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Family limited partnerships and control discounts

John L. Teall

Visiting Professor of Finance, Rensselaer Polytechnic Institute, Troy, NY 12180, USA

Abstract

Distributing assets to Family Limited Partnerships (FLPs) is an estate planning technique designed to reduce assets subject to estate taxation. This paper discusses power index models as measures of power along with extensions of indices to value minority discounts. Power indices and valuation models proposed here are directly applicable to valuing other business entities, particularly where control might be contested. Key among factors affecting control valuation is that voting power among FLP partners is not proportional to ownership. This point is especially important to FLP creation because tax-driven value reductions are directly tied to minority voter discounts. © 2007 Academy of Financial Services. All rights reserved.

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1. Introduction

One of the more vexing problems in appraising family business entities is the valuation of control. In many instances, valuing the assets of the entity is fairly straightforward, but the right to control these assets is itself a valuable asset, and too little attention is paid to control valuation methodologies. In addition, the importance of control valuation is frequently understated in the personal finance literature. U.S. legal and tax authorities increasingly recognize the value of controlling family assets, yet the finance literature provides little systematic guidance for measuring this value. The primary purpose of this paper is to offer an adaptation of the power index model for measuring and valuing control of family assets. Personal financial planning provides a wide array of potential applications for the power

* Tel.: +1-518-276-2812; fax: +1-518-276-2348.
E-mail address: jteall@jteall.com.

index model, including valuation for small businesses, divorce settlements, estates and inheritances, real estate investments and Real Estate Investment Trusts (REITs), etc. Applications of these models can arise in scenarios involving divisions or sales of assets, public offerings of securities, disputes (including divorce and inheritance), classifications of voting securities, tax planning, among many others. The focus of this paper will be on the Family Limited Partnership (FLP), an entity typically created to exploit ambiguities in control valuation.

The Family Limited Partnership (FLP) is a legal structure created to hold business or financial assets of a family. Although there are other applications, the FLP is frequently established to serve as an estate-planning vehicle intended to provide estate tax reductions through the restructuring of the control of family assets. Such family assets might include a family business, real estate, securities, art, or any other asset having business or investment value. Typically, a parent transfers assets to the FLP in exchange for some combination of general and limited partnership interests in the FLP. Children or other prospective estate beneficiaries are given some combination of limited and general partnership interests in the FLP. In one common scenario, the parent transfers assets to the FLP in exchange for a limited partnership interest, giving up control of family assets and relinquishing the ability to dispose of or market those assets.[1] The general partnership interests received from the children are valued at an amount below the federal gifting tax threshold (currently $12,000) or the gift is applied towards the unified exclusion. When the parent dies, the estate claims a minority-interest discount (as well as a discount for reduced marketability) on the value of the limited partnership interest, based on its value having been impaired by the loss of control of the parent. The discount typically claimed by the estate ranges from 20% to 50%. However, the beneficiaries of the estate face capital gains obligations when those assets are eventually liquidated since they will ultimately hold and dispose of both general (control) and limited (cash flow) partnership interests. When the capital gains tax rate is less than the estate tax rate (as is the case for larger estates), the estate beneficiaries realize substantial tax benefits. At present, combined federal and state estate tax rates can range above 70%, substantially more than capital gains rates. Donaldson (2006) provides a nice review of FLP structures and related tax issues.

Minority-interest discounts do reflect important reductions in benefits to parties losing control. Parties obtaining control realize the ability to set policy, hire management, set compensation, buy and sell assets, borrow and pay dividends (see Jensen & Meckling, 1976). Especially in the case of smaller firms with concentrated ownership, the ability to control also confers an ability to extract private benefits. Clearly, such control has value representing a significant proportion of FLP assets, particularly given the small size of the FLP and the probable divergent interests of its partners. Bishop (2006) reviews court cases struggling with control discounts and their valuation. Although the I.R.S. has contested many FLP discounts on numerous grounds, tax courts have usually upheld them. Now, the debate rages over the size of discounts. In many instances, estate planners, appraisers, tax authorities, and tax courts have relied on behavior of public securities, case law, and precedent to set these discounts. A few models have been published devised to value the minority discounts associated with FLPs, but these models do not normally allocate value across more than two partners with varying levels of power. For example, Sansing (1999) developed a model based on the costs of transferring wealth from minority to majority partners, and Graham and Lefanowicz (1997) developed a model based on observed premiums in public securities markets. Neither of these frameworks employs the well-known power index models used in other fields to allocate control value and neither appropriately accounts for the non-proportional relationships between holdings of votes and power.

