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Freedom at 55 or drudgery till 70?

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Abstract

The classic preretirement problem for the financial planner is to advise a client how much to save, how much must be saved each year to reach a specified goal, and how the investment assets should be allocated between fixed income and equity. The traditional solution is to assume a fixed rate of return for each asset class and test scenarios until the mixture of variables yields a solution that meets the stated savings goal and seems feasible for the client. This binary result (accept or reject the plan) ignores the inherent uncertainty. In this paper, we derive a stochastic model in which the rate of return and the rate of increase of annual savings are both variable and calculate the probability that a particular goal will be achieved, given any initial savings endowment, periodic additional savings amount, mixture of assets (represented by the return distribution), and time to the goal. The calculations can be done on an Excel spreadsheet. We illustrate the use and numerical results of the model with a realistic retirement planning scenario and variations on it. While this solution is particularly important for preretirement planning, it applies quite generally to meeting any financial goal, such as saving a down payment for a house.

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1. Introduction

Retirement planning has become the most important topic in personal finance in the developed countries. The baby boom bulge is starting to retire, and many of them are ill-prepared. The disappearance of defined benefit plans places more responsibility for retirement income on the employees at the same time as many social security pension plans

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are underfunded. In some countries, notably the United States with its postretirement health benefits challenges, private company plans are also more dramatically underfunded. Governments do not have the means to take up the slack. The worldwide crash in stock markets in 2008 put most defined benefit pension plans into an underfunded status and left those with defined contribution plans and self-funded retirement plans facing a much poorer future.

The financial planner solves the classic preretirement problem by assuming a rate of return and then determining if the current wealth and planned savings in future years will accumulate to a desired retirement goal. The rate of return chosen may depend on a specified investment mix. This procedure is well-established and gives a reasonable estimate of the feasibility of a given retirement plan. It ignores the significant risk in a world of volatile asset returns. Telling a client the standard deviation of returns utterly fails to portray the risk of falling short of a goal.

One way to show the extent of the risk is a simulation. Vora and McGinnis (2000), Booth (2004), and Schleef and Eisinger (2007) analyze retirement targets and asset allocation using Monte Carlo simulation. The client can see the probability of any particular plan succeeding in reaching the desired goal. For the preretirement plan, simulation is however quite cumbersome, because there are so many possible paths to follow and hence many simulations to be run.

Another way is to develop an analytical solution for the probability of ruin, which is the path that Milevsky and Robinson (2005) and Robinson and Tahani (2010) follow for the postretirement planning problem, how much income is sustainable in retirement?

Ho et al. (1995, 1996) develop an analytical solution for the probability of ruin in a preretirement plan and we label their model HMR. Their solution is useful, but has a significant flaw. To get a solution, they assume that the periodic savings deposits are all certain, and take the present value of them using the risk-free rate. They then add this notional present value of future saving to the existing endowment, and then make this single value subject to a stochastic rate of return from the initial time to the time of the goal. In this paper, we make both the initial wealth and the periodic savings subject to the stochastic rate of return at every period. Thus, the risk that the rate of return will be lower during the early years when savings are lower is incorporated into our model. We also make the savings amount itself stochastic, instead of constant. This more realistic formulation gives us more confidence in general applicability of the result.

2. The model

Our problem then is to calculate the probability that a person who has some initial endowment, plans to save and invest a specified amount annually, and has a goal to retire at a known time with a desired amount (in real dollars), will actually reach that goal.

