.K. and to simplify the analysis we further assume that there is no risk to the accrual of pension benefits arising from unemployment. Fig. 1 shows the age-earnings profile of the employee, scaled to average male earnings (=1.0). The profile incorporates relatively rapid promotion in the earlier years of the employee's career, with a peak in the mid- to late 40s, and below-average promotional increments thereafter. Employees have above average earnings in their midcareers and below average earnings at the beginning and end of their careers. In addition to these promotional increments, the employee will also experience an annual increase in real wages arising from productivity increases over his career. We assume a 2% per annum increase in real wages, in line with the annual increase in national average earnings in the U.K. over the postwar period.<sup>3</sup>

The alternative scenarios we investigate involve delays in joining the pension plan to various ages between 35 and 55. We document the impact of this deferment on the retirement income outcome for a 10% contribution rate, and calculate the contribution rate that would be required to replicate the pension benefits produced by membership from age 25. We also show the impact of deferring retirement to age 70, given growing comment (e.g., Pensions Commission, 2004) that increasing longevity should be matched by higher retirement ages, and of retiring early at age 55, which could happen to an individual as a result of ill health or redundancy. Finally, we investigate some scenarios for a female pension plan member, in which we compare an unbroken pension plan participation record with the case where the woman has a spell out of the labor market to raise a family. To do this, we use the

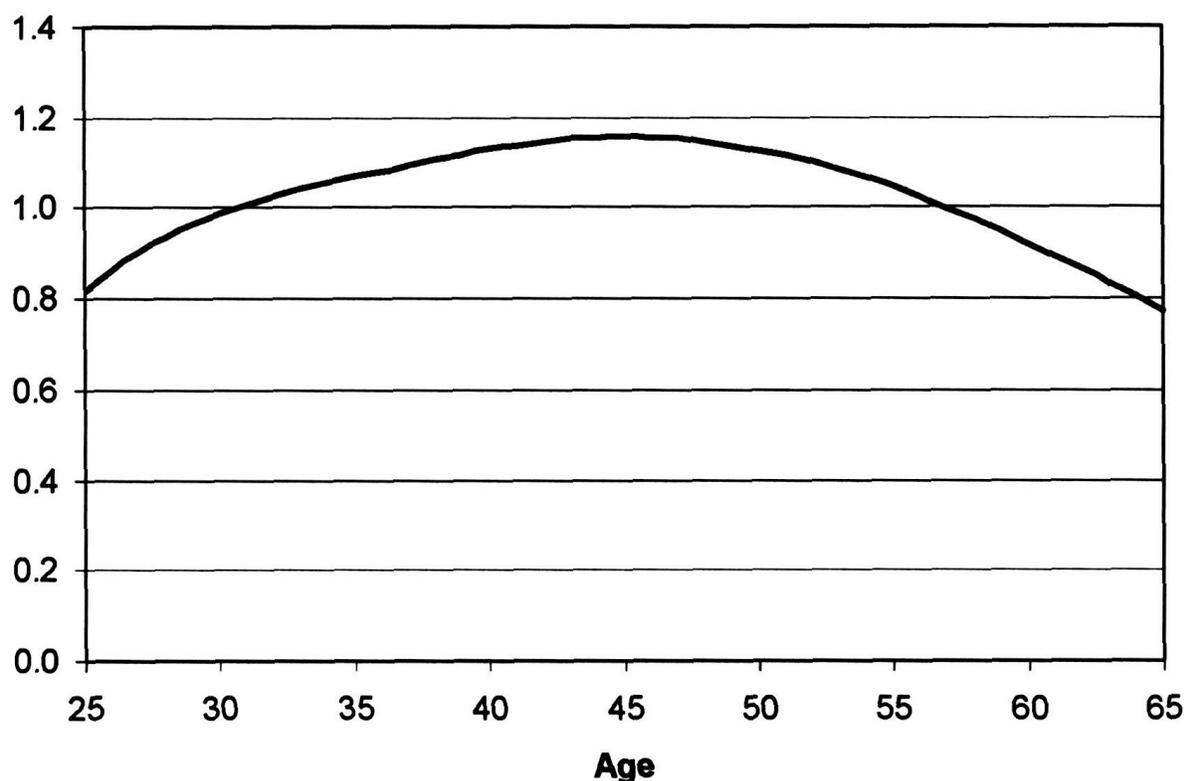


Fig. 1. Chart shows the age-real earnings profile for the average male employee in the UK. Earnings follow the path of the curve as the employee grows older but are also subject to additional real growth on account of economy-wide productivity gains.

age-earnings profile of a typical female worker in the U.K.; such a worker's earnings peak in her early 30s, around a decade before the average male (see Fig. 2).

The model we use to conduct our analysis is the PensionMetrics model described in detail in Blake, Cairns and Dowd (2001) and briefly in the appendix. This model uses stochastic simulation to determine the anticipated distribution of pension outcomes, measured in terms of the ratio of initial pension to final salary (i.e., the replacement ratio), for any given set of input parameters such as contribution rate, asset allocation strategy, anticipated retirement age, and so on, taking into account the stochastic nature of investment returns, interest rates and salary levels.

With respect to the investment of the pension contributions, we consider four alternative stylized asset allocation profiles. We investigate a conservative, low-risk strategy that is 100% bonds and a high-risk strategy invested 100% in UK equities. We also consider a balanced strategy that is 60% equities and 40% bonds, a common asset mix in United States plans, and a deterministic life-cycle strategy that invests a proportion equal to (100-member's age) in equities and the remainder in bonds.<sup>4</sup> This life-cycle strategy is designed to reduce the investment risk borne by the member in the years immediately preceding retirement, on the grounds that it would be difficult to recover from losses sustained at such a late stage in the working life. The balanced and life-cycle strategies use annual rebalancing.