The FLP has grown to be an important estate-planning vehicle among wealthier families and a major part of its usefulness draws from value associated with control by family members. Nonetheless, the academic literature on valuation of FLP control discounts, aside from the papers discussed above is rather sparse. More generally, academic literature concerning estate planning and control valuation in the personal finance field is also sparse. Exceptions include Bost and Cherin (2000), who discuss the dissolution of a life estate trust, and Crabb (1991, 1993/1994) who discuss the construction of efficient frontiers for estates. There clearly exists a need for further academic literature concerning estates in the personal finance realm, particularly where control is an issue.

2. Voting and power

The FLP is structured to separate claims on cash flows from control, providing the source of value that is transferred out of the estate for estate tax purposes. This value is reunited with cash flow claims after partnership assets are ultimately distributed to the estate heirs, to be taxed later at the lower capital gains rate. Thus, the FLP provides an opportunity to both defer the tax until the estate beneficiary liquidates FLP assets and provides an opportunity for taxation at the capital gains rather than the estate tax rate. Valuing the control separated from the cash flow claims might be regarded as a two-step process: valuing the benefits associated with control (the value to hire and fire managers, buy and sell assets, etc.) and determining the extent of control given up as a result of this separation. This paper focuses on the second issue, because measurement of control can be particularly tricky.

It is well known that the one share-one vote rule does not distribute power proportionally to fractional ownership (Shapley & Shubik, 1954; Dubey & Shapley, 1978; Shapiro & Shapley, 1978). For example, consider the obvious case of two opposing voters, where one has 51% of the votes. When a relatively small number of owners hold varying proportions of partnership interests, ownership and control levels are more divergent. Such scenarios are common for FLPs and other family enterprises.

Game theory literature provides important insight into measurement of power (e.g., von Neumann & Morgenstern, 1944; Milnor & Shapley, 1978; Owen, 1972). The Shapley value and its "oceanic" variations (for firms with large numbers of shareholders) have been used extensively in the financial literature, including Rydqvist (1988) and Chung and Kim (1999), and there have been a number of references to the Banzhaf index including Rydqvist (1986). However, the focus of the papers in the finance literature has been oriented to large firms with many shareholders, each holding small proportions of firm stock.

3. Measuring power

Measuring power and valuing control can play an important role in understanding the governance and the valuation of any organization held by multiple partiers for investment purposes. Such organizations include, but are not limited to, corporations, family and closely held firms and partnerships, and publicly held investment institutions. The FLP provides an ideal laboratory for applications of power index models because they are created specifically for the purpose of transferring power for tax purposes. In this section, we introduce the power index as a means of measuring power in an FLP.

Consider the following example where Mr. And Mrs. Smith fund an FLP with 99 voting general partnership shares, giving shares to their three children, Anne, Bert, and Cindy. Suppose that the 99 shares of the FLP are distributed ($\#_i$) as follows:

i	$\#_i$	i	$\#_i$
Mr. Smith	31	Anne	20
Mrs. Smith	30	Bert	17
		Cindy	1

(1)

Assume that a simple majority vote (50 of 99 votes) will determine the outcome of any partnership election with only two possible outcomes (0/1). We wish to determine the relative power levels of the five partners a given 0/1 election. Our index of power will depend on how power is measured and the particular voting coalitions that form among voters. Regardless, we will see that power will not be proportional to shareholdings. An investor is said to have power if his vote has potential to be pivotal in a corporate election or if he has the potential to "swing" the result of the election. We might measure the power of a participant in an election by determining the likelihood that investor will be pivotal or swing election results. The reader may notice immediately that Mr. and Mrs. Smith as well as Anne are capable of influencing election results; Bert and Cindy are not, at least with the current distribution of votes. Thus, despite their partnership shares, Bert and Cindy have no voting power in the partnership elections. This section will discuss three measures of power, the Shapley value, and its oceanic variant, the Banzhaf Index, and the Owen Index.

