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Retirement withdrawals: an analysis of the benefits of periodic "midcourse" adjustments

John J. Spitzer

SUNY-College at Brockport, Department of Business Administration and Economics, Brockport, NY 14420, USA

Abstract

Much research has addressed the question of how much money can safely be withdrawn from a retirement portfolio without prematurely running out of money (shortfall risk). Instead of constant (inflation adjusted) annual withdrawals, this study uses withdrawal amounts (and optionally, asset allocations) that are modified every five years over a 30-year withdrawal horizon. A bootstrap is used initially to obtain the conditional probability rules. Further simulations demonstrate that periodic (every five years) adjustments can decrease the risk of running out of money as well as increase the amount withdrawn, as compared to a "constant withdrawal amount" strategy.

Jel classifications: G10; J26

Keywords: Retirement; Withdrawals; Asset allocation; Bootstrap

1. Introduction

The amount of money that may safely be withdrawn from a retirement portfolio has been the subject of numerous studies. Bengen (1994, 2004) is at the forefront of this research, followed by Cooley, Hubbard and Walz (1998, 1999, 2003). The majority of studies focus on finding a constant withdrawal amount (after adjusting for inflation) and a fixed stock/bond asset allocation that will sustain a retiree for 25 or 30 or 35 years. The consensus outcome is that (inflation adjusted or real) withdrawals of about 4% to 5% of the starting portfolio balance are sustainable for 30 to 35 years with stocks comprising 50% to 75% of the portfolio

* Tel.: +1-585-225-8849; fax: +1-585-395-2542.
E-mail address: jspitzer@brockport.edu.

and bonds the remainder. Spitzer, Strieter and Singh (2007) build on these studies and provide broader guidelines. Others, including Guyton (2004) and Tezel (2004), indicate that by adding more asset types to the existing stock/bond portfolio, it is possible to increase the withdrawal percentage. Stout and Mitchell (2006) develop a dynamic withdrawal system that incorporates retiree mortality. Because the probability of running out of money before death is smaller than the probability of running out of money over 30 years, their method provides larger withdrawals than do fixed withdrawal models. Dus, Maurer and Mitchell (2005) compare and contrast several types of phased withdrawal strategies. Dus et al. and Spitzer et al. have excellent literature reviews on withdrawal strategies.

The constant withdrawal amount strategies tend to provide large "average remaining balances," (although a retiree should not count on this in practice). Many retirees are concerned about their own lifestyle, not that of their progeny; they have little desire to leave a large estate. In relatively good health early in their retirement, many retirees would like to spend larger amounts than the 4% or 5% of starting balance recommended in previous studies. They expect that their consumption needs for entertainment, education, and travel will be much less when they are considerably older. Hence, they anticipate needing less money (in real terms) very late in retirement and more money early in retirement. Basu (2005) looks at the changing components of retiree spending during retirement and finds that leisure spending and living expenses decrease with age while healthcare expenditures increase with age. Bernicke (2005) demonstrates that retirees total spending decreases with age; the traditional constant withdrawal amount takes out too much money, greatly increasing the risk of shortfall.

Guyton (2004), Guyton and Klinger (2006), Stout and Mitchell (2006), and Klinger (2007) each propose methods for varying the withdrawal amounts in a manner that provides more retirement income at the expense of a smaller estate. Each of these papers provides their own set of "rules." Some rules encompass dynamically changing asset allocation, changing asset mixes, setting withdrawal caps and limits, present value analysis, eschewing inflation adjustments under some circumstances, and so forth. Results indicate that these rules can be effective in providing larger withdrawals during retirement with the same level of safety as the constant withdrawal methods. The complexity of the rules, however, makes implementation difficult even for a knowledgeable retiree.

The methods proposed here [based on Stein & DeMuth (2005), see below] provide a new approach that is easily understood and relatively simple to implement. The simplest recommendation, for example, is merely to reset the withdrawal percentage every five years to a prescribed level. Three methods, labeled Cases A, B, and C, will be compared.

Case A: Case A has a constant (inflation adjusted) annual withdrawal amount that will safely sustain the retiree for 25 or 30 or 35 years. This case is the one most commonly associated with retirement withdrawals and provides a benchmark to which the other cases can be compared.

Case B: Stein and DeMuth (2005) introduce the idea of dynamically readjusting the withdrawal amount during retirement, where the readjustment depends on the number of years remaining. Using a constant 50/50 stock/bond portfolio, they suggest, "Every five years, recalculate your retirement withdrawals as if you were starting anew" (p. 14). This sage advice is based on the observation that the conditions under which withdrawals take place keep changing: every five years of withdrawals means that the portfolio needs to last five fewer years! Stein and DeMuth show (pp. 126–129) that dynamic adjustment of withdrawal amounts (in their fixed portfolio) allows the retiree to take more money out "along the way," but leave a smaller estate. This practice of variable withdrawals with a fixed asset allocation will be referred to as Case (B.)