Following Byrne, Blake, Cairns and Dowd (2005) we use a combination of historical

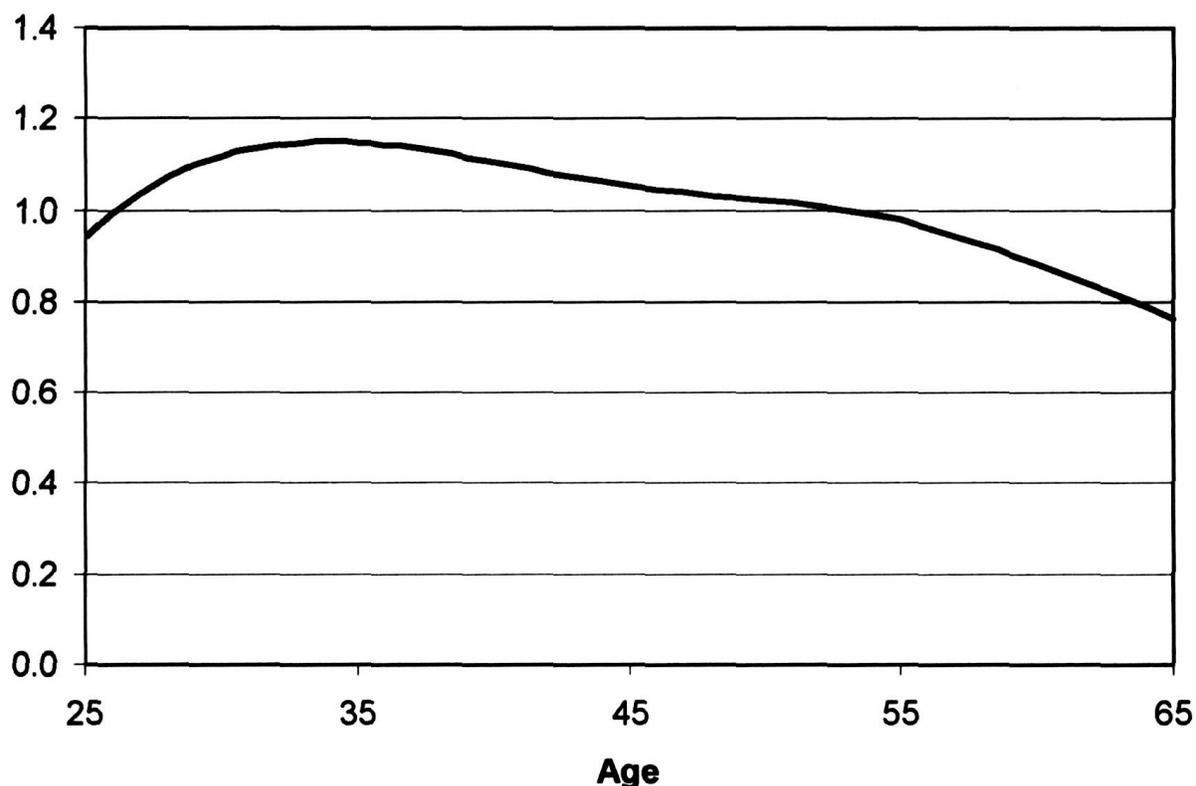


Fig. 2. Chart shows the age-real earnings profile for the average female employee in the UK. Earnings follow the path of the curve as the employee grows older but are also subject to additional real growth on account of economy-wide productivity gains.

returns data and forward-looking return estimates to parameterise the stochastic model. The source for the historical returns is the ABN Amro/LBS data set discussed in Dimson, Marsh and Staunton (2001) and available commercially through Ibbotson Associates. We use data for the postwar period of 1947 to 2003 to estimate the volatility and correlation structure of the asset classes (see Table A1 in the Appendix). However, we do not use the historical mean returns as our estimate of future returns. Instead, we use forward-looking return assumptions to account for the possibility that the historical realized equity risk premium, defined as the difference between the average return on equities and the return on Treasury bills, is larger than can reasonably be expected in future.

We use the forward-looking return estimates because some commentators (e.g., Fama & French, 2002; Arnott & Bernstein, 2002; Dimson et al., 2001) believe that the realized equity risk premium over the postwar period is an upward biased estimate of the likely future risk premium. They argue that high historical equity returns between 1950 and 1999 were mainly because of: (1) unexpectedly high profitability and hence strong dividend growth and (2) an unexpected fall in long-term discount rates, the result of a sustained decline in the volatility of earnings, dividends and returns. Neither of these factors can be relied upon to boost future equity returns. These commentators conclude that the best estimate of a global equity risk premium is about 3.5% relative to U.S. Treasury bills, and we use this suggested equity

Table 2 Forward-looking return parameters

	Nominal Annual return	Real annual Gross return	Real annual return Post charges
Equities (UK & global)	7.5%	5.0%	4.0%
Bonds	4.5%	2.0%	1.0%
Cash	4.0%	1.5%	0.5%

*Note:* Inflation is assumed at 2.5% in line with Bank of England's target for the Retail Price Index (RPIX). The 1.0% annual charge is now typical of DC pensions in the UK. No allowance is made for any excess returns from active management. The bond return assumption reflects current market yields. The equity return figure is based on a market level dividend discount model, where dividends grow in line with GDP, while the cash return is derived by subtracting a 3.5% equity risk premium from the equity return figure. The approach is consistent with that used by the UK's Financial Services Authority (2003).

premium to produce an alternative set of forward-looking nominal return parameters, which we adjust for pension plan charges (1.0%) and expected inflation (2.5%).<sup>5</sup> We conduct a sensitivity analysis of our return assumptions later in the paper.

