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An alternative approach to after-tax valuation

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Abstract

Reichenstein (2001, 2007) argues that the type of savings account in which an asset is held affects the after-tax return received by and after-tax risk borne by investors. He uses this powerful insight to develop the notion of after-tax asset values that are predicated on an asset's current after-tax consumption value. This paper builds on the risk-sharing insight and approaches after-tax asset valuation from an investment perspective based on future benefits. It also extends the model to accommodate a broader array of more realistic taxation environments. Examples of after-tax optimization indicate that the recommended asset disposition depends heavily on the model chosen.

JEL classifications: D91; G2; G23

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1. Introduction

Investors have access to many types of savings accounts unavailable to previous generations. Many of these accounts can be classified generally into three categories. The first type is taxable accounts for which deposits are made on an after-tax basis and investment returns are taxed. A second class of accounts can be called tax-deferred accounts, or TDAs [e.g., traditional IRAs, 401(k) plans, 403(b) plans, 457 plans, and Keogh plans]. Contributions to these accounts may be made on a pre-tax basis (i.e., tax-deductible), and the investment returns accumulate on a tax-free basis until funds are withdrawn at which time they are taxed as ordinary income. As such, these accounts are sometimes said to have front-end loaded tax benefits. A third class of accounts has back-end loaded tax benefits [e.g., Roth IRAs, Roth

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401(k) plans, Roth 403(b) plans, 529 plans]. These accounts can be called tax-exempt on prospective basis because although contributions must be made on an after-tax basis (i.e., not tax-deductible) their earnings can accumulate free of taxation even as funds are withdrawn.[1]

Researchers tend to agree that balances held in these accounts are not economically comparable because of their different tax treatment and have developed models make these balances comparable. This paper has four goals. The first purpose of this paper is to compare and contrast two different classes of models for making account balances comparable, identifying the areas in which they compete and areas in which they do not. The first class of models, developed by Sibley (2002) and extended by Horan (2002) and Poterba (2004), converts balances in TDAs and tax-exempt accounts into values that are comparable to balances in taxable accounts. The second class of models, developed by Reichenstein and Jennings (2003) and Reichenstein (2001, 2007), attempts to model after-tax values appropriate for mean-variance optimization in an after-tax framework.

The second purpose of this paper is to evaluate the Reichenstein model for after-tax valuation and offer an alternative. Reichenstein (2001, 2007) correctly argues that the type of account in which an asset is held affects the return received by and risk borne by the investor. This important insight is a necessary part of any after-tax valuation and portfolio optimization model. The Reichenstein approach views after-tax values from a current consumption perspective based on after-tax liquidation value using the investor's withdrawal tax rate. The third purpose of this paper is to derive an alternative approach based on prospective investment benefits that yields more intuitive results. Finally, I extend and generalize the after-tax value model to accommodate a broader array of more complex taxation schemes. The after-tax optimization examples presented in this paper indicate that investors who rely on an incorrect after-tax optimization framework stand to materially misallocate their assets.

Sorting out the proper treatment of after-tax value is important for several reasons. First, it may provide a superior measure of total wealth and asset allocation than previous measures proffered heretofore. Second, portfolio optimization procedures that use incorrect models for after-tax values will produce incorrect asset allocation results. This paper demonstrates that the differences can be material. Third, the notion of after-tax portfolio optimization is related to the burgeoning literature on asset location (i.e., the study of placing particular assets in particular types of accounts.) Refining our understanding of after-tax valuations advances our understanding of asset location, as well.

The balance of this paper is organized as follows. Section 2 distinguishes between two classes of models designed to compare balances in different types of accounts, specifically taxable equivalent and after-tax values. Section 3 highlights two different approaches to estimate after-tax values. It also extends the model to accommodate more realistic tax environments. Section 4 presents examples of after-tax portfolio optimization, comparing, and contrasting the results from different approaches. It indicates that the particular model selected materially affects the prescribed optimal asset allocation. Finally, Section 5 concludes and offers avenues for future research.

2. Literature review

2.1. Taxable equivalent values

Prior work contains useful reviews of much of the literature on tax-efficient investing [e.g., Horan, Peterson & McLeod (1997), Horan & Peterson (2001), and Horan (2003)]. This paper, however, focuses specifically on models that view balances in different types of accounts in a commonly denominated currency. Sibley (2002) introduces the first approach considered here for making balances in different types of accounts comparable. He develops a model to calculate taxable equivalent values (TEVs) that make balances in non-taxable accounts comparable with taxable accounts. Specifically, a TEV is the amount of assets in a taxable account that would produce the same after-tax cash flow as a balance held in a tax-advantaged account, like a TDA or Roth IRA. Sibley develops this concept in the context of liquidating the account at some future point in time as a single cash flow and uses a simple taxation scheme in which returns are fully taxed annually as ordinary income. Recognizing that some asset classes such as equity have inherent tax deferral characteristics, Horan (2002) develops a generalized version of Sibley's approach that accommodates a broad array of taxation schemes and annuitized cash flow patterns. Horan's model allows a portion of the return to be taxed as ordinary income; a portion to be taxed as realized capital gain; and a portion to be tax-deferred as unrealized capital gain. Poterba (2004) develops a similar framework for taxable equivalents. For simplicity, this paper focuses on models for lump sum distributions rather than annuitized distributions.

