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I Bonds versus TIPS: should individual investors prefer one to the other?

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Abstract

Both TIPS and Series I Bonds are adjusted for inflation, offering a real rate and an inflation adjustment. The inflation adjustment is the same on both securities, but the real portion of the interest rate on TIPS is generally much higher. Despite I Bonds' less attractive real rate, they have several features that add to their value. They may be redeemed before maturity, at par value plus accrued interest, eliminating price risk. In addition, taxes may be deferred until redemption. We estimate the value of these two features, and find that they are substantial and could potentially offset the lower real rate of I Bonds.

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1. Introduction

The United States Treasury offers two types of inflation-indexed securities: Treasury Inflation-Protected Securities, commonly known as "TIPS," and Series I Savings Bonds. Although both types of security use the all-urban consumer price index (CPI-U) to account for inflation, and both are backed by the full faith and credit of the U.S. Treasury, they differ considerably with respect to their target markets, pricing, taxation, and risk characteristics. TIPS, which can be held by institutions as well as individuals, have been a well-received innovation, with $288.7 billion outstanding as of May, 2005. Series I Bonds, which can be

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held only by individuals, are a much smaller presence in the market, with $27.9 billion outstanding as of May, 2005.[1]

The real portions of the yields on I Bonds and TIPS are set by two quite different methods. The real portion of the I Bond yield is fixed at issue by the U.S. Treasury, whereas the market determines the appropriate real yield on TIPS at auction, and subsequently when they are traded in the secondary market. In mid-2005, the I Bond real rate was 1.2%, substantially less than the market-determined yield on TIPS, which on August 5, 2005 was approximately 2.00% for the 20-year maturity (issued July 2005) and 1.80% for a five-year maturity (issued April 2005).

Superficially, it appears that the I Bond is grossly unattractive relative to TIPS. However, the I Bond has two valuable options not included in TIPS:

1. The *option to redeem* the bond before maturity, at par plus accrued interest, providing it has been owned for at least one year (there is a small penalty for redeeming a bond held for less than five years).
2. The *option to defer income taxes* on the accrued interest up until redemption.

Early redemption could be attractive if the Treasury raises the real coupon on new I Bonds; investors could redeem older low coupon I Bonds at par and buy the new higher-coupon I Bond. As a result, I Bonds have no price risk, whereas TIPS, which have large real durations (Vankudre, Lindner and Arora, 1997; Roll, 2004), suffer significant price losses from yield increases.

Although TIPS have received some academic and institutional attention,[2] there is little academic research to date on Savings Bonds. Potts and Reichenstein (1995) and Poole (1995) discuss the redemption and tax options embedded in Series EE Savings Bonds. Arak and Rosenstein (2004) find that the demand for Series EE Savings Bonds is affected by economic variables including the state of the economy and interest rates. In one of the few articles on I Bonds, Boes and Bezik (2004) compare the relative merits of Series EE Bonds to I Bonds.[3]

Our objective in this paper is to evaluate the two special features of I Bonds to determine if they add enough value to make I Bonds an attractive alternative to TIPS despite the current disparity in real coupons. In Section 2, we describe the features of I Bonds and contrast them to TIPS. In Section 3, we develop the methodology for estimating the value of the early redemption option. In Section 4, we present estimated values for this option. In Section 5, we examine the tax-deferral option. We draw conclusions in Section 6.

2. Comparison of I bond and TIPS features

Series I Bonds were first issued in September 1998, about a year after the first TIPS issue in July 1997. I Bonds, issued only to individuals, have a 30-year stated maturity. Individuals may purchase up to $30,000 in paper bonds and up to another $30,000 online per calendar year. They are "accrual securities," whose value increases as interest is earned (interest accrues monthly and is compounded semiannually).

TIPS are now available in a variety of maturities: five, 10, and 20 years (30-year maturities were offered until 1998). Although TIPS trade in the secondary market, I Bonds are non-negotiable and can be redeemed only by the Treasury.