2.1. Cumulative return, initial wealth, and stochastic savings

By analogy to the stochastic present value in Milevsky and Robinson (2005) and Robinson and Tahani (2010), we introduce the notion of the *stochastic future value* (SFV) of the stochastic savings and initial wealth at a known future date (e.g., retirement date). The SFV is a random variable that can only be known through its probability distribution. More specifically, we define it by (technical details are presented in the Appendix):

$$SFV = wR_T + R_T \int_0^T S_t R_t^{-1}\, dt \tag{1}$$

where T denotes the known retirement date (in years), w is the initial wealth, and (S_t) and (R_t) are, respectively, the savings and the cumulative return processes. We assume that the processes (S_t) and (R_t) follow two correlated Geometric Brownian motions given by:

$$\begin{cases} dV_t = \mu V_t dt + \sigma V_t dB_t & ; \quad V_0 = v \\ dR_t = \tilde{\mu} R_t dt + \tilde{\sigma} R_t dB_t & ; \quad R_0 = 1 \\ dS_t = \eta S_t dt + \beta S_t dZ_t & ; \quad S_0 = s \\ d < B,Z >_t = \rho\, dt & \end{cases} \tag{2}$$

where (V_t) is the risky asset process. The constants (μ,η) and (σ,β) denote respectively the drifts and the diffusion parameters of (V_t) and (S_t). The last equation simply states that the correlation between the savings and the cumulative return (or the risky asset) is equal to ρ. Because the cumulative return is based on a portfolio consisting of a fraction of $(1\text{-}\alpha)$ invested in the risk-free asset and a fraction α invested in the risky asset (V), we have:

$$\begin{cases} \tilde{\mu} = (1-\alpha)r + \alpha\mu \\ \tilde{\sigma} = \alpha\sigma \end{cases} \tag{3}$$

where r is the risk-free rate of return. In our retirement problem, the purpose is to determine the optimal value $\hat{\alpha}$ that maximizes the probability of reaching the retirement goal G, that is $\hat{\alpha} = \text{argmax}_\alpha\, P(SFV(\alpha) \geq G)$. Using standard stochastic calculus, it can be shown that the solutions to Eq. (2) are given by:

$$\begin{cases} R_t = R_0 \exp(\bar{\theta} t + \tilde{\sigma} B_t) & ; \quad \bar{\theta} = \tilde{\mu} - \dfrac{1}{2}\tilde{\sigma}^2 \\ S_t = S_0 \exp(\bar{\eta} t + \beta Z_t) & ; \quad \bar{\eta} = \eta - \dfrac{1}{2}\beta^2 \end{cases} \tag{4}$$

To value the probability of reaching the retirement goal, we need to determine the probability distribution of the SFV as given in Eq. (1). Unfortunately, the SFV is similar to a (continuous) sum of lognormal variables for which there is no simple closed-form cumulative distribution function. Thanks to the Asian options (i.e., arithmetic average options) literature, many analytical approximations based on known probability distributions are available. We will approximate the SFV distribution using the lognormal distribution by matching the first two moments.

2.2. Main result: Lognormal approximation

Based on the first two moments E_1 and E_2 of the SFV (presented in the Appendix), we can approximate the probability to reach the retirement goal by:

$$P(SFV \geq G) = 1 - N\left(\frac{\ln(G) - a}{b}\right) \tag{5}$$

where $N(.)$ is the standard normal cumulative distribution and the parameters a and b are given by:

$$\begin{cases} a = 2\ln(E_1) - \dfrac{1}{2}\ln(E_2) \\ b = \sqrt{\ln(E_2) - 2\ln(E_1)} \end{cases} \tag{6}$$

Knowing the properties of the standard normal distribution, we can calculate the probability to reach the retirement goal in Eq. (5) as a function of the fraction α, and then maximize it to determine the optimal allocation $\hat{\alpha}$, the corresponding probability to reach G and the expected wealth at the retirement date. Hereafter, we denote our model by the SFV model, to distinguish it from the HMR model.

3. Numerical examples

The functions in the previous sections can all be programmed into an Excel spreadsheet[1] (Fig. 1 provides two screenshots of the spreadsheet). In this section we provide some numerical examples.

3.1. The base case

Let us first set up a situation and then we will look at variations. A couple aged about 40 would like to retire at age 65. Their goal is to have \$500,000 in today's dollars (i.e., in real dollars) to supplement their basic pension income. They have liquid wealth that is invested towards that goal of \$100,000 now, and they can save \$7,000 p.a. if they are quite frugal. They would like to know if they can meet their goal, and if they could spend more now and manage with only \$5,000 p.a. in savings. All these values are in real dollars. The list of the variables and their values for this first situation, are shown in Table 1.