The Shapley value (Shapley & Shubik, 1954; Shapiro & Shapley, 1978) is the oldest of the power indices and is the foundation for most of the others. The oceanic variation of the Shapley Index (Milnor & Shapley, 1978) has been the most influential in the financial and economics literature. This index is based on an election where n voters queue to vote in any one of n! equiprobable orders or permutations. The Shapley Power Index (S_i) for a particular voter is used to determine the probability that his block of votes will be pivotal, assuming that prior votes in the queue are cast unanimously, in sequence, and that each voter's position in that sequence is random. Thus, this index determines the average marginal contribution of voter Y to any voting coalition Q to which he might belong. A coalition Q is defined as a subset of voters who cast identical votes. The Shapley Index for voter Y is calculated as follows:

Table 1 Pivotal voters in 120 potential election outcomes

ABCDE	BACDE	ACBDE	BCADE	CABDE	CBADE	DEABC	DEBAC	DEACB	BECDA
ABDCE	BADCE	ACDBE	BCDAE	CADBE	CBDAE	DEBCA	DECAB	DECBA	CEADB
ADBCE	BDACE	ADCBE	BDCAE	CDABE	CDBAE	EABCD	EBACD	EACBD	CEBDA
DABCE	DBACE	DACBE	DBCAE	DCABE	DCBAE	EBCAD	ECABD	ECBAD	AEBCD
ABCED	BACED	ACBED	BCAED	CABED	CBAED	EABDC	EBADC	EACDB	BEACD
ABDEC	BADEC	ACDEB	BCDEA	CADEB	CBDEA	EBCDA	ECADB	ECBDA	AECBD
ADBEC	BDAEC	ADCEB	BDCEA	CDAEB	CDBEA	EDABC	EDBAC	EDACB	BECAD
DABEC	DBAEC	DACEB	DBCEA	DCAEB	DCBEA	EDBCA	EDCAB	EDCBA	CEABD
ABECD	BAECD	ACEBD	BCEAD	CAEBD	CBEAD	EADBC	EBDAC	EADCB	CEBAD
ABEDC	BAEDC	ACEDB	BCEDA	CAEDB	CBEDA	EBDCA	ECDAB	ECDBA	AEBDC
ADEBC	BDEAC	ADECB	BDECA	CDEAB	CDEBA	AEDBC	BEDAC	AEDCB	BEADC
DAEBC	DBEAC	DAECB	DBECA	DCEAB	DCEBA	BEDCA	CEDAB	CEDBA	AECDB

Note: The underlined partner is the pivotal voter (with a value of one) for that particular permutation of voters. Here, Mr. Smith is represented as A, Mrs. Smith as B, Anne as C, Bert as D, and Cindy as E.

$$S_Y = \sum_{q=1}^{n} \sum_{Y \in Q} c(Q)^{-1}[v(Q) - v(Q - \{Y\})] \tag{2}$$

where:

n = the number of participants in the election
q = the number of participants in coalition Q
v(Q) = the characteristic function or maximum potential worth of coalition Q; 1 if the coalition wins and 0 if it loses
$c(Q) = [(n-1)!]/[(q-1)!(n-q)!]$

The maximum worth or characteristic function v(Q) of a coalition Q (combination of voters voting identically) might interpreted as the total of its members' benefits of belonging to the coalition. If v(Q) or v(Q-{Y}) are limited to values of either zero or one for the purpose of measuring power, the normalized Shapley Value $S_Y/\Sigma_i S_i$ is taken to be the probability that voter Y is pivotal. Each election will have a single pivotal voter who ultimately decides the election result. In this case, a coalition has a maximum worth v(Q) of one if it wins the election or zero if it loses. Using Eq. (1) (results and computations are summarized in Tables 1 and 2), we find that Shapley Values for Mr. and Mrs. Smith and Anne are 11.53 and zero for Bert and Cindy. Normalized power indices P_x are computed below for each of the potentially pivotal voters x:

$$P_x = \frac{S_x}{\Sigma S_Y} \qquad \text{e.g.,} \quad P_A = \frac{11.53}{11.53 + 11.53 + 11.53 + 0 + 0} = 1/3 \tag{3}$$

Thus, normalized power index values are 1/3 for the three potentially pivotal partners; normalized Shapley Values are 0 for Bert and Cindy. Note that these same normalized index values can be obtained by adding the number of pivots (see Table 1) potentially realized by a given partner and dividing by the total of pivots or queues. For example, each of the three potentially pivotal partners each has 40 pivots, totaling 120. Thus, each of their normalized