Case C: The Stein and DeMuth idea is extended here by dynamically (every five years) changing the withdrawal amount and changing the asset allocation mix as well. When both variables are changed together, it may be possible to provide increased safety and larger withdrawal amounts compared to constant withdrawal and constant asset allocation methods.

2. Bootstrap: obtaining the decision parameters

The study proceeds in two phases: the Bootstrap phase and the simulation phase.[1] Both phases used the same dataset. Annual real (inflation-adjusted) rates of return from 1926 through 2005 for stocks (S&P 500) and bonds (intermediate-term U.S. Treasury bonds) are obtained from *Stocks, Bonds, Bills and Inflation 2005 Yearbook, Ibbotson Associates* (Ibbotson Associates, 2006). Real dollars are used throughout; the extra calculation of adjusting for inflation each period is therefore avoided. Taxes and transaction fees are ignored.

2.1. Bootstrap: finding withdrawal amounts, shortfall risks, allocations

The bootstrap repeatedly samples from the historical data using 21 different asset allocations (from 0%/100% stock/bonds to 100%/0% stock bonds in steps of 5%) and 231 different withdrawal amounts (from 2.0% to 25.0% of the starting balance in steps of 0.1%). The data obtained are used to find the probability of running out of money (shortfall risk) after 5, 10, 15, 20, 25, 30, and 35 years of withdrawals. Under each of the conditions, a single replication finds the amount of the portfolio remaining (if any) as annual withdrawals are made from the portfolio; the portfolio value fluctuates yearly because of the varying stock and bond returns. Ten thousand replications are performed in each of the 21 × 231 asset allocation-withdrawal conditions, in each of the seven periods. The portfolio is rebalanced each year to its starting allocation. The bootstrap keeps track of the number (or alternately the proportion over 10,000 trials) of portfolios that run out of money for *each* of the 33,957 (21 stock/bond allocations × 231 withdrawal rates × 7 withdrawal durations) conditions. A more mathematical description of the model and the bootstrap is contained in the Appendix for the interested reader.

2.2. Finding the conditional probabilities

It is possible to sort through this mass of data and find the largest withdrawal that is sustainable with a given shortfall probability, for a given length of time, and for any asset allocation. The process is described next, starting with the simplest example, Case A.

