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After-tax value of annuities

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Abstract

This paper adds to the growing literature on after-tax asset valuation and asset allocation by developing a model to value annuities on an after-tax basis. Qualified and non-qualified annuities are shown to be equivalent to a tax-deferred account (like a traditional IRA) plus a cost basis tax shield, if any. Viewed in this light, the after-tax value of an annuity decreases as the investment horizon increases, but is independent of the risk of the underlying investment. © 2008 Academy of Financial Services. All rights reserved.

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1. Introduction

As more wealth becomes concentrated with individuals, researchers and practitioners are becoming increasingly aware of the tax implications of managing portfolios for private investors. Among the many issues that arise in the management of private wealth is measuring after-tax asset allocation (e.g., Reichenstein [2000, 2006, 2007] and Horan [2007a, b]), performing after-tax mean-variance optimization (e.g., Wilcox, Horvitz, and DiBartolomeo [2006], Reichenstein [2007], and Horan [2007b]), analyzing asset location (e.g., Dammon, Spatt and Zhang [2004] and Poterba, Shoven and Sialm [2004]), planning tax efficient withdrawal strategies (Spitzer and Singh, 2006; Horan 2006a, b), and measuring after-tax performance measurement (e.g., Stein [1998], Stein, Langstraat and Narasihhan [1999], Horan, Lawton and Johnson [2008]). The issues examined in all these studies are predicated on the notion of valuing assets on an after-tax basis. Although methodological

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differences exist from one study to the next, a common theme among them is that the after-tax value of an asset is generally not equal to its pre-tax market value and depends on the type of account in which it is held.

Previous research has examined accounts that can be classified generally into four categories. The first type is taxable accounts for which deposits are made on an after-tax basis and investment returns are taxed. A second class of accounts can be called tax-deferred accounts, or TDAs (e.g., traditional IRAs, 401(k) plans, 403(b) plans, 457 plans, Keogh plans, and qualified annuities). Contributions to these accounts may be made on a pre-tax basis (i.e., tax-deductible), and the investment returns accumulate on a tax-free basis until funds are withdrawn at which time they are taxed as ordinary income. As such, these accounts are sometimes said to have front-end loaded tax benefits. A third class of accounts has tax-deferral characteristics like TDA but the initial contribution is not deductible (e.g., non-deductible IRAs and non-qualified annuities). A subset of these second and third types of accounts, specifically non-qualified annuities, are the subject of this paper.

A fourth class of accounts has back-end loaded tax benefits (e.g., Roth IRAs, Roth 401(k) plans, Roth 403(b) plans, 529 plans). These accounts can be called tax-exempt on a prospective basis because although contributions must be made on an after-tax basis (i.e., not tax-deductible) their earnings can accumulate free of taxation even as funds are withdrawn.[1] Researchers agree that balances held in these accounts are not economically comparable because their different tax treatment affects the return received by and risk borne by the investor. In other words, an identical asset held in each of these types of accounts is a unique after-tax asset from both a risk and return perspective.

As yet, annuities are unexamined in this context. The purpose of this paper is to develop a model for valuing qualified and non-qualified annuities vehicles on an after-tax basis. It does not address the merits of including or excluding annuities in an investment or retirement plan, which would focus on issues like mortality risk, mortality credits, and longevity risk. Rather, this study provides an analytical framework for comparing the value of annuities relative to other assets in a portfolio after having accounted for taxes whether in the accumulation or distribution phase.

This line of inquiry is important for several reasons. Financial planners traditionally use asset market values (i.e., unadjusted pre-tax market values reported on an account statement) to calculate an investor's asset allocation, an approach that overlooks the fundamental notion that the same asset held in different types of accounts can produce different after-tax cash flows, have different risk profiles, and hence have different after-tax values. Models developed in a pre-tax environment do not necessarily apply in a more economically relevant after-tax environment, and this tenet certainly applies to asset allocation. Advisors relying on pre-tax market values to determine a client's mix of equity, fixed income, and other assets can be misled about a portfolio's true risk profile. Depending on an investment's asset class and the type of account in which it is held, the taxing authority may share in a portion of an investor's return and hence absorbs some investment risk. Therefore, this paper advances the practice of after-tax portfolio management generally.

Second, annuities can be an integral part of a retirement portfolio for many investors and should be evaluated on an after-tax basis like any other asset (e.g., Bodie and Pesando [1983], Brown, Mitchell and Poterba [1999], Ibbotson, Milevsky, Chen and Zhu [2007]).

That they have escaped examination in an after-tax framework up to this point is curious because annuities are becoming an increasing popular tool for retirement planning (e.g., Brown and Poterba [2006] and Viceira [2007]). Perhaps because of their ability to manage longevity risk (and sometimes inflation risk) or perhaps because of the decline of defined benefit plans, the dollar value of annuities sold in the United States has more than tripled from 1990 to 2005 (Viceira, 2007) and grown 14% in 2006, comprising about 10% of retirement assets (Brady and Holden, 2007). The growth of the annuity market may also be related to the impending retirement of baby boomers, the first of which are scheduled to turn 62 years old in 2008. At this point, the first boomers are eligible to start receiving social security benefits. Therefore, developing a model to measure the after-tax value of annuities is a natural response to the increasing interest in after-tax portfolio management, the gap in the literature regarding annuities specifically, and the growing importance of annuities as a financial planning tool.

