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Optimal asset allocation in the presence of nonfinancial assets

Vladyslav Kyrychenko

MFC Global Investment Management, Toronto, Ontario M4W 1E5, Canada

Abstract

In this paper, a comprehensive mean-variance model, which includes all major nonfinancial assets (housing, human capital, and private business) besides financial ones is calibrated with empirical data to generate optimal asset allocations between stocks, bonds, and cash. The model is used to investigate the effect that each of the nonfinancial assets has on the optimal mix of financial assets. Among others, it generates the optimal portfolio allocation for investors working in different industries and living in different cities. The model is also able to rationalize a popular investment advice recommending decreasing share of stocks in financial portfolios with increasing age.

JEL classification: G11

Keywords: Asset allocation; Mean-variance model; Nonfinancial assets

1. Introduction

The importance of nonfinancial assets for optimal portfolio allocation has been known in the financial economics literature at least since the works of Mayers (1972) and Fama and Schwert (1977), who among the first investigated the effect of human capital on portfolio choice. Even though nonfinancial assets are rarely traded (houses or private business) or not traded at all (human capital), they are included into portfolio choice because their returns are correlated with returns on financial assets and because they are by far the largest assets in households' possession. As reported in Table 1 (see Section 3.1.), based on the 2004 Survey

* Tel.: +1-416-852-1173; fax: +1-416-926-5432.
E-mail address: vkyrychenko@schulich.yorku.ca.

of Consumer Finances, three major nonfinancial assets—human capital, housing, and private business—constituted almost 82% of the total assets for the average household in 2003.[1]

It was not until the last decade, however, that the asset allocation models incorporating nonfinancial assets, such as human capital, private business, and housing, became really popular. Human capital (in the form of labor income) is taken into account in the models of Campbell and Viceira (2002), Heaton and Lucas (2000b), Gomes and Michaelides (2004). Human capital combined with life insurance enters in the asset allocation models of Huang, Milevsky and Wang (2005) and Chen, Ibbotson, Milevsky and Zhu (2006). Cocco (2005) and Waggle and Johnson (2003) include house value in their asset allocation models. Yao and Zhang (2005) incorporate both labor income and housing. The value of proprietors' business enters in the models of Heaton and Lucas (2000b) and Willen (2003). Faig and Shum (2002) model portfolio selection for investors with "personal illiquid projects," which can include housing or private businesses.

These models, however, usually operate with a limited set of assets, such as one risky asset (stock), one riskless asset (cash or riskless bond), and one nonfinancial asset. This is understandable since those models are usually dynamic. While being more realistic than the static models of portfolio choice, they would have been extremely difficult or even impossible to solve if they included more asset classes.

In this paper, I employ the mean-variance asset allocation model, augmenting the familiar financial assets—stocks, bonds, and cash—with all major nonfinancial assets, such as housing, human capital, and private business. The model generates the optimal weights of financial assets, while the weights of the nonfinancial assets are taken as given, based on household empirical data from the Survey of Consumer Finances. The model can be readily re-estimated for investors with different weights. Asset returns, the means, standard deviations, and correlation matrix, which enter the model as parameters, are taken from the actual historical data and can vary according to each investor's circumstances. In fact, I do this in sections 4.3 and 4.4. Historical asset returns come from the following sources: (1) Ibbotson Associates—historic returns on financial assets; (2) National Income and Product Accounts (NIPA)—aggregate per-capita wages, salaries, and proprietor income needed to calculate the returns on human capital and private business; (3) the Office of Federal Housing Enterprise Oversight (OFHEO)—quarterly U.S. Housing Price Index for the calculation of the returns on housing.

Although this one-period model is less sophisticated than dynamic asset allocation models, it partially compensates this shortcoming by including all major nonfinancial assets, something that no dynamic model has been able to do. Although mean-variance approach has been used in household portfolio choice models that include nonfinancial assets,[2] this paper is the only study, to my knowledge, which models optimal portfolio allocation for stocks, bonds, and cash, in the presence of *all* major nonfinancial assets.

The augmented model generates optimal portfolio allocations for investors: (1) with and without housing, human capital, and private business; (2) working in different industries; (3) residing in different cities. It is also able to shed some light to the asset allocation puzzle, identified by Canner, Mankiw and Weil (1997). This puzzle refers to the systematic inconsistency between two asset allocation rules: one based on the mutual fund separation theorem (Tobin, 1958), and the other based on the advice of finance practitioners. The mutual fund separation theorem, on which modern portfolio theory is based, says that for investors whose portfolios consist of equity, bonds, and cash (the latter being a proxy for the risk-free asset), equity to bonds ratio should be constant and do not depend on investors' level of risk-aversion. Financial advisors, however, recommend that the proportion of stocks to bonds should decline with investor's increasing degree of risk-aversion. Another common piece of practitioners' advice is to decrease the proportion of stocks in a portfolio with investors' age. The mean-variance model employed in this paper is able to rationalize both pieces of the popular investment advice. However, the recommendation to lower stocks-to-bonds ratio (hereafter SBR) with increasing investors' risk-aversion is supported by the model only when borrowing constraint is binding.

The rest of the paper is organized as follows. Section 2 presents the setup of the mean-variance model. Section 3 describes the empirical data used to calibrate the model. Section 4 presents the results of several versions of the model. Section 5 concludes the paper.

2. Model's setup

According to the mean-variance model, an investor minimizes portfolio variance subject to the target expected return of the portfolio. The investor's whole portfolio is composed of the financial assets: stocks, bonds, and cash, as well as the nonfinancial ones: housing, human capital, and private business. In this model, I assume the shares of nonfinancial assets to be exogenous, because demand for them is primarily formed by factors other than investment goals. For example, the size of investment in residential housing is mainly determined by the agent's family size, location, and individual preferences rather than by investment demand. Some studies report that entrepreneurs overinvest in their private business (Gentry & Hubbard, 2004), given that returns on it are about the same as the returns on public equity (Moskowitz & Vissing-Jorgensen, 2002), while its risk is significantly higher. This suggests that the amount invested in one's business is at least partially determined by motives other than pure investment.