While some pension funds are actively managed, we make no allowance for any (positive or negative) excess returns generated by active management. We also assume that annual returns on the assets in the pension fund follow a multivariate normal process.<sup>6</sup> The return parameters are shown in Table 2.

When the plan member reaches retirement age, the accumulated fund is converted into a single life annuity that provides a level income to him until he dies. The annuity rate is based on a long-term interest rate consistent with the returns on fixed-income assets earned by the fund in the year leading up to retirement and with the PMA92 survival probabilities at the relevant age taken from mortality tables published by the Institute and Faculty of Actuaries: these reflect the mortality experience of males buying pension annuities from UK life offices. Where we analyze scenarios for female plan members, we use the corresponding female mortality tables (PFA92). We do not take account of the possibility of future improvements in longevity.

We present the results of the simulation in terms of the replacement ratio; that is, the ratio of initial pension to salary immediately before retirement. Many final salary, DB pension plans have offered a replacement ratio of 1/60th of final pay for each year worked, up to a maximum of 40/60ths of final pay. Under such a plan, a worker with 40 years' service would therefore get a pension income of two-thirds of final salary. We can use this ratio as a benchmark for the outcomes from the DC plan.

## 5. Results and analysis

### 5.1. The case of an average male

Table 3 shows the distribution of replacement ratios for our benchmark case of a male who contributes 10% of salary to the pension plan from age 25 to age 65. The results are based on 5000 simulations using the PensionMetrics model. The median replacement ratio for that

Table 3 Replacement ratios by age of joining pension plan: male retiring at 65

Start age	Strategy	Median replacement ratio	Mean replacement ratio	Inter-quartile VaR range	5% VaR replacement ratio	Contribution rate to match age 25 start	Contribution rate for 50% replacement ratio
25	100% Bonds	0.29	0.31	0.23–0.37	0.18	—	17.2
	Balanced 60:40	0.37	0.44	0.26–0.53	0.16	—	13.5
	100% Equities	0.39	0.56	0.23–0.65	0.12	—	12.8
	Life-cycle	0.35	0.41	0.26–0.49	0.18	—	14.3
35	100% Bonds	0.25	0.27	0.21–0.31	0.16	11.6	20.0
	Balanced 60:40	0.31	0.36	0.22–0.44	0.15	11.9	16.1
	100% Equities	0.33	0.43	0.20–0.53	0.11	11.8	15.2
	Life-cycle	0.30	0.33	0.23–0.40	0.16	11.7	16.7
45	100% Bonds	0.19	0.20	0.16–0.22	0.13	15.3	26.3
	Balanced 60:40	0.22	0.24	0.16–0.28	0.11	16.8	22.7
	100% Equities	0.23	0.27	0.15–0.33	0.09	17.0	21.7
	Life-cycle	0.21	0.22	0.17–0.26	0.13	16.7	23.8
55	100% Bonds	0.10	0.10	0.09–0.11	0.07	29.0	50.0
	Balanced 60:40	0.11	0.11	0.09–0.13	0.07	33.6	45.5
	100% Equities	0.11	0.12	0.08–0.14	0.05	35.5	45.5
	Life-cycle	0.10	0.11	0.09–0.12	0.07	35.0	50.0

*Note:* Balanced strategy is 60% equities; 40% bonds with annual rebalancing. Life-cycle strategy invests a proportion equal to 100 – member's age in equities with the remainder in bonds, with annual adjustment. Median, mean, inter-quartile VaR range (between 25th and 75th percentiles of the distribution of replacement ratios) and 5% VaR (below which 5% of the distribution of replacement ratios fall) all based on the member contributing 10% of salary throughout pension plan membership. Contribution rate to match age 25 is the contribution rate required from the later age of joining to replicate median replacement ratio estimated for age 25. Contribution rate for 50% replacement ratio is that required to have 50% probability of a replacement ratio of at least 0.50.

individual ranges from 0.29, where the contributions are invested only in bonds, to 0.39 for the 100% equity strategy. The differences in median replacement ratios across the four investment strategies are, of course, explained by the differing levels of equity content and the impact this has on the expected investment return. It follows that the median replacement ratios are also influenced by the expected returns assumed for the various asset classes.

It is also clear from the Table that the pension outcomes are very risky, with the 5% value-at-risk replacement ratios for starting age 25 ranging from 0.12 to 0.18. The interpretation of these figures is that the individual has a 1-in-20 chance of receiving a replacement ratio of that amount or less. The 'low-risk' strategy of 100% bonds limits the downside compared with the 100% equity and balanced strategies, but only at the expense of a significant reduction in the mean and median replacement ratios. This strategy has been called one of 'reckless conservatism' and this description is reinforced by an examination of the interquartile range. For the 100% bonds strategy, the range is 0.23 to 0.37, the interpretation of which is that there is a 50% probability of the outcome lying in this range. By contrast, the 100% equity strategy has the same lower bound as bonds but almost twice the upper bound. The balanced and life-cycle strategies appear to offer relatively attractive risk-return trade-offs compared to the single asset class strategies. Both have relatively high median replacement ratios (0.37 and 0.35, respectively), while having a more limited

downside (5% VaRs of 0.16 and 0.18, respectively.) The life-cycle strategy has better downside results than the balanced strategy, but loses some of the upside potential (75<sup>th</sup> percentile VaRs of 0.49 and 0.53, respectively.) Notably, the life-cycle strategy has the same 5% VaR level as the conservative 100% bonds strategy, but a significantly higher median replacement ratio.