Sibley models the TEV for lump sum distributions from a TDA earning a pre-tax return of r as

$$TEV_{TDA} = \frac{(1 + r)^n(1 - T_n)}{[1 + r(1 - t_{oi})]^n} \tag{1}$$

where n is the investment horizon, T_n is the terminal tax rate when funds are withdrawn, and t_{oi} is the tax rate on ordinary income. The numerator represents the after-tax cash flow generated by each dollar in a TDA after withdrawal taxes are deducted. The denominator is the taxable equivalent discount factor when returns are fully taxed annually as ordinary income.

Horan's more general formulation, which accommodates taxation schemes with tax deferral characteristics, is

$$TEV_{TDA} = \frac{(1 + r)^n(1 - T_n)}{(1 + r^*)^n(1 - T^*) + T^*} \tag{2}$$

where $r^* = r - rp_{oi}t_{oi} - rp_{cg}t_{cg} = r\,(1 - p_{oi}t_{oi} - p_{cg}t_{cg})$ represents the effective annual after-tax return, and $T^* = t_{cg}(1 - p_{oi} - p_{cg})/(1 - p_{oi}t_{oi} - p_{cg}t_{cg})$ is the effective capital gains tax rate after adjusting the basis for previously paid taxes on ordinary income and realized capital gain [see Horan and Peterson (2002) for a more thorough development].[2] In these expressions, p_{oi} and p_{cg} are the proportion of return recognized annually as ordinary income and capital gain, respectively, whereas t_{oi} and t_{cg} represent the associated tax rates. These after-tax accumulations implicitly assume the cost basis of the asset in the taxable account equals its current market value.[3]

The denominator, which is the only distinction between Eqs. (1) and (2), is the taxable equivalent discount factor for a multifaceted taxation scheme in which a portion of returns is taxed as either ordinary income, realized capital gain, or unrealized capital gain. The Horan model reduces to Sibley's more straightforward formulation if the return in the taxable account is taxed entirely as ordinary income (e.g., $p_{oi} = 1$ and $p_{cg} = 0$).[4]

The respective Sibley and Horan TEVs for lump sum distributions from Roth IRAs are

$$TEV_{Roth} = \frac{(1 + r)^n}{[1 + r(1 - t_n)]^n}$$

and (3)

$$TEV_{Roth} = \frac{(1 + r)^n}{(1 + r^*)^n(1 - T^*) + T^*}$$

The numerators and denominators have analogous interpretations as the TDA taxable equivalent values, and the Roth IRA taxable equivalent values differ from TDA taxable equivalents by a factor of $(1 - T_n)$. Again, the Horan model reduces to Sibley's more straightforward formulation if all the return in the taxable account is taxed annually entirely as ordinary income. Because these models seek to convert the nominal balances in TDAs and tax-exempt accounts to values comparable with those in a taxable account, the taxable equivalent value of a dollar in a taxable account is, by definition, equal to one.[5]

Sibley and Horan indicate that TEVs can be used to provide a consistent measure of total wealth accumulation.[6] Horan suggests further that these models can be used for (1) measuring asset allocation more accurately than simple measures using nominal pre-tax account balances, (2) determining the value of securities used as collateral for a loan, (3) calculating the realizable value for estates involved in probate, litigation, or divorce proceedings, and (4) assessing an individual's optimal withdrawal policy or the timing of one's retirement. Notably, they do not suggest their models are suitable for after-tax portfolio optimization.

2.2. After-tax values

A second approach for making nominal balances in different types of accounts comparable to each other was introduced by Reichenstein (1998) and Reichenstein and Jennings (2003) and refined by Reichenstein (2001, 2007), referred to hereafter as the Reichenstein approach. Reichenstein (2001, 2007) argues that properly assessing portfolio risk and asset allocation requires the use of after-tax values (ATVs) because the government, by taxing annual returns or terminal withdrawals, shares investment risk as well as investment returns with the account holder. This insight is particularly important as it pertains to portfolio optimization. Specifically, Reichenstein (2001, 2007) uses an example to shown that if the standard deviation of pre-tax returns is σ and if returns are fully taxed as ordinary income and if all investment losses can be recognized in the year they are incurred, then the standard deviation of after-tax returns for a taxable account is $\sigma(1 - t_{oi})$. That is, an investor bears approximately $(1 - t_{oi})$ of the pre-tax risk.