Therefore, the total portfolio variance can be minimized by selecting the optimal shares of stocks, bonds, and cash. Two restrictive assumptions that reflect the investment reality for most individual investors are imposed on the model. First, it is assumed that an investor cannot go short on bonds and/or stocks. Second, the share of cash in his or her financial portfolio cannot be less than −100%.[3]

The model's set up is as follows:

$$\begin{array}{lll} & & \vec{w} * \vec{u}' = E(R) \\ & & \vec{w} * \vec{1}' = 1 \\ Min_{w_i} \frac{1}{2} \vec{w} V \vec{w}' & \text{s.t.} & w_c \geq -1 \\ i = c,b,s & & w_b \geq 0 \\ & & w_s \geq 0 \end{array} \tag{1}$$

where:

$\vec{w}$ —vector of portfolio weights of cash (c), bonds (b), stocks (s), housing (h), human capital, or capitalized labor income (l), and private business or entrepreneurship (e);

$\vec{\mu}$ —vector of expected returns on the above assets;

V—variance-covariance matrix of assets returns;

$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$, where ρ_{ij}-coefficient of correlation between the returns of asset i and asset j and σ_{ij} is the standard deviation of returns on asset i.

$E(R)$—target level of portfolio expected return.

The model's parameters—weights of nonfinancial assets in the total portfolio, means, standard deviations and correlation coefficients of assets' historic returns are estimated from empirical data, which is described in the next section.

The model is solved by creating the Lagrangian and writing out a set of first order conditions. The resulting system of five linear equations with five unknowns is then solved by Matlab for optimal portfolio weights w_c, w_b, w_s of cash, bonds, and stocks correspondingly (presented in the appendix).

3. Empirical estimation of model's parameters

3.1. Average shares of financial and nonfinancial assets

The relative weights of the nonfinancial assets in the total investors' portfolios are estimated based on the latest (2004) Survey of Consumer Finances (hereafter SCF). The SCF, sponsored by the Federal Reserve Board is a triennial survey of U.S. households' income, assets, liabilities, and demographic characteristics; it is considered the most detailed and representative source of financial information on U.S. population. In particular, it contains the households' holdings of assets, both financial (stocks, bonds, cash, mutual funds, pension plans), and nonfinancial (real estate, private business, vehicles) as well as liabilities. It also provides information on households' income and its sources (salaries, wages, proprietary income, pensions, social security benefits, etc.) as well as individual characteristics (age, occupation, education, degree of risk-aversion, etc.) In the 2004 survey, 4519 households were interviewed. As SCF intentionally oversamples wealthy households, it contains a sample of weights to compensate for this, which are taken into account in calculation of portfolio weights in this paper.

Table 1 presents the average shares of households' major financial and nonfinancial assets in the total assets portfolio.

Because the focus of this article is the effect of nonfinancial assets on financial asset allocation, the sample is restricted to only those households that possess nontrivial amounts of both financial and nonfinancial assets. For this reason, in all subsamples, the average age of the head of household and his or her spouse is restricted to be under 70 and the household's financial net worth has to be at least $1,000. The first column contains the data on the households where, in addition to the aforementioned restrictions, the income from salaries and wages is at least $1,000, the value of their private business is at least $1,000, and their house value is at least $10,000. The next three columns present the average percentage

Table 1 Average shares of financial and nonfinancial assets of U.S. households (2004 Survey of Consumer Finances)

	Have all nonfinancial assets	All but private business	All but human capital	All but house	Age*				
					≤30	31–40	41–50	51–60	61–70
House equity	9.0%	11.2%	22.8%	0.1%	4.6%	6.7%	9.3%	13.9%	21.2%
Private business	29.7%	0.0%	42.6%	20.8%	1.0%	5.4%	7.9%	14.5%	19.3%
Human capital	42.9%	73.3%	0.0%	67.2%	90.5%	82.2%	69.4%	46.5%	13.5%
Total nonfinancial assets	81.5%	84.5%	65.4%	88.0%	96.2%	94.3%	86.6%	75.0%	53.9%
Stocks	10.0%	8.5%	18.8%	4.9%	1.7%	2.9%	7.6%	13.9%	23.6%
Bonds	3.4%	2.4%	7.7%	1.5%	0.5%	0.7%	2.0%	4.1%	10.0%
Cash	5.1%	4.6%	8.0%	5.6%	1.6%	2.0%	3.9%	7.0%	12.4%
Total financial assets	18.5%	15.5%	34.6%	12.0%	3.8%	5.7%	13.4%	25.0%	46.1%
Total assets	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
N of obs.	882	1271	215	43	135	303	459	424	173

*For households with nontrivial amounts of five asset classes (including nonentrepreneurs).

asset shares for the households with: (1) no private business; (2) no human capital; and (3) no house. Finally, the last five columns display the average asset shares for investors of different age groups. All other restrictions mentioned above remain in place except for the business value (this restriction is removed to maintain a reasonable sample size for the age group under 30).

The value of human capital is calculated as the present value of labor income flow discounted at the 5% discount rate.[4] It is assumed that a household will receive current wages and salaries until the average age of the head of household and his or her spouse reaches 65 (or, if their current age is between 65 and 70, till the age of 70).

3.2. Empirical data on assets' returns

Expected returns, standard deviations, and correlation coefficients for all assets in the model are set to their historic values. All returns are converted to their real values by discounting them with the Consumer Price Index.

Monthly returns on the S&P 500 index (stocks), Lehman Brothers aggregate U.S. bond index (bonds), and T-bills (cash), are provided by Ibbotson Associates for the 1926 through 2005 period (except for the bond index, the returns on which are available for 1976 through 2005 period). It is worth noting that cash, as represented by T-bills, has a nonzero standard deviation. Therefore, there is no truly riskless asset in the model presented in this paper. Monthly returns on financial assets are compounded to produce annual returns to match the annual frequency of the available data on labor and entrepreneurial income.

