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Valuation of assets in taxable accounts and annuities

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Abstract

Horan and Robinson (2008) and I agree on several issues including how to calculate the after-tax value of assets held in tax-deferred accounts like a 401(k) and tax-exempt accounts like a Roth IRA. We agree that an asset's after-tax value is its after-tax future value when discounted back to the present by dividing by one plus the risk-appropriate discount rate. However, we disagree about the risk-appropriate discount rate for and, therefore, after-tax value of (1) assets held in taxable accounts and (2) assets that earn tax-deferred returns, where the latter include assets held in non-qualified annuities and passively-held stocks in taxable accounts. This study explains my arguments for calculating the after-tax values of these assets.

Keywords: asset allocation; after-tax asset allocation; after-tax asset valuations; tax-deferred growth

1. Introduction

Horan and Robinson (2008) and I agree on much more than we disagree. We agree that proper management of individuals' portfolios require managing their after-tax asset allocation (e.g., Horan [2007a, b], Reichenstein [1998, 2000, 2006a, 2007a, b, 2008], and Reichenstein and Jennings [2003]). Individuals' portfolios should be evaluated using after-tax performance measures (e.g., Horan, Lawton and Johnson [2008], Stein [1998], and Stein, Langstraat and Narasihhan [1999]). They should be concerned with obtaining an optimal mean-variance after-tax portfolio (e.g., Horan [2007b], Reichenstein [2007a, b], and Wilcox, Horvitz and DiBartolomeo [2006]). Moreover, individuals must recognize that the same asset—say a bond—is effectively a different asset when held in (1) a taxable account, (2) a tax-deferred account such as a 401(k) or tax-exempt account such as a Roth IRA, and (3) a

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non-qualified annuity, because the after-tax return received by the individual and after-tax risk borne by the individual differ with each saving vehicle (e.g., Horan [2007a, b] and Reichenstein [2000, 2006a, 2007a, b]). Individuals should be concerned with asset location, and the argument for asset location follows directly from the after-tax valuation framework (e.g., Dammon, Spatt and Zhang [2004], Poterba, Shoven and Sialm [2004], and Reichenstein [2001]). Finally, we agree that the same concepts can be applied to form tax-efficient withdrawal strategies in retirement (e.g., Horan [2006a, b], Reichenstein [2006b, 2006c), and Spitzer and Singh [2006]).

To manage individuals' after-tax portfolios, we must be able to calculate their after-tax asset allocations, which requires being able to calculate assets' after-tax values when held in different savings vehicles. On this front, we share much agreement. We agree that an asset's after-tax (present) value should be its after-tax future value when discounted by a risk-appropriate discount rate that reflects the portion of an asset's risk that is borne by the individual investor. Furthermore, we agree on how to calculate the after-tax values of assets held in tax-exempt accounts (e.g., Roth IRA, Roth 401(k), Roth 403(b)) and tax-deferred accounts (e.g., 401(k), 403(b), traditional IRA). We do, however, disagree about how to calculate the after-tax value of assets held in taxable accounts and non-qualified annuities. In this article, I will present my approach to valuing these assets. As is the nature of research, much of this paper will focus on areas of disagreement. I hope that the reader does not lose perspective of our broad areas of agreement.

For example, we agree that, when calculating an individual's asset allocation, $1 of *pre-tax* funds in a tax-deferred account is not the same as $1 of *after-tax* funds in a Roth IRA. Moreover, we agree on how advisers should distinguish between these accounts that have equal market values. Today, most financial advisers treat $1 of pre-tax funds as if it is the same as $1 of after-tax funds; that is, they ignore taxes and thus implicitly assume that taxes do not exist. We agree that we should expect more from financial advisers.

2. Principal, risk, and returns sharing across saving vehicles

This section extends the analysis in Reichenstein (2007b) to incorporate assets held in taxable assets and non-qualified annuities with and without built-in (i.e., unrealized) deferred returns. The extension relies on advancements made possible by Horan and Robinson's (2008) models.

Table 1 presents the after-tax future value of bonds and stocks with a market value of $1 when held in tax-exempt account (henceforth, tax-exempt Roth or Roth), tax-deferred account, taxable account, and non-qualified annuity. The pre-tax rate of return is r, the length of the investment horizon is n, the ordinary income tax rate is T in all years and T_n in year n, and the long-term capital gain rate is T_c in all years.

2.1. Roth

From Table 1, when held in a tax-exempt Roth the $1 market value represents after-tax funds. It grows tax exempt at r percentage for n years. Its after-tax future value is $(1 + r)^n$.

Table 1 After-tax future values of bonds and stocks in Roth, tax-deferred account, taxable account, and non-qualified annuity

	Bonds	Stocks
Roth	$(1 + r)^n$	$(1 + r)^n$
Tax-deferred account	$(1 + r)^n (1 - T_n)$	$(1 + r)^n (1 - T_n)$
Taxable account	$(1 + r(1 - T))^n$	
Taxable account		
Day trader:		$(1 - (1 - B)T) (1 + r(1 - T))^n$
Active investor:		$(1 - (1 - B)T_c) (1 + r(1 - T_c))^n$
Passive investor:		$(1 + r)^n (1 - T_c) + BT_c$
Exempt investor:		$(1 + r)^n$
Non-qualified annuity	$(1 + r)^n (1 - T_n) + BT_n$	$(1 + r)^n (1 - T_n) + BT_n$

Note. Today's market value is $1 for each asset held in each savings vehicle. r denotes the pretax return, n the length of the investment horizon, t the ordinary income tax rate in all years with T_n the tax rate in year n, T_c the long-term capital gain tax rate, and B denotes the cost basis as a percentage of market value.