Table 3 also reveals that delaying pension plan membership beyond age 25 has the predictable result of shifting downward the range of likely replacement ratios. For example, deferring membership by ten years to age 35 reduces the median replacement ratio from the balanced managed strategy from 0.37 to 0.31. The corresponding figures for deferment to ages 45 and 55 are 0.22 and 0.11, respectively.

An alternative way to look at this is to calculate what level of contribution rate would have to be paid from these later starting ages to replicate the median replacement ratio generated by a 10% contribution rate paid from age 25. Using the balanced managed strategy, the answer to this is 11.9% from age 35, 16.8% from age 45 and 33.6% from age 55, and the results for the other asset allocation strategies are broadly similar. These results show that anyone delaying contributing to a pension plan beyond their 30s must devote a very significant portion of their income to making these 'catch up' contributions. Indeed, the levels of the contributions required from the later starting ages are such that it might not be possible for individuals on low-to-middle incomes to afford them.<sup>7</sup>

The increase in the contribution rate required as a result of delaying pension plan membership is, at first glance, less than might be expected. For example, reducing the contribution period from 40 to 30 years might be expected to raise the required contribution rate by 33% (i.e., 40 years contributions need to be spread over 30 years) even before taking account of the forgone 10 years of investment returns. That the required contribution rate rises by only 16 to 19% is explained by the employee's age-earnings profile. Real earnings rise through most of the employee's career and thus the percentage increase in contributions is being applied to a higher *level* of salary.

The table also shows that even when starting pension contributions at age 25, the median replacement ratios are well below the ratios targeted by traditional final salary pension plans. This point is confirmed by the final column of the Table, which shows the contributions required to have a 50% chance of achieving a replacement ratio of at least 0.50 of final salary (a common replacement ratio in public sector plans). Again, it is valid to question the affordability of the contribution rates required by individuals starting pension contributions at later ages.

Just about the only positive thing that we can say about delaying pension plan membership is that shorter contribution periods reduce the variability of outcomes. For example, the interquartile VaR range for the replacement ratio for the balanced managed strategy from age 25 is 0.27. By comparison, for the same strategy from age 55, the range is reduced to 0.04. However, being more certain about the value of your (very much smaller) pension does not seem to be a particularly good reason to delay starting to save.

An individual might be able to make a given target replacement ratio more affordable by retiring at a later age. Table 4 gives the outcomes for the same starting ages and investment strategies as Table 3, but this time based on the assumption that employment and saving continue to, and the pension benefits are not taken until, age 70. The age-earnings profile we

Table 4 Replacement ratios by age of joining pension plan: male retiring at 70

Start age	Strategy	Median replacement ratio	Mean replacement ratio	Inter-quartile VaR range	5% VaR replacement ratio	Contribution rate to match age 25 start	Contribution rate for 50% replacement ratio
25	100% Bonds	0.36	0.40	0.28–0.48	0.20	—	13.9
	Balanced 60:40	0.47	0.58	0.32–0.71	0.19	—	10.6
	100% Equities	0.50	0.76	0.29–0.90	0.14	—	10.0
	Life-cycle	0.45	0.53	0.33–0.64	0.21	—	11.1
35	100% Bonds	0.32	0.35	0.26–0.42	0.19	11.3	15.6
	Balanced 60:40	0.41	0.49	0.29–0.60	0.18	11.5	12.2
	100% Equities	0.43	0.61	0.26–0.74	0.13	11.6	11.6
	Life-cycle	0.39	0.44	0.29–0.53	0.20	11.5	12.8
45	100% Bonds	0.25	0.27	0.21–0.31	0.16	14.4	20.0
	Balanced 60:40	0.30	0.34	0.22–0.42	0.15	15.7	16.7
	100% Equities	0.31	0.40	0.20–0.49	0.11	16.1	16.1
	Life-cycle	0.29	0.31	0.23–0.38	0.16	15.5	17.2
55	100% Bonds	0.15	0.16	0.13–0.18	0.11	24.0	33.3
	Balanced 60:40	0.17	0.19	0.14–0.22	0.10	27.6	29.4
	100% Equities	0.18	0.21	0.13–0.25	0.08	27.8	27.8
	Life-cycle	0.17	0.18	0.14–0.20	0.11	26.5	29.4

*Note:* Balanced strategy is 60% equities; 40% bonds with annual rebalancing. Life-cycle strategy invests a proportion equal to 100 – member's age in equities with the remainder in bonds, with annual adjustment. Median, mean, inter-quartile VaR range (between 25th and 75th percentiles of the distribution of replacement ratios) and 5% VaR (below which 5% of the distribution of replacement ratios fall) all based on the member contributing 10% of salary throughout pension plan membership. Contribution rate to match age 25 is the contribution rate required from the later age of joining to replicate median replacement ratio estimated for age 25. Contribution rate for 50% replacement ratio is that required to have 50% probability of a replacement ratio of at least 0.50.

have used implies real wage growth declines from the individual's late-40s until the point of retirement. To enable an easier comparison with the previous results, we assume the decline in real wage growth ends at age 65 and the real growth remains steady for the additional five years of employment. Given that relatively few people currently work beyond age 65, there is little firm evidence on the true age-earnings profile post-65.

Table 4 shows that replacement ratios at age 70 for given starting ages and contribution levels are substantially increased, relative to the equivalent scenarios with retirement at age 65. For example, focusing on the balanced managed strategy, the median replacement ratio increases from 0.37 to 0.47 for someone starting contributing at age 25, while the increase for someone starting at age 55 is from 0.11 to 0.17. The interquartile ranges also shift up, although the improvements in the 5% VaR levels are somewhat lower. We might not like the prospect of working longer and later into life, but it is clearly an effective way of improving the affordability of pensions. The benefit in terms of lower required contributions for any target level of retirement income can be seen in the final column of the Table, which illustrates required contributions for a 0.50 replacement ratio. For an individual starting paying into the pension plan at age 55 and using the balanced managed strategy, the required contribution rate to achieve this replacement ratio for retirement at age 65 is 45.5% of salary, but this falls to 29.4% of salary if retirement is postponed to age 70.

