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Optimal savings liquidation for income replacement in the presence of income uncertainty

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Abstract

The decision to liquidate savings for debt repayment is an important one that most consumers face at some time. Traditional financial analysis of this liquidation decision merely compares the interest paid on the debt to the interest earned on the savings, resulting in an almost universal recommendation to liquidate. This paper proposes that savings valuation should include a financial asset that has not previously been explicitly valued: an income replacement option. Applying an option valuation framework to the savings liquidation decision dramatically reduces the number of situations where liquidation is a proper strategy, particularly as income volatility increases. © 2007 Academy of Financial Services. All rights reserved.

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1. Introduction

A recurring question in financial planning is whether current savings should be liquidated to pay off existing debt or whether the savings has more value as a reserve should difficult economic times arise.[1] As early as 1933, Watkins made the argument that the accumulation of a reserve for potential future expenses during times of economic distress should be the foremost goal of savers. In most situations, the existing debt charges an interest rate higher

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than the rate earned in liquid savings accounts. Therefore, it is not surprising that many people, and some financial planners, see the liquidation of savings to pay off debt as a valuable financial tool. On the other hand, many people maintain large savings accounts despite existing debts, and many financial planners recommend such a strategy. Who has the better of this argument, and is there a logical framework to analyze the situation for different individuals?

Traditional financial analysis of this problem seems to yield a simple answer. If the interest rate on the debt exceeds the interest rate earned on savings, the savings should be liquidated to pay off or reduce the debt. If the savings are not sufficient to pay off the debt entirely, no new savings should be added until the debt is fully paid. The opportunity cost of savings appears to be higher than the opportunity cost of paying off the debt. This traditional financial analysis provides the evidence used by the financial planners who would advise the client to pay off the debt.

For many individuals and financial planners, this advice seems risky or just plain wrong. Perhaps they cannot quite explain why, but somehow it just seems dangerous to completely liquidate a savings account. What will there be to fall back on should something happen? To this group of individuals and planners, the accumulation and maintenance of an emergency fund is such an important signal about overall financial health that it can be used to categorize potential clients into target and nontarget groups, according to Joo and Grable (2006). Although the intuition driving the decision making of the individuals in the Joo and Grable study is sensible, sound financial decisions are not made based on intuition alone. This paper proposes a framework for analytically evaluating the savings maintenance or liquidation decision.

2. Literature review

Greninger, Hampton, Kitt and Achacoso (1996), in a survey of financial planning practitioners and financial planning educators, find that the consensus estimate for the necessary size of an emergency fund is 2.5 to 3 months of living expenses. However, Chang and Huston (1995) find that only three in 10 households have enough money saved to last three or more months. Clearly, the accumulation and maintenance of an emergency fund is a relevant issue to a large number of individuals. The corollary question of whether accumulated savings should be used to repay existing debt should be of equal importance. To date, very little attention has been paid to this corollary, and none has been devoted to an analytical model of the savings liquidation decision.

Klein (1951), using Survey of Consumer Finances data from the Federal Reserve, finds that households with liquid savings of some form that experience an income drop will save less after the drop than households with smaller amounts of savings. He finds that households experiencing a temporary drop in income resort to savings to maintain their customary standard of living, while households experiencing a permanent reduction in income decrease their expenditures and actually increase their savings rate. Finally, Klein (1951) also reports that households with an *expected* income reduction tend to increase their preparatory savings.

Using a much newer Survey of Consumer Finances, along with data from the Consumer Expenditure Survey, Bi and Montalto (2004) report that age, educational attainment, the availability of a home equity line of credit, and the amount of credit available on credit cards all positively affect the likelihood of meeting an objective savings guideline. The presence of a non-Hispanic black household member, overspending, and lack of a savings motive all negatively impact the likelihood of meeting an objective savings guideline. Attitude toward borrowing, risk tolerance, expectation of future income, employment status of partner, and ability to borrow against retirement accounts have no significant impact on the likelihood of meeting the objective savings guideline. When Bi and Montalto (2004) consider a subjective savings guideline, the results differ substantially, suggesting that differences in meeting the objective guideline may be due more to reduced savings opportunities than to a failure to recognize the importance of savings.

Chieffe and Rakes (1999) consider planning for emergency expenditures, including the possible maintenance of an emergency fund, an important enough issue to include it as one of the main components of their Model for Financial Planning. They suggest an open line of credit through a credit card or home equity loan might be an acceptable substitute; this proposition is supported by Bi and Montalto (2004).

Hall (1978) argues that, in a rational expectations framework, consumption changes should occur randomly, and only in response to new information about future income, if the permanent income hypothesis with constant interest rates holds.[2] If consumption changes occur randomly, then so should changes in savings. Alessie and Lusardi (1997), using a survey of Dutch households, find that consumers are in fact able to predict future income changes with a high degree of accuracy. Thus, although the arguments from the Hall line of literature may be applied in macroeconomic terms, the findings from Alessie and Lusardi and Klein would seem to indicate that individuals are quite capable of predicting when they will need additional savings.