Table 1 U.S. Series I Savings Bond fixed rates versus TIPS coupon rates

I Bonds		TIPS				
Issue date	Fixed rate (%)	Issue date	Coupon rate (%)	Price (% of par)	Current yield at issue (%)	Term (years)
01–1999	3.30	01–1999	3.875	99.811	3.882	10
07–1999	3.30	07–1999*	3.875	100.033	3.874	9.5
04–1999	3.30	04–1999	3.875	99.578	3.891	30
10–1999	3.30	10–1999*	3.875	96.989	3.995	29.5
10–2000	3.60	10–2000*	3.875	103.628	3.739	28.5
01–2000	3.40	01–2000	4.25	99.298	4.280	10
07–2000	3.60	07–2000*	4.25	103.539	4.105	9.5
01–2001	3.40	01–2001	3.50	99.818	3.506	10
07–2001	3.00	07–2001*	3.50	101.863	3.436	9.5
10–2001	3.00	10–2001	3.375	98.314	3.433	30.5
01–2002	2.00	01–2002	3.375	99.120	3.405	10
07–2002	2.00	07–2002	3.00	99.154	3.026	10
10–2002	2.00	10–2002*	3.00	106.777	2.810	9.75
01–2003	1.60	01–2003*	3.00	106.474	2.818	9.5
07–2003	1.10	07–2003	1.875	98.881	1.896	10
10–2003	1.10	10–2003*	1.875	97.201	1.929	9.75
01–2004	1.10	01–2004	2.00	99.829	2.003	10

* Bond was reissued on the issue date shown.

Savings Bond rates are taken from wwws.publicdebt.treas.gov/sav/sbirate2.htm, which gives historical Savings Bond issue information. TIPS rates are taken from wwws.publicdebt.treas.gov/AI/OFAuctions, which is a search of Treasury auction results. TIPS current yields are the annual coupon divided by the price as a percentage of par.

The interest accrual on I Bonds is a "composite rate," which combines a "fixed rate" and a variable "inflation rate." The fixed rate is set by the Treasury and applies over the entire life of a bond purchased at that rate. Table 1 shows that, historically, the fixed rate on I Bonds has been below the coupon rate at issue on TIPS that were auctioned at about the same time. The inflation adjustment is made twice a year, on November 1, reflecting the April to September CPI-U percentage change and on May 1, reflecting October through March.

The fixed coupon on TIPS is determined by the real yields bid when the security is auctioned. That fixed coupon is paid on the inflation-adjusted principal, an adjustment made semiannually (the Treasury publishes daily index ratios to enable investors to determine daily accrued interest). As TIPS trade in the secondary market, their prices may deviate from par and fall below their original auction price or their inflation-adjusted principal value.

Investors in I Bonds can begin paying federal taxes on accrued interest at any time, or defer such taxes until redemption. On TIPS, in comparison, both paid interest and accreted (not yet paid) principal are taxed each year.

3. Valuation of the early redemption option

3.1. Option description

I Bonds can be redeemed at par plus accrued interest at any time after the minimum one-year holding period. If the fixed rate on new I Bonds has been increased by the Treasury, a holder can redeem an old bond and use the proceeds to purchase a new I Bond with the higher fixed rate and exactly the same inflation adjustment.[4,5] In effect, the early redemption option is a put option that eliminates the price losses that normal fixed rate instruments (including TIPS) suffer when real yields in the market increase.

The early redemption option is a compound option because if the investor exercises the option and purchases a new bond, the new bond also contains an early redemption option. Thus, it is conceivable that an investor will exercise the option several times if real interest rates rise over long investment horizons. Although it is possible to simulate optimal redemption behavior that incorporates the value of the compound option, we elect a simpler approach that can be more easily replicated and is more likely to represent the typical individual's behavior.

We obtain a lower limit to the option value by estimating the values of simple one-time redemption options (European style) at two of the most obvious redemption times: (1) at the one-year point, when redemption is first allowed, and (2) at the five-year point, when the early redemption penalty no longer applies. If there is a positive gain from switching at the one- or five-year point, the investor redeems and rolls over into a new I Bond. This means we are ignoring the "American" style feature that would permit the investor to wait for the new fixed rate to go even higher before switching. At this point, we ignore the compound feature that allows the investor to roll over into a new I Bond and get a new option; we discuss the value of the compound feature later in the paper.

Our method, which uses the Black-Scholes (1973) option pricing model to value the option to put the I Bond back to the U.S. Treasury, is obviously only an approximation. Our simplifying assumptions include the following: the options are European style, the yield curve is flat with parallel shifts, and yield distribution (for I Bond fixed rates) is roughly lognormal. We discuss our other simplifying assumptions in the later description of our methodology.

3.2. The inputs

As with any option on a dividend or interest-paying security, we require six inputs to obtain an option value: (1) the strike price, (2) the current price, (3) the income yield, (4) the expiration date, (5) the expected volatility, and (6) the risk-free rate.