Table 5 Replacement ratios by age of joining pension plan: male retiring at 55

Start age	Strategy	Median replacement ratio	Mean replacement ratio	Inter-quartile VaR range	5% VaR replacement ratio	Contribution rate to match age 25 start	Contribution rate for 50% replacement ratio
25	100% Bonds	0.14	0.15	0.12–0.17	0.10	—	35.7
	Balanced 60:40	0.17	0.19	0.13–0.22	0.09	—	29.4
	100% Equities	0.17	0.22	0.12–0.27	0.07	—	29.4
	Life-cycle	0.17	0.18	0.13–0.22	0.09	—	29.4
35	100% Bonds	0.11	0.12	0.10–0.13	0.09	12.7	45.5
	Balanced 60:40	0.13	0.14	0.10–0.17	0.08	13.1	38.5
	100% Equities	0.14	0.16	0.09–0.20	0.06	12.1	35.7
	Life-cycle	0.13	0.14	0.10–0.16	0.08	13.1	38.5
45	100% Bonds	0.07	0.07	0.06–0.07	0.05	20.0	71.4
	Balanced 60:40	0.07	0.07	0.06–0.09	0.05	24.3	71.4
	100% Equities	0.07	0.08	0.06–0.10	0.04	24.3	71.4
	Life-cycle	0.07	0.07	0.06–0.08	0.05	24.3	71.4

*Note:* Balanced strategy is 60% equities; 40% bonds with annual rebalancing. Life-cycle strategy invests a proportion equal to 100 – member's age in equities with the remainder in bonds, with annual adjustment. Median, mean, inter-quartile VaR range (between 25th and 75th percentiles of the distribution of replacement ratios) and 5% VaR (below which 5% of the distribution of replacement ratios fall) all based on the member contributing 10% of salary throughout pension plan membership. Contribution rate to match age 25 is the contribution rate required from the later age of joining to replicate median replacement ratio estimated for age 25. Contribution rate for 50% replacement ratio is that required to have 50% probability of a replacement ratio of at least 0.50.

The impact of deferring retirement is the result of contributions being paid for five years longer and the annuity rate increasing to take account of the reduced period for which the pension is expected to be paid given the more advanced age at purchase. However, it is important to note that the life expectancy of a 70 year old is unlikely to be a full five years less than that of a 65 year old. The fact that an individual has survived an extra five years tends to indicate a higher chance of prolonged life, a feature of mortality statistics that is cheerfully known in the annuity industry as “mortality drag.”

Naturally enough, retiring earlier will have the opposite effect. The results for retirement at age 55 are shown in Table 5. The median replacement ratio for someone retiring at age 55, having contributed 10% of salary to the balanced managed strategy since age 25, is 0.17, with an interquartile range of 0.13 to 0.22. The comparable figure for a retirement age of 65 is 0.37, with an interquartile range of 0.26 to 0.53. The required contribution rates for a 0.50 replacement ratio at age 55 are all at least 50% greater than the corresponding rates required for retirement at age 65. Early retirement is, therefore, extremely costly.

The situation is particularly unfortunate for those who start retirement saving late, but then find themselves out of work in their 50s. The only potential mitigating factor is that if an individual is forced to retire on the grounds of ill-health, they may be eligible for an ‘enhanced’ or ‘impaired life’ annuity. This would take account of their reduced life expectancy and offer a higher rate of income than the rates used in our modeling.

### 5.2. The case of an average female

So far, we have assumed that individuals might delay joining the pension plan, but that once they have joined they continue to make contributions to the plan until retirement. However, for some individuals, the contribution record will be interrupted, for example by unemployment or an extended period of illness. An interrupted contribution record is also very common for female workers who leave the labor market for a period to raise a family. We analyze this latter scenario in Table 6, which shows two alternative labor market participation profiles for a female employee: a five-year career break and a 10-year career break, both starting at age 30.<sup>8</sup>

The replacement ratios for pension plan participation from age 25 to age 65 are all marginally lower than the equivalents for males shown in Table 3. This reflects a combination of the different age-wage profile and the longer life expectancy of females, which feeds through into lower annuity rates. The Table also shows that relative to the benchmark of unbroken participation from age 25, a five-year career break reduces the median replacement ratio from 0.36 to 0.33 (under the balanced managed strategy) and the ten-year break reduces it further to 0.28. However, it is worth noting that our analysis assumes the career break has no impact on the subsequent age-earnings profile,<sup>9</sup> and this assumption is likely to be optimistic. Thus, the combination of longer life expectancies and career breaks to raise a family means that women face considerably greater obstacles than men in achieving ade-

Table 6 Replacement ratios by labor market participation experience: female retiring at 65