Table 2 Computing SHAPLEY VALUES

$S_{Mr.S} = 1/5 \cdot \{(1/1)\cdot 0 + (1/4)\cdot 12 + (1/6)\cdot 28 + (1/4)\cdot 40 + (1/1)\cdot 40\} = 11.53$
$S_{Mrs.S} = 1/5 \cdot \{(1/1)\cdot 0 + (1/4)\cdot 12 + (1/6)\cdot 28 + (1/4)\cdot 40 + (1/1)\cdot 40\} = 11.53$
$S_{Anne} = 1/5 \cdot \{(1/1)\cdot 0 + (1/4)\cdot 12 + (1/6)\cdot 28 + (1/4)\cdot 40 + (1/1)\cdot 40\} = 11.53$
$S_{Bert} = 1/5 \cdot \{(1/1)\cdot 0 + (1/4)\cdot 0 + (1/6)\cdot 0 + (1/4)\cdot 0 + (1/1)\cdot 0\} = 0$
$S_{Cindy} = 1/5 \cdot \{(1/1)\cdot 0 + (1/4)\cdot 0 + (1/6)\cdot 0 + (1/4)\cdot 0 + (1/1)\cdot 0\} = 0$

Note: There are 5! = 120 permutations of five voters from the example. Thus, there are (5–1)! = 24 permutations where a given partner votes in a given slot 1 through 5. This table computes the number of times a particular voter will be pivotal given his participation in a coalition with a q or fewer members. Notice that Voters A, B, and C each have 40 pivots, while Voters D and E have none. Fifty votes out of 99 are required for a favorable majority.

power indices is 1/3, reflecting the fact that each of these three partners is equally influential in an election.

Shapley and normalized Shapley Indices suggest that Mr. and Mrs. Smith and Anne share equally the power in the firm (assuming that either all permutations or combinations of partners are equiprobable), though their shareholdings are far from equal. Bert has no power, even though his holdings are significantly larger than those of Cindy and 85% as large of those of Cindy. Now, suppose that Mrs. Smith gives a single partnership share to Cindy. Now the FLP with 99 outstanding shares has five partners (i) whose shareholdings ($\#_i$) are revised as follows: Mr. Smith: 31, Mrs. Smith: 29, Anne: 20, Bert: 17 and Cindy: 2. Because the composition of shareholdings has changed among partners, power indices will change. Because of the diminished power of Mrs. Smith, both Bert and Cindy now have opportunities to become pivotal. This particular shift in power also reduces the power of Anne and enhances the power of voter Mr. Smith. Eq. (1) with revised values indicates that Mr. Smith is now pivotal in 48 rather than in 40 permutations, Mrs. Smith is pivotal in 32 rather than 40, Anne in 32 rather than 40, and Bert and Cindy are pivotal in 8 rather than none. Eq. (1) reveals that the Shapley values for Mr. Smith is 13.80, Mrs. Smith B and Anne is 8.03, and 2.26 for Bert and Cindy. Revised normalized power indices P_x are computed for each of the potentially pivotal partners x:

$$P_x = \frac{S_x}{\Sigma S_Y} \qquad \text{e.g.,} \quad P_A = \frac{13.80}{13.80 + 8.03 + 8.03 + 2.26 + 2.26} = 0.40 \tag{4}$$

Thus, normalized power index values are 0.40 for Mr. Smith, 0.234 for Mrs. Smith and Anne and 0.066 for Bert and Cindy.

Notice how sensitive the power indices for all of the partners are to a seemingly minor shift in partnership stakes. By receiving a single share from her mother, Cindy increased her normalized power index from zero to 0.066. Thus, her probability of controlling (being pivotal in) an election outcome has increased by 6.6%. Because a partner's power index is roughly comparable to his probability of being pivotal in an election, the increase in the power index may be interpreted as the increase in his being pivotal.

The rather striking sensitivity of power indices to the exact distribution of shares can be very useful in determining the appropriate strategy for gifting interests in the FLP. We can infer from the following section of this paper that the value of shares and each participant's shareholdings after a redistribution of claims on income or control in the FLP will be very sensitive to how many shares are held by each participant. Thus, it will matter exactly who gives shares to whom and each participant's proportional wealth in the FLP will depend on the redistribution strategy.