3.2.1. Strike price

If there was no cost to switching, the investor could profit from redeeming the old bond and buying a new one every time the fixed coupon was raised above the fixed coupon on the currently held I Bond. There are costs of switching, however, so the investor needs enough of a yield increase to cover costs. The "strike yield" is the new higher yield that generates enough extra income to just offset the costs of switching. Any yield above this strike yield would produce a net profit for the investor.

We use the following terminology for the formulae describing our calculations:

t = the marginal tax rate, decimal;

f = the inflation rate;

c_0 = the original real rate coupon, called the "fixed rate" by the U.S. Treasury;

S_1 = the sum reinvested at the new higher coupon;

c_1 = the strike yield, the minimum real rate needed for a profitable switch.

To simplify the calculations and presentation, we assume that interest is paid annually rather than semiannually. We provide an example of the calculations in the appendix.

The minimum new coupon, c_1, for a profitable switch can be obtained from the following:

$$(1 - t)S_1[1 + (.01)(f + c_1)]^{h-1} + tS_1 = (1-t)100[1 + (.01)(f + c_0)]^h + t100 \quad (1)$$

$$\text{where } S_1 = 100 + (1 - t)(.01)(.75)(f + c_0)100 \quad (2)$$

In Eq. (1), the right-hand side represents the net value of the *original bond* at redemption at time h, after paying taxes at redemption. The left-hand side represents the final redemption value, with a switch at the end of year 1: that is, (1) the investor redeems the old bond, netting S_1 after taxes and penalty (note that the investor's gross coupon is $[(0.75)*(f + c_0)100]$; (2) the investor invests the proceeds in a new higher coupon bond, receiving c_1 for h-1 years, and paying the taxes due at year h. Note that the new fixed rate of c_1 is earned for (h-1) years because the original bond with a fixed rate of c_0 was held for the first year.

For the five-year option, the strike yield, c_1, is given by:

$$(1 - t)S_5[1 + (.01)(f + c_1)]^{h-5} + tS_5 = (1 - t)100[1 + (.01)(f + c_0)]^h + t100 \quad (3)$$

$$\text{where } S_5 = (1 - t)100[1 + (.01)(f + c_0)]^5 + t100 \quad (4)$$

S_5 is the reinvestable proceeds when the first bond is redeemed at the end of year five and the taxes paid.

Each strike yield can be translated into a "strike price." For example, if the individual had a planned holding period of 10 years and the option has one year to expiration, the residual security has nine years to planned redemption. We solve for the price of a nine-year security that has a coupon of c_0 when the required coupon is c_1. Obviously, the strike price would be less than par because the required yield for a switch is above the initial coupon.

The strike yields and strike prices for the one-year option, assuming an initial fixed coupon of 2% and an inflation rate of 2% are shown in panel A of Table 2. For example, assuming a tax rate of 30% and an investment horizon of 10 years, the new fixed rate (c_1) would need to be at least 2.14% [compared to the original fixed rate (c_0) of 2%] to justify a switch. This strike yield translates into a strike price of 98.77 (see section A.1 of the appendix).

Table 2, panel B shows the strike yields and strike prices for the five-year option. The strike yields are somewhat higher than the strike yields for the one-year option. This reflects: (1) the cost of paying five years of taxes at redemption, rather than at the end of the investment horizon, and (2) that the new fixed rate will be received for only h-5 years, instead of h-1 years.

Table 2 Strike yields, in percentage, and strike prices, as a percentage of par, for the redemption option

A. Strike yields and corresponding strike prices for the one-year option, assuming initial fixed rate is 2%, inflation is 2%

Tax rate %	Holding period			
	10 years		30 years	
	Strike yield	Strike price	Strike yield	Strike price
0	2.1117	99.04	2.0325	99.04
30	2.1427	98.77	2.0562	98.45

B. Strike yields for the five-year option assuming initial fixed rate is 2%, inflation is 2%

	Holding period			
Tax rate-%	10 years		30 years	
	Strike yield	Strike price	Strike yield	Strike price
0	2.0000	100.00	2.0000	100.00
30	2.2082	99.01	2.1443	96.06

C. Strike yields and prices for other combinations of fixed rate (c_0), inflation rate (f); 30% tax rate, one-year option

	Holding period			
c_0,f	10 years		30 years	
	Strike yield	Strike price	Strike yield	Strike price
1,2	1.1029	99.11	1.0396	98.89
2,2	2.1427	98.77	2.1562	98.45
3,3	3.2748	96.70	3.0910	97.54

The strike yield is the new *fixed rate* that generates a final after-tax value, after deducting switching costs, that is identical to the after-tax value of the original I Bond fixed rate. See Eqs. (1) and (2) for the strike yield calculation for the one-year option (Panels A and C), and Eqs. (3) and (4) for the strike yield calculation for the five-year option (Panel B).