Table 2 presents the portion of the market value effectively owned by the investor, portions of returns received by and risk borne by the investor, and the effective tax rate. When held in a Roth the individual effectively owns all of the market value. He receives 100% of pre-tax returns and bears 100% of pre-tax risk. The effective tax rate is 0%.

The after-tax (present) value is its after-tax future value when discounted by the risk-appropriate discount rate. Because the investor bears all risk in a Roth, the appropriate discount rate is r. The after-tax value is thus $(1 + r)^n/(1 + r)^n$ or $1, where the numerator is the after-tax future value and the denominator is one plus the risk-appropriate discount rate.

2.2. Tax-deferred account

From Table 1, when held in the tax-deferred account the $1 market value represents pre-tax funds. It grows tax deferred at r per year and after n years its pre-tax value is (1 +

Table 2 Market value effectively owned by, returns received by, and risk borne by individual investors in Roth, tax-deferred account, taxable account, and non-qualified annuity

	Market value	Returns and risk	Effective tax rate
Roth	100%	100%	0%
Tax-deferred account	$1 - T_n$	100%	0%
Tax acct, bonds	100%	$1 - T$	T
Tax acct, trader stocks (B = 1)	100%	$1 - T$	T
Tax acct, trader stocks (B = 0.8)	$1 - (1 - 0.8)T$	$1 - T$	T
Tax acct, active stocks (B = 1)	100%	$1 - T_c$	T_c
Tax acct active stocks (B = 0.8)	$1 - (1 - 0.8)T_c$	$1 - T_c$	T_c
Tax acct passive stocks (B = 1)	100%	$\geq 1 - T_c$	$\leq T_c$
Tax acct passive stocks (B = 0.8)	$1 - (1 - 0.8)T_c$	$>1 - T_c$	$< T_c$
Tax acct passive stocks (B = 0)	$1 - T_c$	100%	0%
Tax acct exempt stocks (B = all)	100%	100%	0%
Non-qualified annuity (B = 1)	100%	$\geq 1 - T_n$	$\leq T_n$
Non-qualified annuity (B = 0.8)	$1 - (1 - 0.8)T_n$	$>1 - T_n$	$<T_n$
Qualified annuity (B = 0)	$1 - T_n$	100%	0%

Note. B denotes asset's cost basis as percentage of market value.

$r)^n$ and its after-tax future value is $(1 + r)^n (1 - T_n)$. From Table 2, the individual investor effectively owns $(1 - T_n)$ of the market value. He receives 100% of pre-tax returns and bears 100% of pre-tax risk. The effective tax rate is zero.

Generalizing, the tax-deferred account is like a trust, where the individual investor owns $(1 - T_n)$ of the principal (i.e., market value) and the government is a minority partner owning the remaining $T_n$of principal. The individual, as general partner, selects the investment and decides when to withdraw funds. However, the government, as minority partner, gets T_n of each dollar withdrawn, where T_n is the ordinary tax rate in the withdrawal year n years hence.

Each dollar in a tax-deferred account is like $(1 - T_n)$ dollar in a Roth because, when invested in the same asset, they will each buy the same amount of goods and services in retirement. To continue with the trust analogy, each dollar in a tax-deferred account can be separated into $(1 - T_n)$ of the individual's after-tax funds plus T_n, which is the government's share of the principal. The *after-tax* value of each dollar in the tax-deferred account grows from $(1 - T_n)$ today to $(1 - T_n)(1 + r)^n$ in n years, where r is the pre-tax rate of return. Therefore, the individual receives 100% of pre-tax returns and bears 100% of pre-tax risk on his $(1 - T_n)$ of after-tax funds. The effective tax rate is 0%.

The after-tax value of the tax-deferred account is its after-tax future value when discounted by the risk-appropriate discount rate. Because the investor bears all risk, the appropriate discount rate is r. The after-tax value is thus $(1 + r)^n (1 - T_n)/(1 + r)^n$ or $(1 - T_n)$.

2.3. Bonds in taxable account

This section discusses the valuation of bonds held in taxable accounts. A later section discusses the valuation of stocks held in taxable accounts. For bonds, today's \$1 market value represents after-tax funds. From Table 1, it grows at the $r(1 - T)$ after-tax rate of return and after n years its after-tax value is $(1 + r(1 - T))^n$. From Table 2, the individual effectively owns all the market value, receives $(1 - T)$ of pre-tax returns and bears $(1 - T)$ of pre-tax risk, and the effective tax rate is T. Therefore, the appropriate discount rate is $r(1 - T)$. The asset's after-tax value is $(1 + r(1 - T))^n/(1 + r(1 - T))^n$ or \$1, where the numerator is the after-tax future value and the denominator is one plus the risk-appropriate discount rate.