Banzhaf (1965) developed a second power index that estimates the probability that a given voter is a "swinger." A swinger is a voter who can change an election result by changing his vote. The Banzhaf Index may perform better in an FLP setting than the Shapley Indices because it derives from equiprobable voting coalitions or combinations (perhaps generating many "swingers," each of whom are capable of influencing the election result) rather than equiprobable voting permutations (orderings of voters where only one voter in the "queue" can be pivotal). Thus, it is possible in a given election for more than one voter to swing election results. In an election, n voters may form 2^{n+1} coalitions (including the null set), half of which, or 2^n win, assuming $0.5n+1$ votes are required for a majority. The number of swings for a particular voter j, $\mu_j(v)$, equals the number of coalitions that require his participation to win. To determine a voter's relative power, one may compute the normalized Banzhaf Power Index as follows:

$$B(v) = \mu_Y(v) / \sum_{j \in N} \mu_j(v). \quad (5)$$

where

$\mu_Y(v)$ = the number of coalitions that require Voter y to win
$\mu_j(v)$ = the number of coalitions that require Voter j to win
N = the set of all voters

The Banzhaf Index permits multiple swingers in any given election outcome. If an election outcome generates multiple swingers, increments to their power indices are equally distributed. Dubey and Shapley (1978) suggest that Banzhaf indices may be revised to reflect probabilities of a given voter being a swinger:

$$B_Y(v) = \mu_Y(v)/2^{n+1} \quad (6)$$

where n is the number of voters and 2^{n+1} is the number of potential coalitions that may be formed. Table 3 provides an example of applying the Banzhaf Index to our original example.

The Shapley and Banzhaf indices have the advantage, particularly in the regulatory and judicial arenas, of being "sociologically neutral" in that they do not require assumptions regarding the election preferences of any of the contestants in the election. Each voting permutation or combination is regarded as being equally likely to be realized. However, this sociological neutrality may present some disadvantages in the family setting. In many instances a given family member or partner may have known specific preferences regarding the outcome of an election or may be more likely to form coalitions with one partner than another. Furthermore, one or more of his competitors for control may also have indicated preferences or seem likely to form certain coalitions. These stated or implied preferences may drastically alter the partnership's balance of power. Hence, one might value partnership

Table 3 Swingers in 32 potential election outcomes

Voters for	Voters against	Voters for	Voters against	Voters for	Voters against	Voters for	Voters against
A	BCDE	AE	BCD	ABD	CE	CDE	AB
B	ACDE	BC	ADE	ABE	CD	*ABCD	E
C	ABDE	BD	ACE	ACD	BE	*ABCE	D
D	*ABCE	BE	ACD	ACE	BD	ABDE	C
E	*ABCD	CD	ABE	ADE	BC	ACDE	B
AB	CDE	CE	ABD	BCD	AE	BCDE	A
AC	BDE	DE	*ABC	BCE	AD	*ABCDE	
AD	BCE	*ABC	DE	BDE	AC	*ABCDE	

Note: Asterisks indicate that neither coalition has a swinger. Underlined voters are swingers in that potential outcome. Partner A is a swinger in 16 of 32 potential outcomes. Partners B and C are each swingers in 16 potential outcomes. Partners D and E are never swingers. The sum of power indices is 2; normalized power indices are simply raw values divided by 2. Note that the total number of coalitions = 64 = 2^6 = 2^{n+1}.

interests with a power index that reflects contestants' preferences or probabilities of joining particular voting coalitions.

Owen (1972) developed a power index that accounts for contestants' preferences by assigning probabilities p_i to each voter i of voting "1" in an election. Let N be the set of all voters in a corporate election and T be a subset of voters who might form a coalition. The characteristic function (one for winning coalitions and zero for losing coalitions) for coalition T is v(T). Owen's power index for voter Y is simply the sum of his contributions to all coalitions, each weighted for its probability of being formed:

$$O_Y = \sum_{T \in N/Y} \{\prod_{i \in T} p_i\} \quad \{\prod_{i \notin T} [1 - p_i][v(T \cup Y) - v(T)]\}. \tag{7}$$

where

N/Y is the set of all voters, excluding voter Y
v(T 200 Y)-v(T) is voter Y's contribution to coalition T (0 or 1)
p_i is the probability that voter i joins coalition T.