3.2.2. Expiration date

We choose one-year and five-year expirations for the options as plausible benchmarks, as discussed above.

3.2.3. Volatility

As with any option, the value of the early redemption option depends upon the expected volatility. The 3 3/8's of 1/15/12 TIPS issue had an annual yield volatility[6] of 37% from its issue date (January 15, 2002) to November 4, 2005 using weekly data. Constant maturity yields on the 10-year TIPS had an annual volatility of 34% based on weekly from January 3, 2003 to November 4, 2005. Of course, the TIPS real yields are market determined, whereas the I Bond fixed rate is set administratively. The fixed I Bond coupon had a volatility of 22.7% from November 1998 to May 2004.[7]

Based upon the volatility of TIPS and I Bonds real yields, we use 20% and 30% as hypothetical I Bond real yield volatilities.[8] We translate these yield volatilities into price volatilities. The price volatility calculations are illustrated in Section A.2 of the Appendix.

3.2.4. Risk-free rate

We require a risk-free interest rate as one of the inputs to the option-valuation model[9] as well as to value the tax deferral. For tax rates of 0% and 30% and various holding periods, we calculate the after-tax yield (internal rate of return) over the investment horizon. For example, for a 4% pretax yield, 10-year holding period, 30% tax rate, the after-tax future value per hundred dollars invested is $133.61. The IRR of this investment is 2.94%.[10]

3.2.5. Other assumptions for the early redemption option

The current price of the underlying bond is par (100) because the bond can be redeemed at par (plus accrued interest) by the U.S. Treasury The after-tax annual return over the holding period is used as the yield of the security as well as the risk-free rate.[11]

We consider two investment horizons, 10 years and 30 years. When there is a redemption and replacement, the new bond is assumed to be held for the remaining time in the investment horizon.

4. Estimated values for the early redemption option

4.1. Value of the one-year option

Our approach is to value the option in dollars and then convert it to annual yields, in basis points. We also note that the option applies not only to the original 100 par but also to the accrued interest at exercise. Therefore, the option values based on 100 par must be "blown up." Using a 4% nominal yield, 2% real plus 2% inflation, this "blow-up factor" for an investor taxed at 30% is 1.0210 for the one-year option and 1.1517 for the five-year option.[12] (For a nontaxable investor, the blow-up factors are 1.0300 for the one year and 1.2166 for the five year option.)

Values for the option to redeem at the end of year one are shown in Table 3. The values are shown to just two significant figures to avoid giving a false sense of precision. Panel A shows values for an assumed volatility of 30%. For example, the option to redeem at year one is worth 15 bp per year on the original bond for an investor in the 30% tax bracket with a 10-year initial holding period (see Section A.3 of the Appendix). This implies that an investor with a planned holding period of 10 years who could buy a normal I Bond with a 2% fixed rate would require a 2.15% fixed rate on an I Bond that did not have the early redemption option.

A lower tax rate makes the redemption option more valuable because it reduces the "tax penalty" for early redemption. This lowers the strike yield, raises the strike price, and therefore raises the dollar value of the put option.

The option values for a 30-year holding period are larger because there is more time to benefit from the higher coupon obtained from the switch into a new higher yielding I Bond. For example, an individual in the 30% tax bracket has a one-year option worth 20 bp per year.

The volatility assumption is important in these option calculations, as we would expect.

Table 3 Value of the option to redeem and reinvest in a new I Bond at the end of one year, in basis points per year over the holding period

A. Option value, in basis points, 30% volatility

	Investment horizon	
Tax rate (%)	10 years	30 years
0	17	21
30	15	20

B. Option value, in basis points, 20% volatility

	Investment horizon	
Tax rate (%)	10 years	30 years
0	10	14
30	9	13

The initial fixed rate and the annual inflation rate are each assumed to be 2% per year. The volatility is the assumed volatility of the *fixed* (real) rate.

In panel B of Table 3, we can see that, the option values are substantially lower, by 30% to 40%, when the assumed volatility is just 20%.