To illustrate that the government shares in the risk, consider a bond with a 6% pre-tax expected return and 4% pre-tax standard deviation. For a three-year period, the returns are 2%, 6%, and 10%, that is, the mean return and one standard deviation below and above the mean. The standard deviation of these returns is 4%. If the ordinary income tax rate is 25% then the after-tax returns are 1.5%, 4.5%, and 7.5%. Therefore, the after-tax expected return is 4.5% or 6% (1−0.25) and after-tax risk is 3% or 4% (1−0.25). Generalizing, the individual receives approximately $(1 - T)$ of the pre-tax returns and bears approximately $(1 - T)$ of the pre-tax risk, whereas the government takes approximately T of the returns and bears approximately T of the risk.[1]

Horan (2007a, b) and Horan and Robinson (2008) disagree with my approach to calculating the after-tax value of bonds held in taxable accounts. To illustrate the differences in opinions, assume someone invest \$1 today in a risk-free bond earning 3% per year for 20 years and the ordinary income tax rate is 30% each year. We agree that the after-tax future value is \$1.52 or $(1 + 0.03(1 - 0.3))^{20}$. We disagree about the appropriate risk-adjusted discount rate. I believe it should be 2.1% or 3% (1 − 0.3). They say the government cannot share in the risk of a risk-free asset, so the appropriate discount rate is 3%. I believe its after-tax value is $(1 + 0.03(1 - 0.3))^{20}/(1 + 0.03(1 - 0.3))^{20}$ or \$1. They believe the after-tax value is $(1 + 0.03(1 - 0.3))^{20}/(1 + 0.03)^{20}$ or \$0.84. In short, they believe the value of the \$1 invested today depends upon the risk-free rate, the length of the investment horizon, and the tax rate. I believe it is worth \$1.

Consider two investments. The first is \$1 invested in this risk-free asset held in a Roth and the second is \$1 invested in this risk-free asset held in a taxable account. We agree that the after-tax future value of the Roth is certain to be higher than the after-tax future value of the taxable account.[2] Based on this, Horan (2007a, b) and Horan and Robinson (2008) conclude that the Roth must have a higher value today. Clearly, the investor would prefer to have \$1 invested in a Roth than a taxable account. I view this Roth advantage as a tax subsidy that increases with the tax rate, length of investment horizon, and rate of return. By limiting the annual contribution to Roths, the government limits the size of this subsidy. However, I believe the after-tax value of both investments is \$1.

2.4. *Non-qualified annuity*

This section discusses the valuation of non-qualified annuities. The benefit of this savings vehicle is tax-deferred returns, and this section illustrates valuation issues related to deferred returns. Later, we will apply these lessons to the valuation of stocks held in taxable accounts, especially by passive investors.

Let me pause to make a point about terminology. For consistency with Horan and Robinson (2008) and my prior use, I will call this savings vehicle a non-qualified annuity. However, a better name for this savings vehicle might be a nondeductible traditional IRA. In this vehicle, the individual invests after-tax funds, returns grow tax deferred, and at withdrawal deferred returns are taxed as ordinary income. Compared to bonds held in a taxable account, the benefit of this saving vehicle is tax-deferred growth. To understand problems that are *not* addressed in this study, suppose someone invested \$8,000 years ago in a non-qualified annuity, and its account value is \$10,000. If there is a 6% surrender charge, then the surrender value would be \$9,400 if she liquidates the account this year. It is not clear whether the asset's market value should be \$9,400 or \$10,000. Moreover, because of higher expenses, the annuity may pay a lower return than similar-risk market-based assets. A lawsuit may determine that individuals who were sold non-qualified annuities own unsuitable investments. If so, damage estimates would reflect the below-market risk-adjusted returns. This study does not consider surrender fees nor noncompetitive returns. Therefore, the analysis of "non-qualified annuities" in this study deals only with the benefits of tax-deferred growth and the valuation of assets with deferred returns.

Suppose someone originally invested \$B of after-tax funds in bonds or stocks held in a non-qualified annuity and its market value is now \$1. B is the cost basis and (1 − B) is tax-deferred returns as a percentage of market value. From Table 1, its after-tax future value after n years is $(1 + r)^n (1 - T_n) + BT_n$. This is Eq. (3) from Horan and Robinson (2008),

Table 3 Effective tax rates by horizon and cost basis for assets held in annuities for 8% return and eventual tax rate of 25%

Investment horizon (years)	B = 1	B = 0.8	B = 0.4	B = 0
1	25.00%	21.05%	11.76%	0%
3	23.60	19.82	11.00	0
5	22.28	18.67	10.30	0
10	19.31	16.09	8.78	0
15	16.79	13.94	7.54	0
20	14.68	12.15	6.53	0
25	12.91	10.66	5.71	0
30	11.43	9.43	5.03	0
35	10.20	8.40	4.47	0
40	9.16	7.55	4.01	0

Note. B denotes asset's cost basis as percentage of market value.

and it represents a significant improvement compared to prior models. Today's \$1 market value grows to $(1 + r)^n$ before taxes. Taxes at T_n are paid on deferred returns. The after-tax future value is the same as if the market value was taxed at T_n but then we add back BT_n, the tax shield from the cost basis.

Table 3 presents effective tax rates by investment horizon and cost basis for an asset held in an annuity earning a tax-deferred return of 8% per year that will eventually be taxed at 25%. The underlying asset could be stocks or bonds; in the annuity, deferred returns, whether in the form of interest, dividends, or capital gains, are eventually taxed as ordinary income. As the horizon lengthens (i.e., looking down a column), the effective tax rate decreases. This is the benefit of tax-deferred growth. The effective tax rate reflects the percentage of returns received by and risk borne by the individual investor. It will be used to calculate the after-tax value of assets held in an annuity.