Contribution periods	Strategy	Median replacement ratio	Mean replacement ratio	Inter-quartile VaR range	5% VaR replacement ratio	Contribution rate to match full period	Contribution rate for 50% replacement ratio
25–65	100% Bonds	0.28	0.31	0.23–0.36	0.17	—	17.9
	Balanced 60:40	0.36	0.43	0.25–0.52	0.16	—	13.9
	100% Equities	0.38	0.55	0.23–0.65	0.11	—	13.2
	Life-cycle	0.35	0.40	0.26–0.48	0.18	—	14.3
25–30; 35–65	100% Bonds	0.26	0.28	0.21–0.33	0.16	10.8	19.2
	Balanced 60:40	0.33	0.38	0.23–0.46	0.15	10.9	15.2
	100% Equities	0.35	0.48	0.21–0.57	0.11	10.9	14.3
25–30; 40–65	Life-cycle	0.32	0.36	0.24–0.43	0.17	10.9	15.6
	100% Bonds	0.23	0.25	0.19–0.28	0.15	12.2	21.7
	Balanced 60:40	0.28	0.32	0.21–0.39	0.14	12.8	17.9
	100% Equities	0.30	0.40	0.19–0.48	0.10	12.7	16.7
	Life-cycle	0.27	0.30	0.21–0.36	0.15	13.0	18.5

*Note:* Balanced strategy is 60% equities; 40% bonds with annual rebalancing. Life-cycle strategy invests a proportion equal to 100 – member's age in equities with the remainder in bonds, with annual adjustment. Median, mean, inter-quartile VaR range (between 25th and 75th percentiles of the distribution of replacement ratios) and 5% VaR (below which 5% of the distribution of replacement ratios fall) all based on the member contributing 10% of salary throughout pension plan membership. Contribution rate to match full period is the contribution rate required over the interrupted contribution history to replicate the median replacement ratio estimated for an uninterrupted work history. Contribution rate for 50% replacement ratio is that required to have 50% probability of a replacement ratio of at least 0.50.

Table 7 Sensitivity analysis of the equity risk premium: replacement ratios by age of joining pension plan: male retiring at 65

Start age	Strategy	Equity risk premium (relative to T-bills)					
		1.5%		3.5%		5.5%	
		Median replacement ratio	Inter-quartile VaR range	Median replacement ratio	Inter-quartile VaR range	Median replacement ratio	Inter-quartile VaR range
25	100% Bonds	0.29	0.23–0.37	0.29	0.23–0.37	0.29	0.23–0.37
	Balanced 60:40	0.30	0.21–0.42	0.37	0.26–0.53	0.46	0.31–0.67
	100% Equities	0.28	0.17–0.45	0.39	0.23–0.65	0.56	0.33–0.98
	Life-cycle	0.30	0.23–0.41	0.35	0.26–0.49	0.42	0.31–0.58
35	100% Bonds	0.25	0.21–0.31	0.25	0.21–0.31	0.25	0.21–0.31
	Balanced 60:40	0.26	0.19–0.36	0.31	0.22–0.44	0.37	0.27–0.53
	100% Equities	0.25	0.16–0.39	0.33	0.20–0.53	0.44	0.27–0.73
	Life-cycle	0.26	0.20–0.35	0.30	0.23–0.40	0.34	0.26–0.46
45	100% Bonds	0.19	0.16–0.22	0.19	0.16–0.22	0.19	0.16–0.22
	Balanced 60:40	0.19	0.15–0.25	0.22	0.16–0.28	0.25	0.19–0.33
	100% Equities	0.18	0.12–0.27	0.23	0.15–0.33	0.28	0.18–0.42
	Life-cycle	0.19	0.15–0.24	0.21	0.17–0.26	0.23	0.18–0.29
55	100% Bonds	0.10	0.09–0.11	0.10	0.09–0.11	0.10	0.09–0.11
	Balanced 60:40	0.10	0.08–0.12	0.11	0.09–0.13	0.11	0.09–0.14
	100% Equities	0.10	0.07–0.13	0.11	0.08–0.14	0.12	0.09–0.16
	Life-cycle	0.10	0.08–0.11	0.10	0.09–0.12	0.11	0.09–0.13

*Note:* Balanced strategy is 60% equities; 40% bonds with annual rebalancing. Life-cycle strategy invests a proportion equal to 100 – member's age in equities with the remainder in bonds, with annual adjustment. Median, mean, inter-quartile VaR range (between 25th and 75th percentiles of the distribution of replacement ratios) and 5% VaR (below which 5% of the distribution of replacement ratios fall) all based on the member contributing 10% of salary throughout pension plan membership.

quate pension provision. Of course, women often face the additional problem of having lower salary incomes as well.

### 5.3. Sensitivity analysis

#### 5.3.1. Equity risk premium

The results above are conditional on the assumed values for the parameters of the model. The most important of these is the size of the equity risk premium. We selected a value of 3.5% on the grounds that this value is supported by the recent literature as a forward-looking estimate. Naturally, with a more optimistic equity risk premium, the distribution of replacement ratios will be shifted upwards, and vice versa. Table 7 shows the extent of these shifts when the equity risk premium is reduced to 1.5% or raised to 5.5%. Clearly, only the balanced, life cycle and 100% equity strategies are affected, but the effect is significant. With a start age of 25 and a balanced strategy, the median replacement ratio falls by 19% from 0.37 to 0.30 when the equity risk premium drops to 1.5%, and increases by 24% from 0.37 to 0.46 when the equity risk premium rises to 5.5%. The corresponding changes for the 100% equity strategy are a 28% fall and a 44% increase. The percentage changes are smaller at higher starting ages. Nevertheless the Table gives a flavor of the extent of the uncertainty

Table 8 Sensitivity analysis of the contribution rate: replacement ratios by age of joining pension plan: male retiring at 65

