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# Modeling withdrawals from a taxable account

Andrei Shynkevich

*Department of Finance, Kent State University, College of Business Administration, Kent, OH 44242, USA*

**Abstract**

This paper demonstrates how to correctly model withdrawals from a taxable account. The presented framework is largely generalized as it accommodates for varying rates of return, changing tax rates, different time periods, flexible withdrawal amounts, inflation, cost basis, and level of capital gains annual distribution. Closed-form solutions for special cases of constant rate of return, constant annual after-tax withdrawal, constant tax rates and constant share of distributed capital gains as well as the inflation-adjusted after-tax withdrawal are derived and supplemented by illustrative examples. The results have broad applications for the research in financial planning whenever modeling of the withdrawal phase has to be incorporated into the analysis. 

*JEL classification:* G10; J26

*Keywords:* Annuitization; After-tax value; Withdrawal; Taxable equity account

## 1. Introduction

Retirement planning requires a careful consideration of the accumulation phase as well as the withdrawal phase. Much of the literature has focused on the accumulation phase leading to the development of models where different retirement savings alternatives are compared to each other based on their after-tax values at the start of the withdrawal phase. Such models are usually referred as the ending wealth models. The commonly accepted wisdom of such works is neatly summarized in Reichenstein (1999): when saving for the retirement, the better choice between two savings vehicles is the one that produces the greater after-tax ending wealth.

Corresponding author. Tel.: +1-330-672-1226; fax: +1-330-672-9806.
*E-mail address:* ashynkev@kent.edu.

The analysis of the withdrawal phase is equally important but has received less attention despite the fact that a proper analysis of different retirement savings vehicles is predicated upon each vehicle's ability to generate after-tax returns during the accumulation phase as well as its ability to generate higher after-tax values of retirement income during the actual retirement. A higher after-tax value of the accumulated retirement savings at the beginning of the retirement does not necessarily result in a higher after-tax value of the regular withdrawals used by the retiree to finance his or her actual spending during the retirement as argued by Sibley (2002) and Horan (2002, 2007). In other words, more after-tax money at the beginning of retirement does not necessarily imply more after-tax money during retirement.

The accumulated after-tax value of savings invested over a number of years has been explored in the literature in detail (e.g., see Craig and Austin, 1997; Horan, 2002; Reichenstein, 1999; and Poterba, 2004). What is less straightforward is how to model withdrawals from a set of accounts with retirement savings.[1] In particular, a careful modeling of withdrawals from a taxable account such as a brokerage equity account has not yet been attempted. This paper fills the gap in the existing literature by presenting the quantitative side behind the decumulation modeling of the taxable account. The withdrawal modeling of the taxable account presented in this paper can be used to extend the existing retirement planning models by incorporating the withdrawal phase of the retirement portfolio in the analysis when a part of such portfolio is held in the taxable account.

The application of the withdrawal modeling presented in this paper can contribute to the ongoing discussion on the comparison of balances in the accounts with different tax treatment by accommodating multiple withdrawals from the taxable equity account. Two classes of models have been developed to make balances of different accounts comparable to each other: one by Sibley (2002), Horan (2002, 2007), Poterba (2004), and Horan and Robinson (2008), and the other by Reichenstein (1998, 2007, 2008a, 2008b) and Reichenstein and Jennings (2003). The first group of studies calculates the taxable equivalent values that make balances in nontaxable accounts comparable with taxable accounts by providing the equivalent income during retirement, whereas the second group computes after-tax values by applying a tax- and risk-adjusted discount rate to future cash flows. One of the issues the two models disagree on is the valuation of assets held in the taxable account. The idea is to determine the future stream of after-tax cash flows a given tax-advantaged account is expected to produce for the retiree and find the number of current after-tax dollars invested in the taxable account that would produce the same stream of future cash flows. The difficulty of such valuation approach that has been frequently overlooked is that most retirees usually do not liquidate their retirement accounts as well as taxable accounts with a single withdrawal but rather make regular withdrawals over a number of periods. Another limitation that has not been fully addressed is how to model withdrawals from the taxable account when the annual returns in the account are not fully taxed at ordinary income tax rates. The valuation procedure offered in this paper shows how to correctly accommodate multiple withdrawals from the taxable account holding equities whose returns are taxed at capital gains tax rates. Our approach of modeling withdrawals from the taxable account is highly generalized and accommodates for varying rates of return, changing tax rates, different time periods, flexible withdrawal amounts, inflation, presence of built-in, that is, unrealized capital gains and different levels of distribution of annual capital gains.

The rest of the paper is structured as follows. Section 2 provides literature review on the withdrawal modeling at the taxable account. Section 3 describes how the withdrawals from the taxable brokerage account are properly modeled with and without annual distributions of capital gains. Section 4 summarizes.

## 2. Literature review

The literature on the withdrawal modeling from the taxable account is scarce and incomplete. Only a few works attempt to annuitize equity holdings in the taxable account for the withdrawal purposes, but they seem to have fallen short on the matter. Reichenstein (2006a, 2006b) discusses strategies for selecting the sequence of fund withdrawals from savings vehicles during retirement and considers the impact of alternative withdrawal strategies on a portfolio's longevity when a part of the retirement portfolio, in the form of equities, is held in the taxable account. Spitzer and Singh (2006) pose a similar question and compare the longevity of pairs of portfolios that differ in significant ways including a case where funds are held in the accounts with different tax treatment. Sibley (2002) derives a model to determine the current after-tax value of a tax-advantaged retirement account that calculates the number of after-tax dollars held in the taxable account and earning ordinary taxable income necessary to provide the same level of future consumption as the tax-advantaged account via a series of multiple withdrawals. Lewis and Caliendo (2006) annuitize the balance of the taxable account in a manner similar to how it is done for a traditional IRA. They, however, define by the "taxable account" an equivalent of Roth IRA or any other front-loaded account where money grows tax-free. Technically, their derivations are correct, but cannot be applied to regular taxable brokerage accounts where growth is not tax-free but any realized capital gains and/or paid dividends are subject to taxation.