Let us first consider an asset with a cost basis of 1 and a one-year investment horizon. Someone invests \$1 today in a non-qualified annuity and withdraws the funds after one year. For a one-year horizon, the models for bonds held in taxable account and an annuity are the same, that is, $(1 + 0.08(1 - 0.25))^1 = (1.08)^1(1 - 0.25) + 0.25$. There is no benefit for deferring gains for one year. The effective tax rate is 25%. This investor receives 75% of returns and bears 75% of risk. Generalizing, the appropriate discount rate is $r(1 - T_c)$. The after-tax value of this annuity is $(1 + r(1 - T_c))/(1 + r(1 - T_c))$ or \$1.

Suppose the investment horizon is 20 years. From Table 3, for a cost basis of 1 the effective tax rate is 14.68%. Mathematically, $(1 + 0.08(1 - 0.1468))^{20} = (1.08)^{20}(1 - 0.25) + 1(0.25) = \$3.75$; that is, the after-tax future value is the same as if the return was taxable each year at 14.68%. The after-tax value grows from \$1 if withdrawn today to \$3.75 if withdrawn in 20 years. By deferring returns for 20 years, the annuity's after-tax value grows at the 6.83% rate of return, $3.75^{(1/20)} - 1$. The investor receives 85.32% of pre-tax returns, 6.83%/8%. Therefore, the effective tax rate is 14.68%, (8−6.83%)/8%. The appropriate discount rate is 8%(1 − 0.1468) or 6.83%. The after-tax value is $(1 + 0.08(1 - 0.0844))^{20}/(1 + 0.08(1 - 0.0844))^{20}$ or \$1, where the numerator is also $(1.08)^{20}(1 - 0.25) + 1(0.25)$, that is, the after-tax future value and the denominator is one plus the risk-appropriate discount rate.

In their Eq. (4), Horan and Robinson (2008) value all annuities regardless of horizon and cost basis by discounting the after-tax future value at the risk-free rate. I believe this is their mistake. It leads to what they call their "counterintuitive" conclusion that the after-tax value of an annuity decreases with the length of the investment horizon. In my approach, the annuity's after-tax value is independent of the investment horizon. To be more precise, for cost bases exceeding zero the effective tax rate decreases with the length of the investment horizon, but the annuity's after-tax value is $1 - (1 - B)T_n$ regardless of the length of the investment horizon.

As Horan and Robinson (2008) note, the effective tax rate, T*, is calculated as the rate that makes the after-tax future value of $(1.08)^{20}(1 - 0.25) + 1(0.25)$ equal to $(1 + 0.08(1 - T^*))^{20}$. Poterba (2000) calls T* the accrual equivalent tax rate. I call it the effective tax rate. This procedure ensures that its after-tax values is always equal to $1 - (1 - B)T_n$. This point will be demonstrated by valuing a non-qualified annuity with a cost basis of 0.8 for investment horizons of one and 20 years.

Consider a non-qualified annuity with a cost basis of 0.8. It has a cost basis of $0.80 and a market value of $1. Therefore, it has embedded deferred returns of $0.20 or (1 − B). If sold today, its after-tax value would be $0.95. Generalizing, it would be $1 - (1 - B)T_n$. If sold one year hence, its after-tax value would be $1.01, [(1.08)(1 − 0.25) + 0.8(0.25)]. By delaying recognition of the tax-deferred gain for a year, the investor receives a 6.32% after-tax return. That is, she receives 78.95% or 6.32%/8% of pre-tax returns and bears 78.95% of pre-tax risk, where 78.95% is (1 − 0.2105) and 21.05% is the effective tax rate from Table 3. The appropriate discount rate is 8% (1 − 0.2105) or 6.32%. The after-tax value is $0.95(1 + 0.08(1 − 0.2105))/(1 + 0.08(1 − 0.2105)) or $0.95, where the numerator is also (1.08)(1 − 0.25) + 0.8(0.25), that is, the after-tax future value, and the denominator is one plus the risk-appropriate discount rate. She effectively owns 95% of the market value but receives 87.63% of returns and bears 87.63% of risk.

Let's consider the after-tax value of this annuity if she has a 20-year investment horizon. The after-tax value grows from $0.95 if sold today to $3.70 if sold in 20 years, $(1.08)^{20}(1 - 0.25) + 0.80(0.25)$. By deferring gains for 20 years, she receives a 7.03% after-tax rate of return. The investor receives 87.85% or 7.03%/8% of pre-tax returns and bears 87.85% of pre-tax risk. Therefore, the effective tax rate is 12.15% or (1 − 0.8785). The appropriate discount rate is 8%(1 − 0.1215) or 7.03%. The after-tax value is $\$0.95(1 + 0.08(1 - 0.1215))^{20}/(1 + 0.08(1 - 0.1215))^{20}$ or $0.95. She effectively owns 95% of market value, just like with the one-year horizon. Generalizing, the investor effectively owns $1 - (1 - B)T_n$ of the annuity's market value and this is also the after-tax value.

From Tables 2 and 3, for an annuity with a cost basis of B, the investor effectively owns $1 - (1 - B)T_n$ of market value. For basis of 1, she receives $(1 - T_n)$ of returns and bears $(1 - T_n)$ of risk for a one-year horizon, but receives more than $(1 - T_n)$ of returns and bears more than $(1 - T_n)$ of risk for a multiyear horizon. For cost basis less than 1, the effective tax rate is less than T_n for all horizons. The qualified annuity has no cost basis, and we all agree it is simply a tax-deferred account. Therefore, the investor effectively owns $(1 - T_n)$ of market value and has an effective tax rate of 0%.