For the measure of human capital returns I use the approach of Heaton and Lucas (2000a), with the returns on human capital approximated by the rate of labor income change:

$$R_{L,t} = L_t/L_{t-1}$$

Table 2 Means, standard deviations and correlation coefficients of historic returns on financial and nonfinancial assets (1976–2004 annual returns)

	Mean	Standard deviation	Correlation coefficients					
			Stocks	Bonds	Cash	Housing	Human capital	Private business
Stocks	9.4%	15.4%	1	0.359	0.274	−0.014	0.155	0.145
Bonds	4.8%	8.3%		1	0.653	0.157	0.599	0.185
Cash	1.8%	2.4%			1	−0.183	0.414	0.064
Housing	1.6%	3.4%				1	0.412	0.353
Human capital	0.8%	1.7%					1	0.402
Private business	1.7%	6.1%						1

Similarly, returns on private business are assumed to be equal to the rate of entrepreneurial income change:

$$R_{E,t}=E_t/E_{t-1}$$

The source for aggregate per-capita wages, salaries, and proprietor income for 1929 through 2004 period is the U.S. Department of Commerce, Bureau of Economic Analysis, NIPA. NIPA provides income data for the whole U.S. economy as well as for specific industries. Finally, the returns on housing are calculated using the quarterly U.S. Housing Price Index from the OFHEO for the period of 1975 through 2005.

3.3. Statistical properties of historic returns on financial and nonfinancial assets

The means, standard deviations, and correlation matrix of returns on stocks, bonds, cash, housing, human capital, and private business are presented in Table 2.

Real returns on human capital have a mean of 0.8% and standard deviation of 1.7%. The coefficient of its correlation with returns on stocks, bonds, and T-bills are $\rho_{ls} = 0.155$, $\rho_{lb} = 0.599$, and $\rho_{lc} = 0.414$, respectively. Private business, on average, generate real returns of 1.7% with standard deviation of 6.1% and correlations with stocks, bonds, and T-bills of $\rho_{es} = 0.145$, $\rho_{eb} = 0.185$, and $\rho_{lc} = 0.065$. The relatively low volatility of human capital and entrepreneurial returns is because of the fact that these returns are estimated by the aggregate per-capita income process as opposed to the individual one. Judging from the correlation matrix, human capital and private business returns have more resemblance with returns on bonds than with returns on stocks. This characteristic proves to be crucial when modeling the optimal stocks-to-bonds ratio. It is also notable that entrepreneurial returns are much less correlated with returns on bonds and cash than returns on human capital.

Real annual returns on the average U.S. housing price index have the mean of 1.6% and standard deviation of 3.4%. It is the only nonfinancial asset whose returns are negatively correlated with returns on stocks ($\rho_{hs} = -0.014$) and cash ($\rho_{hc} = -0.183$). It can be concluded from this data that average homeowner, ceteris paribus, should favor stocks and cash more than bonds.

Table 3 Optimal composition of financial portfolio–the effects of nonfinancial assets

		Expected return of the total asset portfolio					
		1.5%	2.0%	2.5%	3.0%	3.5%	4.0%
Base model (all assets)	% cash	86%	48%	−9%	−67%	−100%	−100%
	% bonds	0%	2%	37%	72%	67%	7%
	% stocks	14%	50%	72%	95%	133%	193%
	Stocks-to-bonds ratio	N/A	25.1	2.0	1.3	2.0	25.8
No housing	% cash	55%	−12%	−99%	−100%	N/A	N/A
	% bonds	0%	22%	75%	0%	N/A	N/A
	% stocks	45%	91%	124%	200%	N/A	N/A
	Stocks-to-bonds ratio	N/A	4.1	1.7	N/A	N/A	N/A
No human capital	% cash	N/A	88%	65%	34%	4%	−27%
	% bonds	N/A	0%	7%	26%	44%	63%
	% stocks	N/A	12%	28%	40%	52%	64%
	Stocks-to-bonds ratio	N/A	N/A	4.0	1.6	1.2	1.0
No private business	% cash	62%	12%	−53%	−100%	−100%	N/A
	% bonds	0%	16%	55%	66%	0%	N/A
	% stocks	38%	72%	98%	134%	200%	N/A
	Stocks-to-bonds ratio	N/A	4.6	1.8	2.1	N/A	N/A

4. Model's results

4.1. Base model results and marginal effects of nonfinancial assets

This section presents the output of several versions of the mean-variance model. The base model generates the optimal asset allocation for a household that has positive amounts of all major nonfinancial assets, that is, housing, human capital, and private business. Exogenous shares of these assets that are inputs to the model, are those reported in the first column of Table 1. The model produces the optimal portfolio weights of stocks, bonds, and cash for a range of expected returns of the total asset portfolio.

To assess the marginal effect of each nonfinancial asset, as an illustrative example, the model generates the optimal portfolio weights for households who do not possess that asset. The portfolio weights of the remaining nonfinancial assets are reported in the Columns 2–4 of Table 1. The model's results are exhibited in Table 3, Fig. 1 and Fig. 2. Table 3 presents the optimal composition of financial portfolio for the four groups of households: with all assets (base model), no house, no human capital, and no private business. Portfolios are ranged according to the target expected returns of the total asset portfolio. Fig. 1 displays optimal stock-to-bond ratios for all the above groups, while Fig. 2 presents the optimal portfolio composition for the base model.

Based on the above charts and table, several observations can be made. The relationship between the expected returns of the total asset portfolio and the stocks-to-bonds ratio is U-shaped. Because higher expected returns mean higher standard deviation of the portfolio, and more risk-averse investors prefer portfolios with lower standard deviations (and lower expected returns), the model predicts that the optimal SBR first decreases with declining risk-aversion and then increases. The second (increasing) part of SBR profile is in line with popular investment advice and corresponds to the range of portfolio allocations where the