Reichenstein (2006a, 2006b) specifies the value of after-tax funds held in the taxable account as well as the effective tax rate on assets held in such account. It is unclear how the withdrawals of funds from the taxable account are subsequently modeled in those papers. The missing link is a clear distinction between the cost basis of the investment held in the taxable account and the accumulated unrealized capital gains. Recall that the after-tax value of the portfolio of stocks held in the taxable account is computed as the difference between the before-tax value of the portfolio and the product of the unrealized capital gains and the tax rate on capital gains. Therefore, to have an unambiguous description of the portfolio held in the taxable account, apart from its after-tax value and the applied tax rate, one has to know either its before-tax value or the amount of unrealized capital gains or the portfolio's cost basis. In the example considered in Reichenstein (2006a, 2006b), the higher wealth couple has $800,000 of after-tax funds in the taxable account and the applied tax rate on capital gains is assumed at 15%. There are an infinite number of before-tax values of the portfolio satisfying these two conditions. One of such portfolios has the cost basis of $375,000 and the unrealized capital gains of $500,000; another portfolio consists of the cost basis of $630,000 and the unrealized capital gains of $200,000. Each portfolio is likely to result in a different amount of the constant real annual withdrawal because every time funds are withdrawn from the taxable account, a portion of the cost basis is taken out together with a portion of the unrealized capital gains. Therefore, different ratios between the accumulated capital gains and the cost basis will result in different combinations of the withdrawn principal and the withdrawn gains, and ultimately, in different amounts of after-tax funds available for the retiree's consumption upon every scheduled withdrawal.

Spitzer and Singh (2006) provide information about the tax rate on returns in the taxable account as well as the account value at the beginning of the retirement. The annuitization of the taxable account is performed in a trivial fashion: the beginning value of the account is taken as the present value of a regular annuity and the rate used to compute the annual withdrawal amount is taken as the after-tax rate of growth. For example, with the value of $50, the after-tax rate of 4.968% (rate of growth before tax is 7.2% and tax rate on returns is 31%), and the annual payment of $7.50, the number of years until depletion is found equal to 8.3 years, as can be seen from their Table 1. Such a result will be unambiguously correct under two conditions (assumptions): (1) all capital gains are realized at the end of each year, and (2) the beginning value of the portfolio ($50) has no unrealized capital gains. Sibley (2002) also assumes that the return in the taxable account is fully taxed every year at a single, constant ordinary income tax rate and the cost basis is therefore always equal to one. Such assumptions, however, would represent a significant deviation from the conventional approach of employing a tax-efficient low-turnover buy-and-hold strategy for the purposes of holding equities in the taxable account. Horan (2002) introduces an approach that incorporates a more complex yet lighter tax structure where only a portion of an annual return is taxed annually at a higher tax rate while the remainder of the return is taxed at a lower tax rate when the investor actually sells the shares, but he stops short of modeling the withdrawals and limits his computations to a seemingly circuitous derivation of the present value interest factor of an annuity using an after-tax cost of capital associated with an equity fund held in the taxable account.

## 3. Withdrawal modeling

### 3.1. Case 1: No short-term capital gain distributions

The major difference between withdrawing funds from the tax-shielded account and the taxable account is that in the latter case, every time a withdrawal is made, a portion of the cost basis and a portion of the accumulated unrealized capital gains are withdrawn together but only the part representing capital gains is taxed. The value of the withdrawn cost basis and the value of the realized capital gains are proportional to each other according to a predetermined relationship. Cleary, if an individual sells 100 shares of an equity fund in his or her portfolio, he or she withdraws the amount of cost basis assigned to those 100 shares, and the rest of the withdrawal represents the realization of capital gains. Hence, there is a unique relationship between the amount of withdrawn cost basis and the value of realized capital gains.

Suppose $S$ is the current before-tax value of the investment in an equity fund or a stock and that the asset held does not distribute any portion of capital gains on a regular basis. Suppose further that the investment in the equity fund or the stock consists of $N$ shares and each share is assigned the same cost basis,[2] where $aS$, $0 < a \leq 1$, is the cost basis of the entire investment (hence, the cost basis of each share is $aS/N$ and the price of each share is $S/N$). Then $(1 - a)S$ is the before-tax value of the unrealized capital gains. For simplicity, assume that all unrealized capital gains are composed of long-term capital gains. If the individual wants to have after-tax dollars in the amount $w_0$ available for consumption today, where the consumption is financed by the equity holdings in the taxable account, how many shares of the equity fund does he or she have to sell?

The ratio of the unrealized long-term capital gains to the current before-tax value of the investment is equal to $(1 - a)$. The long-term capital gains tax would comprise the percentage of the withdrawn account's value equal to $t_{cg0}(1 - a)$, where $t_{cg0}$ is the current tax rate on long-term capital gains. The after-tax value of equity investment is therefore equal to $S(1 - t_{cg0} + at_{cg0})$.

The before-tax-to-after-tax ratio of the investment is therefore

$$\frac{1}{1 - t_{cg0} + at_{cg0}}. \tag{1}$$

Suppose that one wants to have $w_0$ in after-tax dollars available for consumption after the withdrawal, where $w_0$ is strictly less than the current after-tax value of the investment in the taxable account. This implies that a partial withdrawal takes place, that is, the investor plans to sell a portion of its equity holdings. The actual amount of the withdrawal must be equal to the product of $w_0$ and the existing before-tax-to-after-tax ratio of the investment. Denote the amount of the immediate withdrawal by $W_0$:

$$W_0 = \frac{w_0}{1 - t_{cg0} + at_{cg0}}. \tag{2}$$

Such a withdrawal guarantees that the funds left after paying the capital gains tax will be equal to $w_0$ and the amount of capital gains tax on the withdrawn portion of capital gains will therefore be equal to $W_0 - w_0$. The before-tax amount of remaining investment left in the account after the withdrawal will be equal to

$$S - W_0 = S - \frac{w_0}{A_0}, \tag{3}$$

where $A_0 = 1 - t_{cg0} + at_{cg0}$. The number of sold shares is therefore equal to $W_0/(S/N) = w_0N/(A_0S)$ and the number of the remaining shares is $N[1 - w_0/(A_0S)]$. The new cost basis is the product of the old cost basis and the share of the remaining before-tax value of the investment after the withdrawal:

$$aS\left[1 - \frac{W_0}{S}\right] = a\left(S - \frac{w_0}{A_0}\right). \tag{4}$$

This is where the relationship between the withdrawn portion of cost basis and the withdrawn portion of capital gains is accounted for: the cost basis is reduced proportionally to the reduction of the total before-tax value of the investment and so are the remaining unrealized capital gains. To see that, compute the remaining amount of capital gains that is the difference of the account's new before-tax value and its new cost basis:

$$S - \frac{w_0}{A_0} - a\left(S - \frac{W_0}{A_0}\right) = (1 - a)\left(S - \frac{w_0}{A_0}\right). \tag{5}$$

Compare the remaining before-tax value of the investment, the new cost basis and the remaining value of capital gains to their respective original values to see that the total before-tax value of the investment as well as the value of each of its two components has been reduced proportionally by the factor of $(S - w_0/A_0)/S$.