The next section reviews the after-tax valuation literature, which serves as the foundation for the model that is developed in section three. Section three incorporates both qualified and non-qualified annuities into a single integrated model. We consider alternative approaches but find that they are incongruent with accepted dogma. Section four illustrates how the annuity's after-tax value varies with the four factors the impact its value and shows how these factors interact with each other. Section five concludes.

2. Literature review

The notion of after-tax asset valuation has been developing for some time. Reichenstein (1998) and Reichenstein and Jennings (2003) note that individuals are unable to consume pre-tax dollars. As a result, they argue that measures of after-tax value can provide a more accurate measure of total wealth and how that wealth is distributed among different asset classes than traditional pre-tax models. In a further refinement, Reichenstein (2001, 2007) argues that properly assessing portfolio risk and asset allocation requires the use of after-tax values (ATVs) because the government, by taxing annual returns or terminal withdrawals on assets held in taxable accounts, shares investment returns as well as investment risk with the account holder.[22] As a result, the after-tax risk borne by the investor must be less than or equal to the pre-tax risk.

Horan (2007a, b) refines the ATV concept and provides a specific definition of the idea, which had escaped precise definition until that time. Specifically, the ATV of an asset is the present value of future after-tax cash flows discounted using a proper tax-adjusted and risk-adjusted discount rate. The discount rate adjustment in this definition is important and has been the subject of much attention in the corporate finance literature (e.g., Miles and Ezzell [1980], Sick [1990], Taggart [1991], Fernandez [2004], Cooper and Nyborg [2006, 2008]) and the after-tax portfolio performance literature (e.g., Stein [1998], Stein, Langstraat and Narasihhan [1999], and Horan, Lawton and Johnson [2008]). The taxation of TDAs does not change the volatility of after-tax cash flows, so the investor retains all investment risk and the discount rate should not be adjusted for taxes (e.g., Reichenstein [2006, 2007] and Horan [2007a, b]).

These authors also show, however, that the taxation of taxable accounts does change the risk profile of taxable investments. Horan's (2007a, b) major insight is that, although taxation provides government a claim on a proportion of the investor's *entire* return, its risk-sharing role relates only to the asset's *risk premium.* Consider a risk-free security. Although taxes reduce the return of a riskless investment, they cannot reduce its risk. Therefore, when identifying a risk-adjusted rate to discount future cash flows, the risk-free portion of an asset's expected return should not be adjusted for taxes. The government's risk-sharing role relates to the risk premium, which should be should be adjusted for taxes (see Cooper and Nyborg [2006, 2008]). The Appendix develops the notion of after-tax value and related concepts more fully.

The ATV concept applies to many portfolio management issues, including after-tax asset allocation. Reichenstein (2001, 2007) points out that the same pre-tax asset is effectively a different after-tax asset depending on the type of account in which it is held. One can then implement mean-variance optimization treating each asset in each type of account as a distinct after-tax asset, an approach that effectively incorporates many of the ideas in the nascent literature on asset location (e.g., Dammon, Spatt and Zhang [2004], Poterba, Shoven and Sialm [2004]). See Horan (2005) and Jennings and Reichenstein (2006) for a review of the asset location literature and much of the other literature on tax-efficient wealth accumulation. Valuing annuities on an after-tax basis is a perquisite to incorporating them into an after-tax mean-variance optimization and asset location framework.

3. After-tax value of annuities

3.1. Qualified and non-qualified annuities

In this paper, we assume that annuities accumulate returns in a tax-deferred manner and that at least a portion of withdrawals are taxed as ordinary income much the same as a TDA, such as a traditional IRA. Qualified annuities (QAs) are functionally equivalent to a TDA from a tax perspective. Contributions are made on a pre-tax basis (i.e., tax-deductible), returns accumulate in a tax deferred manner, and the entire withdrawal is taxed as ordinary income. Non-qualified annuities (NQAs) also accumulate returns on a tax-deferred basis. The initial contribution, however, is not tax-deductible but is also not taxed when withdrawn. So, only a portion of the withdrawal of an NQA is subject to tax. In this sense, an NQA is similar to a non-deductible IRA.

Once funds are deposited into an annuity, the tax deductibility of the contribution or lack thereof is a sunk cost or benefit. That tax differential is certainly relevant for comparing the relative attractiveness of different investment options to be sure. Once that choice is made, however, the tax treatment of the initial contribution is no longer a marginal cash flow and does not therefore affect the after-tax valuation of the annuity except as it relates to the annuity's cost basis, which is an important point discussed below.