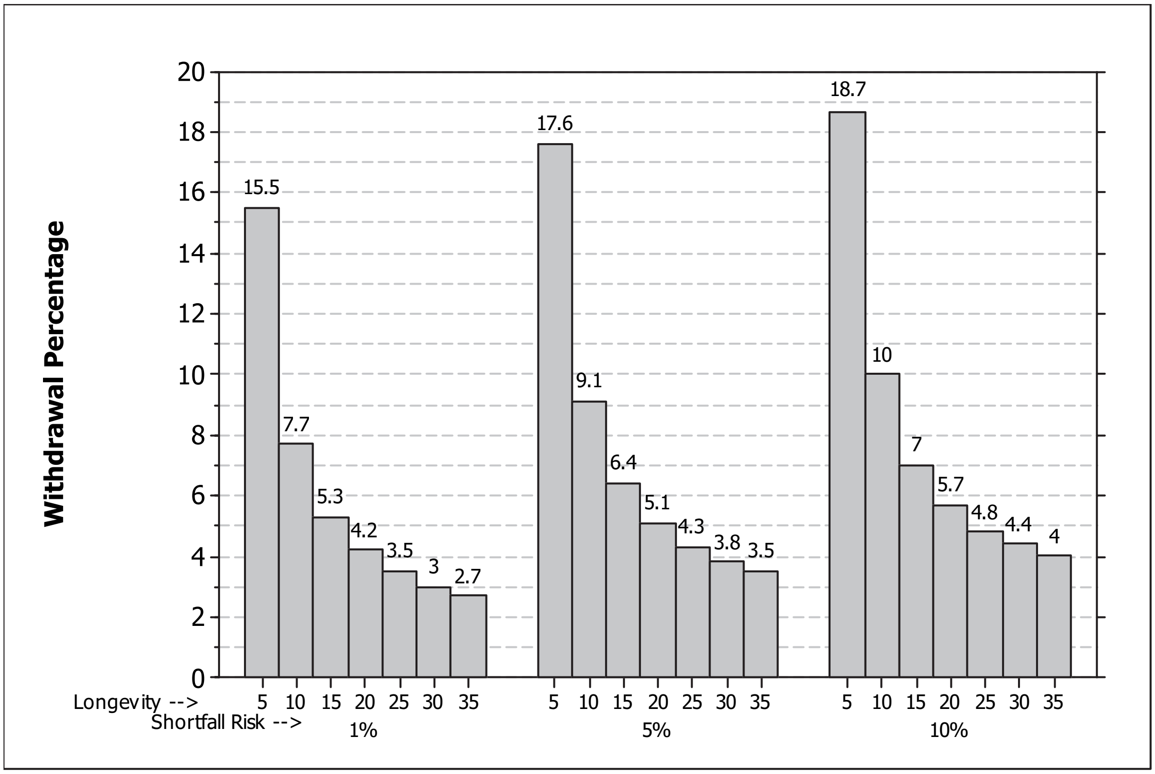


Fig. 1. Maximum sustainable withdrawal percentages and corresponding portfolio stock percentages for shortfall risks of 1%, 5%, and 10% for each of seven withdrawal durations: 5, 10, 15, 20, 25, 30, and 35 years. Withdrawal percentages at top of bar. Stocks and bonds are each 50% of the portfolio.

Case A: Assume a withdrawal period of 30 years with a constant asset allocation of 50%/50% stocks and bonds. From the previously obtained bootstraps results, first remove all asset allocations except 50% stocks and 50% bonds. Next, remove all withdrawal periods other than 30-years. Lastly, remove all portfolio outcomes with more than 1% (100) [or 5% (500), or 10% (1000)] shortfalls. From the portfolios that remain, search for the largest withdrawal amount. If there is more than one such portfolio (ties), select the combination with the fewest shortfalls. The amounts found (3%, 3.8%, and 4.4% of the starting balance in this example) are the Case A amounts for 30-year withdrawals and represent the maximum withdrawal amount that can be sustained for 30-years with a fixed 50/50 stock-bond allocation with 1%, 5%, and 10% shortfall risk.

Case B: The process for Case B is similar, except that withdrawal amounts for 5, 10, 15, 20, 25, 30, and 35 years are retained. Fig. 1 shows maximum withdrawal amounts with a fixed 50/50 asset allocation over the seven different longevities at 1%, 5%, 10% shortfall risk. Fig. 1 is consistent with the scenario presented in Stein and Demuth (2005), where the withdrawal amount changes every five years, but the asset allocation remains fixed. Note that the longer the portfolio needs to last, the smaller the withdrawal percentage is; that is, the bar height falls as the length of the withdrawal period increases. Also, the bars, for any given withdrawal length, tend to be taller as the shortfall risk increases. If the retiree is willing to take a 10% risk of running out of money over 20 years, 5.7% can be