Our principal objective with this paper is to demonstrate the validity of this model and its usefulness to financial planners and their clients. The spreadsheet will function with any rate of return that the user wants. We have taken our rates of return and the standard deviation of equity as the long-term arithmetic averages for Canadian T-bills and Canadian and United States equity (in Canadian dollars) from Ho and Robinson (2005), Appendix E. We use the same rates throughout the paper.

We show three different solutions to the basic question in Table 2. Panel A shows financial planner's deterministic solution. If the portfolio is invested wholly in the riskless asset, the

Panel A: Constant Savings

Input		Output		Moments calculation			
Initial Wealth (W_0)	100,000.00 $	Optimal Allocation	84.94%	μ_tilda	5.44%	Moments	
Initial Savings (S_0)	7,000.00 $	Prob(Wealth > G)	67.78%	σ_tilda	13.59%	E1	762,710.31 $
Goal (G)	500,000.00 $	Expected Wealth	762,710.31 $	A1	-0.0544	E2	8.02114E+11
r (in real terms)	2.30%	Std Dev Wealth	469,453.50 $	A2	0.1273	Std Dev	469,453.50 $
T	25			A3	-0.0729		
μ (in real terms)	6.00%	Solve		A4	-0.1273	Normal distribution parameters	
σ	16.00%			A5	-0.0544	a	13.3840
η	0.00%					b	5.6679E-01
β	0.00%						
ρ	0.00%						

Panel B: Stochastic Savings

Input		Output		Moments calculation			
Initial Wealth (W_0)	100,000.00 $	Optimal Allocation	54.57%	μ_tilda	4.32%	Moments	
Initial Savings (S_0)	5,000.00 $	Prob(Wealth > G)	78.13%	σ_tilda	8.73%	E1	695,561.14 $
Goal (G)	500,000.00 $	Expected Wealth	695,561.14 $	A1	0.0068	E2	5.45849E+11
r (in real terms)	2.30%	Std Dev Wealth	249,085.15 $	A2	0.0940	Std Dev	249,085.15 $
T	25			A3	-0.0002		
μ (in real terms)	6.00%	Solve		A4	0.0109	Normal distribution parameters	
σ	16.00%			A5	0.0111	a	13.3921
η	5.00%					b	3.4736E-01
β	7.00%						
ρ	10.00%						

Fig. 1. Screenshots of the Excel spreadsheet. Panel A: Constant savings. Panel B: Stochastic savings.

goal cannot be reached. A balanced portfolio or a 90% equity investment will do the job. Even in practice, we know that this is not a wholly satisfactory answer. We know that the equity return is uncertain, and therefore we cannot say for sure that the goal can be met.

Panel B displays the fundamental contribution of the paper. The optimal asset allocation is solved, while at the same time the probability of reaching the goal calculated. The expected wealth and standard deviation are also shown, which illustrates in another way the riskiness of the scenario. The financial planner's deterministic solution in Panel A says that the goal is reached with a largely equity portfolio, whereas both the HMR and SFV models say that there is a substantial risk of shortfall. We might well have expected a significant possibility of shortfall with a balanced portfolio, because deterministic expected value is only $79,000

Table 1 Variables and initial values

S_0: Initial annual savings	$7,000
W: Wealth at any time, context makes the implicit subscript clear	Initial wealth = $100,000
G: Goal, the desired value of wealth at T	$500,000
T: The number of years to the goal	25
r: Riskless rate (*in real terms*)	2.3%
μ: Drift term for equity portfolio (*in real terms*)	6%
σ: Diffusion of risky portfolio	16%
η: Drift term for S	0
β: Diffusion of S	0
ρ: Correlation of S and R (*cumulative return*)	0