However, none of these studies directly addresses the problem of whether or not to liquidate a savings account to pay off existing debt. Fortunately, the intuitive reaction by many people helps reveal what the traditional financial analysis is missing. The existence of savings mandates further analysis before a decision should be made on whether to repay the debt, because liquid savings provide the saver with an unrecognized financial asset: an option. The money contained in a savings account can be used to replace income at the discretion of the savings account holder, essentially performing the role of an option security. Like all options, this option has value, and this value should be included when making the decision of whether to save money or liquidate savings to pay off debt. The traditional financial analysis, which only values the interest that can be earned on savings versus the interest charged on the debt, ignores this option's value when evaluating the decision to pay off an existing debt. Certainly, the interest differential is part of the value of savings, but it is only a component part. The option value of the savings account must be evaluated as well for a full and complete analysis.

3. Theory

The true value of the savings account, in addition to the obvious cash balance, can be broken down into two parts. One part is the accruing interest that may be earned on the cash balance, which is the value normally considered when making the decision to liquidate savings for debt repayment.[3] The second portion of the value is the option provided by the savings. To value the option, precise definition of the terms of the option is necessary. The first concern to be addressed is the type of option involved in the analysis. This paper argues that the option provided by savings is best viewed as an American put option.

A long position in an American put option provides the owner with the right, but not the obligation, to sell the underlying asset at any point until the expiration of the option. The most common underlying asset is the stock of a company; therefore, this well-known option will provide the basis for comparison. To obtain the long position, the owner must pay the option premium. The option is usually priced as if it were for one share of stock, and then multiplied by the number of shares actually controlled to determine the total price for a traded option. After this, the owner of the option may sell his stock for the strike price stated in the put option.

Savings provides the same sort of option to savers. The saver "pays" for his option by foregoing current consumption. The current income is saved, and becomes a "share" of current income much like a share of stock. The main difference in this procedure from the stock example is that money is converted to shares of stock when stock is purchased. When current income is saved, there is no formal exchange of cash for stock, but rather of cash for a credit in a savings account. When the holder of the option chooses to exercise his option, he "sells" his "share" of current income for cash just like the holder of stock sells his shares of stock for cash. In other words, the savings account holder exchanges the savings account credit for cash, rather than exchanging stock for cash. As with American put options, the savings account credit may be exchanged for cash at any point before the expiration of the debt, at which time the exchange would no longer be necessary.

As previously mentioned, the example assumes that savings are accumulated in a traditional passbook savings account, providing near instantaneous access to the funds deposited.[4] The underlying asset for this put option is income. Standard option pricing models for American options, such as the binomial option pricing model, indicate that as the value of the underlying asset decreases, the value of the put option increases. In this case, as income decreases, the put option on savings becomes relatively more valuable. This result is intuitively appealing. Large amounts of savings are generally more valuable when one's income is low relative to its historic level, or when income goes to zero. As income rises, relative to its historic level, a given amount of savings will generally have less value to the saver.

The strike price for this put option is also straightforward. The strike price of a put option on stock is the price the underlying stock may be sold for. For a put option on savings, the appropriate strike price is the price one share of savings can be sold for. For simplicity, one share of income is initially defined to be three months of income. Three months of income is often cited by financial planners as the appropriate size of an emergency fund. Because the savings option will be exercised to replace income, the strike price is equal to three months of income.

For a standard put option, the value of the option increases as the strike price increases.

Once again, this result is intuitively appealing, because the option value of the savings increases as the amount saved increases. As the amount saved increases, the saver has more shares of income to sell, and, therefore, the saver has more options. In other words, there is more ability to replace income with savings as savings increases.

As with options on stocks, an appropriate risk free rate may be used to value the savings account option. The London Inter-Bank Offered Rate (LIBOR) will be used in this analysis, although short-term U.S. Treasury rates are also popular in other circumstances. The use of a risk-free rate may also be model dependent.

The volatility of this option can be represented by the volatility of income. Thus, a more volatile income increases the value of the option. This makes intuitive sense under these circumstances. Someone with highly variable income is more likely to need his savings to live than someone with a stable income. Therefore, it appears that an equal dollar amount of savings should have more value to someone with unstable income. There is a long line of literature dating back at least to Leland (1968) providing support for this proposition under a variety of assumptions about the economic environment.[5]

Perhaps the most problematic aspect of this option approach is determining the expiration date of the option. For a stock option, the underlying asset is a share of stock, which theoretically never expires. The option contract itself specifies a time period during which the stock may be sold at the stated price. If the option expires unused, it is worthless.

For a savings account, no such time period is explicitly stated. The underlying asset is income, which also may never expire. The savings may be "sold" to replace this income, and the savings account itself will not lose this ability by expiration of the account. Closer examination reveals that the option has value when it may be used to replace income that is necessary to pay off a debt. If there are no debts to be paid, the absence of income is not problematic. In this case, debt refers to all obligations that the holder of the account may owe, including day to day expenses that are debts until paid. When the debt is paid, the option to use the savings to pay off debt has indirectly expired. Therefore, the maturity on this option should be expressed as the time remaining on the loan.