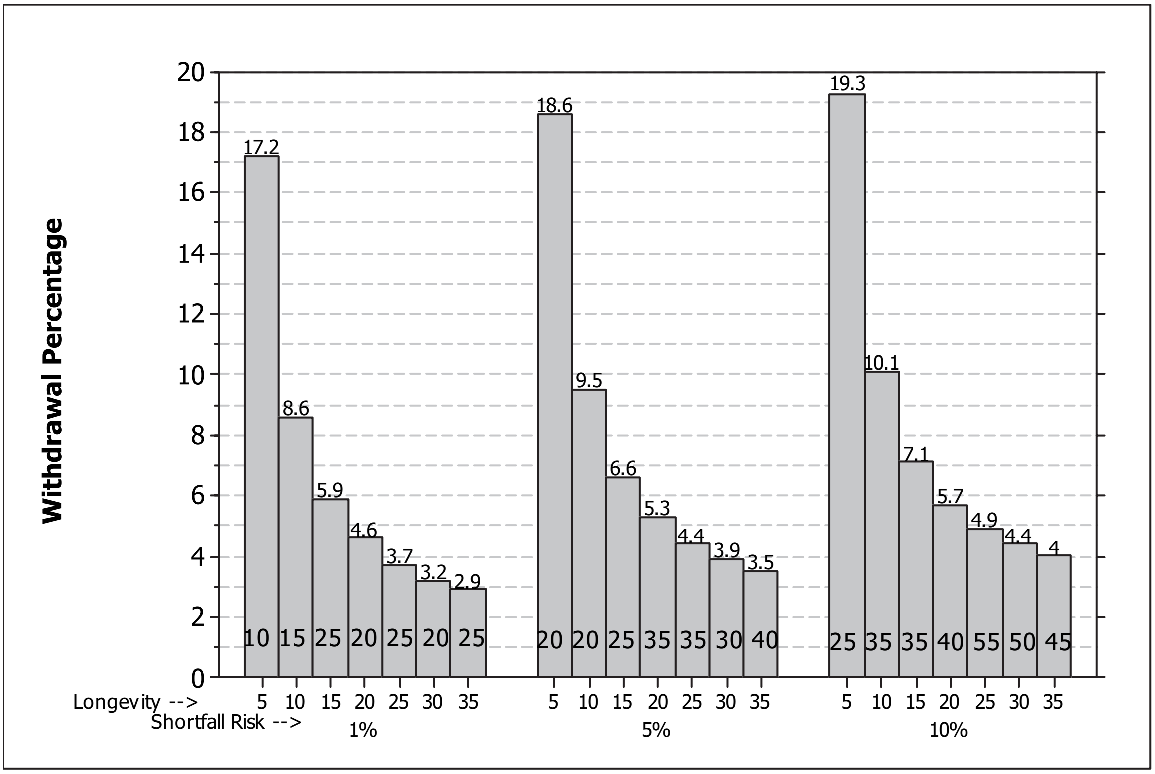


Fig. 2. Maximum sustainable withdrawal percentages and corresponding portfolio stock percentages for shortfall probabilities of 1%, 5%, and 10% for each of seven withdrawal durations: 5, 10, 15, 20, 25, 30, and 35 years. Withdrawal percentages at top of bar. Number inside a bar shows the percentage of stock required.

withdrawn; if the retiree is only willing to take a 1% shortfall risk, only 4.2% of the portfolio can be withdrawn.

Case C: The investigation is extended by looking as the optimal withdrawal amount across many different asset allocations, not a fixed 50/50 allocation. For example, (1) select the subset of portfolios for which 5% or less ran out of money after 25 years. (2) Find the largest withdrawal amount for these surviving portfolio(s). (3) If several portfolios are in this subset, choose the outcome with the fewest shortfalls and note its stock/bond allocation. This process provides the maximum withdrawal amount (and the matching asset allocation) with a probability of shortfall of *5% or less*. The resulting withdrawal amount, with its corresponding stock/bond allocation, is the highest withdrawal that can be maintained without running out of money after 25 years with at least 95% success.

Fig. 2 shows the withdrawal amount for each of the five-year longevities at each shortfall risk. There are three clusters (groups) of seven bars. Each cluster represents 1%, 5%, and 10% shortfall risk, respectively. The leftmost bar in each group represents successful withdrawals for 5 years; the next bar represents successful withdrawals for 10 years, and so on. The height of the bar (shown at the top of each bar) shows the largest sustainable withdrawal percentage (inflation adjusted constant amount as a percentage of the starting portfolio). Lastly, the number inside each bar shows the proportion of stock required to achieve this withdrawal rate at the given shortfall risk.[2]

> ***Example 1:*** The first bar in the graph indicates that if the portfolio needs to last five more years, then by allocating 10% of the portfolio to stocks (and 90% to bonds), withdrawing 17.2% of the current portfolio (in real terms) each year is sustainable 99% of the time.
> ***Example 2:*** The fifth bar from the left in the middle cluster indicates that if the portfolio needs to last 25 more years, then by allocating 35% of the portfolio to stocks (and 65% to bonds), withdrawing 4.4% of the current portfolio (in real terms) each year is sustainable 95% of the time.

These results are consistent with those found in the various studies of Bengen, Cooley et al., Spitzer et al., and others. Other studies often used different data, so there will not be a perfect correspondence among these results and all other studies on the topic. For example, Cooley et al. used corporate bonds in their studies, not intermediate term U.S. Government bonds. Generally, if a retiree wishes larger withdrawal amounts (given the same withdrawal longevity), shortfall risk increases and so does the stock proportion to accommodate the larger withdrawal amount. For example, with a 30-year withdrawal, 3.2% can be withdrawn with a 1% shortfall risk, 3.9% with a 5% shortfall risk, and 4.4% with a 10% shortfall risk. The proportion of stock required to sustain each withdrawal rate increases from 20% to 30% to 50% respectively. There is a tendency for the optimal portfolio to contain more stock the longer it needs to last. The optimal proportion of stock also rises with increases in shortfall risk.