The modeling of the withdrawals from the investment held in the taxable account can be carried on to future periods in the same fashion. Suppose the withdrawals take place annually. In a year, the account's before-tax value will be $(1 + r_1)(S - w_0/A_0)$, where $r_1$ is the annual rate of return on the equity fund during year one. The amount of the cost basis is unchanged from what it was a year ago meaning that it is still equal to $a(S - w_0/A_0)$. The new amount of the unrealized capital gains is the difference between the former and the latter:

$$(1 + r_1)\left(S - \frac{w_0}{A_0}\right) - a\left(S - \frac{w_0}{A_0}\right) = (1 + r_1 - a)\left(S - \frac{w_0}{A_0}\right). \tag{6}$$

Suppose the account holder wants to have $w_1$ in after-tax dollars available after the withdrawal. To figure out the actual amount of the withdrawal at the end of year one, the new before-tax-to-after-tax ratio must be found first. The new ratio of the unrealized capital gains to the investment's total before-tax value is

$$1 - \frac{a\left(S - \dfrac{w_0}{A_0}\right)}{(1 + r_1)\left(S - \dfrac{w_0}{A_0}\right)} = 1 - \frac{a}{1 + r_1}. \tag{7}$$

The long-term capital gains tax as a percentage of the withdrawn portion of the account's value would be the equivalent of $t_{cg1}[1 - a/(1 + r_1)]$, where $t_{cg1}$ is the tax rate on long-term capital gains in year one. The after-tax value of the equity investment at the account is therefore

$$(1 + r_1)\left(S - \frac{w_0}{A_0}\right)\left[1 - t_{cg1}\left(1 - \frac{a}{1 + r_1}\right)\right] = \left(S - \frac{w_0}{A_0}\right)[(1 + r_1)(1 - t_{cg1}) + at_{cg1}]. \tag{8}$$

The new before-tax-to-after-tax ratio is therefore

$$\frac{1 + r_1}{(1 + r_1)(1 - t_{cg1}) + at_{cg1}}, \tag{9}$$

and the amount of the withdrawal at the end of year one, $W_1$, is equal to

$$W_1 = \frac{w_1(1 + r_1)}{(1 + r_1)(1 - t_{cg1}) + at_{cg1}}. \tag{10}$$

Such a withdrawal at the end of year one implies that the after-tax dollars available for consumption will be equal to $w_1$ and the amount of capital gains tax on the withdrawn portion of capital gains will be equal to $W_1 - w_1$. The before-tax value of the remaining portion of the investment left in the account at the end of year one after the withdrawal will be equal to

$$\left(S - \frac{w_0}{A_0}\right)(1 + r_1) - W_1 = (1 + r_1)\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right), \tag{11}$$

where $A_I = (1 + r_1)(1 - t_{cg1}) + at_{cg1}$. The number of sold shares is equal to $W_1/[S(1 + r_1)/N] = w_1N/(A_1S)$ and the number of the remaining shares is $N[1 - w_0/(A_0S) - w_1/(A_1S)]$. The new cost basis is the product of the old cost basis and the share of the remaining before-tax value of the investment after the withdrawal:

$$a\left(S - \frac{w_0}{A_0}\right)\left[1 - \frac{W_1}{\left(S - \frac{w_0}{A_0}\right)(1 + r_1)}\right] = a\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right). \tag{12}$$

The computations can be carried on to the next year. Suppose that during year two, the investment earns rate or return $r_2$. The investment's total value will therefore be

$$\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right)(1 + r_1)(1 + r_2), \tag{13}$$

while the before-tax value of the unrealized capital gains is equal to

$$\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right)(1 + r_1)(1 + r_2) - a\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right)$$
$$= [(1 + r_1)(1 + r_2) - a]\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right). \tag{14}$$

Suppose the account holder wants to have $w_2$ in after-tax dollars available for consumption after the withdrawal. The new ratio of the unrealized capital gains to the investment's total before-tax value at the end of year two is

$$1 - \frac{a\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right)}{(1 + r_1)(1 + r_2)\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right)} = 1 - \frac{a}{(1 + r_1)(1+r_2)}. \tag{15}$$

The long-term capital gains tax as a percentage of the withdrawn account's value would be the equivalent of $t_{cg2}[1 - a/((1 + r_1)(1 + r_2))]$, where $t_{cg2}$ is the tax rate on long-term capital gains in year two. The after-tax value of the equity investments at the account is therefore

$$(1 + r_1)(1 + r_2)\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right)\left[1 - t_{cg2}\left(1 - \frac{a}{(1 + r_1)(1 + r_2)}\right)\right]$$
$$= \left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right)[(1 + r_1)(1 + r_2)(1 - t_{cg2}) + at_{cg2}]. \quad (16)$$

The new before-tax-to-after-tax ratio is therefore

$$\frac{(1 + r_1)(1 + r_2)}{(1 + r_1)(1 + r_2)(1 - t_{cg2}) + at_{cg2}}, \quad (17)$$

and the amount of the withdrawal at the end of year two, $W_2$, is equal to

$$W_2 = \frac{w_2(1 + r_1)(1 + r_2)}{(1 + r_1)(1 + r_2)(1 - t_{cg2}) + at_{cg2}}. \quad (18)$$

Such a withdrawal at the end of year two implies that the amount of after-tax dollars available for consumption will be $w_2$ and the amount of capital gains tax on the withdrawn portion of capital gains will be equal to $W_2 - w_2$. The before-tax value of the remaining investments left in the account at the end of year two after the withdrawal is therefore

$$\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1}\right)(1 + r_1)(1 + r_2) - W_2 = (1 + r_1)(1 + r_2)\left(S - \frac{w_0}{A_0} - \frac{w_1}{A_1} - \frac{w_2}{A_2}\right), \quad (19)$$

where $A_2 = (1 + r_1)(1 + r_2)(1 - t_{cg2}) + at_{cg2}$. The number of shares sold at the end of year two is equal to $W_2/[S(1 + r_1)(1 + r_2)/N] = w_2N/(A_1S)$ and the number of the remaining shares will be $N[1 - w_0/(A_0S) - w_1/(A_1S) - w_2/(A_2S)]$.