There is one remaining issue unique to the savings analysis. Savings can be used to replace income, but replace income over what time period? This analysis examines three months of income, because this amount of income is regarded by some as an appropriate emergency fund to provide for living expenses during a job search. The choice of replacement period will obviously affect the current asset value. In other words, one month of income is worth less than three months of income. This will affect perceived option value. In addition, this choice may also affect the volatility of income. Because this work analyzes income replacement over a three month period, the three month volatility of income is utilized.[6]

4. Application

4.1. Normal savings levels

To illustrate how the analysis may be applied in this new fashion, an example will be used. Before this application may begin, it is necessary to choose a model to apply to this problem.

Because of its general acceptance in both academia and practice, this work values the savings put option using the binomial option pricing model (BOPM). Chance and Brooks (2007) provide a detailed description of the BOPM in their book; a very simple example is provided in the Appendix of this work. To employ the BOPM, the time to maturity of the option is divided into a series of time steps. In this example, the maturity of the option is defined as the maturity of the debt obligation, and this time frame is divided into one thousand time steps. This number is sufficient for a reasonable degree of accuracy without imposing significant computing problems.

At each time step, there are two possibilities. The underlying asset price may go up, or it may go down. The current asset price is used to compute the future asset price for either the up or down scenarios with the amount of the up or down price movement determined by the volatility of the underlying asset. The strike price minus the underlying asset is used to compute the value of the put option for both the up and down scenarios, if the option is exercised at that moment. This is referred to as the option's intrinsic value. Additionally, if there are future time steps beyond this one, the option's price must be calculated based on its value at those time periods as well. This value will often be larger than the intrinsic value, and the maximum of these two values is used for the option value at this node of the binomial tree. Because the investor knows in advance that these two possibilities, up or down, are the only possibilities, the investor can form risk free portfolios of options and the underlying asset to guarantee a certain return in the future. Because this portfolio is risk free, it must earn the risk free rate, and the value of the portfolio is discounted back to the initial time step at the risk free rate. As Hull (2006) discusses in his book, an equivalent procedure is to value the option using risk neutral probabilities. Because this procedure is easier to program, it is used for the actual calculations of the example results. This process is repeated for each time step, working backward through the binomial tree, until the beginning price is reached.

There is no closed form equation for the binomial price of an American Put Option, because at some nodes in the binomial tree, the intrinsic value may exceed the calculated option price. At those nodes, the option value must be replaced with the put option's intrinsic value. The only way to determine whether replacement is necessary is to construct the entire binomial tree and test the option values at each node. Brooks (2000) is an excellent source for C++ computer code to program this procedure. A version of this computer program is used to estimate our binomial model.

By way of example, suppose the debt in question is \$10,000 payable over five years and the savings account has a value of \$10,000. Obviously, the savings account is sufficient to pay off the entire debt. Further, suppose the \$10,000 debt charges an interest rate of 5.75%, whereas the savings account pays 2.75%. These rates correspond to readily available rates on automobile loans and savings accounts at the time of this writing. For simplicity, assume that both loans compound monthly.

A traditional analysis would say that the savings account should be liquidated to pay the debt, because the debt costs more in interest payments than the savings account earns. The difference in interest paid and interest earned is computed for the life of the loan, and then the difference is discounted back to the present using the savings rate, as the savings rate represents the investor's opportunity cost of funds. Using the following net present value (NPV) formula,

Table 1
Option value of savings for varying levels of volatility and income

Income	Volatility		
	10	30	50
$0.01	$9,999.99	$9,999.99	$9,999.99
$4,500	$5,500.00	$5,500.00	$5,651.14
$7,500	$2,500.00	$2,875.51	$4,024.85
$10,000	$342.32	$1,745.26	$3,195.42
$15,000	$1.95	$729.22	$2,179.03

Note: This table shows the option value of savings for varying levels of volatility and income. The strike price, or replacement income, for this option is $10,000 throughout, the maturity is five years, and the risk free rate is 4.77%. The savings option is valued as a put option on income. As the table illustrates, the value of savings increases as income decreases, and the value of savings increases as the volatility of income increases.

$$NPV = \sum_{t=1}^{n} \frac{(r_D \times D_t) - (r_S \times S_t)}{(1 + r_S)^t} \tag{1}$$

completely paying off the loan will have an NPV of $86.70 today, numerically confirming the intuition of the traditional analysis. In this formula, r_D represents the rate charged on outstanding debt, r_S represents the rate paid on savings, D_t is the outstanding balance owed at time t, and S_t is the outstanding balance in savings at time t.