With information in Figs. 1 and 2, it is possible to obtain "conditional probabilities" along the following lines: "If my portfolio needs to last for 10 more years, how much can I withdraw (and what asset allocation do I need) with at least 95% success?" Suppose that the retiree's current portfolio is valued at $500,000 and needs to last 10 more years; the retiree elects a 5% chance of running out of money within 10 years. In Fig. 2 [Cluster 2, Bar 2], the retiree can be 95% certain of not running out of money in the next 10 years by withdrawing $47,500 (9.5%) each year for the next 10 years using the 20% stock/80% bond allocation. Alternately using Case B (Fig. 1, Cluster 2, Bar 2), the retiree would choose to withdraw only $45,500 (9.1%).

Given these conditional probabilities, it is possible to make midcourse corrections every five years up to the last five years of the portfolio. Another example might help to illustrate the principle. Based on a retiree's genes and general health, her expected longevity is 25 years. Let's add on five more years for safety's sake and assume that she will need money for 30 years. Leaving a sizable estate is *not* her goal; safely withdrawing as much money as possible each year is. Using Case C (Fig. 2), she initially accepts a 5% chance of running out of money. She will maintain a 30/70 stock/bond allocation and begin her retirement by withdrawing 3.9% of her initial balance (Cluster 2, Bar 6), continuing to take this same inflation-adjusted amount for five years. At the end of five-years, she evaluates her current situation; she only has to maintain her portfolio for 25 more years, not 30. She modifies the stock/bond allocation to 35%/65% and now withdraws 4.4% of the remaining balance (Fig. 2, Cluster 2, Bar 5) (adjusting this amount for inflation each year) for the next five years. (The dollar amount of her withdrawal could be smaller or larger in the second step compared to the first step. Even though the percentage amount has risen, the portfolio amount has changed. 4.4% of her current portfolio balance could be more or less than 3.9% of her initial balance.) Every five years, she corrects her asset allocation (if necessary) and adjusts her withdrawal amount. If her health condition or her risk tolerance changes, she can modify her behavior accordingly. If instead she chooses to maintain a constant 50/50 asset allocation throughout (Case B), she would initially withdraw 3.8% of the starting balance; with 25 years remaining in step 2, she would change her withdrawals to 4.3% of the current balance.

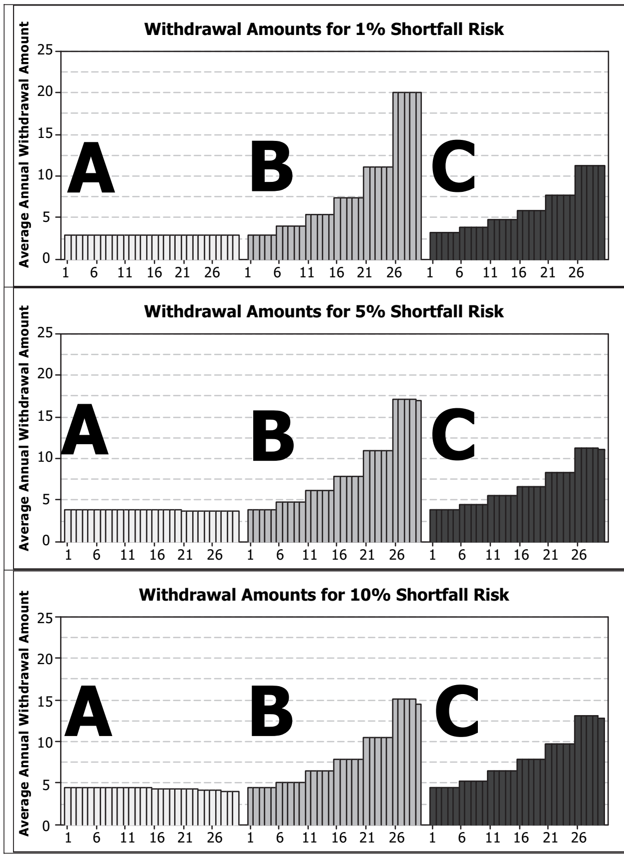
3. Simulation I: testing the strategies

Do these Variable Withdrawal strategies (with or without asset allocation changes) afford more safety and/or provide greater total withdrawals than the usual strategy of withdrawing a constant (real) amount (Case A)? This question will be considered by simulating 10,000 30-year sequences of the historical data and noting (1) the average withdrawal amount in each year, (2) the percentage of shortfalls, (3) the timing of shortfalls (whether they occur in year 12, say, vs. year 30), and (4) the balance remaining. Three withdrawal strategies (Cases A, B, and C) for three shortfall risks levels (1%, 5%, and 10%) will be examined. Case A is, of course, the benchmark case, representing the usual constant (real) withdrawal. Case B is a variant of the Stein-Demuth withdrawal suggestion. Case C is an extension that allows not only the withdrawal amount to be dynamically modified, but also the underlying asset allocation.