The computations can be continued further in the same manner to find out that at the end of year $k$, $k > 1$, the investor sells the number of shares $N_k$ with the before-tax value $W_k$ leaving the account's before-tax balance at the level $AB_k$ with the cost basis $CB_k$, where

$$N_k = N \min\left[\frac{w_k}{A_kS}, 1 - \frac{W_0}{S} - \sum_{i=1}^{k-1} \frac{W_i}{S\prod_{j=1}^{i}(1 + r_j)}\right], \quad (20)$$

$$W_k = \min\left[\frac{w_k \prod_{i=1}^{k}(1+r_i)}{A_k}, \left(S - W_0 - \sum_{i=1}^{k-1}\frac{W_i}{\prod_{j=1}^{i}(1+r_j)}\right)\prod_{i=1}^{k}(1+r_i)\right], \tag{21}$$

$$AB_k = \left(S - \sum_{i=0}^{k-1}\frac{w_i}{A_i}\right)\prod_{i=1}^{k}(1+r_i) - W_k, \tag{22}$$

$$CB_k = a\left(S - \sum_{i=0}^{k-1}\frac{w_i}{A_i} - \frac{W_k}{\prod_{i=1}^{k}(1+r_i)}\right), \tag{23}$$

$$A_k = (1 - t_{cgk})\prod_{i=1}^{k}(1+r_i) + at_{cgk}. \tag{24}$$

The use of minimum terms in Expressions (20) and (21) assures that the investor cannot withdraw more than the amount of the funds left at the account. If the desired after-tax consumption $w_k$ exceeds the after-tax value of the investment at that point, then the investor sells the remaining number of shares and receives their after-tax value. After that the account is fully depleted and all subsequent withdrawals are automatically equal to zero as implied in (20) and (21).

There is one exception to the above modeling of the withdrawals caused by the opportunity of tax loss harvesting. If in a particular year the cost basis of the investment exceeds the total before-tax value of that investment, the investor can sell the asset at a loss and claim a tax credit. In compliance with the wash sale rule, the investor can buy the same asset back 31 days after the sale. The new value of the investment will be equal to its old value plus the amount of the tax credit and the new cost basis will be set equal to the investment's new value, that is, the value of $a$ is reset to one. After the sale of the asset for a loss and then buying it back, the withdrawal modeling from the taxable account proceeds using the framework described above.

The examples of withdrawal modeling from the taxable account are illustrated next for two special cases. In the first case, the annual rate of return as well as the tax rate on long-term capital gains is assumed constant, and the after-tax amount of withdrawn funds available for consumption annually is also assumed constant. In the second case, the rate of return and the tax rate are constant whereas the amount of consumption is adjusted annually by the constant level of inflation. The withdrawals from the taxable account holding an equity fund are annuitized over the time period of length $T$, where $T$ is the number of years. Hence, the total number of annual withdrawals is $T + 1$, which includes the immediate withdrawal.

For $w$ to be the constant annual after-tax consumption over the period of length $T$

Table 1 The year-by-year withdrawal picture from the taxable account, where $S$ = \$100,000, $a$ = 0.4, $r$ = 0.08, $t_{cg}$ = 0.2, and $T$ = 10 (values are rounded)

| Year (end of) | Balance before with-drawal | Cost basis | Unrealized capital gains | Number of shares sold | With-drawal, total | Withdrawn cost basis | Realized capital gains | After-tax allowance ($w_i$) | Balance after with-drawal |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 100000 | 40000 | 60000 | 126.63 | 12663 | 5065 | 7598 | 11143 | 87337 |
| 1 | 94324 | 34935 | 59389 | 118.04 | 12748 | 4721 | 8027 | 11143 | 81576 |
| 2 | 88102 | 30213 | 57889 | 109.99 | 12829 | 4400 | 8429 | 11143 | 75273 |
| 3 | 81295 | 25814 | 55481 | 102.44 | 12904 | 4097 | 8807 | 11143 | 68391 |
| 4 | 73862 | 21716 | 52146 | 95.37 | 12975 | 3815 | 9160 | 11143 | 60887 |
| 5 | 65758 | 17901 | 47856 | 88.76 | 13041 | 3550 | 9491 | 11143 | 52717 |
| 6 | 56934 | 14351 | 42583 | 82.57 | 13103 | 3303 | 9800 | 11143 | 43831 |
| 7 | 47337 | 11048 | 36289 | 76.79 | 13161 | 3072 | 10089 | 11143 | 34176 |
| 8 | 36910 | 7976 | 28934 | 71.40 | 13215 | 2856 | 10359 | 11143 | 23696 |
| 9 | 25591 | 5121 | 20470 | 66.36 | 13265 | 2654 | 10611 | 11143 | 12326 |
| 10 | 13312 | 2466 | 10846 | 61.66 | 13312 | 2466 | 10846 | 11143 | 0 |

generated by the withdrawals from the equity holding taxable account, it must be that the account is entirely exhausted by the end of year $T$ following the $(T+1)th$ scheduled withdrawal. Hence, to find $w$ one has to solve $AB_T = 0$. From (22),

$$S - \sum_{i=0}^{T} \frac{w}{A_i} = 0 \Rightarrow w = \frac{S}{\sum_{i=0}^{T} \frac{1}{A_i}}. \tag{25}$$

Table 1 demonstrates the year-by-year accumulations and withdrawals for the taxable account holding an equity fund with a current before-tax value of \$100,000. The cost basis of the investment is \$40,000. The investor holds 1,000 shares of the fund, where each share has the same cost basis and is currently priced at \$100. The annual rate of return is assumed at 8%, tax rate on capital gains is 20% and the annuitization period $T$ is 10 years. Using expression (25), the constant annual after-tax allowance $w$ is found to be \$11,143.