However, inclusion of the option value into the analysis requires computation of the option value of the savings account. Initially, the example assumes that $10,000 represents exactly three months of income, so that the strike price and stock price are equal. LIBOR is used to represent the risk free rate, because this rate is commonly used to represent the actual dealer cost of funds. According to Hull (2006), many dealers regard LIBOR as a better risk free rate proxy than Treasury Bills because of the favorable tax treatment Treasury Bills receive in the United States, and the regulatory holding requirements that influence Treasury Bill rates.

The volatility of the underlying asset is used to represent the variability of income, and as Table 1 clearly indicates, the option value of savings is heavily influenced by volatility at most income levels. The one exception is for near-zero income, because the option is so likely to be exercised in this unemployment scenario that the volatility becomes less significant. At all other income levels, the volatility has a significant impact on the option value of savings. In addition, Table 1 clearly shows the increasing option value for lower income levels. As income drops, the option becomes more valuable.

Throughout the analysis, volatility represents the volatility of earnings; therefore, the stability of job income is of critical importance in the savings liquidation decision. Because of this, it seems prudent to explore the meaning of the volatility measure used. As an aid to understanding the volatility measure, suppose the probabilities were expressed as part of a binomial model. A simple one-period binomial model contains only two possibilities. One possible outcome might be continued employment at the same salary. Another possibility is the complete loss of all income. Using these two outcomes as the only possible ones, an annual volatility of 30% indicates a roughly 98% chance of remaining employed at the same salary over the next three months. An annual volatility of 50% indicates a roughly 93% chance of remaining employed at the same salary. These probabilities are calculated using the following formula for standard deviation:

$$\sigma = \sqrt{\sum_{i=1}^{n} (r_i - \hat{r})^2 P_i} \tag{2}$$

In this formula, r_i represents the return in a specific state of the world, $\hat{r}$ represents the expected return, and P_i represents the probability of each state of the world. This formula results in the standard deviation over the three month period, and is converted to an annual figure via the following formula:

$$\sigma_{Annual} = \sigma_{Period}\,(\sqrt{T}) \tag{3}$$

In this formula, T represents the number of years, or if the period is for less than a year, it represents the fraction of a year. In our example, T is equal to one quarter of a year. Examples of jobs with the proposed volatility levels are easily envisioned. Perhaps the relatively higher volatility represents a seasonal laborer, whereas the relatively lower volatility represents the income of a straight-commissioned salesperson.

Another way of envisioning the income volatilities chosen is to think of a binomial model with an income uptick or an income downtick as the possible outcomes, rather than keeping the same salary or becoming unemployed as the possible outcomes. Starting with a base three month salary of \$10,000, a 10% volatility level implies that the higher salary possibility is \$10,290.86 and the lower salary possibility is \$9,709.14. Similarly, a 30% income volatility implies that the higher salary possibility is \$10,866.03 and the lower salary possibility is \$9,133.97. Finally, at 50% income volatility, the higher salary possibility is \$11,444.81 and the lower salary possibility is \$8,555.19. These numbers are all based on the binomial model proposed by Trigeorgis (1991). Other outcomes are also possible, as are longer term models. This work does not seek to exhaustively explore all possibilities, but merely to provide a context for understanding the volatility measure. In addition, an individual's future income volatility cannot be known with certainty, but the examples outlined hopefully provide a framework for the remaining discussion.

Table 2 illustrates the impact that different levels of current income may have on the decision to liquidate savings. The NPV for paying off the loan is calculated as before. The option-adjusted NPV (OANPV) is NPV after subtracting the option value. The option value is subtracted because, upon exercise of the option, the holder loses the option value. The first three income levels are the average three month incomes for service workers, blue collar employees, and white collar employees, respectively, from the National Compensation Survey of the Bureau of Labor Statistics. The final income level is added for comparison purposes. Panel A shows results computed at 10% income volatility. For the three lowest income levels, the OANPV actually reverses the decision reached from the traditional analysis; specifically, savings should not be liquidated to pay off the debt in these cases. For the highest income level, the traditional analysis and the OANPV both indicate liquidation would be optimal. Panels B and C show the results using 30% and 50% income volatilities,

Table 2
NPV and OANPV for paying off a 5-year $10,000 loan at varying current income levels

	Income			
	$4,500	$7,500	$10,000	$15,000
Panel A: Income Volatility at 10%				
NPV no option	$86.70	$86.70	$86.70	$86.70
Pay off loan?	Yes	Yes	Yes	Yes
OANPV	($5,413.30)	($2,413.30)	($255.62)	$84.75
Pay off loan?	No	No	No	Yes
Panel B: Income Volatility at 30%				
NPV no option	$86.70	$86.70	$86.70	$86.70
Pay off loan?	Yes	Yes	Yes	Yes
OANPV	($5,413.30)	($2,788.81)	($1,658.56)	($642.52)
Pay off loan?	No	No	No	No
Panel C: Income Volatility at 50%				
NPV no option	$86.70	$86.70	$86.70	$86.70
Pay off loan?	Yes	Yes	Yes	Yes
OANPV	($5,564.43)	($3,938.14)	($3,108.71)	($2,092.33)
Pay off loan?	No	No	No	No