The distinction between TEV and ATV is understandably elusive because Reichenstein (2001, 2007) provides no definition of "after-tax" value. One might characterize ATVs, however, as the value of future after-tax cash flows discounted to the present using a tax-adjusted and risk-adjusted discount rate. This definition reflects Reichenstein's premise that the government alters an investor's risk exposure through taxation. TEV and ATV are distinguished by the discount rate. The former discounts cash flows using the after-tax return available through a taxable account. The latter discounts cash flows using a tax-adjusted discount rate appropriate for the particular account. Generally speaking, both approaches are valid for certain purposes. They simply answer different questions.

For Reichenstein, the after-tax value of a dollar in tax-exempt account, like a Roth IRA, is equal to one dollar. At time n, a dollar in a Roth IRA produces an after-tax cash flow of $(1 + r)^n$. Because these returns accumulate in a tax-exempt manner, the appropriate discount factor is $(1 + r)^n$, making the after-tax value equal to one. In other words, a dollar in a tax-exempt account produces the same after-tax cash flow as a dollar in a Roth IRA.

He derives the after-tax value of a TDA in a similar fashion, arguing that investors still bear all the investment risk in TDAs.[7] The future after-tax cash flow produced is $(1 + r)^n(1 - T_n)$. Because the investor bears all the investment risk, the appropriate discount factor remains $(1 + r)^n$. Therefore, Reichenstein models the ATV of assets held in a TDA as

$$ATV_{TDA} = \frac{(1 + r)^n(1 - T_n)}{(1 + r)^n} = (1 - T_n) \tag{4}$$

For Reichenstein, the discount factor to calculate the ATV of taxable assets must be adjusted to reflect the fact that investors bear only $(1 - t_{oi})$ of the pre-tax risk (assuming returns are fully taxed as ordinary income and that losses are fully deductible.) Therefore, he concludes that the appropriate discount rate to calculate ATVs for taxable accounts is $r(1 - t_{oi})$. Because taxable assets in this simple tax environment will accumulate to $[1 + r(1 - t_{oi})]^n$, the ATV of a taxable account for Reichenstein is one,

$$ATV_{Taxable} = \frac{[1 + r(1 - t_{oi})]^n}{[1 + r(1 - t_{oi})]^n} = 1 \tag{5}$$

For Reichenstein, the same result holds assuming Horan's more generalized taxation scheme,

$$ATV_{Taxable} = \frac{(1 + r^*)^n(1 - T^*) + T^*}{(1 + r^*)^n(1 - T^*) + T^*} = 1 \tag{6}$$

Notice that assets in a Roth IRA and taxable account have the same after-tax value in the Reichenstein model, an anomaly discussed below. Reichenstein then applies this framework to mean-variance portfolio optimization, an application for which the TEV models of Horan and Sibley were never intended.

3. More on after-tax values

3.1. Taxable equivalent versus after-tax values

Reichenstein (2007) argues that Sibley's and Horan's taxable equivalent models are wrong because the discount rate in their models do not reflect the fact that investors bear all the risk of investment returns in TDAs and Roth IRAs. It is true that investors bear all the investment risk in TDAs and Roth IRAs, but not in taxable accounts. It is also true that neither Sibley nor Horan nor Poterba accommodates this fact. This criticism is curious, however, because TEVs estimate something different from ATVs in the Reichenstein sense.[8] Sibley and Horan accomplish their goal of making balances in tax-preferred accounts comparable to balances in taxable accounts using "taxable equivalents." Reichenstein attempts to apply a tax-adjusted and risk-adjusted discount rate to future after-tax cash flows to estimate "after-tax values." The two classes of models bear some relation to one another, but are not comparable because they seek to accomplish different goals.

Although the Sibley and Horan models provide better measures of total wealth accumulation and assets allocation than those simply based on nominal market values, they never claim TEVs are suitable for after-tax portfolio optimization, which is Reichenstein's ultimate intent. In short, although TEVs and ATVs represent viable alternatives to calculating asset allocation in a commonly denominated currency, they do not compete with each other as alternatives to performing after-tax portfolio optimization. After-tax values are suitable for that purpose.

3.2. An anomaly

Reichenstein's ATV approach, although generally valid, produces an uncomfortable anomaly that can be remedied. As noted above, taxable accounts (with cost basis equal to market value) and Roth IRAs have equivalent after-tax values in his model. By implication, an investor would be indifferent between owning an identical asset in either a taxable account or a Roth IRA. This conclusion is reasonable if one focuses on current after-tax liquidation values. That is, the two accounts will produce the same after-tax liquidation value for an investor wishing to use funds for current consumption purposes today. If the assets are to be used for investment purposes, however, the Roth IRA is clearly preferred to the taxable account.[9] In the words of Dammon, Spatt, and Zhang (2004), ". . . wealth in the tax-deferred account is more valuable than wealth in the taxable account. . . because of the ability to earn pre-tax returns" (p. 1004). The problem with predicating after-tax values on current consumption value is that funds used for consumption cannot be used for investment and should not therefore be incorporated into a mean-variance optimization framework. For optimization and asset allocation purposes, calculating after-tax values based on prospective investment value makes more sense.