Consider next the second special case when the after-tax dollars available for consumption are adjusted annually by the constant level of inflation. The amount of the after-tax consumption at the end of year $k$ can be denoted as $w_k = w(1+\pi)^k$, where $w$ is the after-tax amount of the current withdrawal and $\pi$ is the constant annual level of inflation. Given the annuitization period of $T$ years and using (22), the condition for $w$ becomes:

$$S - \sum_{i=0}^{T} \frac{w(1+\pi)^i}{A_i} = 0 \Rightarrow w = \frac{S}{\sum_{i=0}^{T} \frac{(1+\pi)^i}{A_i}}. \tag{26}$$

Table 2 demonstrates the year-by-year accumulations and withdrawals for the taxable account holding an equity fund with a current before-tax value of \$100,000. The cost

Table 2 The year-by-year withdrawal picture from the taxable account, where $S$ = \$100,000, $a$ = 0.4, $r$ = 0.08, $\pi$ = 0.02, $t_{cg}$ = 0.2, and $T$ = 10 (values are rounded)

| Year (end of) | Balance before with-drawal | Cost basis | Unrealized capital gains | Number of shares sold | With-drawal, total | Withdrawn cost basis | Realized capital gains | After-tax allowance ($w_i$) | Balance after with-drawal |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 100000 | 40000 | 60000 | 116.09 | 11609 | 4644 | 6965 | 10216 | 88391 |
| 1 | 95462 | 35356 | 60106 | 110.39 | 11922 | 4416 | 7506 | 10421 | 83540 |
| 2 | 90223 | 30941 | 59282 | 104.91 | 12237 | 4196 | 8041 | 10629 | 77986 |
| 3 | 84225 | 26744 | 57481 | 99.69 | 12555 | 3987 | 8568 | 10842 | 71670 |
| 4 | 77403 | 22757 | 54646 | 94.65 | 12877 | 3786 | 9091 | 11058 | 64527 |
| 5 | 69689 | 18972 | 50717 | 89.84 | 13201 | 3594 | 9607 | 11280 | 56488 |
| 6 | 61007 | 15378 | 45629 | 85.26 | 13529 | 3410 | 10119 | 11505 | 47478 |
| 7 | 51276 | 11968 | 39308 | 80.87 | 13860 | 3235 | 10625 | 11735 | 37416 |
| 8 | 40409 | 8733 | 31676 | 76.69 | 14196 | 3068 | 11128 | 11970 | 26213 |
| 9 | 28310 | 5664 | 22646 | 72.71 | 14535 | 2909 | 11626 | 12209 | 13776 |
| 10 | 14878 | 2757 | 12121 | 68.91 | 14878 | 2757 | 12121 | 12454 | 0 |

basis of the investment is \$40,000. The investor holds 1,000 shares of the fund, where each share has the same cost basis and is currently priced at \$100. The annual rate of return is assumed at 8%, inflation is 2%, tax rate on capital gains is 20% and the annuitization period $T$ is 10 years. Using Expression (26), the initial after-tax allowance $w$ is found to be \$10,216, and the value of each subsequent annual after-tax allowance is adjusted up by 2%.

### 3.2. Case 2: Accounting for short-term capital gain distributions

Suppose that a part of the annual return on the equity fund is distributed to investors in the form of short-term capital gains that are taxed at the appropriate ordinary income tax rate. As in the previous case, $S$ is the before-tax value of the investment in an equity fund at the beginning of the retirement where the investment in the equity fund consists of $N$ shares and each share has the same cost basis, where $aS$, $0 < a \leq 1$, is the cost basis of the entire investment. The cost basis of each share is $aS/N$ and the price of each share is $S/N$. The before-tax value of the unrealized long-term capital gains is $(1 - a)S$. Suppose further that the fund has just distributed a portion of last year's capital gains and the after-tax value of such distribution is $B_0$.

The first step in the withdrawal modeling is identical to that for the case with no distributions. If one wants to have $w_0$ in after-tax dollars after the immediate withdrawal, he or she has to withdraw the amount $W_0$ that will have the after-tax value of $w_0 - B_0$. The withdrawal amount is then determined similarly to the way it is done in (2):

$$W_0 = \frac{w_0 - B_0}{1 - t_{cg0} + at_{cg0}}. \tag{27}$$

The before-tax amount of remaining investment left in the account after the withdrawal, the new cost basis and the remaining amount of capital gains are found in similar fashion as (3) through (5) and are shown next in (28) through (30), respectively:

$$S - W_0 = S - \frac{w_0 - B_0}{A_0}, \tag{28}$$

$$aS\left[1 - \frac{W_0}{S}\right] = a\left(S - \frac{w_0 - B_0}{A_0}\right), \tag{29}$$

$$S - \frac{w_0 - B_0}{A_0} - a\left(S - \frac{w_0 - B_0}{A_0}\right) = (1 - a)\left(S - \frac{w_0 - B_0}{A_0}\right). \tag{30}$$

The number of sold shares is $W_0/(S/N) = N(w_0 - B_0)/(A_0S)$ and the number of the remaining shares is $N[1 - (w_0 - B_0)/(A_0S)]$. Suppose that the equity fund distributes a $d_k$ share of its annual return at the end of year $k$, $0 \le d_k \le 1$. At the end of year one, the investor receives the after-tax value of the first distribution equal to $d_1r_1[S - (w_0 - B_0)/A_0](1 - t_{d1})$, where $t_{d1}$ is the tax rate on the distributed portion of short-term capital gains in year one and the investment's remaining before-tax value will be $(1 + r_{1d})[S - (w_0 - B_0)/A_0]$, where $r_{1d} = r_1(1 - d_1)$. The amount of the cost basis is unchanged from what it was a year ago meaning that it is still equal to $a[S - (w_0 - B_0)/A_0]$. The new amount of the unrealized capital gains is the difference between the former and the latter:

$$[1 + r_{1d}]\left(S - \frac{w_0 - B_0}{A_0}\right) - a\left(S - \frac{w_0 - B_0}{A_0}\right) = [1 + r_{1d} - a]\left(S - \frac{w_0 - B_0}{A_0}\right). \tag{31}$$

Suppose the account holder wants to have $w_1$ in the after-tax funds available at the end of the first year and suppose that this amount exceeds the after-tax value of the distributed capital gains. The after-tax amount of the withdrawal must therefore be equal to the difference between the desired total amount of available funds and the funds that became available as a result of the gains distribution, $w_1 - d_1r_1[S - (w_0 - B_0)/A_0](1 - t_{d1})$. To figure out the amount of withdrawal at the end of year one, the new before-tax-to-after-tax ratio must be found first. The new ratio of the unrealized capital gains to the investment's total before-tax value is

$$1 - \frac{a\left(S - \frac{w_0 - B_0}{A_0}\right)}{(1 + r_{1d})\left(S - \frac{w_0 - B_0}{A_0}\right)} = 1 - \frac{a}{1 + r_{1d}}. \tag{32}$$