Following the development in prior studies, we project the future after-tax cash flow of an annuity and discount it to the present using the appropriate discount rate. Because initial investments in NQAs are not tax deductible, the initial contribution represents the cost basis of the annuity and is not taxed on withdrawal. For QAs, however, the cost basis is effectively zero, and the entire withdrawal is taxed. The future after-tax cash flow of each dollar in the annuity can be written as

$$FV_{Annuity,AfterTax} = [(1 + r)(1 - f)]^n - T_n\{[(1 + r)(1 - f)]^n - 1\} - (1 - B)T_n \quad (1)$$

where B is the cost basis expressed as a percentage of market value, and T_n is the tax rate on ordinary income at withdrawal. The first term is the pre-tax accumulation after earning a return of r that is eroded by annual costs of financial intermediation associated with the annuity wrapper of f (e.g., fees, surrender charges) for n periods. The second term is the tax liability associated with the gains above the initial investment from time 0 to time n as if only the capital appreciation were taxed. The third term is the tax liability embedded in the annuity accrued up to time 0 resulting from the basis. If the initial investment is tax-deductible as is the case of the QA, then B equals zero. In the case of the NQA where the initial investment is not tax-deductible, the initial basis equals one but declines over time, assuming the market value of the annuity grows over time.

Eq. (1) can be rearranged into a potentially more intuitive form as follows

$$FV_{Annuity,AfterTax} = [(1 + r)(1 - f)]^n(1 - T_n) + T_n - (1 - B)T_n \quad (2)$$

In this form, the first two terms represent the after-tax accumulation ignoring the embedded tax liability. Together, the first two terms have the familiar look of the future value when the investment is taxed as a capital gain deferred until the end of the investment horizon.[3] Further manipulation, produces

$$FV_{Annuity,AfterTax} = [(1 + r)(1 - f)]^n(1 - T_n) + T_nB \quad (3)$$

that is perhaps more straightforward, yet. For QAs, the cost basis is equal to zero ($B = 0$), and the expression reduces to $[(1 + r)(1 - f)]^n(1 - T_n)$. In the absence of costs of financial intermediation associated with the annuity wrapper (i.e., $f = 0$), this cash flow is identical to the future after-tax cash flow of a TDA, like a traditional IRA, which is consistent with the functional equivalence of the two accounts described earlier. The after-tax cash flow for a NQA with a cost basis greater than zero is greater than that for a QA because a portion of the withdrawal is not taxed upon withdrawal. Recall that this incremental cash flow relates to the taxability of the initial investment. According to this expression, the cash flows associated with an annuity are equal to those associated with a TDA plus a tax shield derived from the cost basis less costs of financial intermediation.

3.2. *After-tax value of annuities*

To determine the after-tax value of an annuity, the after-tax cash flow must be discounted to the present at an appropriate discount rate. It is easiest to begin by considering a risk-free annuity. Because the cash flows are riskless, they should be discounted at the risk-free rate, r_f. Therefore, the after-tax value (ATV) of each dollar in an annuity would be

$$ATV_{Annuity} = \frac{[(1 + r_f)(1 - f)]^n(1 - T_n) + T_nB}{(1 + r_f)^n}, \tag{4}$$

that reduces to

$$ATV_{Annuity} = (1 - f)^n(1 - T_n) + \frac{T_nB}{(1 + r_f)^n}. \tag{5}$$

This approach is analogous to valuing a firm by discounting its after-tax cash flows at the appropriate cost of capital, which in this case is riskless (e.g., Modigliani and Millier (1958, 1963)). For QAs with zero cost basis, the ATV reduces to $(1 - f)^n(1 - T_n)$. In the absence of costs of financial intermediation (i.e., $f = 0$), the ATV of a QA with zero cost basis equals $(1 - T_n)$, which prior studies unanimously agree is also the ATV of a TDA (e.g., Reichenstein [1998, 2001, 2006, 2007] and Horan [2002, 2005, 2007a, b]). This is a natural equivalence on which we elaborate more fully below. The second term of Eq. (5) represents the present value of the tax benefit of the basis not being subject to taxation upon withdrawal. This value can be called the present value of the *cost basis tax shield*. Therefore, an annuity is equivalent to a TDA plus the cost basis tax shield, if any.

Interestingly, Eq. (5) implies that the ATV decreases as the time horizon, n, increases (see the Appendix). At first glance, this result may seem counterintuitive. Upon further reflection, it is sensible when one views the annuity as a TDA plus the present value of the cost basis tax shield. The ATV of a TDA is independent of the time horizon, but the present value of the cost basis tax shield decreases the longer it is delayed.

Introducing risk into the annuity's underlying investment could conceivably change the ATV (see, e.g., Horan [2007a, b]). Interestingly, however, the expression in Eq. (5) applies to annuities with risky underlying investments, as well. The intuition for this result again stems from viewing an annuity as a TDA plus the present value of the cost basis tax shield. The ATV of a TDA represented by the first term is independent of the risk of the underlying investment (e.g., Reichenstein [1998, 2001, 2006, 2007] and Horan [2002, 2005, 2007a, b]). In the second term, the cash flow associated with the cost basis tax shield is fixed and is therefore riskless. It should be discounted at the risk-free rate, as well. Consequently, the ATV of an annuity is unaffected by the risk of the underlying investment.