There are three graph/table pairs in Fig. 3, representing 1% shortfall risk, 5% shortfall risk, and 10% shortfall risk, respectively. Each graph shows the average withdrawal amount for each year obtained during the 30-year period for Case A, Case B, and Case C. Each table shows the Maximum Average Withdrawal achieved, the Average Withdrawal amount over 30 years, the percentage (out of 10,000 sequences) that ran out of money (shortfall %), the earliest year that a portfolio was observed to fail, and the Average Balance Remaining at the end of 30 years. Because the starting balance is $100, all the withdrawal amounts may be interpreted as a percentage of the starting balance; for example, $3.70 is equivalent to 3.7% of the starting amount.

Several general observations can be made:

1. The *calculated* shortfall risks for all three cases agree with the a priori risk; that is, approximately 1% of the portfolios in the "1% Shortfall Risk" category run out of money, 5% in the "5% shortfall risk" category run out of money and the same for the 10% example.
2. Case A provides the smallest average withdrawal amounts and Case B always provides the largest average withdrawal amount for any given shortfall risk.
3. Cases B and C tend to *increase* the withdrawal amount over time.
4. Case A always ends the 30-years with an average Balance Remaining of at least twice the size of the starting balance. To the extent that spending the money and not



Statistics	A	B	C
Maximum:	3.00	20.04	11.27
Average	3.00	8.51	6.13
Shortfall %	0.77%	0.80%	0.85%
Earliest Runout Year	17	30	30
Balance Remaining	309.51	59.05	16.42

A – Fixed Withdrawal and Fixed Allocation (50/50)
B – Variable Withdrawal and Fixed Allocation (50/50)
C – Variable Withdrawal and Variable Allocation

Statistics	A	B	C
Maximum:	3.80	17.17	11.29
Average	3.77	8.43	6.71
Shortfall %	4.46%	4.78%	5.25%
Earliest Runout Year	14	29	29
Balance Remaining	249.42	33.25	12.32

A – Fixed Withdrawal and Fixed Allocation (50/50)
B – Variable Withdrawal and Fixed Allocation (50/50)
C – Variable Withdrawal and Variable Allocation

Statistics	A	B	C
Maximum:	4.40	15.09	13.17
Average	4.31	8.24	7.81
Shortfall %	9.99%	9.50%	9.61%
Earliest Runout Year	12	29	29
Balance Remaining	205.90	22.82	12.36

A – Fixed Withdrawal and Fixed Allocation (50/50)
B – Variable Withdrawal and Fixed Allocation (50/50)
C – Variable Withdrawal and Variable Allocation

Fig. 3. Average withdrawal amounts by year over 30 years for strategies A, B, and C at shortfall risks of 1%, 5%, and 10%. Data obtained by simulating 10,000 30-year periods. X-axis numbers are ordinal years.

building an estate was the goal, Case A is not a success. Cases B and C do better, providing more money *during* retirement but providing a smaller estate.

5. The reader's eye cannot discern in the graph the earliest years in which runouts occur, so that information is provided on the table. For "1% shortfall risk," Case A had a shortfall as early as the 17th year of withdrawals, leaving 13 years unfunded. Cases B and C did not have their first shortfall until the last year of withdrawals. The frequency of shortfalls for Case A increases over time, with most shortfalls occurring near the end of the withdrawal process. For Case A, as shortfall risk increases from 5% to 10%, the first shortfall occurs earlier and earlier, in year 14 and in year 12, respectively. Cases B and C both experience their *first* shortfall in the 29th year of withdrawals for these shortfall risks.
6. As shortfall risk increases from 1% to 5% to 10%, the average withdrawal amounts for Cases A and C rise, but for Case B, the average amount falls slightly.
7. The average withdrawal amounts are smaller for Case C than for Case B. Case C has allocations of more than 50% stock only once; for 10% shortfall risk at 25 years. Because Case C portfolios have less stock, Case C portfolios tend to be less volatile and consequently the variation in withdrawal amounts during each of the withdrawal

years tends to be smaller for C than for B. Case C tends to provide smaller withdrawals than B but with less variability from one five-year switching point to the next five-year switching point.

8. The last five years of withdrawals are the riskiest. The portfolio size at the beginning of the last five-year withdrawal period may be much larger or much smaller than its initial $100 starting value. Additionally, the withdrawal percentage for both Case B and Case C is large. These combined effects imply that there will be considerable variation in withdrawal amounts among the 10,000 samples during the last five years. The standard deviation of withdrawals for 1% and 5% shortfall is more than twice as big for Case B compared to Case C. For 10% shortfall risk, Case B withdrawal standard deviation is 29% larger than for Case C during the last years. Case C may not provide as large withdrawals, but it does provide less extreme amounts.