Table 2 Can you reach a savings goal? Constant real saving

Panel A: Financial planner's deterministic solution						
Allocation to Equity	0%	50%	90%			
Rate of return	2.3%	4.15%	5.63%			
Future value ($000)	$410	$574	$758			
> Goal of $500?	No	Yes	Yes			
Panel B: Stochastic return solutions, parameters given in Table 1. Note: The result for HMR uses their formulation, which gives a less accurate result than our model using the SFV						
	HMR model	SFV model				
E(W): Expected wealth ($000)	842	763				
SD(W): Standard deviation ($000)	569	469				
Optimal allocation α	77%	85%				
Probability ($W > G$)	71%	68%				
Panel C: Stochastic return solutions, varying value of annual savings, SFV model						
Annual savings, S_0	$5,000	$6,000	$7,000	$8,000	$9,000	$9,500
E(W) ($000)	738	796	763	714	633	536
SD(W) ($000)	586	619	469	321	160	36
Optimal allocation α	100%	100%	85%	67%	40%	11%
Probability ($W > G$)	58%	63%	68%	73%	79%	85%

over the goal, or a cushion of only 16%. However, the 90% equity produces an expected value of $758,000, or more than 50% over the required goal. Nonetheless, the optimal portfolio of 85%, according to the SFV model, still runs a risk of falling short almost one-third of the time.

The stochastic models provide two very important insights for the planner. First, they reveal the substantial risk of relying on the deterministic solution. Second, they provide an intuitively useful quantification of the risk. The client can understand the risk and decide accordingly. We have not postulated what the acceptable level of shortfall risk is, nor the tradeoff between that risk and the higher expected wealth of higher equity allocations. That is a decision for the investor.

We explained the difference between the HMR and SFV models earlier. Panel B shows an example of the effect of the difference on the results. With many reasonable combinations of the variables, which we do not show here, the HMR model will normally give a lower estimate of the optimal equity allocation and a higher, or more optimistic, estimate of the probability of reaching the goal. The HMR model did not allow for stochastic savings, and so no comparisons are possible in the remaining tables.

Families rarely save at a constant rate. The usual family life cycle involves little saving in the early years, except perhaps through mortgage payments and mandatory retirement savings. Sometime in the middle of the family life cycle, as real family income rises, and the mortgage payments are declining in real dollars, the family can start discretionary saving. We normally expect this saving to rise over time, but also to fluctuate. The SFV model incorporates this fluctuation by allowing for a stochastic S, with drift η and diffusion β. Examples of the optimal portfolios and shortfalls are shown in Table 3.

Panel A of Table 3 shows results with *S* increasing at different constant rates, that is, $\beta = 0$. The initial annual savings, S_0, is $5,000, instead of the $7,000 of the initial case. We do

Table 3 Probability of reaching a savings goal with increasing annual savings (in this table, S_0 = \$5,000)

Panel A: Stochastic return, savings increase at a constant rate, η; $\beta = 0$						
η	1%	2%	3%	4%	5%	5.115%
$E(W)$ (\$000)	768	777	742	690	568	500
Optimal allocation α	100%	96%	83%	64%	23%	0%
Probability ($W > G$)	61%	64%	68%	74%	83%	100%
Panel B: Stochastic return, stochastic savings						
η	1%	2%	3%	5%	7%	9%
β	2%	4%	5%	7%	8%	10%
$E(W)$ (\$000)	768	780	751	688	722	907
Optimal allocation α	100%	96%	84%	53%	33%	37%
Probability ($W > G$)	61%	64%	68%	79%	92%	96%

not have an independent method of determining reasonable values for the annual growth rate of savings. The values cannot be very large when T is large, because the compounding effect is too fast when we use real dollars. At a 1% growth rate in this example, the optimal allocation to equity is still 100% and the success probability has risen only three percentage points to 61%, compared with zero growth, S_0 = \$5,000, in Table 2, Panel C. At 5.115%, the investor needs no equity, because the growth rate of savings is fast enough to reach the goal of \$500,000 for sure using only the riskless asset. As a matter of practice, there is no truly riskless asset, and so we must recognize that there is a bit of uncertainty in this plan. Furthermore, no-one has a perfectly fixed growth rate in savings.