Table 3 presents effective tax rates by investment horizon for cost bases of 1, 0.8, 0.4, and 0. For assets with positive cost bases, as the horizon lengthens the effective tax rate decreases. This is the benefit of tax-deferred growth. For horizons that are appropriate for individual investors, the effective tax rates are substantially above zero. For example, for basis of 1 and a 25-year horizon, the effective tax rate is 12.91%, or more than half the eventual tax rate. Therefore, in my valuation approach, I discount the after-tax future value back to the present by dividing by $(1 + 0.08(1 - 0.1291))^{25}$. In contrast, in Horan and Robinson's model, they would divide by $(1.08)^{25}$.

From Table 3, as the cost basis decreases (i.e., looking across a row) the effective tax rate decreases. For example, for a one-year horizon the effective tax rate is 25% for a cost basis of 1, 21.05% for a basis of 0.8, 11.76% for a basis of 0.4, and 0% for a basis of zero.

2.5. *Stocks held in taxable account*

For stocks held in a taxable account, the after-tax future value depends upon the individual's stock management style. We consider individuals with one of four management styles: those of a trader, an active investor, a passive investor, and an exempt investor. The passive investor is of special interest in this study because we are concerned about the valuation of assets with tax-deferred returns. For simplicity, I assume all returns are in the form of capital gains. For more detailed models that include dividends, see Reichenstein (2007a, 2008).

The trader realized all gains within a year. Therefore, returns are taxed at the ordinary income tax rate, T. The ending after-tax wealth model is $(1 - (1 - B)T)\,(1 + r(1 - T))^n$. Table 2 considers traders with cost bases of 1 and 0.8.

When the cost basis is 1 then the ending wealth model is $(1 + r(1 - T))^n$, which is identical to the ending wealth model for bonds held in a taxable account. From Table 2, this investor owns all of the market value, receives $(1 - T)$ of returns and bears $(1 - T)$ of risk. Therefore, the appropriate discount rate is $r(1 - T)$. The after-tax value is $(1 + r(1 - T))^n/(1 + r(1 - T))^n$ or \$1.

When the cost basis is 0.8, the cost basis may be \$0.80 and the market value \$1. This trader has a built-in short-term gain representing 20% of the market value. If this year's ordinary income tax rate, T, is 25% then this trader will soon realize the short-term gain and pay \$0.05 in taxes. Therefore, he effectively owns 95% or $1 - (1 - B)T$ of the market value. After paying taxes, he will realize $(1 - T)$ of returns and bear $(1 - T)$ of risk. The effective tax rate is T. Generalizing, the appropriate discount rate is $r(1 - T)$. Therefore, the after-tax value is $(1 - (1 - B)T)\,(1 + r(1 - T))^n/(1 + r(1 - T))^n$ or $(1 - (1 - B)T)$.

The active stock investor realizes all gains as soon as they are eligible for long-term capital gain tax treatment. Therefore, returns are taxed at the capital gain tax rate, T_c. The after-tax future value model is $(1 - (1 - B)T_c)\,(1 + r(1 - T_c))^n$. Table 2 considers active investors with cost bases of 1 and 0.8.

If the cost basis is 1 then the model simplifies to $(1 + r(1 - T_c))^n$. From Table 2, this investor owns all of the market value, receives $(1 - T_c)$ of returns and bears $(1 - T_c)$ of risk. The effective tax rate is T_c. The appropriate discount rate is $r(1 - T_c)$. Therefore, the after-tax value is $(1 + r(1 - T_c))^n/(1 + r(1 - T_c))^n$ or \$1.

When cost basis is 0.8, the cost basis may be \$0.80 and the market value \$1. The active investor will soon realize the gain and pay \$0.03 in taxes. Therefore, he effectively owns 97% of the market value. After paying taxes, he will realize $(1 - T_c)$ of pre-tax returns and

Table 4 Effective tax rates by horizon and cost basis for appreciating assets held in taxable accounts for 8% return and eventual tax rate of 15%

Investment horizon (years)	B = 1	B = 0.8	B = 0.4	B = 0
1	15.00%	12.37%	6.59%	0%
3	14.06	11.58	6.14	0
5	13.19	10.84	5.73	0
10	11.29	9.24	4.86	0
15	9.72	7.94	4.15	0
20	8.44	6.88	3.58	0
25	7.38	6.02	3.13	0
30	6.52	5.31	2.75	0
35	5.80	4.72	2.44	0
40	5.20	4.23	2.19	0

Note. B denotes asset's cost basis as percentage of market value.

bear $(1 - T_c)$ of pre-tax risk. The effective tax rate is T_c. Generalizing, the appropriate discount rate is $r(1 - T_c)$. Therefore, the after-tax value is $(1 - (1 - B)T_c)\,(1 + r(1 - T_c))^n/(1 + r(1 - T_c))^n$ or $(1 - (1 - B)T_c)$.

The passive investor allows returns to grow tax deferred. After n years, she realizes all gains and pays taxes at T_c. From Table 1, the after-tax future value is $(1 + r)^n\,(1 - T_c) + BT_c$, where this model comes from Horan and Robinson (2008). Suppose the cost basis is \$B and market value is \$1. The after-tax future value is the same as if the pre-tax future value, $(1 + r)^n$, is taxed at T_c. We then add back BT_c, which is the tax shield from being able to withdraw the \$B cost basis tax free.