If the characteristic function results in a value of one for a winning coalition and zero for a losing coalition, then O_Y might be interpreted as the probability that voter Y will be a "swinger" in a winning coalition.

If partners are equally likely to form any coalition (p_i = .5), the Owen Indices will be identical to the Banzhaf Indices. The relative strength of the Owen index in measuring power in the FLP is that it allows for varying uncertainties with regard to formation of coalitions. This may be quite useful when clear alliances exist in families or when family dynamics are fluid. Thus, one uses the Owen Index when participants vary in their probabilities (p_i) in joining a given coalition of voters. In our example, Shapley, Banzhaf, and Owen Power Indices will equal one another when Owen probabilities equal 0.5.

Table 4 Computing share values

Partner	Before gifting 1 share to Cindy				After gifting 1 share to Cindy			
	Number of shares	Cash flow value	Votes value	Total share value	Number of shares	Cash flow value	Votes value	Total share value
Mr. Smith	31	$469,697	$166,667	$636,364	31	$469,697	$200,000	$669,697
Mrs. Smith	30	$454,545	$166,667	$621,212	29	$439,394	$117,000	$556,394
Anne	20	$303,030	$166,667	$469,697	20	$303,030	$117,000	$420,030
Bert	17	$257,576	0	$257,576	17	$257,576	$33,000	$290,576
Cindy	1	$15,152	0	$15,152	2	$30,303	$33,000	$63,303
Totals	99	$1,500,000	$500,000	$2,000,000	99	$1,500,000	$500,000	$2,000,000

Share values are computed based on values of claims on cash flows and power indices associated with voting rights, before and after gifting. Note how Mrs. Smith's gifting of a single share to Cindy affects overall share values held by partners.

4. Valuing control and partnership shares

Shares held by the various partners can be valued after measuring power with an appropriate index. However, the overall value of control must be determined before share values can be derived from power indices. This overall control value, the sum of all benefits obtained by a partner from having control, results from gaps in the specificity of FLP contractual structure and contests among partners. Measurement of overall control value is a function of an appraiser or valuation expert. Each partner's power index can usually be roughly interpreted as a probability that he will be able to influence the distribution of those contestable assets whose control is not contractually pre-specified. Hence, the product of the power index and overall value of control (contestable asset value) can be interpreted as the expected value of control associated with that particular partnership interest.

In our original example, suppose that the appraiser (or other interested party) determined that the ability to control the FLPs assets totaled $500,000. Based on their power index values, Mr. Smith, Mrs. Smith, and Anne would have control valued at $166,667 each, based on their equal power index values. Bert and Cindy's shares would be allocated no control value since they cannot influence election results. Suppose, the sum value of assets in the FLP was $2,000,000 before any valuation discounts were taken and $1,500,000 after control discounts are applied (see Table 4). Partner X share value V_x based on cash flow rights and voting rights are determined from proportional ownership and the power index P_x as follows:

$$(Sx/\Sigma S_Y) \cong (\$2{,}000{,}000 - \$500{,}000) + P_x \cong \$500{,}000 = V_x \qquad (8)$$

Total share values for the five partners are computed on the left side of Table 4.

Alternative methodologies in the tax literature for valuing control discounts do not appropriately account for the important effect of share redistributions on controlling election outcomes. Consider the impact of a gift of a single share by Mrs. Smith to Cindy. As we noted in Section 3, power index values for all partners were dramatically affected by this seemingly small gift. The right side values of Table 4 associate new monetary values to the shares held by each of the partners. Note that such gifting, by affecting all of the power indices, affects values of shares held by all of the partners. Similarly, a child's "cashing out" of his partnership interest would affect values of other partner shares. The power index models appropriately account for the relative shifting of values.

5. Conclusions

The Family Limited Partnership provides opportunities to reduce estate values by separating control rights from rights to receive income. However, the literature provides scant guidance on valuation of FLP control rights. This paper discusses a power index approach to valuing voting control in the FLP. Equally important, this approach can be applied to more general family enterprise settings or in any organizational control contests are to be decided by vote. For example, the power index models can also be applied to valuation of corporate control rights. The numerical example offered here depicts how sensitive partners' power is to the distribution of partnership shares and how this sensitivity affects partnership share values. Such dramatic effects on partners' relative control should be reflected in values associated with partner shares. Extant models do not account for this sensitivity; in many instances, current minority discounts taken by estates are unaffected by the distribution of smaller shares.