Reichenstein uses the withdrawal tax rate prevailing at time n to calculate the after-tax value of a TDA to emphasize the fact that the after-tax value of the TDA is based on future cash flows. So characterizing his model as based on "current consumption value" may initially seem misleading. However, in his model taxable accounts and Roth IRAs have equivalent after-tax values despite the fact that after-tax cash flows from a dollar in a taxable account are always less than after-tax cash flows generated from a dollar in a Roth IRA at any future date (assuming $r > 0$) and the difference grows over time. Assigning equivalent values to assets in taxable accounts and Roth IRAs necessarily requires one to view them in terms of current liquidation value at time zero, the only point at which their after-tax cash flows are identical.

A simple example illustrates why taxable accounts and Roth IRAs should be assigned different after-tax values. Consider an investor in the 30% tax bracket with \$1 invested in a risk-free asset with a 10% rate of return held in a taxable account. Suppose further the investor also has \$1 invested in a risk-free asset held in a Roth IRA. One year from now, their after-tax accumulations are \$1.07 and \$1.10, respectively. Because both investments have the same risk (i.e., none), their future after-tax values should be discounted at the same risk-free rate, namely 10%, and their present after-tax values will be different from each other. They cannot have the same after-tax value because they have different future after-tax purchasing power and identical risk. Recall Reichenstein (2001, 2007) argues that the savings vehicle affects the return received by and the risk borne by the investor. It is important to note, however, that for risk-free investments, the savings vehicle only affects the return received by the investor, not the risk. This statement remains consistent with the notion that the savings vehicle affects the return received by and risk borne by the investor because the risk-free investment has no risk.

In summary, Reichenstein's model implies that investors are indifferent between funds in a taxable account and funds in a tax-free account for current consumption purposes. From an investment perspective, investors prefer the latter because ending wealth for the Roth IRA stochastically dominates ending wealth for the taxable account for any return or time horizon for both riskless and risky investments.

One can capture the investment perspective by reformulating the risk-adjusted discount rate for future after-tax cash flows from taxable accounts. Reichenstein (2001, 2007) uses an example to show that the standard deviation of after-tax returns for asset i in a taxable account is $\sigma_i(1 - t_{oi})$ where σ_i is the standard deviation of pre-tax returns for an investment fully taxed as ordinary income, a result shown formally in the Appendix. He concludes that the proper risk-adjusted after-tax discount rate is then $(1 - t_{oi})r_i$, where r_i is the pre-tax return for asset i. However, this is no truer than concluding that a discount rate doubles when standard deviation doubles. Although standard deviation and discount rates are directly related, they do not move in exact proportion. A formal demonstration follows.

3.3. A formal derivation of taxable risk-adjusted discount rates

This section offers a different approach to calculate the ATV of a taxable account based on the future benefits of expected investment value, rather than current after-tax liquidation value. Consider a risky asset i with a standard deviation of pre-tax returns of σ_i and correlation with market returns of ρ. The standard deviation of market returns is σ_m. In a CAPM world, the pre-tax beta is given by $\beta_i = \rho(\sigma_i/\sigma_m)$. Substituting the standard deviation of after-tax returns when returns are fully taxed as ordinary income is $\sigma_i(1 - t_{oi})$ for the total after-tax risk of security i produces the after-tax beta

$$\beta_{After\ Tax,i} = \rho\left(\frac{\sigma_i(1 - t_{oi})}{\sigma_m}\right) = (1 - t_{oi})\beta_i \tag{7}$$

Generally, beta is a measure of risk relative to the market. In this case, the after-tax risk is being measured relative to pre-tax market risk. As a result, market risk, σ_m, is not scaled by $(1 - t_{oi})$ in the same way as the individual asset's risk. Because investors bear all the risk for assets held in TDAs and Roth IRAs, this approach permits comparability of risk relative to a common investable benchmark across different types of accounts. If $(1 - t_{oi})$ were applied to σ_m for the taxable account, it would need to be applied to σ_m for TDAs and Roth IRAs as well, producing something that could be called a taxable equivalent beta, the development of which is beyond the scope of this paper.

The appropriate after-tax discount rate then for risky assets in taxable accounts that are fully taxed as ordinary income is

$$k_{After\ Tax,i} = r_f + (1 - t_{oi})\beta_i[E(r_m) - r_f] \tag{8}$$

This result suggests that the tax adjustment to the discount rate applies not to the entire pre-tax return as Reichenstein advocates. Rather, the tax adjustment applies only to the risk *premium*.[10] Consequently, Reichenstein underestimates the proper discount rate for after-tax values of taxable accounts and therefore overestimates their after-tax value. Applying the tax adjustment only to the risk premium more precisely reflects the notion that the government shares in the investment risk of taxable investments. To be sure, government shares in the entire return, not just the risk premium. However, its risk-sharing role, which is reflected in the discount rate, can relate only to the risk premium by definition.