4.2. Value of the five-year option

Table 4, Panel A shows that the option values for redemption at the five-year point. For example, for a 10-year holding period, the option is worth 22 bp to an individual in a 30% tax bracket. That means that a 2% fixed rate on the I Bond with the early redemption option

Table 4 Value of the option to redeem and reinvest in a new I Bond at the end of five years, in basis points per year

A. Option value, in basis points, assuming 30% volatility

	Investment horizon	
Tax rate (%)	10 years	30 years
0	27	45
30	22	37

B. Option value, in basis points, assuming 20% volatility

	Investment horizon	
Tax rate (%)	10 years	30 years
0	18	30
30	13	23

The initial fixed rate and the annual inflation rate are each assumed to be 2% per year. The volatility is the assumed volatility of the *fixed* (real) rate.

is equivalent to an I Bond with a 2.22% fixed rate that did not allow early redemption. Table 4 Panel B shows that the option values are lower when the assumed volatilities are lower.

For a 30-year holding period, the options are worth substantially more. For example, for the 30% bracket investor, this option is worth about 37 basis points per year over the 30-year holding period.

4.3. Additional considerations

4.3.1. Compound option feature

The option values that we discussed above are lower bounds because of our simplifying assumption that an investor can make just one early redemption over her investment horizon. In fact, if interest rates were to rise continually over some period of time, the investor could exercise the option every 12 months.

To illustrate the possible value of the compound option feature, we calculate the value of the following compound option: An investor with a 30-year horizon has a five-year option and if redemption occurs at the five-year point, a second five-year option is obtained. The second five-year option is valued using the expected yield associated with redemption at the five-year mark as the spot yield for that second option: The strike yield for the second option is computed from Eqs. (3) and (4) with $h = 20$ the residual holding period after exercising the second option, replacing $h = 25$ and the expected spot yield associated with redemption at the five-year mark.[13]

The compound option's value can be calculated as the value of the first option plus the value of that second option multiplied by the probability that the first option is exercised.[14] For the 30-year initial holding period and 30% tax rate, the compound option is worth 59 bp, whereas the one time European option is worth 37 bp.

4.3.2. Other parameters

Our tables are based upon a 2% fixed rate and 2% inflation. We also did calculations for different values of the parameters. The value of the option depends positively on the level of rates and its value is roughly proportional to the fixed rate. For the case of 1% fixed, 2% inflation, which is close to the environment in mid-2005, the redemption option is worth about 7 bp for an investor with a 10-year holding period.

5. The value of the tax deferral feature

5.1 Valuing tax deferral

Savings bonds give the holder the choice of when to begin paying taxes on the accrued interest. Except in unusual cases (such as where the investor's marginal tax rate is expected to increase substantially in the future), individuals should postpone the tax liability as long as possible, until maturity or redemption. We value the tax-deferral option under the assumption that the tax rate and the investor's marginal tax rate remain constant, comparing after-tax returns on I Bonds and TIPS.

The after-tax return on an I Bond is calculated as an internal rate of return with the purchase price as the present value and the after-tax redemption proceeds as the future value (FV). The after-tax redemption value per one hundred dollars invested is:

$$FV = 100[1 + (.01)(f + c_0)]^h(1 - t) + t], \tag{5}$$

with the terms defined as for Eqs. (1) through (4). As noted previously, for a 4% pretax yield, 10-year holding period, 30% tax rate, the after-tax future internal rate of return is 2.94%.

On TIPS, federal taxes are paid each year on both the cash interest and the inflation-adjustment of the principal, although the principal adjustment accrues but is not received in cash until the bond is sold (or redeemed). The annual cash flow per hundred dollar initial investment is:

$$CF_i = 100[(1 + (.01)(f)]^i c_0(1 - t) - [1 + (.01)f]^{i-1}(.01)(ft), \tag{6}$$

for any year i, through maturity. The first term is the after-tax value of the base interest coupon applied to the adjusted principal (the adjusted principal reflects each year's inflation adjustment; we assume all years' inflation rates are 'f') The second term reflects the tax paid on the change in principal value because of that year's inflation adjustment. At maturity, year n, the inflation-adjusted principal $100[(1 + 0.01)(f)]^n$ is also received; this is not a taxable event. The after-tax IRR is that which makes the present value of the after-tax cash flows equal to $100, the initial investment.

TIPS returns depend not only on the sum of the fixed rate and the inflation adjustment, but also on the individual values of each. For example, after-tax returns are lower for a fixed rate of 3% and an inflation adjustment of 1% than for a fixed rate of 1% and an inflation rate of 3%.