Note: This table shows the NPV and OANPV for paying off a five year, $10,000 loan at varying current income levels. The three-month replacement income is held constant at $10,000 and the risk-free rate is held constant at 4.77% for this table. The option value is subtracted because this option is lost if the savings are liquidated. The volatility of income is 10% in Panel A, 30% in Panel B, and 50% in Panel C. This table shows how recognition of the option value of savings may change the decision to liquidate savings and pay off a debt with the proceeds. In particular, those with less current income relative to their normal three-month income should be less willing to liquidate savings. Also, those with more volatile incomes should be less willing to liquidate savings.

respectively. Across all income levels at these higher volatilities, the OPNPV leads to a different conclusion than does the traditional analysis: the savings should not be liquidated to pay off the debt. The difference from the new analysis is not merely theoretical; consideration of the option value changes the decision that should be made as to savings liquidation. A comparison among the three panels shows that, as predicted, the savings liquidation option is more valuable for earners with less stable income, making it less likely that individuals with lower incomes should pay off the debt early.

Table 3 highlights the effect that different levels of replacement income, or savings, can have on the savings liquidation decision. As defined for this analysis, replacement income is the equivalent of the strike price in more familiar uses of the BOPM. The replacement income in Table 3 can be thought of as three months of income at the earnings level of the individual when the money was originally saved. Thus, for the $20,000 and $30,000 replacement income levels, a current three month income level represents a substantial decrease in income for the individual. Panels A, B, and C present results assuming 10%, 30%, and 50% income volatilities, respectively. At all tested levels of replacement income, the OANPV results in the opposite conclusion from the traditional analysis. In other words, in no case should the individual liquidate savings to pay off the debt early, for the income volatilities tested. As expected, for higher levels of replacement income, the option value of savings becomes quite large indeed. Again, comparison among the three panels indicates that individuals with more volatile incomes should be less willing to liquidate savings.

Table 3
NPV and OANPV for paying off a 5-year $10,000 loan at varying replacement income, or strike price, levels

	Replacement Income (Strike Price)		
	$10,000	$20,000	$30,000
Panel A: Income Volatility at 10%			
NPV no option	$86.70	$86.70	$86.70
Pay off loan?	Yes	Yes	Yes
OANPV	($255.62)	($9,913.30)	($19,913.30)
Pay off loan?	No	No	No
Panel B: Income Volatility at 30%			
NPV no option	$86.70	$86.70	$86.70
Pay off loan?	Yes	Yes	Yes
OANPV	($1,658.56)	($9,913.30)	($19,913.30)
Pay off loan?	No	No	No
Panel C: Income Volatility at 50%			
NPV no option	$86.70	$86.70	$86.70
Pay off loan?	Yes	Yes	Yes
OANPV	($3,108.71)	($10,520.00)	($19,913.30)
Pay off loan?	No	No	No

Note: This table shows the NPV and OANPV for paying off a five year, $10,000 loan at varying replacement income, or strike price, levels. The current income is held constant at $10,000 and the risk-free rate is held constant at 4.77% for this table. The option value is subtracted because this option is lost if the savings are liquidated. The volatility of income is 10% in Panel A, 30% in Panel B, and 50% in Panel C. This table shows that the option to replace income becomes much more valuable as the amount of income to be replaced increases.

Table 4 emphasizes the influence that different loan maturities, or option expirations, have on the decision to liquidate savings. Again, the results differ somewhat depending on the income volatility level being tested. Panel A shows results using a 10% income volatility. As the expiration of the option increases, the option value increases, but at a slower rate than the traditional NPV increases. In this sensitivity test, the traditional NPV changes values because of the longer time allowed for interest to accrue on the savings account if the option were not exercised. Despite the traditional NPV increasing in value more quickly, the OANPV remains negative for the range of maturities utilized, such that the OANPV still reverses the decision from the traditional analysis by indicating that savings should not be liquidated. Panels B and C give results using 30% and 50% income volatilities, respectively. At these higher income volatilities, increasing the option expiration increases the value of the savings option more quickly than the traditional NPV, making exercise unlikely for these cases as well.[7]

5. Higher savings levels

This method of valuation can also be applied to situations involving larger amounts of savings. In this case, a large amount of savings means "large" relative to the income of the individual. Recall that one share of the underlying asset is defined as three months of income. Therefore, savings in excess of three months income represent additional options on income.