The model in Eq. (5) assumes that the accumulation in an annuity is withdrawn as a lump sum. Typically, investors draw down annuity accumulations over time. The model can be easily modified to reflect this reality by assuming that the non-deductible contribution (represented by B) is withdrawn over m years beginning at time n. It would generally be advisable to withdraw the non-deductible contribution first, which is the tax treatment for annuities purchased in the U.S. before August 14, 1982. This method accelerates the receipt of the cost basis tax shield. Withdrawals from annuities purchased after this date are treated on an interest-first basis, which delays receipt of the cost basis tax shield.

If withdrawals are annuitized rather than taken as a lump sum, a portion of each payment is considered a return of principal and the remainder taxable as interest according to an amortization table. As a simple approximation, one can assume the cost basis tax shield is received as an annuity over m periods, rather than as a lump sum. Therefore, Eq. (5) can be modified to reflect an annuitized withdrawal pattern in the following way,

$$ATV_{Annuity} = (1 - f)^n(1 - T_n) + \frac{T_n B}{(1 + r_f)^n}\left[\frac{1}{r_f} - \frac{1}{r_f(1 + r_f)^m}\right]. \tag{6}$$

The present value of the cost basis tax shield is simply adjusted to reflect the fact that it will be received over m years beginning n years from now. Because the value of the interest tax shield decreases as the accumulation period n increases, this adjustment becomes less meaningful over long accumulation periods.

3.3. A consideration of alternative approaches

The preceding approach of discounting riskless future after-tax cash flows such as those in Eq. (3) at the risk-free rate is consistent with how Modigliani and Miller (1958, 1963) value after-tax cash flows of a firm in a taxable environment. It is also consistent with a wealth of more recent literature on the value of the corporate interest tax shield (e.g., Miles and Ezzell [1980], Sick [1990], Taggart [1991], Fernandez [2004], Cooper and Nyborg [2006, 2008] and Horan [2007a]) and after-tax portfolio performance measurement (e.g., Stein [1998], Stein, Langstraat and Narasihhan [1999] and Horan, Lawton and Johnson [2008]).

Nonetheless, we consider in this section two possible alternatives and prove by way of contradiction that they are not appropriate for this application. For example, consider discounting riskless cash flows at the after-tax risk-free rate, $r_f(1 - t)$. After all, the opportunity cost of capital could conceivably be the after-tax return on a similar investment held in a taxable account. The problem with this approach is that it conflicts with accepted dogma. If it were true, the ATV of a QA would be greater than $(1 - T_n)$. The QA is functionally equivalent to a TDA, however, and previous research speaks with one voice that the ATV of a TDA is equal to $(1 - T_n)$. This approach is not entirely without merit. It represents an annuity's taxable equivalent value (TEV), which in this context equals the amount of assets that, if held in a taxable account, would produce the same after-tax cash flow as the annuity (e.g., Sibley [2002], Horan [2002], and Poterba [2004]). The Appendix develops the idea of TEV and its relation to ATV more thoroughly.

This approach would also be inconsistent with an analogous problem borrowed from the corporate finance literature. Specifically, Modigliani and Miller (1958, 1963) value a levered firm in the presence of corporate taxes. In doing so, they discount the future after-tax cash flows generated by a firm using the pre-tax return rather than the after-tax return. If the cash flows were discounted using the after-tax return, the firm would have identical values with and without corporate taxes, which is of course non-sensical.

Alternatively, one may be inclined to discount the future after-tax cash flows at the "accrual equivalent" after-tax return.[4] The after-tax accrual equivalent is the return that if earned on the portfolio every year would produce the same after-tax portfolio value at the end of the investment horizon after annually paid taxes and capital gains or withdrawal taxes deferred until the end of the investment horizon. Discounting the future after-tax cash flows at the accrual equivalent after-tax return will always yield the annuity's pre-tax value because algebraically the accrual equivalent return is derived by solving for the rate that equates the pre-tax value with future after-tax cash flow, much like an internal rate of return. In this sense, it is tautological to discount an annuity's future after-tax cash flow by the after-tax accrual equivalent return because this rate is determined by setting pre-tax value and after-tax value equal to each other in the first place.

This approach produces what can be characterized as an annuity equivalent value; that is, the amount of assets that, if held in an annuity, would produce the same after-tax cash flow as a balance held in some other type of account. If, for example, one were to discount the future after-tax cash flow of a taxable account by the annuity accrual equivalent rate, the result would be the amount of cash that would produce the same after-tax cash flow as an annuity.