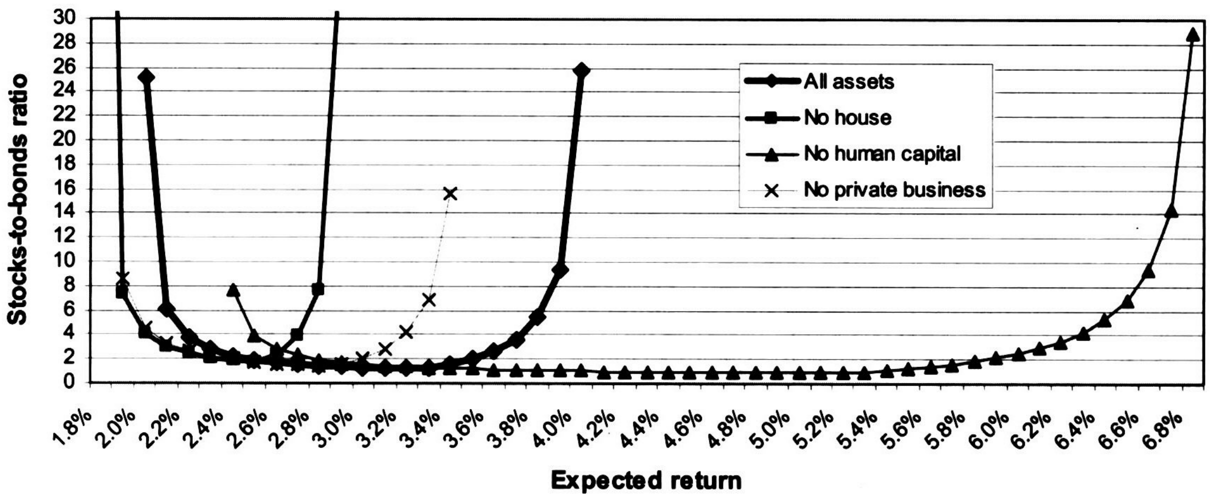

Fig. 1. Optimal stocks-to-bonds ratio and the effects of nonfinancial assets.

constraint on cash content being no less than −100% is binding. This result can be easily explained: when the portfolio weight of cash is fixed at −100%, the only way to raise portfolio expected return is to increase the stocks-to-bonds ratio.

It is less intuitive to understand why the stocks-to-bonds ratio decreases with increasing expected returns and risk of the portfolio, when the cash constraint is not binding. The rationale behind this interesting result lies in the correlation between assets' returns.

Consider the simplest case of asset allocation that does not take into account nonfinancial assets. The unrestricted mean-variance model with zero weights of nonfinancial assets produces the same qualitative result: declining SBR with increasing expected returns of the portfolio. The intuition behind this result can be explained as follows. To increase the expected return (and standard deviation) of the portfolio, it is necessary to reduce the portion of cash in it and increase the share of stocks and bonds, since cash has the lowest expected return and standard deviation among the three assets. Given that, the correlation between the

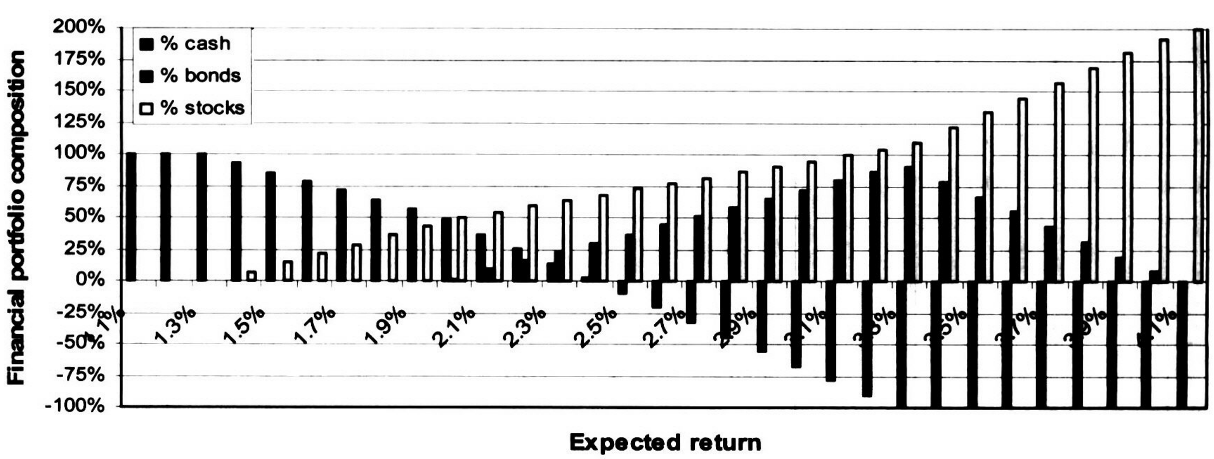

Fig. 2. Composition of financial portfolio (base model).

returns on stocks and the returns on cash ρ_{sc} is lower than the correlation between the returns on bonds and the returns on cash ρ_{bc} (Table 2), the share of bonds should increase more than the share of stocks, bringing the stocks-to-bonds ratio down.[5] In fact, exactly this effect can be observed when looking at Fig. 2. In the range of expected returns where the portfolio share of bonds is positive and the cash constraint is not binding, the shares of stocks and bonds increase with expected returns; however, the share of bonds increases faster, bringing the SBR down. Adding housing, human capital, and private business to the picture brings more complex interactions into the model, with the resulting effect of further reduction in SBR, which strengthens the effect described above.

Some more observations can be made from Table 3. It is notable that financial portfolios of nonentrepreneurs and those without a house contain less cash than the portfolios of households with full set of nonfinancial assets for the same level of total assets' expected returns. The portfolios of the first two groups also have lower SBR than the latter group in the unrestricted range and higher SBR in the range where the cash constraint is binding. On the contrary, for investors without human capital the model recommends significantly higher SBR (in the unrestricted range) but also a much higher share of cash than for those with positive amounts of all nonfinancial assets. These results are because of the higher expected returns on private business and housing compared to returns on human capital. Therefore, to achieve the same level of *total* portfolio returns, nonentrepreneurs and those who do not own a house have to go short on cash and increase the shares of stocks and bonds. Investors with no human capital partially replace it by cash, which also crowd out bonds, increasing both the SBR and the cash share of their portfolio. In the restricted range, however, the only way to achieve higher expected returns is to increase the stocks-to-bonds ratio.