An important question is whether the after-tax discount rate in Eq. (8) holds in equilibrium; that is, whether the after-tax risk-return trade-off is the same for all assets. This equilibrium condition can be demonstrated in an unreported proof. In this framework, the ATV for taxable accounts when returns are fully taxed as ordinary income replaces the discount rate in Eq. (5) with Eq. (8), yielding

$$ATV_{Taxable} = \frac{[1 + r(1 - t_{oi})]^n}{(1 + k_{After\ Tax,\ i})^n} = \frac{[1 + r(1 - t_{oi})]^n}{(1 + r_f + (1 - t_{oi})\beta_i[E(r_m) - r_f])^n} < 1 \tag{9}$$

This result can also be generalized to derive the after-tax discount rate for risky assets in taxable accounts with more complex taxing schemes. The Appendix shows that, more generally, the after-tax discount rate for assets with tax deferral characteristics is

$$k_{After\ Tax,i} \approx r_f + (1 - p_{oi}t_{oi} - p_{cg}t_{cg})\beta_i[E(r_m) - r_f] \tag{10}$$

so that

$$ATV_{Taxable} \approx \frac{(1 + r^*)^n(1 - T^*) + T^*}{(1 + r_f + (1 - p_{oi}t_{oi} - p_{cg}t_{cg})\beta_i[E(r_m) - r_f])^n} < 1. \tag{11}$$

Equations (10) and (11) reduce to Eqs. (8) and (9), respectively, when returns are fully taxed as ordinary income (i.e., when $p_{oi} = 1$ and $p_{cg} = 0$). In fact, Eq. (10) represents the proper discount rate for all three types of accounts if the parameters are specified correctly. When

Table 1 Traditional mean-variance optimization example from Reichenstein (2007)

Account type	Nominal market value	Pre-tax expected return	Pre-tax standard deviation	Pre-tax optimal weights
TDA–stock	$600,000	8%	15%	52.2%
Taxable–bonds	550,000	4%	6%	47.8%
Total portfolio	$1,150,000	6.1%	8.6%	

returns are tax-exempt or the tax on returns is completely deferred as unrealized capital gains until time n, $p_{oi} = p_{cg} = 0$, which implies that $r^* = r$. Consequently, the after-tax discount rate equals the pre-tax discount rate for TDAs and tax-exempt accounts.

4. The impact on after-tax mean-variance portfolio optimization

Reichenstein (2001, 2007) provides an example of after-tax mean-variance portfolio optimization using the after-tax value approach based on the current consumption premise. This section replicates his results and contrasts them with those produced using after-tax values based on investment value. The following section shows that the choice of methodology substantially changes the optimal disposition of assets.

Table 1 presents the Reichenstein example for an investor named Susan, who is in and expects to remain in the 25% tax bracket. Nominally, she has a $600,000 balance in a TDA, such as a traditional IRA, and a $550,000 balance in a taxable account. Market value equals the cost basis in the taxable account. Stock offers a pre-tax return of 8% and a pre-tax standard deviation of 15%. Bonds offer a pre-tax return of 4% and a pre-tax standard deviation of 6%.[11] The correlation coefficient between stocks and bonds is 0.1. Susan has a utility function in the form suggested by Sharpe (1990) of $U = E(r_p) - \sigma_p^2/RT$ where $E(r_p)$ is the expected return on the portfolio, σ_p is the standard deviation of the portfolio, and RT denotes Susan's risk tolerance. Her implied risk tolerance, assuming her current asset allocation is optimal in a pre-tax framework, is 49.9.

Reichenstein (2001, 2007) points out that the same pre-tax asset is effectively a different after-tax asset depending on the type of account in which it is held. One can then implement mean-variance optimization analysis treating each asset in each type of account as a distinct after-tax asset, an approach that effectively incorporates many of the ideas in the nascent literature on asset location [e.g., Dammon, Spatt, and Zhang (2004)]. See Horan (2005) for a review of the asset location literature. Reichenstein's after-tax mean-variance optimization is presented in Panel A of Table 2. Notice that there are now effectively four distinct assets rather than two. For the TDA, the after-tax market value is $(1 - t_{oi})$ times nominal pre-tax market value, and the after-tax return and standard deviation on stocks and bonds are the pre-tax return and standard deviation from Table 1. For the taxable account, however, the after-tax and nominal pre-tax values are equal to each other, as Reichenstein would suggest. The after-tax return is either $r(1 - t_{oi})$ for bonds or $r(1 - t_{cg})$ for stock, according to the simplified tax structure in which returns are fully taxed annually as either ordinary income

Table 2 After-tax mean-variance optimization example from Reichenstein (2007) based on current consumption value

Panel A: After-tax mean-variance optimization

Asset-account type	Nominal market value	After-tax market value	After-tax expected return	After-tax standard deviation	After-tax optimal weights	Pre-tax optimal weights
TDA–stock	$60,000	$45,000	8.00%	15.00%	4.5%	5.2%
TDA–bonds	540,000	405,000	4.00%	6.00%	40.5%	46.9%
Taxable–stock	550,000	550,000	6.80%	12.75%	55.0%	47.8%
Taxable–bond	0	0	3.00%	4.50%	0.0%	0.0%
Total portfolio	$1,150,000	$1,000,000	5.72%	8.29%	100.0%	100.0%

Panel B: Summary asset allocation

Asset type	Nominal market value	After-tax market value	After-tax optimal weights	Pre-tax optimal weights
Stock	$610,000	$450,000	59.5%	53.0%
Bonds	540,000	405,000	40.5%	47.0%
Total portfolio	$1,150,000	$1,000,000	100.0%	100.0%

Note: Some figures do not total because of rounding error.