This paper focused on the Shapley value largely because of its applications to other areas in finance. However, the Banzhaf Index may have an advantage in an FLP and other family settings over the Shapley Indices in that it is based on equiprobable voting coalitions or combinations (perhaps generating many "swingers" in a single winning coalition) rather than equiprobable voting permutations (orderings that will generate only one pivotal vote). This feature of the Banzhaf index may seem more intuitive since it is quite possible that more than one partner can influence results in a given election. On the other hand, Banzhaf values may behave rather oddly or bear little relationship to desirable characteristics of a power-weighting scheme when the number of voters is large, such as in a public corporation setting (see Dubey & Shapley, 1978). Fortunately, this is not normally the case in the FLP context. The Owen index is particularly useful when non-sociological neutral power measures are warranted, such as when certain coalitions of voters are more likely to form than others. Such coalitions are quite likely to be observed in FLP scenarios as well as in divorce and other dispute settings.

Notes

1. It is more common for the parent to exchange assets for a general partnership interest, enabling the parent to retain control while gifting limited partnership interests to the children. This enables the parent to give children a larger proportion of income-producing assets without incurring gift tax liability, since the value of those assets has been impaired by the children's lack of control. The same minority-interest issues discussed in this paper still apply since control is separated from cash flow benefits.

References

Banzhaf, J. F. (1965). Weighted voting does not work: a mathematical analysis. *Rutgers Law Review, 19,* 317–343.

Bishop, C. G. (2006). The ebb and flow of the federal tax role of fiduciary duties in family limited partnerships: from Byrum to Bongard. *Capital University Law Review, 35,* 61.

Bost, J. C., & Cherin, T. (2000). Liquidating a remainder interest: simplifying personal finance. *Financial Services Review, 9,* 183–195.

Chung, K. H., & Kim, J. K. (1999). Corporate ownership and the value of a vote in an emerging market. *Journal of Corporate Finance, 5,* 35–54.

Crabb, R. R. (1991). Probabilistic estate planning. *Financial Services Review, 1,* 143–158.

Crabb, R. R. (1993/1994). Efficient frontiers in estate planning. *Financial Services Review, 3,* 1–27.

Donaldson, S. A. (2006). Super-recognition and the return-to-sender exception: the federal income tax problems of liquidating the family limited partnership. *Capital University Law Review, 35,* 1.

Dubey, P., & Shapley, L. S. (1978). Mathematical properties of the Banzhaf power index. *Mathematics of Operations Research, 4,* 99–131.

Graham, R. C., & C. E. Lefanowicz. (1997). Valuation of companies for the estate and gift tax: evidence on minority interests discounts. *Advances in Taxation, 9,* 77–102.

Jensen, M., & Meckling, W. (1976). Theory of the firm: managerial behavior, agency costs and ownership structure. *Journal of Financial Economics, 3,* 395–360.

Milnor, J. W., & Shapley, L. S. (1978). Values of large games ii: oceanic games. *Mathematics of Operations Research, 3,* 290–307.

Owen, G. (1972). Multilinear extensions of games. *Management Science, 18,* 64–79.

Rydqvist, K. (1986). The Pricing of Shares with Differential Voting Power and the Theory of Oceanic Games. Unpublished Ph.D. dissertation, The Stockholm School of Economics.

Rydqvist, K. (1988). Empirical Investigation of the Voting Premium. Unpublished Working Paper, Northwestern University.

Sansing, R. C. (1999). Economic valuations of valuation discounts. *Journal of the American Tax Association, 21,* 28–38.

Shapiro, N. Z., & Shapley, L. S. (1978). Values of large games I: a limit theorem. *Mathematics of Operations Research, 3,* 1–9.

Shapley, L. S., & Shubik, M. (1954). A method for evaluating the distribution of power in a committee system. *American Political Science Review, 48,* 787–792.

von Neumann, J., & Morgenstern, O. (1944). *The Theory of Games and Economic Behavior.* Princeton, NJ: Princeton University Press.