On a regular fixed-coupon bond without inflation adjustment (called a plain bond hereafter), a taxable investor pays taxes annually on the nominal coupon. The after-tax return on these plain bonds is simply the pretax return multiplied by one minus the tax rate. On a 4% plain par bond, an investor in a 30% marginal bracket earns an after-tax return of 2.80%.

In Table 5, we show the after-tax returns for I Bonds, TIPS, and plain bonds for two interest rate regimes, 4% and 6% before taxes, two investment horizons, 10 and 30 years, and tax rates of 0% and 30%. Comparing after tax returns on the I Bond and TIPS, each of which have a 2% fixed coupon and a 2% inflation adjustment, for a 30% tax rate and the 10-year holding period, the tax-deferral on I Bonds produces an annual benefit of 11 bp. For the 30-year horizon, the value of the tax-deferral is 37 bp per year.

The 6% total return columns of Table 5 show that higher interest rates result in more substantial benefits to tax deferral. For a 30-year holding period and 30% tax bracket, the I Bond deferral option is worth 105 bp relative to TIPS.

It is notable that TIPS have an advantage over plain bonds even though taxes are paid annually for principal adjustments. The reason is that the coupon payment is calculated on the principal, including the gross inflation adjustment, rather than on the after-tax inflation adjustment. This results in a slightly higher return.

Table 5 After-tax returns, in percent, for plain bonds, I Bonds, and TIPS: Plain bonds, TIPS, and I Bonds are issued at par

		Pre-tax composite yield			
		4%		6%	
		Holding period			
Tax rate	Bond type	10 years	30 years	10 years	30 years
0%	Plain	4.00	4.00	6.00	6.00
	I Bond	4.00	4.00	6.00	6.00
	TIPS	4.00	4.00	6.00	6.00
I Bond-TIPS (diff, bp)		0	0	0	0
30%	Plain	2.80	2.80	4.20	4.20
	I Bond	2.94	3.20	4.50	5.31
	TIPS	2.83	2.83	4.26	4.26
I Bond-TIPS (diff, bp)		11	37	24	105

Total pre-tax composite yields are 4% or 6%. Holding periods and investment horizons are 10 or 30 years. Tax rates are 0%, and 30%. After-tax returns are the internal rate of return of the after-tax cash flows for each security.

5.2. The tax option for holders of tips

Constantinides and Ingersoll (1984) showed that taxable bonds provide the investor with an opportunity to reduce their tax bill if interest rates rise. If yields rise and bond prices drop relative to their levels at the purchase date, investors can sell their original security, realize a capital loss, and reduce their tax bill. TIPS also provide taxable investors with that opportunity, which enhances the value of TIPS relative to I Bonds.[15] However, we expect this "tax-loss harvesting option" to be smaller for TIPS because a rise in inflation reflected in nominal yields will not cause a price decline in TIPS.

There are costs involved in taking a tax loss and reinvesting. Investors who are considering I Bond versus TIPS are "retail" investors, because the maximum holding of I Bonds is \$60,000 and many of them would be much smaller buyers. Some retail brokers charge about \$50 to buy or sell Treasury securities, or \$100 for a round trip. In addition, there is a bid-offer spread. For the most liquid issues, it is about 2/32 per 100 par or \$12.50 for \$20,000; older issues would have wider bid-offer spreads. Assuming a \$20,000 investment, the total of \$112.50 amounts to \$0.56 per \$100 par.

The transactions cost, after tax would be (0.85)(0.56), assuming a 15% capital gains tax rate. The tax savings have to be large enough to cover that after-tax transactions cost. The tax saving on a long-term loss of S-100 would be (.15)(100-S). Thus, the maximum price sufficient to justify taking the tax loss satisfies:

$$(0.85)(0.56) = (0.15)(100\text{-}S). \tag{7}$$

Thus, the price must fall to 96.83 or lower for the investor to profit from realizing the tax loss, that is, the tax loss option has a strike price of 96.83.

We assume that volatility on the TIPS real rate is 35%; in line with the data described in the text.

Table 6 Combined value of the I Bond early redemption and tax deferral options and the TIPS tax-loss option

A. One-year option

Holding period	10 years	30 years
Early redemption-tax loss	14	18
Tax deferral	11	37
Total	25	55

B. Five-year option

Holding period	10 years	30 years
Early redemption-tax loss	20	33
Tax deferral	11	37
Total	31	70

The tax rate is assumed to be 30%; *fixed* (real) interest rate volatility is assumed to be 30%. The estimated value of the TIPS tax-loss option is one-tenth of the I Bond early redemption option.