(for bonds) or capital gain (for stock). The after-tax standard deviations for the taxable account are similarly computed as $\sigma\,(1 - t_{oi})$ for bonds or $\sigma(1 - t_{cg})$ for stock.

If Susan were to optimize her after-tax asset allocation, her optimal disposition of assets changes substantially. As Table 2 indicates, the after-tax optimization tends to locate stock in the taxable account and bonds in the TDA as the asset location literature suggests. Although the overall optimal pre-tax weights of stock and bonds (see Panel B) are similar to Table 1 (i.e., 52.2% stock vs. 53.0% stock), the location of the assets has changed. Second, optimal asset allocation appears different when expressed in after-tax terms rather than pre-tax terms (i.e., 59.5% stock vs. 53.0% stock). Heuristically, Table 2 indicates that after-tax optimization changes the optimal disposition of assets.

Table 3 presents the after-tax mean-variance optimization using the after-tax value discount rate in Eq. (10) based on investment value rather than current consumption value. Following Reichenstein (2001, 2007), the entire return for bonds is assumed to be taxed as ordinary income such that $p_{cg} = 0$ and $p_{oi} = 1$, whereas the entire return for stock is assumed to be taxed annually as capital gain such that $p_{cg} = 1$ and $p_{oi} = 0$. The generalized expression for after-tax value in Eq. (11) requires an input for the length of the investment horizon, n, and the risk-free rate, r_f. The time horizon is assumed to be 30 years, which approximates remaining life expectancies for many investors approaching retirement, and the risk-free rate is assumed to be 3% because the bonds in Reichenstein's example are risky and carry a 4% expected return. The choice of n and r_f do not affect the qualitative conclusions presented in this paper although longer time horizons and higher risk-free rates accentuate the results. Note that the ATV of assets in the taxable account is less than (or equal to, in the case of zero value) their nominal pre-tax values.[12] The after-tax expected returns and standard deviation remain unchanged from Table 2. The optimal after-tax asset allocation differs, however.

Table 3 After-tax mean-variance optimization example based in investment value

Panel A: After-tax mean-variance optimization

Asset-account type	Nominal market value	After-tax market value	After-tax expected return	After-tax standard deviation	After-tax optimal weights	Pre-tax optimal weights
TDA–stock	$90,073	$67,555	8.00%	15.00%	7.2%	7.8%
TDA–bonds	509,927	382,445	4.00%	6.00%	40.9%	44.3%
Taxable–stock	550,000	484,821	6.80%	12.75%	51.9%	47.8%
Taxable–bond	0	0	3.00%	4.50%	0.0%	0.0%
Total portfolio	$1,150,000	$934,821	5.74%	8.31%	100.0%	100.0%

Panel B: Summary asset allocation

Asset type	Nominal market value	After-tax market value	After-tax optimal weights	Pre-tax optimal weights
Stock	$640,073	$552,375	59.1%	55.7%
Bonds	509,927	382,445	40.9%	44.3%
Total portfolio	$1,150,000	$934,821	100.0%	100.0%

Note: Some figures do not total because of rounding error.

Like the Reichenstein approach, the after-tax mean-variance optimization based on investment value allocates the entire taxable account to stock. It locates more stock in the TDA than the Reichenstein approach, however, because it recognizes the after-tax investment value of stock in the taxable account is less than its pre-tax value, allocating additional stock to the TDA to makes up the difference. As a result, the optimal overall after-tax weights are similar to the Reichenstein approach (i.e., 59.1% stock vs. 59.5% stock, see Panel B), but the optimal pre-tax weights differ somewhat (i.e., 55.7% stock vs. 53.0%). Viewed differently, Table 3 shifts approximately $30,000 from bonds to stock in the TDA. The difference is less pronounced for shorter time horizons, but greater for higher risk-free rates.

Reichenstein (2001, 2007) and the examples to this point assume that all stock returns are taxed annually as long-term capital gain, that is, all returns are realized completely in one year and one day. This restrictive assumption is represented algebraically as $p_{cg} = 1$ and $p_{oi} = 0$. The return from a diversified equity portfolio typically has a portion of return that is recognized as ordinary income (e.g., dividends and realized short-term capital gains) or deferred as unrealized. To incorporate this reality, Table 4 reports the after-tax optimization using average distribution rates for ordinary income and realized capital gains as reported by Crain and Austin (1997), specifically $p_{cg} = 0.2046$ and $p_{oi} = 0.4536$. This incremental assumption has the effect of increasing the tax efficiency of equity compared to equity returns being fully taxed as capital gain. As a result, the after-tax return and after-tax risk of equity in the taxable account increases.