The long-term capital gains tax as a percentage of the withdrawn portion of the investment's total value would be the equivalent of $t_{cg1}[1 - a/(1 + r_{1d})]$. The after-tax value of the equity investments at the account is therefore

$$(1 + r_{1d})\left(S - \frac{w_0 - B_0}{A_0}\right)\left[1 - t_{cg1}\left(1 - \frac{a}{1 + r_{1d}}\right)\right] = \left(S - \frac{w_0 - B_0}{A_0}\right)[(1 + r_{1d})(1 - t_{cg1}) + \mathrm{at}_{cg1}]. \quad (33)$$

The new before-tax-to-after-tax ratio is therefore

$$\frac{1 + r_{1d}}{(1 + r_{1d})(1 - t_{cg1}) + at_{cg1}}, \quad (34)$$

and the amount of the withdrawal at the end of year one, $W_1$, is equal to

$$W_1 = \frac{[w_1 - d_1 r_1[S - (w_0 - B_0/A_0)](1 - t_{d1})](1 + r_{1d})}{(1 + r_{1d})(1 - t_{cg1}) + at_{cg1}}. \quad (35)$$

Such a withdrawal at the end of year one implies that after-tax dollars available for consumption will be equal to $w_1$ and the amount of capital gains tax on the withdrawn portion of capital gains will be equal to $W_1 - w_1 + d_1 r_1[S - (w_0 - B_0)/A_0](1 - t_{d1})$. The before-tax value of the remaining investment left in the account at the end of year one after the withdrawal will be equal to

$$\left(S - \frac{w_0 - B_0}{A_0}\right)(1 + r_{1d}) - W_1 = (1 + r_{1d})\left(S - \frac{w_0 - B_0}{A_0} - \frac{w_1 - B_1}{A_1}\right), \quad (36)$$

where $A_1 = (1 + r_{1d})(1 - t_{cg1}) + at_{cg1}$ and $B_1 = d_1 r_1[S - (w_0 - B_0)/A_0](1 - t_{d1})$. The number of sold shares is equal to $W_1/[S(1 + r_{1d})/N] = N(w_1 - B_1)/(A_1 S)$ and the number of the remaining shares is $N[1 - (w_0 - B_0)/(A_0 S) - (w_1 - B_1)/(A_1 S)]$. The new cost basis is the product of the old cost basis and the share of the remaining before-tax value of the investment after the withdrawal:

$$a\left(S - \frac{w_0 - B_0}{A_0}\right)\left[1 - \frac{W_1}{\left(S - \frac{w_0 - B_0}{A_0}\right)(1 + r_{1d})}\right] = a\left(S - \frac{w_0 - B_0}{A_0} - \frac{w_1 - B_1}{A_1}\right). \quad (37)$$

The computations are carried further into year two. At the end of year two, the investor receives the after-tax value of the distribution equal to $d_2 r_2(1 + r_{1d})[S - (w_0 - B_0)/A_0 - (w_1 - B_1)/A_1](1 - t_{d2})$ and the investment's remaining before-tax value will be $(1 + r_{1d})(1 + r_{2d})[S - (w_0 - B_0)/A_0 - (w_1 - B_1)/A_1]$, where $r_{2d} = r_2(1 - d_2)$. If the account holder wants to have $w_2$ in after-tax funds available at the end of the year two, assuming that this amount exceeds the after-tax value of the distributed capital gains, the after-tax amount of the withdrawal must therefore be equal to

$w_2 - d_2r_2(1 + r_{1d})[S - (w_0 - B_0)/A_0 - (w_1 - B_1)/A_1](1 - t_{d2})$. In the fashion similar to how the first-year results were obtained, the following answers are recorded for year two. The amount of withdrawal at the end of year two, $W_2$, is equal to

$$W_2 = \frac{[w_2 - d_2r_2(1 + r_{1d})[S - w_0 - B_0)/A_0 - (w_1 - B_1)/A_1](1 - t_{d2})](1 + r_{1d})(1 + r_{2d})}{(1 + r_{1d})(1 + r_{2d})(1 - t_{cg}) + at_{cg}}, \tag{38}$$

and the before-tax value of the remaining investment left in the account at the end of year two after the withdrawal will be equal to

$$(1 + r_{1d})(1 + r_{2d})\left(S - \frac{w_0 - B_0}{A_0} - \frac{w_1 - B_1}{A_1} - \frac{w_2 - B_2}{A_2}\right), \tag{39}$$

where $A_2 = (1 + r_{1d})(1 + r_{2d})(1 - t_{cg2}) + at_{cg2}$ and $B_2 = d_2r_2(1 + r_{1d})[S - (w_0 - B_0)/A_0 - (w_1 - B_1)/A_1](1 - t_{d2})$. The number of sold shares is equal to $N(w_2 - B_2)/(A_2S)$ and the number of the remaining shares is $N[1 - (w_0 - B_0)/(A_0S) - (w_1 - B_1)/(A_1S) - (w_2 - B_2)/(A_2S)]$.

The computations can be continued further to find out that at the end of year $k$, $k > 1$, the investor receives a before-tax distribution in the amount $D_k$ and sells the number of shares $N_k$ with the before-tax value $W_k$ leaving the account's before-tax balance at the level of $AB_k$ with the cost basis of $CB_k$, where

$$D_k = d_kr_k\left[S - W_0 - \sum_{i=1}^{k-1} \frac{W_i}{\prod_{j=1}^{i}(1 + r_{jd})}\right]\prod_{i=1}^{k-1}(r + 4_{id}), \tag{40}$$

$$N_k = N \min\left[\frac{w_k - B_k}{A_kS}, 1 - \frac{W_0}{S} - \sum_{i=1}^{k-1}\frac{W_i}{S\prod_{j=1}^{i}(1 + r_{jd})}\right], \tag{41}$$

$$W_k = \min\left[\frac{(w_k - B_k)\prod_{i=1}^{k}(1 + r_{id})}{A_k}, \left(S - W_0 - \sum_{i=1}^{k-1}\frac{W_i}{\prod_{j=1}^{i}(1 + r_j)}\right)\prod_{i=1}^{k}(1 + r_i)\right], \tag{42}$$

$$AB_k = \left(S - \sum_{i=0}^{k-1}\frac{w_i - B_i}{A_i}\right)\prod_{i=1}^{k}(1 + r_{id}) - W_k, \tag{43}$$

$$CB_k = a\left(S - \sum_{i=0}^{k-1} \frac{w_i - B_i}{A_i} - \frac{W_k}{\prod_{i=1}^{k}(1 + r_{id})}\right), \tag{44}$$

$$A_k = (1 - t_{cgk})\prod_{i=1}^{k}(1 + r_{id}) + at_{cgk}, \tag{45}$$

with $B_k = D_k(1 - t_{dk})$ and $r_{kd} = r_k(1 - d_k)$. By making $d_i$ for every $i$ equal to zero (no distributions of capital gains), one would arrive to the solution in (20) through (24). By making $d_i$ for every $i$ equal to one (100% distribution of capital gains) and assuming constant rate of return and constant tax rate on distributions, one would obtain the solution in Spitzer and Singh (2006) and Sibley (2002).