Start age	Strategy	Contribution rate (% of salary)					
		5%		10%		15%	
		Median replacement ratio	Inter-quartile VaR range	Median replacement ratio	Inter-quartile VaR range	Median replacement ratio	Inter-quartile VaR range
25	100% Bonds	0.15	0.12–0.19	0.29	0.23–0.37	0.44	0.35–0.56
	Balanced 60:40	0.19	0.13–0.27	0.37	0.26–0.53	0.56	0.39–0.80
	100% Equities	0.20	0.12–0.33	0.39	0.23–0.65	0.59	0.35–0.98
	Life-cycle	0.18	0.13–0.25	0.35	0.26–0.49	0.53	0.39–0.74
35	100% Bonds	0.13	0.11–0.16	0.25	0.21–0.31	0.38	0.32–0.47
	Balanced 60:40	0.16	0.11–0.22	0.31	0.22–0.44	0.47	0.33–0.66
	100% Equities	0.17	0.10–0.27	0.33	0.20–0.53	0.50	0.30–0.80
	Life-cycle	0.15	0.12–0.20	0.30	0.23–0.40	0.45	0.35–0.60
45	100% Bonds	0.10	0.08–0.11	0.19	0.16–0.22	0.29	0.24–0.33
	Balanced 60:40	0.11	0.08–0.14	0.22	0.16–0.28	0.33	0.24–0.42
	100% Equities	0.12	0.08–0.17	0.23	0.15–0.33	0.35	0.23–0.50
	Life-cycle	0.11	0.09–0.13	0.21	0.17–0.26	0.32	0.26–0.39
55	100% Bonds	0.05	0.05–0.06	0.10	0.09–0.11	0.15	0.14–0.17
	Balanced 60:40	0.06	0.05–0.07	0.11	0.09–0.13	0.17	0.14–0.20
	100% Equities	0.06	0.04–0.07	0.11	0.08–0.14	0.17	0.12–0.21
	Life-cycle	0.05	0.05–0.06	0.10	0.09–0.12	0.15	0.14–0.18

*Note:* Balanced strategy is 60% equities; 40% bonds with annual rebalancing. Life-cycle strategy invests a proportion equal to 100 – member's age in equities with the remainder in bonds, with annual adjustment. Median and inter-quartile VaR range all based on the member contributing the specified percentage of salary throughout pension plan membership. The analysis uses an equity risk premium of 3.5% relative to T-bills.

attached to the pension from DC plans as a consequence of one of the key determinants of differential returns.

### 5.3.2. Contribution rates

Our final sensitivity analysis focuses on the member's contribution rate. We have assumed in previous analyses that the member contributes a constant 10% of salary throughout his or her period of scheme membership. This assumption was based on survey evidence that 10% is close to the average contribution rate for these types of schemes. However, many scheme members save less than 10%, while it is also open to most members to raise their contributions above 10%, assuming they have sufficient spare income to do so. Table 8 shows replacement ratios (median and interquartile VaR ranges) for retirement at 65 with various joining ages based on contribution rates of 5%, 10%, and 15% of salary. Raising the contribution level has the predictable result of raising the median replacement ratio and all of the distribution values listed in the Table. In fact, the values scale perfectly, a 50% higher contribution rate produces a 50% higher replacement ratio. The interquartile range shifts up and widens by a factor of 50%, as the 25th and 75th percentiles rise by 50%. Raising contribution levels is, like working longer, a powerful way to improve the adequacy of an individual's retirement income provision, albeit one which may not be viewed as attractive by the individual.

## 6. Conclusions

Many individuals delay starting to contribute to a pension plan until well into their working lives. This might be because they are waiting for their income to increase above their perceived permanent income (as in 'life-cycle' theory), or because they are paying off university or mortgage loans or raising a family, or because of procrastination. They may simply be risk averse and prefer the certainty of consumption now to the *possibility* of consumption in the future. In this paper, we have analyzed likely retirement incomes for someone starting to contribute to a pension plan from age 25 and calculated the additional contributions that someone starting saving later in life has to make to 'catch up'. These catch-up contributions are large, and in some cases, so large that it is difficult to imagine that they are affordable for anyone not on a high income. Deferring retirement to a later age goes some way to easing the burden of annual contributions, but is not always possible; in any case, working longer is a solution that many people would rather avoid.

The results suggest people should start saving for a pension as soon as possible and raise the question of how they can be helped to do so. In the UK, the Pensions Commission (2004, 2005) has published extensive analysis of possible approaches to improve pension saving. Compulsion, simply requiring by law that everyone saves for a pension, is a relatively extreme answer that may bring its own problems. Less rigid 'behavioral' mechanisms, such as automatically enrolling employees in their employer's pension plan while continuing to give them the right to opt out, have been shown to be effective in raising pension plan participation rates (Choi, Laibson, Madrian & Metrick, 2002) and may represent a better alternative. The Pensions Commission (2005) has suggested that automatic enrollment should be used in a planned new national pension savings scheme, for employees without adequate existing pension provision.

Education and advice for employees will probably also be required. It seems reasonable to believe that at least some employees would be spurred into action if they were aware of, and understood the implications of, statistics like the dismal ones we have presented in this paper. Financial planners and advisors have a key role to play in this education process.

### Appendix: The PensionMetrics simulation model

The PensionMetrics (PM) accumulation model is a stochastic simulation model whose purpose is to investigate the design of DC pension plans. In a DC scheme, pension contributions from the plan member and his or her employer are invested in a portfolio of assets. The returns on the assets will be stochastic and some assets will have more volatile returns than others. The DC pension fund will therefore grow in a stochastic fashion too.