Table 4
NPV and OANPV for paying off a $10,000 loan of varying maturities

	Maturity of Debt			
	3 Years	5 Years	10 Years	20 Years
Panel A: Income Volatility at 10%				
NPV no option	$61.83	$86.70	$140.64	$165.97
Pay off loan?	Yes	Yes	Yes	Yes
OANPV	($257.27)	($255.62)	($219.83)	($200.60)
Pay off loan?	No	No	No	No
Panel B: Income Volatility at 30%				
NPV no option	$61.83	$86.70	$140.64	$165.97
Pay off loan?	Yes	Yes	Yes	Yes
OANPV	($1,432.33)	($1,658.56)	($1,914.95)	($2,108.11)
Pay off loan?	No	No	No	No
Panel C: Income Volatility at 50%				
NPV no option	$61.83	$86.70	$140.64	$165.97
Pay off loan?	Yes	Yes	Yes	Yes
OANPV	($2,641.71)	($3,108.71)	($3,667.24)	($4,059.38)
Pay off loan?	No	No	No	No

Note: This table shows the NPV and OANPV for paying off a $10,000 loan of varying maturities. The original setup is retained, with three-month replacement income, original debt and savings balances, and current income all held constant at $10,000. The risk free rate is also held constant at 4.77%. The option value is subtracted because this option is lost if the savings are liquidated. The volatility of income is 10% in Panel A, 30% in Panel B, and 50% in Panel C. Panel A shows that, for individuals with low income volatilities, increasing the maturity of the debt, or the expiration of the option, actually makes liquidation more attractive. Panels B and C show that, for individuals with relatively higher income volatilities, increasing the maturity of the debt makes liquidation less attractive.

However, these options are not all identical. Each new option has an asset value equal to the asset value for the previous option, plus the strike price of the previous option.

A simple example will clarify the logic of this analysis. Assume the setup from the basic example still holds, except that the individual is now assumed to have $20,000 in savings to go with $10,000 in three month income. The first $10,000 in savings should be valued precisely as before; however, the second $10,000 in savings should be valued as though the current income is $20,000. In the event of a job loss, it is unlikely that the individual will spend all $20,000 in savings at once. Instead, he will first spend his current $10,000 income on living expenses. If that proves insufficient, he will next spend the first $10,000 in savings on living expenses, effectively exercising his first savings option. Only after this option is exercised will it become necessary to spend the last $10,000 in savings. The initial $10,000 in savings effectively provides another source of current income before this option will be exercised.

Once again, this analysis is intuitively appealing. As savings exceeds the income replacement threshold, the option value of the savings declines. The initial savings has a relatively high value, because it is more likely that this savings will be needed to replace income. Additional savings is less likely to be needed for this purpose, so it is less valuable. Table 5 compares the traditional and OANPV of savings liquidation for loan repayment given various initial levels of savings. Panel A provides results assuming a 10% income volatility.

Table 5
NPV and OANPV for paying off a 5-year $10,000 loan at varying savings levels, using a constant risk free rate of 4.77%

	Savings Levels		
	$10,000	$20,000	$30,000
Panel A: Income Volatility at 10%			
NPV no option	$86.70	$86.70	$86.70
Pay off loan?	Yes	Yes	Yes
OANPV	($255.62)	$86.69	$86.70
Pay off loan?	No	Yes	Yes
Panel B: Income Volatility at 30%			
NPV no option	$86.70	$86.70	$86.70
Pay off loan?	Yes	Yes	Yes
OANPV	($1,658.56)	($253.80)	($5.98)
Pay off loan?	No	No	No
Panel C: Income Volatility at 50%			
NPV no option	$86.70	$86.70	$86.70
Pay off loan?	Yes	Yes	Yes
OANPV	($3,108.71)	($1,498.53)	($855.61)
Pay off loan?	No	No	No

Note: This table shows the NPV and OANPV for paying off a 5-year $10,000 loan at varying savings levels, using a constant risk-free rate of 4.77%. Panels A, B, and C show the results with income volatilities of 10%, 30%, and 50%, respectively. At savings levels greater than three months of income, the effective asset price for the option becomes the current asset price plus the first level of savings. Therefore, as savings increases, each additional unit of savings is less valuable as an option. This table shows how the decision to pay off a $10,000 loan from savings might change depending on the amount of savings. At low savings levels, the OANPV of liquidating $10,000 in savings may be negative, but at higher savings levels the OANPV may become positive. Again, it can be seen that individuals with more volatile incomes should be less willing to liquidate savings.

At the "normal" savings level of $10,000, use of the OANPV reverses the decision from the traditional analysis. At increased initial savings levels, the traditional and OANPV arrive at the same decision: liquidate the savings. The conflicting liquidation decisions at different initial savings levels are not surprising, given the above explanation that the additional savings options are less likely to be exercised and are therefore less valuable. Panels B and C display results obtained using 30% and 50% income volatilities, respectively. The increased volatility proves more important in the option valuation than does the presence of additional savings. In all cases at the higher income volatility levels, the decision supported by the OANPV is opposite that of the traditional analysis. Specifically, individuals with more volatile incomes should not liquidate their savings to repay the debt.

6. Conclusion

The traditional analysis of the decision to pay off a loan early by withdrawing savings is dangerously incomplete. The results systematically underestimate the real value of savings. Based solely on a traditional NPV analysis, individuals with extremely unstable jobs may be told to liquidate savings to pay off a loan. This decision could prove catastrophic if income drops dramatically because of unemployment or cyclical variations. By recognizing the option value of savings, the current work hopes to adjust the analysis to reflect this additional value. The resulting option-adjusted NPV should allow better decision-making than utilization of the traditional NPV approach alone. Indeed, the findings indicate that only individuals with very low income volatilities should ever rely on a traditional NPV approach to the liquidation decision. Individuals with higher income volatilities should carefully weigh the option value lost by savings liquidation for debt reduction.