The examples of withdrawal modeling from the taxable account in the presence of annual distributions are illustrated next for two special cases. In the first case, the annual rate of return and the tax rates on short-term and long-term capital gains are assumed constant, the distribution share of annual capital gains is unchanged, and the after-tax amount of withdrawn funds used for consumption annually is also assumed constant. In the second case, the rate of return, the two tax rates and the distribution share are all constant whereas the amount of consumption is adjusted annually by the constant level of inflation. The withdrawals from the taxable account holding an equity fund are annuitized over the period of length $T$, where $T$ is the number of years. Hence, the total number of annual withdrawals is $T + 1$, which includes the immediate withdrawal.

For $w$ to be the constant annual after-tax consumption over the period of length $T$ generated by the withdrawals from the equity holding taxable account, it must be that the account is entirely exhausted by the end of year $T$ following the $(T + 1)th$ scheduled withdrawal. Hence, to find $w$ one has to solve $AB_T = 0$. From (43),

$$S - \sum_{i=0}^{T} \frac{w - B_i}{A_i} = 0 \Rightarrow w = \frac{S + \sum_{i=0}^{T} \frac{B_i}{A_i}}{\sum_{i=0}^{T} \frac{1}{A_i}}. \tag{46}$$

Because each $B_i = D_i(1 - t_{di})$ is a function of $w$ as can be seen in (40), the resulting expression for the closed-form solution for $w$ will be cumbersome, but the above linear equation can be easily solved for the sole unknown variable.

Table 3 depicts the year-by-year accumulations and withdrawals for the account invested in an equity fund that currently has a before-tax value of \$100,000 and a cost basis of \$40,000. The investor holds 1,000 shares of the fund, where each share has the same cost basis and is currently priced at \$100. No portion of last year's capital gains has just been distributed to the shareholders ($B_0 = 0$). The annual rate of return is assumed at 8%, share of the distributed future annual return is 30%, tax rate on capital gains is 20%, tax rate on

Table 3 The year-by-year withdrawal picture from the taxable account, where $S$ = \$100,000, $a$ = 0.4, $r$ = 0.08, $d$ = 0.3, $t_{cg}$ = 0.2, $t_d$ = 0.35, and $T$ = 10 (values are rounded)

| Year (end of) | Balance before with-drawal | Cost basis | Unrealized capital gains | Distributed capital gains | Number of shares sold | With-drawal, total | Withdrawn cost basis | Realized capital gains | After-tax allowance ($w_i$) | Balance after with-drawal |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 100000 | 40000 | 60000 | 0.0 | 124.37 | 12437 | 4975 | 7463 | 10945 | 87563 |
| 1 | 92466 | 35025 | 57441 | 2101.5 | 103.58 | 10938 | 4143 | 6795 | 10945 | 81528 |
| 2 | 86094 | 30882 | 55212 | 1956.7 | 99.51 | 11096 | 3980 | 7116 | 10945 | 74997 |
| 3 | 79197 | 26902 | 52296 | 1799.9 | 95.64 | 11262 | 3826 | 7437 | 10945 | 67935 |
| 4 | 71739 | 23076 | 48663 | 1630.4 | 91.97 | 11437 | 3679 | 7758 | 10945 | 60302 |
| 5 | 63679 | 19397 | 44282 | 1447.3 | 88.49 | 11620 | 3540 | 8081 | 10945 | 52059 |
| 6 | 54974 | 15857 | 39117 | 1249.4 | 85.20 | 11814 | 3408 | 8406 | 10945 | 43160 |
| 7 | 45577 | 12450 | 33127 | 1035.8 | 82.08 | 12019 | 3283 | 8736 | 10945 | 33558 |
| 8 | 35437 | 9167 | 26271 | 805.4 | 79.13 | 12236 | 3165 | 9071 | 10945 | 23202 |
| 9 | 24501 | 6002 | 18499 | 556.8 | 76.34 | 12465 | 3054 | 9412 | 10945 | 12036 |
| 10 | 12710 | 2948 | 9762 | 288.9 | 73.70 | 12710 | 2948 | 9761 | 10945 | 0 |

distributed capital gains is 35% and the annuitization period $T$ is 10 years. By solving equation (46), the amount of the constant annual after-tax allowance is found to equal \$10,945.

If the annual after-tax consumption is adjusted by the level of inflation, that is, $w_k = w(1 + \pi)^k$, where $w$ is the after-tax amount of the current withdrawal, then given the annuitization period of $T$ years and using (43), the condition for $w$ becomes:

$$S - \sum_{i=0}^{T} \frac{w(1 + \pi)^i - B_i}{A_i} = 0 \Rightarrow w = \frac{S + \sum_{i=0}^{T} \frac{B_i}{A_i}}{\sum_{i=0}^{T} \frac{(1 + \pi)^i}{A_i}}. \tag{47}$$

Table 4 demonstrates the year-by-year accumulations and withdrawals for the taxable account holding an equity fund with a current before-tax value of \$100,000 where the cost basis of the investment is \$40,000. The investor holds 1,000 shares of the fund, where each share has the same cost basis and is currently priced at \$100. No portion of last year's capital gains has just been distributed to the shareholders ($B_0 = 0$). The annual rate of return is assumed at 8%, share of the distributed future annual return is 30%, annual inflation is 2%, tax rate on capital gains is 20%, tax rate on distributed capital gains is 35% and the annuitization period $T$ is 10 years. Using equation (47), the initial after-tax allowance $w$ is found to be \$10,026, and each subsequent value of the annual after-tax allowance is adjusted by the level of inflation.