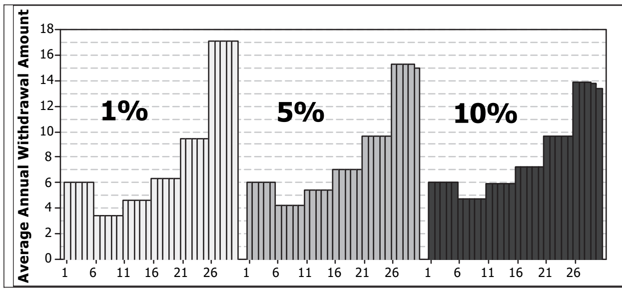
On average, the variable withdrawal methods B and C provide *much* more money during retirement than Case A and, *if* they run out of money, they tend to do so very late in the withdrawal process.

One final note: the data presented in Fig. 3 represent *average* withdrawals, not actual withdrawals. The graphs and statistics shown are calculated over 10,000 30-year cycles. Cases B and C make withdrawals based on a percentage of the portfolio value at the beginning of each five-year period. As the portfolio value fluctuates up and down, so does the withdrawal amount. Withdrawals using Case B or Case C may be very large if the portfolio is doing well, and at other times very small. The retiree must be apprised that while the *average* withdrawal amount using Case B or Case C is higher than Case A, the withdrawal amounts they experience may be markedly different from the average. While the average withdrawal amount using Case B or Case C is higher than Case A, actual withdrawal amounts experienced may be markedly different from the average.

4. Simulation II: big withdrawals upfront?

The results from Simulation I suggest that Case B results are in some sense better than Case C and are easier to implement since the asset allocations do not change. While Case B is a clear improvement over Case A and forcefully shows that more money can be withdrawn during retirement than is generally recommended, Case B withdraws increasing amounts over the retirement horizon, not decreasing amounts. Many retirees have the desire to "enjoy" retirement with travel and entertainment early in retirement when their health and energy allow it. They anticipate that their spending needs will decline over time, barring extended (insurable) health problems. What many retirees would like to do is reverse the sequence of the bars in Fig. 3 and have large withdrawals early with the amounts tapering off over time.

The conditional probabilities inherent in Case Bs methodology can be used to accomplish this very desirable outcome! The following simulation is an example only. It is not meant to be an exhaustive study, merely to suggest that larger withdrawals early on are possible and survivable. Suppose that the retiree wants to withdraw 6% during the first five years of



Statistics	1%	5%	10%
Maximum:	17.11	15.25	13.83
Average	7.84	7.92	7.88
Shortfall %	0.80%	4.78%	9.50%
Earliest Runout Year	30	29	29
Balance Remaining	50.41	29.54	20.91

Fig. 4. Case B withdrawals for 1%, 5%, and 10% shortfall risk. There were 6% withdrawals during the first five years.

retirement . . . much too much by current safety standards and almost certainly not sustainable over 30 years.

Simulation II generates 10,000 30-year withdrawals for Case B only, for the three shortfall risks. Six percentage withdrawals were implemented during the first five years and then the withdrawal amounts revert to those shown in Fig. 1 for each succeeding five-year period. Fig. 4 and its accompanying table demonstrate that safely withdrawing large amounts of money early in retirement is possible. While the average withdrawal amount has declined compared to Case B in Fig. 3, the retiree has managed to increase withdrawals in the first five-years by a significant amount. (In the 1% shortfall case, the withdrawal amount doubled.) Retirees, possibly with help from a financial advisor, can determine the trade-off of other early withdrawals. *The essential point is that the trade-off exists*.

5. Summary and conclusions

First, a bootstrap is used to obtain the probabilities of running out of money during retirement under a wide set of circumstances: varying retirement lengths, varying withdrawal amounts, and varying asset allocations. Probabilities are also obtained given varying retirement lengths, varying withdrawal amounts and a *fixed* asset allocation of half stocks and half bonds.

Second, a simulation determines what happens to the pattern of withdrawals over a 30-year period for three cases, A, B, and C. Case A withdraws a constant amount from a portfolio with 50% stocks and 50% bonds. Case B allows the retiree to re-evaluate and readjust withdrawal amounts every five years from a stock/bond allocation fixed at 50%/50%. Case C is an extension of Case B, with asset allocations that also change.

Third, a further simulation demonstrates that it is possible to increase the amount of early retirement withdrawals without increasing shortfall risk. The cost of doing so is a reduction in average withdrawals over the 30-year span and a reduction in Balance Remaining.