This approach has meaning and context, but it does not produce an annuity's ATV because it too leads to several inconsistencies. Consider a QA with zero cost basis. Discounting the expression in Eq. (3) at the annuity's accrual equivalent rate produces an after-tax value equal to a dollar. The literature unanimously agrees that the after-tax value of each dollar in a Roth IRA is equal to one because returns are not taxed so the government shares in neither the return nor the risk of the investment. From a tax perspective, however, a QA is functionally equivalent to a TDA, which has an ATV equal to $(1 - T_n)$. Discounting the after-tax cash flow of a QA at the annuity accrual equivalent rate yields an ATV equal to that of a Roth IRA rather than a TDA, its functional equivalent. Therefore, the annuity's accrual equivalent rate cannot be the proper discount rate to compute ATV.

4. Some examples

This section illustrates how the ATV of an annuity varies with four parameters that determine its value. One way to accomplish this is perform a comparative statics analysis, which is presented in the Appendix. Another method is to perform a sensitivity analysis using the four inputs. Table 1 presents various ATVs using Eq. (5) for each dollar held in an annuity for various time horizons assuming a lump sum withdrawal and no costs of financial intermediation associated with the annuity wrapper. Unless otherwise noted, the base case assumes the risk-free rate is 5%, the terminal tax rate is 28%, and the cost basis is 50% of pre-tax market value.

One pattern is immediately obvious in Panel A. Annuity ATVs decrease as the time horizon increases, and the effect is more pronounced for high cost bases. The intuition for this relationship is that an annuity can be viewed as a TDA (which has an ATV that is invariant to the time horizon) plus the cost basis tax shield (which becomes less valuable the longer it is delayed). Panel A also illustrates that the ATV approaches $(1 - T_n)$ as the time horizon increases, which can be verified by taking the limit of Eq. (5) as n approaches infinity (see the Appendix). The ATV approaches $(1 - T_n)$ more quickly at lower costs bases (e.g., for non-qualified annuities), which can also be inferred by examining the limit in the Appendix.

Panel B shows ATV for different terminal tax rates. The same patterns emerge. ATV

Table 1
After-tax value (ATV) per pre-tax dollar held in an annuity for various time horizons, cost.bases, tax rates, and risk-free rates

	Investment horizon in years (n)							
	0	5	10	15	20	25	30	35
Panel A: Cost basis (B)								
10%	0.748	0.742	0.737	0.733	0.731	0.728	0.726	0.725
20%	0.776	0.764	0.754	0.747	0.741	0.737	0.733	0.730
30%	0.804	0.786	0.772	0.760	0.752	0.745	0.739	0.735
40%	0.832	0.808	0.789	0.774	0.762	0.753	0.746	0.740
50%	0.860	0.830	0.806	0.787	0.773	0.761	0.752	0.745
60%	0.888	0.852	0.823	0.801	0.783	0.770	0.759	0.750
70%	0.916	0.874	0.840	0.814	0.794	0.778	0.765	0.756
80%	0.944	0.896	0.858	0.828	0.804	0.786	0.772	0.761
90%	0.972	0.917	0.875	0.841	0.815	0.794	0.778	0.766
100%	1.000	0.939	0.892	0.855	0.826	0.803	0.785	0.771
Panel B: Terminal tax rate (T_n)								
0%	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5%	0.975	0.970	0.965	0.962	0.959	0.957	0.956	0.955
15%	0.925	0.909	0.896	0.886	0.878	0.872	0.867	0.864
25%	0.875	0.848	0.827	0.810	0.797	0.787	0.779	0.773
28%	0.860	0.830	0.806	0.787	0.773	0.761	0.752	0.745
33%	0.835	0.799	0.771	0.749	0.732	0.719	0.708	0.700
35%	0.825	0.787	0.757	0.734	0.716	0.702	0.690	0.682
40%	0.800	0.757	0.723	0.696	0.675	0.659	0.646	0.636
50%	0.750	0.696	0.653	0.620	0.594	0.574	0.558	0.545
Panel C: Risk-free rate (r_f)								
0%	0.860	0.860	0.860	0.860	0.860	0.860	0.860	0.860
1%	0.860	0.853	0.847	0.841	0.835	0.829	0.824	0.819
2%	0.860	0.847	0.835	0.824	0.814	0.805	0.797	0.790
3%	0.860	0.841	0.824	0.810	0.798	0.787	0.778	0.770
4%	0.860	0.835	0.815	0.798	0.784	0.773	0.763	0.755
5%	0.860	0.830	0.806	0.787	0.773	0.761	0.752	0.745
6%	0.860	0.825	0.798	0.778	0.764	0.753	0.744	0.738
7%	0.860	0.820	0.791	0.771	0.756	0.746	0.738	0.733
8%	0.860	0.815	0.785	0.764	0.750	0.740	0.734	0.729
9%	0.860	0.811	0.779	0.758	0.745	0.736	0.731	0.727
10%	0.860	0.807	0.774	0.754	0.741	0.733	0.728	0.725

Note. The base case assumes the risk-free rate is 5%, the terminal tax rate is 28%, the cost basis is 50% of pre-tax market value, and no costs of financial intermediation associated with the annuity wrapper.

decreases and approaches $(1 - T_n)$ as the investment horizon increases. Moreover, the ATV decreases as tax rates increases, which is quite intuitive. One may also notice that the value approaches the limit more quickly at lower tax rates, which is borne out by examining the limit in the Appendix. Panel C displays the effect of the risk-free rate. ATVs decrease as the tax rate increases and the impact becomes more pronounced for longer investment horizons. Again, this result is intuitive when the ATV is seen as a TDA plus the cost basis tax shield.