4.2. Optimal asset allocation for investors of different ages

Besides anything else, people of different age groups have different shares of nonfinancial assets in their total asset portfolio. Most notably, the proportion of human capital declines with age, while the proportion of housing rises. The weight of financial assets in the total portfolio also increases with age at the expense of the weight of the nonfinancial assets. The portfolio weights of housing, human capital, and private business, which are used as inputs in the mean-variance model, are estimated from the 2004 Survey of Consumer Finances for each age group (below 30, 30–40, 40–50, 50–60, and 60–70) and reported in Table 1, Columns 5 through 9. The resulting financial asset allocations for these age groups are exhibited in Figs. 3 and 4. Fig. 3 presents the optimal stocks-to-bonds ratio while Fig. 4 displays the proportion of stocks in the financial portfolio.

In Fig. 3, the optimal stocks-to-bonds ratios for only 3 age groups of 5 are displayed: 40–50, 50–60, and 60–70 years of age. This is because for the groups below 30 and 30 to 40 years of age, the model recommends zero bond holdings for any level of target expected returns of the total asset portfolio. The recommended SBR for the investors from the 40 to 50 age group is lower than for older investors if the cash constraint is not binding. If the constraint is binding, the SBR for the 40 to 50 age group grows with expected returns and becomes higher than the recommended SBR for the other two groups. Similarly, the SBR for

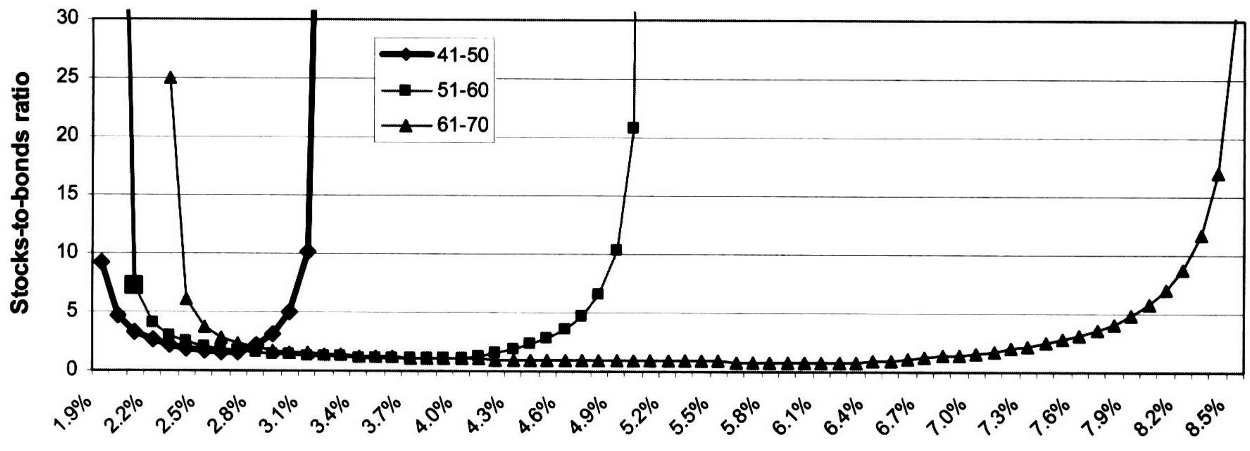

Fig. 3. Optimal stocks-to-bonds ratio for different age groups.

the 50 to 60 age group is lower than that of the 60 to 70 age group in the unrestricted range but then surpasses the latter in the restricted range.

The rationale behind the SBR increasing with age where the cash constraint is not binding can be explained as follows. When age increases, the share of human capital goes down and the shares of housing and private business go up. To compensate for an increase of total portfolio expected returns and risk caused by the replacement of human capital by housing and private business, the composition of the financial portfolio tilts towards cash, lowering the share of stocks and bonds. Given that $\rho_{sc} < \rho_{bc}$, the share of bonds should decline more than the share of stocks, which raises the stocks-to-bonds ratio. When the cash constraint is binding, the only way to lower the risk of financial portfolio is to decrease the SBR.

It is interesting to see what the model recommends with respect to the popular advice of lowering the stocks content in the financial portfolio with investors' age. Judging from Fig. 4, the model clearly advocates decreasing the share of stocks in the financial portfolio with increasing age, which is completely in line with the popular investment advice.

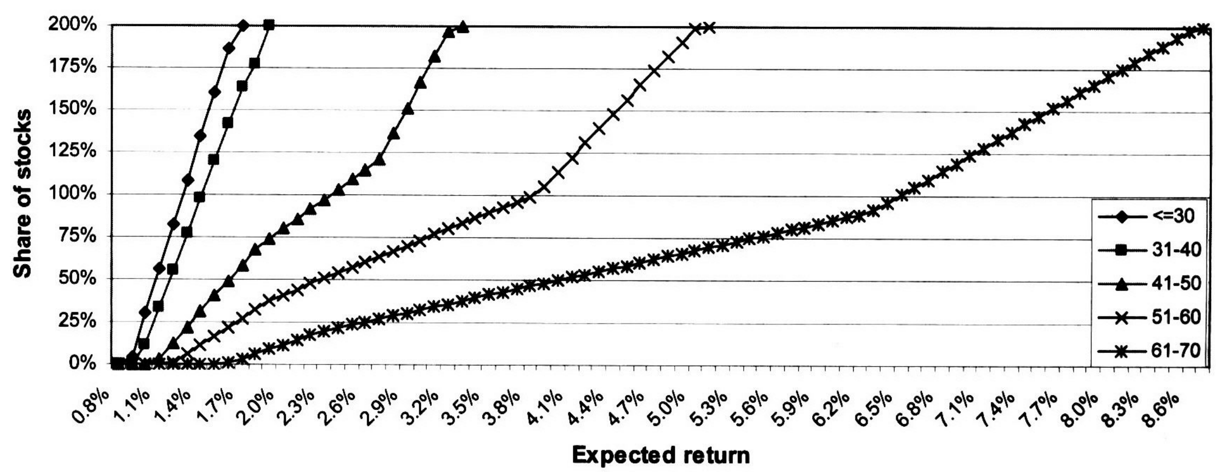

Fig. 4. Optimal proportion of stocks in the financial portfolio for different age groups.