Results from the simulations strongly suggest that Case B and Case C, the "course correction strategies," are much less likely to run out of money early in retirement than the constant withdrawal-strategy. Cases B and C are more likely to provide larger total withdrawals over the retirement span than is Case A. Case B and Case C are *not* designed to provide a large estate; they are designed to provide withdrawals in amounts as large as possible while making it very unlikely that the portfolio will prematurely run dry. One of the advantages of Case B and Case C lies in the simplicity of implementation. For Case B, every five years, the retiree need only modify the withdrawal amount based on the current portfolio size.

The process described here can undoubtedly be improved upon by investigating the effect on withdrawal amounts when (1) adding more varied assets to the mix (e.g., small-caps, midcaps, international stocks, TIPS), (2) investigating the affect of not rebalancing the portfolio, ala Spitzer and Singh (2007), (3) incorporating adjustments for mortality as suggested by Stout and Mitchell (2006), (4) looking at the effect of other fixed stock/bond allocations, such as 60/40 or 70/30, and (5) further refining the early withdrawal amounts to provide larger withdrawals earlier in retirement.

Appendix

The following variables are used to define the model and the bootstrap process:

t = the year in which the withdrawal occurs; $t = 1, 2, \ldots T$,
S_t = the size of the portfolio in year t. The starting amount of the portfolio is \$100. (S_0 = \$100)
w = the withdrawal amount as a percentage of S_0 at the end of each year (231 different rates, ranging from 2.0% to 25% in steps of 0.1)
r_{st} = annual real (inflation-adjusted) rate of return on stocks at period t,
r_{bt} = annual real (inflation-adjusted) rate of return on bonds at period t,
λ = the proportion of the portfolio designated for stocks (21 different allocations, ranging from 0 to 1.00 in steps of 0.05).

Allocations are made only between stocks and bonds, thus $(1-\lambda)$ is the proportion of the portfolio allocated to bonds. For example, a λ of 0.30 means that 30% of the portfolio is allocated to stocks and 70% is allocated to bonds. Rates of return r_{st} and r_{bt} vary with t and are selected from the Ibbotson data described above.

The value of portfolio at any time $t > 0$ is given by:

$$S_t = S_{t-1}\{1 + \lambda r_{st} + (1-\lambda) r_{bt}\} - w \tag{1}$$

that depends on (1) how long the withdrawal process lasts (t), (2) the amount of the withdrawal (w), (3) the rates of return for stocks and bonds (r_{st} and r_{bt}), (4) the stock/bond allocation (λ), and (5) (trivially) the starting amount, (S_0). Rebalancing the portfolio to λ stock and $(1-\lambda)$ bond proportions is implicit in Eq (1). Success is defined as $S_t > 0$. If S_t is negative (a shortfall), S_t is set equal to zero, and the shortfall count for that (λ, t, w) condition is incremented by 1.

There are 21 different asset allocations and 231 different withdrawal rates for a total of (21 asset allocations $\times$ 231 withdrawal rates =) 4851 asset allocation/withdrawal combinations for T = 5, 10, 15, 20, 25, 30, 35. The following steps are repeated 10,000 times for each of these combinations.

Set t = 1, S_0 = \$100. Select the starting value of "w," the withdrawal amount.

a. Randomly generate a number between 1926 and 2005 (inclusive), which is the "current year" subscript, "t." Obtain r_b and r_s for this "year." (This retains the asset class cross-correlations.)
b. Compute S_t of Eq. (1). If $S_t \leq 0$, increment the shortfall count for this (λ, w) combination.
c. Increment t by 1. If t = 5, 10, 15, 20, 25, 30, or 35, save the count of shortfalls for later analysis, otherwise, go to step a.

The steps above constitute a single iteration.

Notes

1. Bootstrapping is a Monte Carlo-like method; samples (with replacement) are repeatedly taken from an available dataset.
2. Each bar in Fig. 2 comes from a three-dimensional (withdrawal percentage, asset allocation, shortfall risk) "solid" object. Because each of the three dimensions was generated in discrete increments, the object is rather a meshwork or wirework than a solid. The surface is not smooth, but rather dimpled or warty with bumps and cavities. For a shortfall risk of 5%, the optimal stock percentage seems to increase, decrease, and then increase again as the longevity requirement increases. This outcome may only be depicting the bumpiness of the surface. The asset allocation percentages shown in each bar may not be the *optimum optimorum*, but they serve as useful guidelines regardless.

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John J. Spitzer, Ph.D., is emeritus professor of economics at the State University of New York, Brockport, NY, where he taught courses in economics and statistics. His research interests include econometric modeling, asset allocation, and retirement issues.

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