Table 2 shows how the cost basis interacts with the terminal tax rate and the risk-free rate. The investment horizon is assumed to be 10 years. Predictably, the ATV increases as the cost basis increases. The relationship is more pronounced, however, at high tax rates. This result

Table 2
After-tax value (ATV) per pre-tax dollar held in an annuity for various cost.bases and tax rates

	Cost basis (B)							
	30%	40%	50%	60%	70%	80%	90%	100%
Panel A: Terminal tax rate (T_n)								
0%	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5%	0.959	0.962	0.965	0.968	0.971	0.975	0.978	0.981
15%	0.878	0.887	0.896	0.905	0.914	0.924	0.933	0.942
25%	0.796	0.811	0.827	0.842	0.857	0.873	0.888	0.903
28%	0.772	0.789	0.806	0.823	0.840	0.858	0.875	0.892
33%	0.731	0.751	0.771	0.792	0.812	0.832	0.852	0.873
35%	0.714	0.736	0.757	0.779	0.800	0.822	0.843	0.865
40%	0.674	0.698	0.723	0.747	0.772	0.796	0.821	0.846
50%	0.592	0.623	0.653	0.684	0.715	0.746	0.776	0.807
Panel B: Risk-free rate (r_f)								
0%	0.804	0.832	0.860	0.888	0.916	0.944	0.972	1.000
1%	0.796	0.821	0.847	0.872	0.897	0.923	0.948	0.973
2%	0.789	0.812	0.835	0.858	0.881	0.904	0.927	0.950
3%	0.783	0.803	0.824	0.845	0.866	0.887	0.908	0.928
4%	0.777	0.796	0.815	0.833	0.852	0.871	0.890	0.909
5%	0.772	0.789	0.806	0.823	0.840	0.858	0.875	0.892
6%	0.767	0.783	0.798	0.814	0.829	0.845	0.861	0.876
7%	0.763	0.777	0.791	0.805	0.820	0.834	0.848	0.862
8%	0.759	0.772	0.785	0.798	0.811	0.824	0.837	0.850
9%	0.755	0.767	0.779	0.791	0.803	0.815	0.826	0.838
10%	0.752	0.763	0.774	0.785	0.796	0.806	0.817	0.828

Note. The base case assumes the risk-free rate is 5%, the terminal tax rate is 28%, the investment horizon is 10 years, and no costs of financial intermediation associated with the annuity wrapper.

is driven by the product of these two variables in the second term of Eq. (5). Panel B displays the interaction between cost basis and interest rates. The ATV again increases with the cost basis, but the effect is diminished somewhat at high interest rates because high interest rates erode the present value of the cost basis tax shield.

5. Conclusion

The importance of viewing asset allocation and other portfolio management analyses on an after-tax basis is gaining acceptance. At present, the literature is silent about the after-tax value of annuities. This paper seeks to fill that void. We find that annuities can be viewed as TDAs plus the present value of the cost basis tax shield. Put differently, qualified annuities that received a tax deduction for initial investments have no cost basis and are functionally equivalent to a TDA from a tax perspective save for any costs of financial intermediation associated with the annuity wrapper. Consequently, the ATV for qualified annuities, like their TDAs counterparts, is equal to $(1 - T_n)$. However, when the initial investment (or a portion thereof) is not tax-deductible as in the case of a non-qualified annuity, the annuity has a cost basis that will escape taxation when funds are withdrawn. This tax shield increases the after-tax value of the annuity, but its value decreases as the investment horizon increases because its receipt is delayed.

Interestingly, the ATV of annuities is unaffected by the risk of the underlying investment. Intuitively, this insight can be explained by recalling that an annuity is functionally equivalent to a TDA plus the present value of the cost basis tax shield, if any. The risk of the underlying investment affects the value of neither component.

The ATV of annuity can be completely described by five factors–the cost basis, the withdrawal tax rate, the investment horizon, the risk-free rate, and any costs of financial intermediation. We show how the ATV varies with respect to these variables and how these variables interact with each other. We also consider, but dismiss, other approaches that are incongruent with accepted methodologies. Finally, we modify the model to accommodate annuitized, rather than lump sum, distributions.

Valuing assets on an after-tax basis is becoming increasingly important. Future research might consider how to incorporate annuities into an after-tax mean-variance optimization analysis. Alternatively, authors may direct their attention to incorporating annuities into an after-tax performance measurement scheme. In any case, this article advances the profession toward after-tax investment solutions that heretofore have been largely overlooked by private client advisors.