Appendix

The appendix provides a detailed example of the put option valuation methodology, using the same hypothetical example discussed in the main body of the paper. The strike price of the option is \$10,000, the underlying asset price is \$10,000, the risk free rate is 4.77%, the maturity of the debt is five years, and the volatility of income is 30%. To make the example tractable, only two time steps are used in this calculation, rather than the 1,000 time steps used throughout the rest of the paper.

As the name implies, a binomial model consists of two possibilities at each point in time. From any given point, the asset value may either go up or down, with the amount of the increase or decrease determined by the volatility of the asset value. Trigeorgis (1991) develops a method to calculate the size of the up or down movements and the resulting risk neutral probabilities that may be used to price the option. Specifically, the size of an increase in value for any asset y is expressed as (1)

$$\Delta y_u = \sqrt{\sigma^2 \Delta t + \left(\mu - \frac{\sigma^2}{2}\right)^2 \Delta t^2} \tag{1}$$

and the size of a decrease in value for asset y is expressed as

$$\Delta y_d = -\Delta y_u \tag{2}$$

where σ^2 equals the volatility of the asset, Δt equals the time between steps, and μ equals the risk free rate. The resulting risk neutral probability of an up move is expressed as

$$p = \frac{1}{2} + \frac{1}{2} \frac{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t}{\Delta y_u} \tag{3}$$

Applying Eqs. (1) through (3) to the values for the example results in the binomial tree for the asset price given in Fig. 1.

The first node represents the beginning asset price at time 0, whereas the next two nodes represent the two possible prices one time step later (i.e., after 2.5 years). The final three nodes are the possible asset prices in five years. Fig. 2 gives the initial put option values based on the asset prices illustrated Fig. 1.

The put option has no value when the asset price equals or exceeds \$10,000. At expiration,

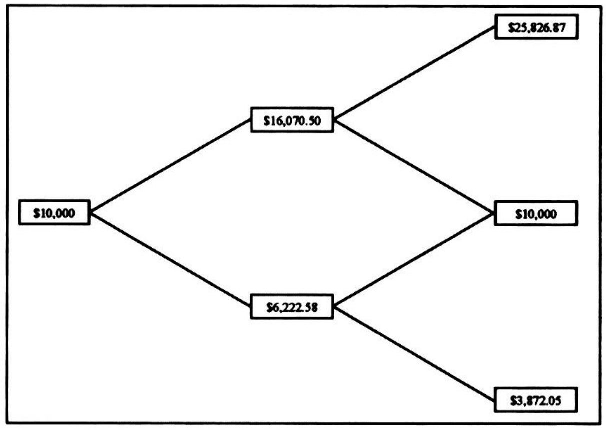

Fig. 1.

the put option has an intrinsic value equal to the strike price minus the asset price, or $6,127.95. The values at nodes A, B, and C cannot be computed without reference to the ending nodes. Once the values of the ending nodes are known, the computation of the option value at node B becomes quite simple. Because the put option value will be 0 no matter what happens after node B, the value of the put option at node B must be 0 as well.

The computation of the value at node C is a two step procedure. First, the value of the put option must be calculated based on its possible future values. To do this, the risk neutral probability of an increase in asset value is calculated from Eq. (3) as 0.5071, making the corresponding risk neutral probability of a decrease in asset value 0.4929. Next, these probabilities are multiplied by the put option value corresponding to either an increase or decrease in asset price. Discounting the resulting value back one time step at the risk free rate gives the preliminary put option value at node C of $2,680.92. The second step is to compare the preliminary put option value to its intrinsic value, which is the exercise price minus the strike price, or $3,777.42. Because the intrinsic value is greater than the preliminary option value, a rational investor would immediately exercise the option at node C; hence, this larger intrinsic value is the appropriate put option value at node C. Fig. 3 presents the new binomial tree.

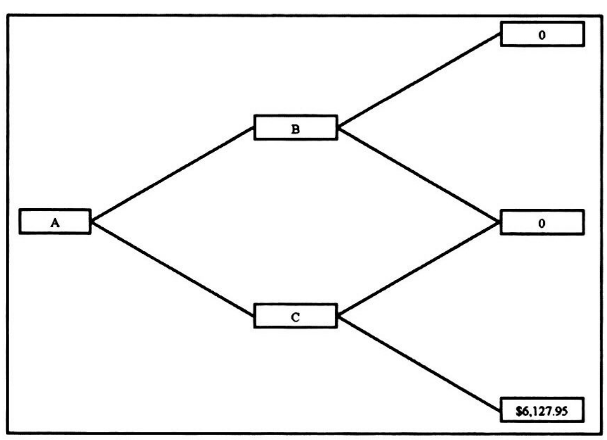

Fig. 2.