The withdrawal modeling from the taxable equity holding account as presented here allows for the estimation of the effect of capital gains distributions on the level of annual after-tax withdrawals. Such distributions represent a source of tax inefficiency since they result in a loss of income because of the higher rate of taxation. For the example above that assumes zero inflation, the distribution ratio of 30% results in a loss of \$198

Table 4 The year-by-year withdrawal picture from the taxable account, where $S$ = \$100,000, $a$ = 0.4, $r$ = 0.08, $d$ = 0.3, $\pi$ = 0.02, $t_{cg}$ = 0.2, $t_d$ = 0.35, and $T$ = 10 (values are rounded)

| Year (end of) | Balance before with-drawal | Cost basis | Unrealized capital gains | Distributed capital gains | Number of shares sold | With-drawal, total | With-drawn cost basis | Realized capital gains | After-tax allowance ($w_i$) | Balance after with-drawal |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 100000 | 40000 | 60000 | 0.0 | 113.94 | 11394 | 4557 | 6836 | 10026 | 88606 |
| 1 | 93568 | 35443 | 58126 | 2126.6 | 95.64 | 10099 | 3826 | 6274 | 10227 | 83469 |
| 2 | 88143 | 31617 | 56526 | 2003.3 | 93.91 | 10472 | 3757 | 6716 | 10431 | 77671 |
| 3 | 82020 | 27861 | 54160 | 1864.1 | 92.25 | 10863 | 3690 | 7173 | 10640 | 71158 |
| 4 | 75142 | 24171 | 50972 | 1707.8 | 90.65 | 11272 | 3626 | 7646 | 10853 | 63870 |
| 5 | 67447 | 20545 | 46902 | 1532.9 | 89.10 | 11701 | 3564 | 8137 | 11070 | 55746 |
| 6 | 58868 | 16981 | 41887 | 1337.9 | 87.62 | 12151 | 3505 | 8646 | 11291 | 46717 |
| 7 | 49334 | 13476 | 35858 | 1121.2 | 86.20 | 12623 | 3448 | 9175 | 11517 | 36710 |
| 8 | 38766 | 10028 | 28738 | 881.1 | 84.84 | 13120 | 3394 | 9726 | 11747 | 25646 |
| 9 | 27082 | 6634 | 20448 | 615.5 | 83.54 | 13642 | 3342 | 10301 | 11982 | 13440 |
| 10 | 14192 | 3292 | 10900 | 322.6 | 82.30 | 14192 | 3292 | 10900 | 12222 | 0 |

(11,143 − 10,945) in the annual after-tax income, the equivalent of the income loss of 1.78%. If the distribution ratio is assumed at 100% like in Sibley (2002) and Spitzer and Singh (2006), the annual loss will grow to \$657 or the equivalent of 5.90%. In general, the relative size of the loss increases not only with the size of capital gains distributions $d$, but also with the rate of return $r$, annuitization period $T$, difference between the tax rate on such distributions $t_d$ and the tax rate on long-term capital gains $t_{cg}$, and the share of cost basis $a$. For example, when $a = 1$, $r = 0.1$, $t_{cg} = 0.2$, $t_d = 0.4$, and $T = 20$, the annual after-tax allowance when $d = 0$ is 20.1% higher than for the case when annual capital gains are fully distributed!

## 4. Summary

A common wisdom accepted in the field of financial planning is that most retirees usually do not liquidate holdings in their retirement accounts and taxable accounts with a single withdrawal but rather make regular withdrawals over a number of periods. The existing literature on modeling withdrawals from the taxable account tends to assume that annual returns generated by the assets held in the account are fully taxed as ordinary income. Such assumption is reasonable if the underlying asset is a bond but for the case of the equity holdings and a buy-and-hold portfolio, capital gains are taxed only upon their realization and the taxes are effectively deferred until then. This paper fills the gap in the literature and demonstrates how to model multiple withdrawals from the equity holding taxable account whose returns are taxed at capital gains tax rates by employing a very generalized setup. Additionally, several special cases are illustrated where a set of parameters is assumed constant and the value of the taxable account is annuitized over a number of years.

A portion of annual capital gains can be distributed by equity funds to their shareholders. The withdrawal modeling presented in this paper allows the incorporation of such distributions into the analysis and to estimate the effect of such distributions on the level of annual consumption financed by the withdrawals from the taxable account. The loss of income because of tax inefficiency can be substantial and increases with the rate of return, annuitization period, difference between the tax rate applied on the distributions and the tax rate on long-term capital gains, and the share of cost basis in the current value of the investment.

The presented withdrawal modeling technique is applicable in a wide range of models that try to compare balances in the accounts with different tax treatment by accommodating multiple withdrawals through the retirement period. It can be further used to determine the future stream of after-tax cash flows a given tax-advantaged account is expected to produce for the retiree and find the number of current after-tax dollars invested in the taxable account that would produce the same stream of future cash flows.

The withdrawal modeling from the taxable account that uses either a FIFO (first-in-first-out) or a LIFO (last-in-first-out) method to determine the cost basis of sold shares will not differ conceptually from the methodology described here. The investment's entire value will be split into a number of lots and these lots will be liquidated sequentially. If only a partial liquidation of a particular lot is required, the remaining value as well as the remaining cost basis of such lot must be computed in such a manner that preserves the existing ratio of the cost basis to the value of the unrealized capital gains, which is the core element of the withdrawal modeling presented in this work. In other words, the cost basis is reduced proportionally to the reduction of the lot's before-tax value and so are its accumulated capital gains. This is equivalent to selling a portion of shares in the lot thus realizing capital gains on those shares and paying the capital gains tax on the sale.

## Notes

1. Notable exceptions are Horan (2006a, 2006b) where optimal withdrawal strategies from the deductible individual retirement account (IRA) and Roth IRA are explored.
2. The assumption about the same cost basis for each share is both realistic and convenient to arrive to tractable results. In its Publication 550 "Investment Income and Expenses," the IRS states that "you can choose to use the average basis of mutual fund shares if you acquired the shares at various times and prices and left them on deposit in an account kept by a custodian or agent." This method is usually referred as the average basis, the average cost, or the average cost single category and it is more tax efficient than first-in first-out (FIFO), which is used by default. Under the new policies issued by the IRS in late 2009 that will become effective as of January 1, 2011, the accepted cost-basis methodologies include first-in first-out, average cost, and tax lot reporting.

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