Table 4 Means, standard deviations and correlation coefficients of historic returns on human capital for different industries (1976–2004 annual returns)

Industries	Mean	Standard deviation	Correlation coefficients with returns on			
			Stocks	Bonds	Cash	Housing
Finance, insurance, real estate	2.6%	3.1%	0.207	0.572	0.452	0.382
Public administration	1.1%	1.9%	0.036	0.615	0.497	0.263
Manufacturing	1.0%	2.5%	0.166	0.404	0.170	0.281
Wholesale and retail trade	0.7%	3.0%	−0.074	0.315	0.230	0.453

4.3. Optimal asset allocation for investors employed in different industries

Because the labor income streams of people working in various industries are different, this should have an implication for the optimal financial asset allocations generated by the model that includes human capital. In this subsection, the optimal shares of stocks, bonds, and cash for investors employed in manufacturing, wholesale and retail trade (hereafter trade), public administration, as well as finance, insurance, and real estate (hereafter finance) are produced by the mean-variance model. The average labor income process used in the calibration of the base model is replaced with those representing the above-mentioned industries. Labor income data on these industries is available in the NIPA database. The statistical properties of the returns on human capital for these four industries are exhibited in Table 4.[6]

According to Table 4, the returns on human capital in trade as well as in finance, insurance and real estate have the highest standard deviations. The expected returns in finance, however, are much higher than in any other industry in the group while trade has the lowest expected returns.

The returns to human capital in trade has a small negative (−0.074) correlation with stock returns and a smaller correlation with bonds returns (0.315) than returns in the other three industries. The returns in finance have the highest correlation with stock market returns (0.207) but also a fairly high correlation with bonds returns (0.572). Finally, as can be expected, the labor income of government employees is the most stable, has low correlation with the stock market (0.036) and the highest correlation with bond returns (0.615).

The resulting financial asset allocations for people working in these industries are presented in Table 5.

According to Table 5, individuals employed in finance, insurance, and real estate industry are advised to keep the highest share of cash among all four industries for the same level of total expected returns. This is not surprising given that the returns on human capital in that industry are significantly higher than in the other three. Government employees are recommended to hold the lowest share of bonds. Finally, individuals working in wholesale and retail trade should hold the most aggressive portfolios with the lowest share of cash and the highest stocks content.

The stocks-to-bonds ratio for investors employed in finance, insurance, and real estate cannot be compared to the SBRs of individuals working in the other three industries for the same level of expected returns. In the group of public administration, trade, and manufac-

Table 5 Optimal composition of financial portfolio for investors employed in different industries

		Expected return of the total asset portfolio					
		1.5%	2.0%	2.5%	3.0%	3.5%	4.0%
Finance, insurance, real estate	% cash	N/A	N/A	89%	48%	3%	−61%
	% bonds	N/A	N/A	0%	0%	7%	46%
	% stocks	N/A	N/A	11%	52%	90%	115%
	Stocks-to-bonds ratio	N/A	N/A	N/A	N/A	13.2	2.5
Public administration	% cash	79%	38%	−17%	−82%	−100%	N/A
	% bonds	0%	0%	23%	63%	26%	N/A
	% stocks	21%	62%	94%	119%	174%	N/A
	Stocks-to-bonds ratio	N/A	N/A	4.0	1.9	6.7	N/A
Manufacturing	% cash	77%	32%	−32%	−97%	−100%	N/A
	% bonds	0%	6%	46%	85%	23%	N/A
	% stocks	23%	61%	87%	112%	177%	N/A
	Stocks-to-bonds ratio	N/A	10.1	1.9	1.3	7.7	N/A
Wholesale and retail trade	% cash	56%	13%	−52%	−100%	−100%	N/A
	% bonds	0%	4%	43%	56%	0%	N/A
	% stocks	44%	83%	108%	144%	200%	N/A
	Stocks-to-bonds ratio	N/A	21.2	2.5	2.6	N/A	N/A

turing, when the cash constraint is not binding, the model suggests the highest stocks-to-bonds ratio for investors employed in public administration and the lowest SBR is recommended for manufacturing employees. In the range of expected returns where the cash constraint is binding, the order of SBRs is reversed, with the highest ratio being recommended in manufacturing and the lowest ratio in public administration. The advice to keep high ratio of stocks to bonds for public sector employees is quite intuitive as the correlation between human capital returns and the returns on stocks ρ_{ls} in that industry is relatively low whereas the correlation with the returns on bonds ρ_{lb} is high.

When the cash constraint is binding, the model advocates the highest SBR for the trade employees for the following reason. If someone switches his or her occupation from manufacturing or public administration to trade, the expected return of his or her total portfolio will go down as the average return on human capital in trade is the lowest among the group. For the same level of portfolio expected returns, the financial portfolio should become more aggressive, which means increasing the ratio of stocks to bonds with cash content being fixed at −100%.

4.4. Optimal asset allocation for investors residing in different cities

Similar to substituting the average human capital returns with returns in specific industries in the base model, the returns on the general U.S. housing index can be replaced by returns on housing indices for a certain areas or cities. This way, the model can produce more specific recommendations on asset allocation for the residents of that area or city. The last set of models uses the returns for regional housing indices, available from OFHEO, as an input, to generate the optimal asset allocation for residents of New York, Chicago, and San Francisco. Statistical properties of the returns on housing in these cities are reported in Table 6.

Table 6 Means, standard deviations and correlation coefficients of historic returns on housing for different cities (1976–2004 annual returns)

Cities	Mean	Standard deviation	Correlation coefficients with returns on				
			Stocks	Bonds	Cash	Human capital	Private business
Chicago	1.8%	4.9%	−0.105	0.158	−0.034	0.471	0.549
New York	3.9%	7.5%	0.011	0.223	0.206	0.337	0.094
San Francisco	4.9%	8.0%	0.039	−0.087	−0.200	0.102	0.328

As can be seen, the returns on the San-Francisco housing index are the highest with a mean of 4.9% and are also the most volatile (standard deviation of 8.0%). They are followed by the returns on the New York index (mean of 3.9% and standard deviation of 7.5%). Housing returns in Chicago are the lowest among the three cities (mean of 1.8%) and the least volatile (standard deviation of 4.9%). The optimal asset allocations for house owners living in these cities are generated by the mean-variance model and can be found in Table 7.