Table 4 presents effective tax rates by horizon and cost basis assuming an 8% pre-tax return and eventual tax rate of 15%. The patterns of effective tax rates are the same as in Table 3. As the horizon lengthens, the effective tax rate decreases. As the cost basis decreases, the effective tax rate decreases. For a basis of zero, the effective tax rate is zero. Although the cost basis cannot be zero for stock held in taxable accounts, it can approach zero. As the basis approaches zero, this model approaches the model for a tax-deferred account except the eventual tax rate is the long-term gain tax rate, T_c.

Let us first consider how to calculate the after-tax value of stock (or any other appreciating asset) with a cost basis of 1. For a one-year horizon, the effective tax rate is 15% or T_c. This investor receives 85% of pre-tax returns and bears 85% of pre-tax risk. Generalizing, the appropriate discount rate is $r(1 - T_c)$. The after-tax value is $(1 + r(1 - T_c))/(1 + r(1 - T_c))$ or \$1.

From Table 4, for a 20-year horizon and cost basis of 1 the effective tax rate is 8.44%. Mathematically, $(1 + 0.08(1 - 0.0844))^{20} = (1.08)^{20}(1 - 0.15) + \$1(0.15)$; that is, the after-tax future value is the same as if the return was taxable each year at 8.44%. The after-tax value grows from \$1 if withdrawn today to \$4.11 if withdrawn in 20 years. By deferring returns for 20 years, the investor's after-tax value grows at the 7.33% rate of return. The investor receives 91.56% of pre-tax returns, 7.33%/8%. Therefore, the effective tax rate is 8.44%, (8–7.33%)/8%. The appropriate discount rate is 8%(1 − 0.0844). The after-tax value is $(1 + 0.08(1 - 0.0844))^{20}/(1 + 0.08(1 - 0.0844))^{20}$ or \$1.

Now, let us consider how to calculate the after-tax value of passively held stocks when the cost basis is less than 1. From Table 2, the passive stock investor effectively owns $1 - (1 - B)T_c$ of the market value. For example, when the cost basis is 0.8 and T_c is 0.15, he effectively owns 97% of market value. We will demonstrate this conclusion by showing that he effectively owns 97% for two investment horizons and, by extension, for any horizon.

Consider a one-year horizon when the cost basis is \$0.80 and the market value is \$1. If sold today, its after-tax value would be \$0.97. If sold one year hence, its after-tax value would be \$1.038. By delaying recognition of the tax-deferred gain for a year, the investor receives a 7.01% after-tax return. That is, she receives 87.63% or 7.01%/8% of pre-tax returns and bears 87.63% of pre-tax risk, where 87.63% is (1 − 0.1237) and 12.37 is the effective tax rate. The appropriate discount rate is 8%(1 − 0.1237) or 7.01%. The after-tax value is \$0.97(1 + 0.08(1 − 0.1237))/ (1 + 0.08(1 − 0.1237)) or \$0.97. She effectively owns 97% of the market value.

Now, let's consider a 20-year horizon. The after-tax value grows from \$0.97 if sold today to \$4.08 if sold in 20 years, $(1.08)^{20}(1 - 0.15) + 0.80(0.15)$. By deferring gains for 20 years, she receives a 7.45% after-tax rate of return. The investor receives 93.12% or 7.45%/8% of returns and bears 93.12% of risk. Therefore, the effective tax rate is 6.88% or (1 − 0.9312). The appropriate discount rate is 8%(1 − 0.0688). The after-tax value is $\$0.97(1 + 0.08(1 - 0.0688))^{20}/(1 + 0.08(1 - 0.0688))^{20}$ or \$0.97. The investor effectively owns 97% of the market value, just like with a one-year horizon, and this is also its after-tax value.

This method of valuation ensures that the passive investor with basis of B who will eventually pay taxes on the embedded gains at T_c effectively owns $1 - (1 - B)T_c$ of the market value regardless of the actual holding period. That is, the effective tax rates are calculated in a manner to ensure this outcome. However, this reflects the reality that the passive investor with cost basis of 0.8 who defers recognition of the gain for 20 years bears 93.12% of the risk assuming an 8% pre-tax return. Moreover, we could change the assumed return that would change the effective tax rate, but as long as the embedded gains will eventually be realized and taxed at 15%, the investor effectively owns 97% of market value. Generalizing, if the gain will eventually be taxed at T_c then the after-tax value of stocks held in a taxable account with cost basis of B is $1 - (1 - B)T_c$ of market value. The after-tax value is the value as if the embedded gains were sold today *and taxes paid at* T_c. The next example shows that T_c should be the applicable tax rate when sold. If the stock is sold immediately after receiving the step-up in basis then the applicable tax rate is zero.

The exempt investor never pays taxes on unrealized gains. She either awaits the step-up in basis at death or donates the appreciated asset to a qualified charity. From Table 1, the after-tax future value is $(1 + r)^n$; because she will not pay taxes on the gain, the after-tax value is the same as the market value. This is the same as the model for the passive investor when T_c is zero. Moreover, for the exempt investor, the eventual tax rate is zero. From Table 2, she effectively owns all market value, receives all returns and bears all risk. The effective tax rate is zero.