Appendix

A1. After-tax values, the discount rate, and tax equivalent values

The notion of after-tax value (ATV) has been develop in the literature, but deserves attention here. Consider an investor willing to reduce spending by $7,000 and can invest at a risk free rate of 3% for 20 years. Consider a tax-free world with no capital market imperfections in the Modigliani and Miller (1958) sense. The investor expects to accumulate $12,643 in 20 years. How much is this worth today? It seems obvious that it is worth the $7,000 beginning value. Alternatively, one can calculate the present value of $12,643 received 20 years from now at 3% to get the same $7,000. In a world without taxes or other capital market imperfections, the account clearly has a value of $7,000. This tax-free nirvana serves as useful reference point in the following discussion.

Now, assume a 30% tax rate payable on all income (including the risk-free rate) and different types of investment accounts. Consider the following types of accounts:

- Taxable Account (TA): Investments into this account are not deductible and income is taxed annually. No taxes are paid upon withdrawal. The investor can deposit their $7,000 into this account out of after tax funds (funds on which any accumulated taxes have been paid).
- Non-Qualified Annuity (NQA): Investments in this account are not deductible and no taxes are paid until withdrawn. At withdrawal, the investor must pay taxes on the

Table 3
Future values and present values in different tax environments

Account	Future accumulation before withdrawal taxes (FV)	Future after-tax cash flow ($FV_{AfterTax}$)
Tax-free world	$12,643	$12,643
Taxable world		
Taxable account	$10,608	$10,608
Non-qualified Annuity	$12,643	$10,950
Qualified annuity	$18,061	$12,643
	PV using after-tax risk-free discount rate (2.1%) (taxable equivalent values)	PV using pre-tax risk-free discount rate (3.0%) (after-tax values)
Tax-free world	$8,343	$7,000
Taxable world		
Taxable account	$7,000	$5,873
Non-qualified Annuity	$7,226	$6,062
Qualified annuity	$8,343	$7,000

excess over their original contribution. The investor can deposit their $7,000 into this account out of after tax funds (funds on which any accumulated taxes have been paid).

- Qualified-Annuity (QA): Investments in this account are deductible and no taxes are paid until withdrawn. At withdrawal, the investor must pay taxes on the full amount. The investor can deposit their $10,000 into this account and achieves a tax savings of $3,000 for an after-tax cost of $7,000.

Intuitively, the value today of the non-qualified annuity (V_{NQA}) to the investor should be higher than the value of a taxable account (V_{TA}) because the same amount is invested and the investor earns interest on funds that would have been otherwise paid in taxes each year.

Similarly, the value today of the qualified annuity (V_{QA}) should be higher than both of the other accounts since more money can be invested initially and interest is earned each year on these excess funds plus interest on the funds that would have otherwise been paid out in taxes. We should therefore expect $V_{QA} > V_{NQA} > V_{TA}$.

Now, how do the values of all of these accounts compare to the value of the account in a tax-free world in the Modigliani and Miller (1958) sense, $V_{TaxFree}$? Clearly, the value in a tax-free world dominates the value of a non-qualified annuity and taxable account since the same investment amount is made in all three cases and no taxes are paid at all in the tax-free world, so we should expect $V_{TaxFree} > V_{NQA} > V_{TA}$.

It is not immediately obvious whether the value in the tax-free world is greater than the value of a qualified annuity because the initial tax deduction associated with the qualified annuity allows the investor to put more pre-tax capital to work. Let us use numerical examples. Table 3, Panel A displays the future accumulation in the account before withdrawal taxes (FV) and the future after-tax value after any withdrawal taxes ($FV_{AfterTax}$).

Note that the relative magnitudes of the future after-tax cash flow ($FV_{AfterTax}$) conform to our prior expectations. Note, as well, that the future after-tax cash flow of the account in a tax-free world is the same as the future after-tax cash flow of the qualified annuity. The value of the initial tax deduction from the contribution is exactly offset by the withdrawal tax associated with the qualified annuity.

We can compute the present value of these accounts by discounting these future cash flows using the risk free rate in Panel B of Table 3. One might ask whether we value these accounts by discounting with the pre-tax risk free rate (3%) or the after-tax risk free rate (2.1%). On one hand, we traditionally value assets such as stock in companies using a pre-tax discount rate (e.g., the dividend discount model or discounted free cash flow model). On the other hand, our intent is to develop a model that can be used to compare after-tax values of accounts. Perhaps we should discount at the after tax risk free rate?

Discounting at 2.1%, corresponds to calculating taxable equivalent values (TEV) as described by Sibley (2002) and Horan (2002, 2005). TEV is the amount of assets in a taxable account that produces the same after-tax cash flow as a balance held in another type of account, such as a QA or NQA. Discounting at 3.0%, corresponds to after-tax value (ATV). Let us compare present values for both.