The put represents the expected value of the loss write off; the tax saving would be the calculated put value multiplied by the capital gains tax rate of 15%. Under these assumptions, the value of the tax loss option on a 10-year TIPS at the one-year mark is about $.13 per $100 par which is about 1½ bp. This represents about 10% of the value of the I Bond redemption option. Of course, if the investor sells in time to get the short-term capital gains treatment, the tax savings would be considerably larger.

5.3. Combining all of the options (early redemption, tax deferral, and tax-loss harvesting)

An investor in a zero tax bracket would gain the value of the early redemption option alone; there would be no offset for the tax write off on TIPS and none of the tax deferral benefit on I Bonds. On a single five-year option for an investor with a 30-year planned holding period, the value would be 45 bp (see Table 4, Panel A); on the compound five-year option it would be worth roughly 70 bp.

An I Bond investor who pays taxes would gain from the redemption option plus the tax deferral. They would forgo the TIPS tax loss option that is worth about 10% of the redemption option on the I Bond. Thus, the sum of 90% of the redemption option plus the tax deferral gives an idea of the TIPS-I Bond gap that is justified by the net effect of the embedded options.

As summarized in Table 6, I Bond investors with the 10 year horizon obtain modest values from the embedded options: $(15*0.9) + 11 = 25$ bp for the one-year option and 31 bp for the five-year. For 30-year holding period investors, however, the total value of the options is much larger: 55 bp for the one-year option and 70 bp for the five year. The compound options that are available can raise this sum substantially. For example, counting the second five-year option after the first redemption would raise the total value of taxes plus redemption to 90 bp.

The calculations discussed above focus on the case of 2% fixed rate and 2% inflation rate.

In the environment of mid-2005, I Bonds offered a fixed rate of 1.2% and the inflation rate was about 2%. The early redemption options are approximately proportional to the real rate. Therefore, the investor with a 30-year horizon and a 30% tax rate holding a 1.2% real rate I Bond had a five-year option worth about 22 bp (and a compound option worth about 35 bp).

The tax savings, about 75% of the value in table 6, are about 27 bp. The total of the two options would then be 62 bp.

Thus, in the environment of mid-2005, where the I Bond real rate was 1.2% and TIPS real rate was about 2%, these options would not entirely eliminate the 80 bp gap. However, in higher rate environments, as we discussed above, the options could eliminate the entire gap between I Bonds and TIPS and could actually make I Bonds preferable.

6. Conclusions

Although Series I Savings Bonds and TIPS both offer inflation protection, they differ considerably with respect to target markets, pricing, taxation, and risk characteristics. Series I Bonds are available only to individual investors, are priced by the Treasury, rather than at auction, give investors the option to redeem them at face value plus accrued interest at any time after one year, and allow for deferral of taxes until redemption. TIPS have substantial real durations and their prices can increase or decrease substantially as interest rates decline or rise.

The real portions of the yields have consistently been higher for TIPS than for Series I Bonds. Our objective in this paper was to determine whether the early redemption option and the tax-deferral option have sufficient value to offset the higher real yield provided by TIPS. The results indicate that although the early redemption option and the tax deferral add considerable value to the I Bond, it is not enough to offset the yield differential if we assume a one-time redemption option. However, for those investors alert for multiple switching opportunities, the I Bond's lower yield can be offset. In addition, the options would be more valuable in higher interest rate regimes. In summary, our numbers suggest that for long-term holders, I Bonds merit consideration even when their real coupons are considerably lower than TIPS.

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Notes

1. Source for both TIPS and I Bond information: U.S. Treasury Department, Bureau of the Public Debt, Monthly Statement of the Public Debt of the United States, May 31, 2005.
2. See, for example, Arnott (2003), Jarrow and Yildiram (2003), Roll (1996, 2004), and Vankudre et al. (1997).