In short, Table 2 indicates that the after-tax value of stocks held in a taxable account is the market value less the tax liability on the deferred returns. The stock's after-tax value is $1 - (1 - B)T$ of market value if the gain will be realized as a short-term gain this year, $1 - (1 - B)T_c$ of market value if the gain eventually will be realized and taxed at T_c, and 100%

Table 5 After-tax future values and after-tax values of $7,000 by savings vehicle

	After-tax future value	After-tax value[a]	Risk-adjusted discount rate	Agreement
Roth IRA	$12,643	$7,000	3%	Agree
Tax-deferred account	$ 8,850	$4,900	3%	Agree
Taxable account (B = 1)	$10,607	$7,000	3%(1 − 0.3)	Disagree
NQA (B = 1)	$10,950	$7,000	3%(1 − 0.2459)	Disagree
NQA (B = 0.6)	$10,110	$6,160	3%(1 − 0.1639)	Disagree
QA (B = 0)	$ 8,850	$4,900	3%	Agree

Notes. Assumptions: $r = 0.03$, $n = 20$, $t = 0.3$. TDA denotes tax-deferred account. B denotes cost basis as percentage of market value. NQA denotes non-qualified annuity and QA denotes qualified annuity.

[a] After-tax values are also after-tax future values when discounted by 1 plus the risk-appropriate discount rate: Roth IRA: $\$7{,}000(1.03)^{20}/(1.03)^{20} = \$7{,}000$; TDA: $\$7{,}000(1-0.3)(1.03)^{20}/(1.03)^{20} = \$4{,}900$; Taxable account: $\$7{,}000(1 + 0.03(1-0.3))^{20}/(1 + 0.03(1-0.3))^{20} = \$7{,}000$.

of market value if the investor will await the step-up in basis or donate the appreciated asset to a qualified charity.[3]

3. Horan and Robinson's example

Horan and Robinson (2008) present a valuation example in their appendix. This section repeats their example except it uses my valuation approach. The goal of this section is to calculate the after-tax value of assets held in each savings vehicle. In their example, the asset is the risk-free asset. It earns 3% per year and the investment horizon is 20 years. The ordinary income tax rate is 30% for all years, $T = T_n = 0.3$. The market value of the asset is $7,000. Although their example considers a risk-free asset, the framework illustrates the differences in our methods.

From Table 5, the after-tax future value of $7,000 held in a Roth IRA is $\$7{,}000(1.03)^{20}$ or $12,643. The investor bears all risk in a Roth. Therefore, the appropriate discount rate is 0.03, the asset's pre-tax return. So, Horan and Robinson (2008) and I (henceforth, we) agree that the after-tax value is $\$7{,}000(1.03)^{20}/(1.03)^{20}$ or $7,000.

The future after-tax value of $7,000 of pre-tax funds held in a tax-deferred account is $\$7{,}000(1.03)^{20}\ (1 - 0.3)$ or $8,850. The investor bears all risk in a tax-deferred account. Therefore, the appropriate discount rate is 0.03, the asset's expected return. The after-tax value is $\$7{,}000(1.03)^{20}\ (1 - 0.3)/(1.03)^{20}$ or $4,900. Generalizing, we agree that each dollar of pre-tax funds has an after-tax value of $(1\text{-}T_n)$.[4]

Suppose an individual invests $7,000 today in the risk-free asset held in a taxable account. The cost basis equals the market value, that is, B is 1. We agree that the after-tax future value after 20 years is $\$7{,}000(1 + 0.03(1 - 0.3))^{20} = \$7{,}000(1.021)^{20} = \$10{,}607$. We disagree about the appropriate risk-adjusted discount rate. I conclude that the appropriate discount rate is 3%(1 − 0.3) or 2.1%, and the after-tax value is $\$7{,}000\ (1.021)^{20}/(1.021)^{20}$ or $7,000. They conclude that the appropriate discount rate is 3%, and the after-tax value is $\$7{,}000\ (1.021)^{20}/(1.03)^{20}$ or $5,873. In their model, the after-tax value depends of the rate of return, the length of the investment horizon, and the tax rate. To repeat, they conclude that the after-tax value of \$7,000 invested today in a taxable account is less than \$7,000. In my approach, its after-tax value is \$7,000.

Next, let's consider a non-qualified annuity with cost basis of 1 and then consider an annuity with basis less than 1. For a cost basis of 1, the individual invests \$7,000 today in a non-qualified annuity. After 20 years, we agree that its after-tax value is $\$7{,}000[(1.03)^{20}(1 - 0.3) + 0.3]$ or \$10,950. The effective tax rate is 0.2459. That is, 0.2459 is the value of T* that solves the equality $(1.03)^{20}(1 - 0.3) + 0.3 = (1 + 0.03(1\text{-}T^*))^{20}$. Therefore, I conclude that the risk-appropriate discount rate is 0.03(1 − 0.2459) and the after-tax value is $\$7{,}000[(1.03)^{20}(1 - 0.3) + 0.3]/(1 + 0.03(1 - 0.2459))^{20}$ or \$7,000. The after-tax value of this \$7,000 that was just invested in the non-qualified annuity is \$7,000.

According to Horan and Robinson, its after-tax value is $\$7{,}000[(1.03)^{20}(1 - 0.3) + 0.3]/(1.03)^{20}$ or \$6,063. I believe the 3% discount rate is wrong because this investor only bears 75.41% of the asset's risk, where 75.41% is (1 − 0.2459).