In both approaches the relative relationships of the values for the taxable account, non-qualified annuity and qualified annuity are retained. Comparisons to a tax-free world are conceptually problematic for the after-tax risk free rate, however. If we discount the future cash flow in a tax-free world at 2.1%, the account is valued at more than the initial deposit.

Discounting at the before-tax risk free rate not only retains the intuitive relationships, but also makes sense as the after-tax value of the account in a tax free world is the same as the amount that is deposited in the account ($7,000). This approach is consistent with the proposition in Horan (2007a) that the appropriate rate for discounting a risk-free asset is the risk-free rate itself rather than an after-tax rate since the government does not share in any risk. This approach is also consistent with how Modigliani and Miller (1958, 1963) value after-tax cash flows of a firm in a taxable environment. It is also consistent with more recent literature on the value of the corporate interest tax shield (e.g., Miles and Ezzell [1980], Sick [1990], Taggart [1991], Fernandez [2004], Cooper and Nyborg [2006, 2008]) and after-tax performance measurement (e.g., Stein [1998] and Stein, Langstraat and Narasihhan [1999], Horan, Lawton and Johnson [2008]).

We therefore define after-tax value (ATV) of an asset as the present value of future after tax cash flows with no tax adjustment to the risk free rate. The benchmark for comparison is what the present value of the account would have been in a tax-free world.

A2. Comparative statics for the ATV of annuities

The partial derivative of ATV in Eq. (5) with respect to the cost basis, B, is

$$\frac{\partial ATV}{\partial B} = \frac{T_n}{(1 + r_f)^n} > 0. \tag{A1}$$

Therefore, the ATV of an annuity increases as the cost basis increases, and the rate of increase depends on the other factors. Specifically, the positive relation between ATV and cost basis increases as the tax rate increases and decreases with the risk-free rate and the investment horizon.

The partial derivative of Eq. (5) with respect to the terminal tax rate, Tn, is

$$\frac{\partial ATV}{\partial T_n} = -(1-f)^n + \frac{B}{(1+r_f)^n}. \tag{A2}$$

This value will generally be negative for reasonable cost bases and time horizons, and again interacts with the other variables. It becomes less negative as the cost basis increases and more negative as the risk-free rate and the investment horizon increase.

ATV also decreases as the investment horizon increases for the reasons stated in the main text. According to the quotient rule,

$$\begin{aligned}\frac{\partial ATV}{\partial n} &= (1-T_n)[(1-f)^n \ln(1-f)^n] - \frac{T_n B(1+r_f)^n \ln(1+r_f)^n}{(1+r_f)^{2n}} \\ &= (1-T_n)[(1-f)^n \ln(1-f)^n] - T_n B\frac{\ln(1+r_f)^n}{(1+r_f)^n}.\end{aligned} \tag{A3}$$

The first term is negative for positive values of f, and second term is less than zero, as well. The relation becomes more negative as the tax rate, cost basis, and risk-free rate increase.

Finally, the limit of annuity ATV as the investment horizon approaches infinity in the absence of costs of financial intermediation (i.e., f = 0) can be expressed as

$$\lim_{n\to\infty}[ATV_{Annuity}] = \lim_{n\to\infty}\left[(1-T_n) + \frac{T_n B}{(1+r_f)^n}\right] = (1-T_n). \tag{A4}$$

Interestingly, for positive costs of financial intermediation, the expense drag reduces the limit to zero albeit only over very long time horizons. More specifically,

$$\lim_{n\to\infty}[ATV_{Annuity}] = \lim_{n\to\infty}\left[(1-f)^n(1-T_n) + \frac{T_n B}{(1+r_f)^n}\right] = 0. \tag{A5}$$

Notes

1. The analysis in this paper focuses on qualified withdrawals; that is, withdrawals after age 59½ and, in the case of the Roth IRA, after five years of contribution. It also ignores restrictions on qualified withdrawals and early withdrawal penalties. An example of a fifth type of account is healthcare savings accounts, HSAs, which offer triple tax savings. Contributions are tax-deductible, earnings accumulate on a tax-free basis, and withdrawals are tax-exempt. Eligibility and withdrawal requirements are more restrictive, however, than those listed above.
2. A similar but distinct class of models developed by Sibley (2002) and extended by Horan (2002) and Poterba (2004), converts balances in TDAs and tax-exempt accounts into values that are comparable to balances in taxable accounts. These approaches, although sometimes labeled by the authors as after-tax values, are more

accurately described as taxable equivalent values (TEV). For a time, the two terms were used interchangeably. See Horan (2007b) for a comparative analysis between taxable equivalent and after-tax models.

3. The after-tax future value of an investment taxed entirely as a deferred capital gain is $(1 + r)^n - [(1 + r)^n - 1]t_{cg} = (1 + r)^n(1 - t_{cg}) + t_{cg}$. Note the similarity.
4. This term is used by Poterba (2000), but the same concept has been used by other authors.

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