The PM model generates a range of outcomes (i.e., a probability distribution function) for the value of the replacement pension from the accrued DC pension fund on the retirement date of the plan member. The replacement ratio is calculated as the ratio of the pension from the DC fund to the plan member's final salary. The pension from the DC fund is, in turn, calculated as the ratio of the value of the DC fund to the annuity factor. The annuity factor is the expected present value of an annual pension of one unit from retirement until death and

Table A1 Real returns and earnings growth 1947 to 2003

	UK T-bills	UK Equities	UK Bonds	US Equities	UK real Earnings Growth
Mean return (arithmetic %)	1.19%	9.18%	1.79%	8.71%	2.07%
Standard deviation (annual %)	3.99%	23.22%	13.31%	21.04%	2.00%
Correlation matrix					
UK T-bills	1.000				
UK equities	0.051	1.000			
UK bonds	0.465	0.513	1.000		
US equities	0.136	0.576	0.253	1.000	
UK real earnings	0.049	-0.026	-0.347	0.045	1.000

Source: returns from ABN Amro/LBS data from Ibbotson Associates (Dimson et al., 2001). Earnings data from the Office for National Statistics.

depends on both the interest rates ruling at the time of retirement and estimates of the survival probabilities of the plan member for each year after retirement. These survival probabilities are taken from the PMA92 and PFA92 tables of mortality rates produced by the Institute and Faculty of Actuaries that are based on the mortality experience of respectively male and female pensioner annuitants in the UK in the early 1990s.

The model requires assumptions about both risk factors and control factors. The first risk factor relates to real (i.e., inflation-adjusted) asset returns. The benchmark asset returns model we use is a multivariate normal model, with the variance-covariance matrix calibrated using time series returns on assets over the postwar period. Experimentation has shown that the particular asset returns model used makes little difference to the distribution of pension outcomes, except in the extreme tails of the distribution. In this study, therefore, we just report results from the benchmark multivariate normal model. The historical return and correlation parameters are shown in Table A1 below. The mean returns we use are based on a forward-looking analysis of the equity risk premium, which is described in Section 4, with the parameter values shown in Table 2.

The second risk factor relates to interest rates. We need to model the evolution of interest rates over time to forecast the annuity factor at retirement. The interest rate model that we use is based on the Vasicek (1977) model that links bond returns and bond yields in a consistent manner.

The third risk factor is earnings. Earnings are modeled using the age-earnings profile (or salary scale) of the plan member. This shows how salary varies with age in the plan member's profession at a given point in time. We assume that an individual's salary over his or her career follows the same profile as the rest of his or her profession (that, in general, will be gender specific), but is subject to annual uprating in line with the real growth in national average earnings. In this study, we use the profile of the average male and female employee in the UK.

The final risk factor is unemployment. This is modeled as a binary variable (1: employed, 0: unemployed) for each period, with an age-dependent probability of unemployment, for

example, taken from national average unemployment rates at different ages. However, in this study, for reasons of simplicity, we assume the unemployment probability is zero.

There are three control variables; variables that are set by either the pension plan member or the pension plan provider in each period of the model. The first is the pension fund contribution rate, which we assume to be a constant proportion of the plan member's income for the whole period. We use 10% in this study.

The second is the asset allocation that is the key control variable in the model, since experiments show that it dominates the distribution of pension outcomes. This study investigates four stylized asset allocation profiles: a 100% bonds strategy; a 100% UK equities strategy; a 'balanced managed' strategy, invested in equities (60%) and bonds (40%); and a deterministic life-cycle strategy that invests a proportion (100-member's age) in equities and the remainder in bonds. The latter two strategies use annual rebalancing.

The third control variable is the retirement age. The base retirement age is set at 65. However, we experiment with different retirement ages.

Having specified all the risk and control factors, we use the model to perform thousands of simulations of the stochastic variables, such as the asset returns and interest rates, and then generate an empirical distribution of possible replacement ratios for the plan member's selected retirement date.

A replacement ratio of unity implies that the particular DC pension plan has fully replicated the plan member's final salary. However, the generated distribution of replacement ratios will typically be quite wide. To make a suitable comparison, we need to specify one or more percentiles from the distribution and then compare these values with the target pension ratio of unity. The  $i$ th percentile of this distribution is also known as the value-at-risk (VaR) at the  $(100 - i)$ th confidence level.

## Notes

1. However, this tells us nothing about the behavior of the 34% of employees who are either not eligible to join their employer's plan, or who work for an employer which does not offer a pension plan.
2. In other words, individuals use hyperbolic rather than exponential discounting.
3. The impact of differing career salary profiles, by gender and by type of occupation, on the retirement income from DC pensions is discussed in detail in Blake, Cairns and Dowd (2004). For simplicity, in this paper we focus on one age-earnings profile, namely that of a typical male, but it is also clear that many of our results carry over to female workers, or workers in specific occupations. Some results for females are also presented in Table 6.
4. We thank the reviewer for suggesting the latter two strategies. Notably, in the UK "balanced" strategies tend to have higher equity content (80%+) than in the United States, while life-cycle products tend to switch from equity to bonds only in the last five years before planned retirement. See Byrne et al. (2005) for a discussion.
5. UK legislation for 'stakeholder' DC pensions capped annual charges at 1% per annum and this has become a common charge rate for DC plans in the UK. The inflation rate

we assume is consistent with the target set by the government for the Bank of England.

6. This was the simplest of the seven asset return models used in Blake et al. (2001). That study showed that the specification of the asset-return process had relatively little impact on the estimated pension outcome.
7. In some cases, legislative limits may also limit contributions into tax-favored pensions vehicles to an amount less than the rates we have calculated. However, recent changes in UK regulations mean that individuals can now pay up to 100% of salary each year into a pension plan, subject to an annual contribution limit of £215,000 and an overall pension fund cap of £1.5m. Hence, most individuals in the UK are not now constrained, other than by their income, in what they can pay into their pension plan.
8. ONS statistics show that the mean age amongst married women in the UK for the birth of a first child is 29.9 years, and for the second child 31.5 years.
9. The woman rejoins at the salary she would have received had she remained in continuous employment.

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