3. These papers deal with floating rate I and EE bonds. The floating interest rate on EE Bonds was changed to a fixed rate as of May 1, 2005.
4. Savings bonds can be bought in denominations as small as $25. Therefore, a small portion of the redeemed bond, less than $25, might not be able to be rolled into a new bond.
5. The holder of an I Bond could switch into some other instrument whose real rate has risen, but we focus on the narrower option to switch into another I Bond.
6. Volatility is computed as is standard for options: (1) the real yield in week i is divided by the real yield in week i-1; (2) the natural log of the ratios is computed; (3) the standard deviation is calculated; and (4) the standard deviation is annualized.
7. We use semiannual fixed rates over the history of I Bonds for this calculation, whose methodology is similar to that of the previous footnote except that to annualize the standard deviation, we multiply by the square root of 2.
8. An administered rate lacks the frequent tiny changes that a market rate would show. Thus, the assumption about the probability distribution while convenient is not accurate. This assumption combined with a number of others means that the reader should treat our numbers as rough estimates.
9. The typical option pricing formula is based on the assumption that an investor is able to form a risk-free portfolio of the option and a position in the underlying and therefore being entitled to the risk-free rate on their investment.
10. If the yield curve is flat, the level of the interest rate at the outset generally does not have a large influence on the value of a typical bond option. For the redemption option, the interest rate also influences the strike price on the option.
11. See the previous footnote. The appropriate risk-free rate is that of the investor constructing the riskless portfolio of the option and the underlying.
12. For the one-year option, the pre-tax return is 3% because of the early redemption penalty. For five-year options, the pre-tax return is 4% per year.
13. The expected real yield if the first five-year option is exercised is 3.03%. The strike yield for the second option is 3.20%.
14. The probability of exercise in a risk-free world is the term $N(d_2)$ in the Black-Scholes formula. See Hull (2003), p. 247.
15. We are grateful to the referee for this important insight.
16. See for example, Livingston (2005), p. 98.

Appendix

This appendix illustrates calculations of the inputs to value a one-year option for an investor with a 10-year investment horizon. The real rate and the inflation rate are each assumed to be 2%. The tax rate is assumed to be 30%.

A.1. Strike price

We solve for S_1, the reinvestable proceeds from a redemption, using Eq. (2). We then plug this S_1 into Eq. (1). For a 10-year investor, we obtain $c_1 = 2.1427$.

We use this strike yield to determine the strike price. For the 10-year holding period, at the expiration date of the one-year option, the security has a remaining life of nine years. The residual security, a nine-year with a coupon of 2% real plus 2% inflation, must be priced in a market that demands a yield of 2.1427% real plus 2% inflation. Its price in this higher yield environment is calculated as:

$$100(1.04)^9/(1.041427)^9 = 98.77$$

This is the strike price on the option.

A.2. Price volatility

We seek the price volatility for the residual security, which is the security that would remain at the option's expiration. We assume a volatility of 30% on the real rate. If the expected holding period is 10 years and the time to option expiration is one year, the bond's "residual tenor" is nine years. If we knew the duration of this security, we could convert a yield change into a price change.

The duration of accrual bonds such as an I Bond are not the same as a bond that pays a regular coupon. With a regular coupon bond, interest is paid semiannually as it accrues. However, I Bond coupons are reinvested at the coupon rate. This automatic reinvestment raises the implicit duration of these bonds. For example, at a par value of 100, an I Bond with a 4% coupon and nine years to maturity will have a final value of 142.3312. A 10 basis point increase in interest rates would reduce the present value of the bond to:

$$100(1.04)^9/(1.041)^9 = 99.139.$$

Thus, the price change for a 10 bp increase in yield would be $100 - 99.139 = -0.861$, resulting in an implicit modified duration of 8.61 years.

To translate yield changes into price volatility, notice that % change in price = [yield change (in percentage points)](duration of the residual security), or % change in price = [yield volatility (as a decimal)](initial yield)(duration of the residual security).[16]

Assuming a yield volatility of 30%, an initial real rate of 2%, and a residual tenor of nine years, our estimate of the price volatility is:

$$(0.30)(2)(8.61) = 5.17\%.$$

A.3. Converting the dollar option value into basis points per year

The above calculations give us the following inputs for determining the option value:

Underlying asset price: $100.00
Strike price: 98.77
Volatility: 5.17%
Risk-free rate: 2.94%
Time to expiration: 1 year
Earned yield 2.94%

Using the Black-Scholes option-pricing model (1973), the put option value is $1.45.

We convert the option price in dollars into basis points per year by dividing the dollar value by the modified duration of the original security. The duration of the 10-year with the assumed parameters is 9.56 years, as calculated in Section A.2. The option value, in bp, is 1.45/9.56 = 15.16 bp. Multiplying by the blow-up factor of 1.021, the option value is 15.48 bp.

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