In Table 4, the after-tax expected return and after-tax standard deviation for stock in the taxable account are computed using the generalized after-tax expressions in equations (A4) and (A5) in the Appendix. Similarly, the after-tax value of stock in the taxable account is computed using Eq. (11) assuming a 30-year investment horizon and risk-free rate of 3%. As before, the optimal disposition of assets locates stock in the taxable account entirely.

Table 4 After-tax mean-variance optimization example based on investment value using average distribution rates of ordinary income and capital gain for growth and income mutual funds

Panel A: After-tax mean-variance optimization

Asset-account type	Nominal market value	After-tax market value	After-tax expected return	After-tax standard deviation	After-tax optimal weights	Pre-tax optimal weights
TDA–stock	$78,521	$58,891	8.00%	15.00%	6.4%	6.8%
TDA–bonds	521,479	391,109	4.00%	6.00%	42.4%	45.3%
Taxable–stock	550,000	472,429	7.05%	13.21%	51.2%	47.8%
Taxable–bond	0	0	3.00%	4.50%	0.0%	0.0%
Total portfolio	$1,150,000	$922,429	5.82%	8.37%	100.0%	100.0%

Panel B: Summary asset allocation

Asset type	Nominal market value	After-tax market value	After-tax optimal weights	Pre-tax optimal weights
Stock	$628,521	$531,319	57.6%	54.7%
Bonds	521,479	391,109	42.4%	45.3%
Total portfolio	$1,150,000	$922,429	100.0%	100.0%

Note: Some figures do not total because of rounding error.

However, less stock is allocated to the TDA compared with Table 3 because the after-tax return and after-tax risk of equity in the taxable account has increased, allowing Susan to increase the expected return on her overall portfolio with less equity in the TDA. As a result, Susan's optimal overall stock allocation decreases. Notice, however, that the nominal stock allocation in the TDA remains approximately $18,000 greater than the stock allocation proposed by Reichenstein's model, but this difference grows for greater risk-free rates. These results demonstrate that investors and financial planners will allocate assets differently depending on the after-tax optimization model chosen.

5. Conclusion

This paper conceptually distinguishes between the taxable equivalent values developed by Sibley (2002), Horan (2002), and Poterba (2004) from the after-tax values advocated by Reichenstein and Jennings (2003) and Reichenstein (2001, 2007). Taxable equivalent value represents the amount of taxable assets required to produce the same after-tax cash flow as a balance held in another account, like a TDA or tax-exempt account. After-tax value is the value of future after-tax cash flows discounted to the present at a tax-adjusted and risk-adjusted discount rate. Although they have some common applications, such as producing better measures of assets allocation than traditional pre-tax values, after-tax values are better suited for after-tax portfolio optimization.

Reichenstein (2001, 2007) argues that the type of account in which an asset is held affects the after-tax returns received by the investors as well as the after-tax risk borne by the investor. This paper embraces that notion and offers several refinements, which affects the discount rate to compute the after-tax value of taxable accounts. Specifically, this paper derives after-tax values based on expected investment value rather than current consumption value. The framework is then extended to a more realistic taxation environment in which a portion of returns is taxed as either ordinary income, realized capital gain, or deferred capital gain.

The after-tax framework in this paper substantially changes the optimal disposition of assets compared to Reichenstein (2007). Therefore, investors may improve the efficiency of their asset allocation using the model presented here.

Many questions remain unanswered. Opportunities for future research include applying after-tax valuation and optimization models to other types of tax-preferred accounts, such as health care savings accounts (HSAs), non-deductible IRAs, or non-qualified tax-deferred annuities. Alternatively, future research might develop after-tax valuation models for progressive tax rate environments rather than uniform marginal tax rate environments such as the one considered in this paper. Research might also consider the effect of stochastic tax rates on after-tax values. In any case, the literature on tax efficient investing generally and on after-tax portfolio optimization specifically can improve awareness of the issues related to tax efficient portfolio management.

Notes

1. The analysis in this paper focuses on qualified withdrawals; that is, withdrawals after age 59.5 years and, in the case of the Roth IRA, after five years of contribution. It also ignores restrictions on qualified withdrawals and early withdrawal penalties. An example of a fourth type of account is healthcare savings accounts, HSAs, which offer triple tax savings. Contributions are tax-deductible, earnings accumulate on a tax-free basis, and withdrawals are tax-exempt. Eligibility and withdrawal requirements are more restrictive, however, than those listed above.
2. The effective annual after-tax return (r^*) reflects the tax erosion caused by a portion of the return being taxed as ordinary income and another portion being taxed as realized capital gain or dividend. It does not capture tax effects of deferred unrealized capital gains. The value T^* represents the effective capital gains tax rate, and does capture the impact of gains that are deferred until the end of the time horizon, n. When liquidating a portfolio in which all dividends and interim realized capital gains are reinvested, only a portion of the appreciation is taxed as capital gain because some appreciation has previously been taxed as ordinary income, dividends, or capital gain.
3. They also assume symmetry in the tax code that is not always present, especially regarding the netting of gains and losses. Wilcox, Horvitz, and di Bartolomeo (2006) discuss in more detail the disadvantage of having to offset lightly taxed long-term gains with heavily tax short-term losses. We abstract from this nuance here, however.
4. Poterba (2004) develops his model in a continuous time framework.
5. Notably, one can derive analogous concepts for TDA equivalent values and Roth IRA equivalent values, the interpretation and application of which would be similar to taxable equivalent values.