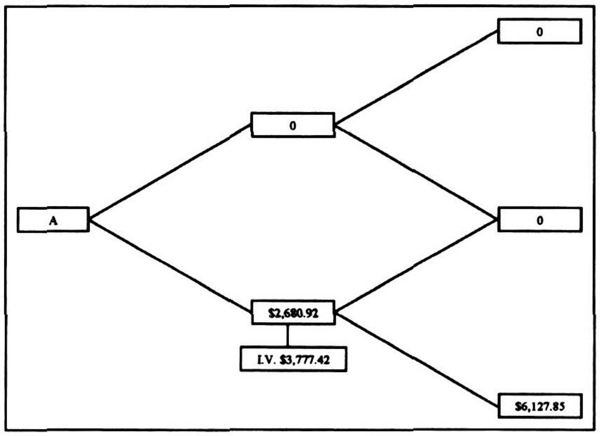

Fig. 3.

Finally, the value of the put option at node A, or the present value of the put option, can be computed. The risk neutral probability of an increase in asset value is multiplied by the value of node B, and the risk neutral probability of a decrease in asset value is multiplied by the value of node C. After discounting this result one time step, the preliminary put option value of $1,652.59 is obtained. Because the intrinsic value at node A is $0, the preliminary value is not replaced, and the put option value today is $1,652.59. Fig. 4 presents the completed binomial tree.

As mentioned above, the binomial tree utilized in this example incorporates only two time steps to allow for better exposition. The trees used to calculate the put option values throughout the paper use one thousand time steps. To illustrate the impact that additional steps may have, the put option value with 100 time steps is $1,743.00, the put option value with 1,000 time steps is $1,745.26, and the put option value with 2,000 time steps is $1,745.38. More time steps obviously should lead to more accuracy, but at the expense of significantly increased computing time. One thousand time steps seems a reasonable compromise between these competing factors.

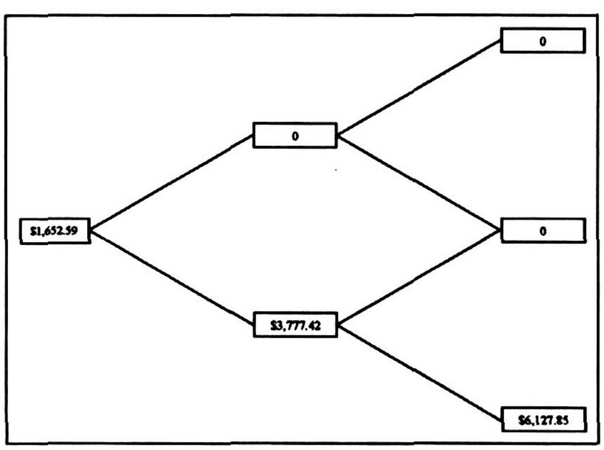

Fig. 4.

Notes

1. The savings referred to here are liquid savings in a traditional demand deposit account or a close substitute, such as a money market deposit account. It is assumed that more liquid forms of saving would be exhausted before less liquid forms, such as an IRA account, are utilized. For a more thorough discussion of liquidity considerations when choosing between these various types of savings plans, see Cho (1986).
2. Hall (1978) spawned a great deal of debate for and against the permanent income hypothesis under rational expectations. Although a complete survey of this line of literature is beyond the scope of the current paper, Flavin (1981), Hayashi (1982), Muellbauer (1983), Bernanke (1985), Campbell (1987), and Tanner (1997) all find some measure of support for the permanent income hypothesis. Conversely, Deaton (1991), Dardanoni (1991), Carroll (1992), Carroll (1994), Carroll (1997), and Carroll and Samwick (1997) find support for an alternative model of household saving, the "buffer stock" model.
3. We are taking a beginning level of savings as a given in the argument presented here. For a more complete discussion of savings accumulation and savings rates, see Ramsey (1928), Uzawa (1961), Srinivasan (1964), or Howard (1978).
4. This assumption is not necessary to the results, but less liquid savings, such as certificates of deposit or IRA accounts, would add unnecessary complexity to the option valuation.
5. Other papers from this line of literature include Sandmo (1970), Rothschild and Stiglitz (1971), Dreze and Modigliani (1972), Juster and Wachtel (1972), Sibley (1975), Miller (1976), Mendelson and Amihud (1982), Skinner (1988), Kimball (1990), Caballero (1991), Weil (1993), Huggett (1996), Hahm and Steigerwald (1999), and Huggett (2004).
6. This is not the first application of option valuation techniques to personal finance. Hamill and Sternberg (1995) apply the Black-Scholes Option Pricing Model to the valuation of junior equity interests in family businesses that are transferred to subsequent generations of family members.
7. Several values of LIBOR from the full range of LIBOR values between January 1, 2000 and May 15, 2007 were tested. However, the results across various LIBOR values were so similar that inclusion was deemed unwarranted.

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