As can be observed from the above table, when the cash constraint is not binding, New Yorkers are recommended the highest stocks-to-bonds ratio among the three cities, while Chicago residents should hold the lowest SBR. This may seem counterintuitive at first glance given that the correlation of Chicago housing index returns with the stock returns is lower than those of New York and San Francisco and is, in fact, negative 0.105. Moreover, the correlation between Chicago housing index returns and bond index returns is higher than that of San Francisco. However, as well as in the cases considered previously, it is necessary to look at the correlation structure of returns on all the assets in the search for an answer.

Here is the explanation of the above result. Residents of Chicago moving to New York or San Francisco will have to lower the return and risk of their financial portfolios to compensate for the increase of risk and return of their housing. To do this, they would increase the weight of cash at the expense of stocks and bonds. As cash has much higher correlation with

Table 7 Optimal composition of financial portfolio for investors residing in different cities

		Expected return of the total asset portfolio					
		1.5%	2.0%	2.5%	3.0%	3.5%	4.0%
Chicago	% cash	84%	47%	−10%	−68%	−100%	−100%
	% bonds	0%	0%	35%	70%	64%	5%
	% stocks	16%	52%	75%	97%	136%	195%
	Stocks-to-bonds ratio	N/A	N/A	2.1	1.4	2.1	42.7
New York	% cash	97%	61%	12%	−45%	−100%	−100%
	% bonds	0%	0%	21%	56%	86%	27%
	% stocks	3%	39%	67%	89%	114%	173%
	Stocks-to-bonds ratio	N/A	N/A	3.2	1.6	1.3	6.4
San Francisco	% cash	100%	68%	16%	−42%	−99%	−100%
	% bonds	0%	0%	26%	61%	96%	38%
	% stocks	0%	32%	58%	81%	103%	162%
	Stocks-to-bonds ratio	N/A	N/A	2.2	1.3	1.1	4.3

the returns on bonds than on stocks, the share of bonds will decline proportionally more than the share of stocks with the result being a declining stocks-to-bonds ratio. In fact, as one can observe from the Table 7, the optimal portfolios of Chicago residents contain the least proportion of cash. This effect is further strengthened by the negative 0.200 correlation between housing and cash in San Francisco and weakened by the positive 0.206 correlation in New York. However, the higher correlation between housing and bonds in New York (0.223) and a lower one in San Francisco (negative 0.087), all relative to the corresponding correlations in Chicago, work in the opposite direction.

In the range of expected returns where the cash constraint is binding, Chicago residents are advised to hold the highest stocks-to-bonds ratio and San-Francisco residents, the lowest. This is the result of expected returns on housing in Chicago being the lowest and in San Francisco being the highest among the three cities. With the cash content being fixed at −100%, the only way for a Chicago resident to have the same expected return of the total asset portfolio is to hold significantly higher fraction of stocks and, consequently, the higher SBR.

5. Conclusions

In this paper, I derive the optimal allocations of stocks, bonds, and cash with an enhanced mean-variance model, which includes major nonfinancial assets (housing, human capital, and private business), in addition to financial assets. Producing the optimal portfolio allocations for investors having positive amounts of housing, human capital, and private business versus those with zero value of one of these assets, demonstrates the effect that each of the nonfinancial assets have on the optimal mix of financial assets. The model also generates the relative shares of stocks, bonds, and cash for investors employed in different industries or living in different cities. Finally, generating the optimal portfolio weights of financial assets for investors of different ages and for the different levels of portfolio expected returns and risk, allows one to see whether the model's recommendations are in line with the popular investment advice.

The main findings of this paper are the following.

The nonfinancial assets have the following effects to the relative allocations to stocks, bonds, and cash. According to the model, financial portfolios of nonentrepreneurs and those without a house should contain less cash than the portfolios of households with the full set of nonfinancial assets for the same level of total assets' expected returns. The portfolios of the first two groups also have lower stocks-to-bonds ratios than the latter group in the unrestricted range and higher SBRs in the range where the cash constraint is binding. On the contrary, for investors without human capital, the model recommends a significantly higher SBR (in the unrestricted range) but also a much higher share of cash than for those with positive amounts of all nonfinancial assets.

The optimal asset allocations generated by the mean-variance model confirm the popular investment advice that recommends a decreasing share of stocks in financial portfolios as age increases. As for another popular piece of advice—increasing the SBR with declining risk-aversion—the model's recommendation depends on whether the constraint on the share of cash being no less than −100% of the financial portfolio is binding. In the range of target expected returns where the constraint is not binding, the model gives the opposite recommendation. The main reasons behinds this unexpected result are: (1) cash in the model is a risky asset, with returns that are more highly correlated with returns on bonds than on stocks; (2) the returns on human capital, the largest asset in most people's total portfolios, have higher correlations with the returns on bonds than on stocks. When the cash constraint is binding, the optimal SBR generated by the model does increase with decreasing risk-aversion, which is in line with practitioners' recommendation.

Finally, the model is able to provide some recommendations on asset allocations for investors employed in a particular industry or living in a certain city. In particular, it advocates safer portfolios for investors employed in finance, insurance, and real estate than for those working in public administration, manufacturing, or trade, whereas for those working in trade, it recommends the most aggressive portfolios. Government employees are advised to hold a lower share of bonds than those working in manufacturing or trade.

When the cash constraint is not binding, New Yorkers are recommended higher stocks-to-bonds ratios than the residents of San Francisco or Chicago, whereas the latter are advised to keep less cash in their portfolios than those residing in the other two cities. When the constraint is binding, however, it is Chicago residents who are advised to have the highest SBR in their portfolios. A lesson from the model's results is that looking just at the stocks-to-bonds ratio is deceiving, as the above examples with zero human capital investors, New York residents, and finance industry employees have shown. One also needs to consider the shares of all the assets in the financial portfolio, which include cash as well as stocks and bonds.