For cost basis of 0.6, consider an individual who invested \$4,200 years ago in a non-qualified annuity and its market value is now \$7,000. If liquidated today, its after-tax value would be \$6,160 or \$7000 less the \$840 taxes on the \$2,800 of deferred returns. If withdrawn in 20 years, its after-tax value would be $\$7{,}000[(1.03)^{20}(1 - 0.3) + (0.6)0.3]$ or \$10,110. By deferring gains for 20 years, the after-tax value grows from \$6,160 today to \$10,110 or at a 2.508% after-tax rate of return. The investor receives 83.61% or 2.508%/3% of returns and bears 83.61% of risk. The effective tax rate is 16.39%. Therefore, the appropriate discount rate is 3% (1 − 0.1639). The after-tax value is $\$7{,}000[(1.03)^{20}(1 - 0.3) + (0.6)0.3]/(1 + 0.03(1 - 0.1639))^{20}$ or \$6,160. This is the same as the after-tax value if liquidated today with deferred returns taxed at 30%.

For cost basis of 0, the individual invests \$7,000 of *pre-tax* funds today in a qualified annuity, which is the same as a tax-deferred annuity. After 20 years, its after-tax value will be $\$7{,}000[(1.03)^{20}(1 - 0.3)]$ or \$8,850. Because the effective tax rate is zero, the appropriate discount rate is 3%. The after-tax value is $\$7{,}000[(1.03)^{20}(1 - 0.3)]/(1.03)^{20}$ or \$4,900.

4. Summary and conclusion

Horan and Robinson (2008) made a significant contribution by generalizing the after-tax future value model for a non-qualified annuity across cost bases. In addition, they showed that this model can be easily extended to accommodate assets earning tax-deferred capital gains. The model says that an asset's after-tax future value is the same as that of a tax-deferred account plus the value of the tax shield of the cost basis. However, the risk borne by this investor is not the same as that of an investor in a tax-deferred account.

We agree that an asset's after-tax value is its after-tax future value when discounted back to the present by dividing by one plus the risk-appropriate discount rate. We disagree about the risk-appropriate discount rate for (1) assets held in taxable accounts and (2) for assets that earn tax-deferred returns, where the latter include assets held in non-qualified annuities and passively held stocks in taxable accounts.

Let's review the differences for bonds held in taxable accounts. In my model, the risk-appropriate discount rate is r(1 − T), where r is the pre-tax rate of return and T is the ordinary income tax rate. Therefore, if someone invests $100 in a risk-free bond today, its after-tax value would be $100. In Horan and Robinson's (2008) model, the discount rate for this bond would be r, the pre-tax rate of return. Therefore, the after-tax value of this $100 would depend on the rate of return, length of investment horizon, and tax rate, but it would be less than $100.

Let's review the differences for assets that earn tax-deferred returns (that are eventually taxed). In my valuation framework, the effective tax rate decreases with the tax-deferral period. Therefore, the risk-appropriate discount rate increases with the length of the investment horizon. The after-tax value is the same as if the asset was liquidated today and taxes paid on embedded deferred returns at the eventual tax rate. For a non-qualified annuity, this tax rate is T_n, the ordinary income tax rate at withdrawal n years hence. For stocks passively held in a taxable account when the tax-deferred gains will eventually be realized and taxed, this tax rate is T_c, the long-term capital gain tax rate. If the individual will refrain from selling the appreciated asset until after it receives the step-up in basis at death or will donate the appreciated asset to charity then there is no embedded tax liability, and the asset's after-tax value is the same as its market value.

To reemphasize the point made in the introduction, Horan and Robinson and I agree on more than we disagree. We and the other scholars who have discussed the after-tax value of assets held in various savings vehicles agree that taxes matter! It is time that the profession abandons the traditional approach to calculation an individual's asset allocation that ignores taxes and thus treats $1 of pre-tax funds in a tax-deferred account as being equivalent to $1 of after-tax funds in a Roth IRA or another savings vehicle. The profession has benefited from the many work of Horan and from Horan and Robinson (2008). However, in my opinion, their recommendations for valuing assets held in taxable accounts and non-qualified annuities are faulty.

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Notes

1. To understand why the statement that the investor bears (1-T) of risk is only an approximation, suppose the pre-tax returns are -2%, 4%, and 10%. If the 2% loss is used that year to offset long-term gains and the long-term gain tax rate was 15% then the after-tax returns would be -1.7%, 3%, and 7.5%. In this case, the investor bears approximately (1 − T) of the risk. See Reichenstein (2007b) for a discussion of this and other situations where Table 2 presents only an approximation of the risks borne by the investor. In the remainder of this study, the term "approximate" is dropped.
2. We ignore liquidity issues related to withdrawals from a Roth before age 59.5.
3. In the lead article in the inaugural issue of *Financial Services Review*, Harry Markow-

itz (1991) said "it is not always clear . . . how to treat unrealized gains." He reached this conclusion after spending one night thinking about the differences between individual and institutional investment management. It took me years to reach the same conclusion. Some minds are better than others.

4. In their Appendix, Horan and Robinson assume the beginning market value of the tax-deferred account is $10,000 of pre-tax funds. An investor in the 30% tax bracket this year could invest $10,000 of pre-tax funds and it would reduce this year's spending by $7,000 or $10,000(1 − 0.3). Therefore, the $10,000 investment of *pre-tax* funds in the tax-deferred annuity would be equivalent to a $7,000 investment of *after-tax* funds in any other savings vehicle because they each reduce this year's spending by $7,000. Their framework shows that, when tax rates in the contribution year, T, and in the withdrawal year, T_n, are the same, the after-tax future value is the same from investing $1 in a tax-deferred account or ($1-T_n$) in a tax-exempt Roth. In contrast, Table 5 in my study is intended to compare the after-tax value of saving vehicles with the same market value today. Again, we are in complete agreement with respect to how to value assets held in tax-deferred accounts.

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