6. Sibley (2002), Horan (2002), and Poterba (2004) used the terms "taxable equivalent" and "after-tax" synonymously. To avoid confusing their work with Reichenstein and Jennings (2003) and Reichenstein (2007) who use the latter term exclusively, only the former term is used to describe Sibley's and Horan's models in this paper.
7. For TDAs, this view is based on using the account's after-tax value as its effective principal. For example, consider a TDA with a $1,000 pre-tax market value and a 25% withdrawal tax rate. Its after-tax value is $750. If the possible pre-tax returns are −100% and +100%, the investor will have either $0 or $2,000 to withdrawal before taxes, yielding $0 or $1,500 available after taxes. This range represents a 100% standard deviation based on after-tax principal of $750, i.e., the investors bear all the risk. By contrast, the return distribution based on the pre-tax $1,000 as the principal value produces a standard deviation of 75%, i.e., the government shares in 25% of the risk.
8. As noted earlier Sibley, Horan, and Poterba use the terms taxable equivalent and after-tax value interchangeably, but they articulate clearly what the meaning is and for what purposes these values are useful. The meaning and application are distinct from Reichenstein's latest study.
9. For simplicity, I ignore restrictions on qualified withdrawals.
10. This result can be generalized for other factor models that separate the risk-free rate from the risk premium, such as the APT or the Fama-French three factor model.
11. By implication, the risk-free rate is less than 4%.
12. The after-tax value of the taxable account is calculated as $550,000[{1 + 0.08(1 – 0.15)}^{30}/{1 + 0.03 + (1 – 0.15)(0.05)}^{30}].

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Appendix

Let the after-tax return equal $r(1 - t_{oi})$ and σ equal the standard deviation of pre-tax returns. Then, the variance of after-tax returns in a taxable account can be written as

$$\sigma^2_{After\ Tax,Taxable} = \frac{1}{m}\sum_{j=1}^{m}[r_j(1 - t_{oi}) - \bar{r}(1 - t_{oi})]^2 \tag{A1}$$

where r_j is the return in period j and $r_j(1 - t_{oi})$ is the after-tax return. Factoring out $(1 - t_{oi})$ and taking the square root yields, the standard deviation of after-tax returns.

$$\sigma^2_{After\ Tax,Taxable} = (1 - t_{oi})^2\frac{1}{m}\sum_{j=1}^{m}[r_j - \bar{r}]^2 = (1 - t_{oi})^2\sigma^2 \tag{A2}$$

$$\sigma_{After\ Tax,Taxable} = (1 - t_{oi})\sigma$$

This relationship can be generalized to more typical and complex taxation schemes using similar logic. For environments in which a portion of returns are taxed as ordinary income, a portion is taxed as realized capital gain, and a portion deferred from taxation as unrealized capital gain, the effective after-tax return can be expressed as

$$(1 + r_{After\ Tax,Taxable})^n = (1 + r^*)^n(1 - T^*) + T^* \tag{A3}$$

Taking the n^{th} root of both sides and rearranging terms yields

$$r_{After\ Tax,Taxable} = (1 + r^*)\left[1 - T^* + \frac{T^*}{(1 + r^*)^n}\right]^{1/n} - 1 \tag{A4}$$

$$\approx r^* = r(1 - p_{oi}t_{oi} - p_{cb}t_{cg})$$

The coefficient to the right of $(1 + r^*)$ is close to one for reasonable values of r^* and T^* and approaches one further as n increases. Making this approximation greatly simplifies the expression with little loss of precision. Replacing the after-tax return in Eq. (A1) with the more general expression in Eq. (A4) produces a generalized standard deviation of after-tax returns

$$\sigma_{After\ Tax,Taxable} \approx (1 - p_{oi}t_{oi} - p_{cg}t_{cg})\sigma \tag{A5}$$

In a CAPM world, it follows that the after-tax beta equals $(1 - p_{oi}t_{oi} - p_{cg}t_{cg})\beta$ and that the generalized discount rate for risky assets in taxable accounts is

$$k_{AfterTax,i} \approx r_f + (1 - p_{oi}t_{oi} - p_{cg}t_{cg})\beta_i[E(r_m) - r_f] \tag{A6}$$

where $\beta_i = \rho\sigma_i/\sigma_m$ is the pre-tax beta.

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