Notes

1. The 2004 Survey of Consumer Finances reports households' assets as of the end of 2003.
2. See, for example, Boyle and Guthrie (2005) or Flavin and Yamashita (2002).
3. The 100% corresponds to a common 50% margin requirement on equity.
4. Estimating the discount rate for future labor income is difficult. The 5% rate is chosen based on Heaton and Lucas (2000a).
5. Changing the period of historical returns to 1926 through 2004 and their frequency to quarterly and monthly produces similar correlation structure. The model based on that data produce the same qualitative results.
6. The correlations between returns on human capital and returns on private business are not included in the table and the subsequent model since it is not clear whether typical employees of the above mentioned industries have the same stream of entrepreneurial income as the average household, which is referred to in the base model.

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Appendix

Solution for the mean variance model with exogenous shares of nonfinancial assets

$$Min_{w_i} \frac{1}{2} \vec{w} V \vec{w}' \quad \text{s.t.} \quad \begin{array}{l} \vec{w} * \vec{u}' = E(R) \\ \vec{w} * \vec{1}' = 1 \\ w_c \geq -1 \\ w_b \geq 0 \\ w_s \geq 0 \end{array} \qquad i = c,b,s \tag{2}$$

$\vec{w}' = [w_c w_b w_s w_h w_l w_e]$-vector of portfolio weights of cash (c), bonds (b), stocks (s), housing (h), human capital [capitalized labor income (l)], and private business [entrepreneurship (e)];

$\vec{\mu}' = [\mu_c \mu_b \mu_s \mu_h \mu_l \mu_l]$-vector of expected returns on the above assets;

$$V = \begin{bmatrix} \sigma_c^2 & \sigma_{cb} & \sigma_{cs} & \sigma_{ch} & \sigma_{cl} & \sigma_{ce} \\ \sigma_{be} & \sigma_b^2 & \sigma_{bs} & \sigma_{bh} & \sigma_{bl} & \sigma_{be} \\ \sigma_{sc} & \sigma_{sb} & \sigma_s^2 & \sigma_{sh} & \sigma_{sl} & \sigma_{se} \\ \sigma_{hc} & \sigma_{hb} & \sigma_{hs} & \sigma_h^2 & \sigma_{hl} & \sigma_{he} \\ \sigma_{lc} & \sigma_{lb} & \sigma_{ls} & \sigma_{lh} & \sigma_l^2 & \sigma_{le} \\ \sigma_{ec} & \sigma_{eb} & \sigma_{es} & \sigma_{eh} & \sigma_{el} & \sigma_e^2 \end{bmatrix} \tag{3}$$

- variance-covariance matrix of assets returns;

$\sigma_{ij} = \sigma_i\, \sigma_j\, \rho_{ij}$, where ρ_{ij} is the coefficient of correlation between the returns of asset i and asset j and σ_{ij} is the standard deviation of returns on asset i.

E(R)—target level of portfolio expected return.

Solution:

The lagrangian:

$$L = \frac{1}{2}\vec{w}V\vec{w}' + \lambda_1(E(R) - \vec{w} * \vec{\mu}' + \lambda_2(1 - \vec{w} * \vec{1}') \tag{4}$$

First order conditions:

$$\frac{\partial L}{\partial w_c} = w_c\sigma_c^2 + w_b\sigma_{cb} + w_s\sigma_{cs} + w_h\sigma_{ch} + w_l\sigma_{cl} + w_e\sigma_{ce} - \lambda_1\mu_c - \lambda_2 = 0 \quad (5)$$

$$\frac{\partial L}{\partial w_b} = w_b\sigma_b^2 + w_c\sigma_{cb} + w_s\sigma_{bs} + w_h\sigma_{bh} + w_l\sigma_{bl} + w_e\sigma_{be} - \lambda_1\mu_b - \lambda_2 = 0 \quad (6)$$

$$\frac{\partial L}{\partial w_s} = w_s\sigma_s^2 + w_c\sigma_{cs} + w_b\sigma_{bs} + w_h\sigma_{sh} + w_l\sigma_{sl} + w_e\sigma_{se} - \lambda_1\mu_s - \lambda_2 = 0 \quad (7)$$

$$\frac{\partial L}{\partial \lambda_1} = E(R) - (w_c\mu_c + w_b\mu_b + w_s\mu_s + w_h\mu_h + w_l\mu_l + w_e\mu_e) = 0 \quad (8)$$

$$\frac{\partial L}{\partial \lambda_2} = 1 - (w_c + w_b + w_s + w_h + w_l + w_e) = 0 \quad (9)$$

which is equivalent to

$$w_c\sigma_c^2 + w_b\sigma_{cb} + w_s\sigma_{cs} - \lambda_1\mu_c - \lambda_2 = -w_h\sigma_{ch} - w_l\sigma_{cl} - w_e\sigma_{ce} \quad (10)$$

$$w_b\sigma_b^2 + w_c\sigma_{cb} + w_s\sigma_{bs} - \lambda_1\mu_b - \lambda_2 = -w_h\sigma_{bh} - w_l\sigma_{bl} - w_e\sigma_{be} \quad (11)$$

$$w_s\sigma_s^2 + w_c\sigma_{cs} + w_b\sigma_{bs} - \lambda_1\mu_s - \lambda_2 = -w_h\sigma_{sh} - w_l\sigma_{sl} - w_e\sigma_{se} \quad (12)$$

$$w_c\mu_c + w_b\mu_b + w_s\mu_s = E(R) - (w_h\mu_h + w_l\mu_l + w_e\mu_e) \quad (13)$$

$$w_c + w_b + w_s = 1 - (w_h + w_l + w_e) \quad (14)$$

This system of five linear equations with five unknowns (w_c, w_b, w_s, λ_1, λ_2) can also be written in matrix form: $\mathbf{A}\vec{x} = \vec{b}$, where **A** is the coefficient matrix of the left-hand side, $\mathbf{A}\vec{x} = [w_c, w_b, w_s, \lambda_1, \lambda_2]'$ - vector of unknowns, and $\vec{b}$ is the vector of the right-hand side. Then, $\vec{x} = A/\vec{b}$ s.t. restrictions $w_c \geq -1$, $w_b \geq 0$, $w_s \geq 0$, is the solution of the above system of equations. This system was solved in Matlab for a range of target expected returns *E(R)*.

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