Fortunately, Panel B provides an even more realistic solution. Once we make S stochastic, the solution converges a lot more slowly to the riskless rate. We have no empirical evidence to guide us on suitable values of β; the ones shown seem to give reasonable answers. An interesting result is that when volatility is introduced, it is possible to have the optimal equity allocation rise with a rising drift value. We see that the optimal equity at (η, β) = (7%, 8%) is 33%, and it rises to 37% for (9%, 10%). The expected wealth, $E(W)$, rises a great deal, but there is still a small probability of failing to reach the goal of \$500,000. We think that if most investors were presented with all of Table 3, and could understand it, they would choose the sort of balanced portfolio in the right hand three columns of Panel B, if they expected to increase their real savings per annum every year, on average.

We also added a positive correlation, ρ, to the scenarios of Table 3. It affects the probabilities only a small amount, and would not make any practical difference in a planner's recommendations or an investor's decisions; so we did not include the results.

We could also have shown results for a decreasing rate of S, but such cases are counter to the normal family life cycle. The model will also work with negative values of W_0 and it seems a natural extension, to represent a family with mortgage debt and immaterial liquid savings. This is a bit of a trap, however. A mortgage is nominal, whereas we are working with real dollars. Furthermore, the rate of return on paying off a mortgage is not all the same as the rate of return on an investment portfolio, even though it is an investment. To reflect mortgages using our model here, the planner has to think of the mortgage payments as reducing the potential S, and then when the mortgage is paid, allowing a much higher and a fast growing S.

Table 4 Start at age 40 with no accumulated savings. How much S_0 is needed to retire at 65 with $500,000?

S_0	$7,000	$10,000	$12,000	$15,000
Panel A: Deterministic, no growth in savings				
FV ($000) at 4.15%	297	425	510	637
FV ($000) at 6%	384	549	658	823
Panel B: Stochastic return and savings, η = 2%, β = 4%, ρ = 10%				
$E(W)$ ($000)	496	707	633	674
Optimal allocation α	100%	100%	43%	10%
Probability ($W > G$)	39%	64%	80%	99%

Finally, we illustrate what savings level would be needed if our 40 year old couple has no savings to start with, which is a more likely situation for most people at that age. How much do they need to save to reach their goal at age 65? Table 4 displays some results.

The deterministic constant savings model says that at $10,000 p.a. of real savings invested 100% in equity, the family will more than reach its goal. If the family wants the lower risk of a balanced portfolio, it will need to save $12,000 p.a. in real dollars.

The stochastic model in Panel B, with growth of 2% p.a. and 4% volatility in S, and with 10% correlation between S and the rate of return, paints a less optimistic picture. Remember that Panel B is including growth in S; so the family will actually save more and consume less before retirement, than they would with the plan in Panel A. Nonetheless, the probability of reaching the goal is only 64% with savings starting at $10,000 p.a., invested 100% in equity. The expected value is much higher than in Panel A, but the risk introduced by stochastic returns has a large effect. The family has to go to a savings rate of $15,000 p.a. to get a really secure aim on the goal, and at that rate, with growing savings every year on average, the probability of reaching the goal hits 99%.

4. Conclusion

Financial planners will find the SFV model very useful in helping clients plan their retirement savings programs. The SFV model quantifies the risk inherent in any long run investment plan and allows more informed choices.

The numerical examples are but a few illustrations, since the number of possible situations is enormous. Nonetheless, we can see from the significant savings rates required to reach relatively modest goals that a great number of people are not well-prepared for eventual retirement. For example, take Panel C of Table 2. Suppose a family earning $100,000 per annum were to set that goal of $500,000 in 25 years. Without doing any calculations, we can see that they must also have some pension income in retirement, because $500,000 would not begin to provide the same standard of living as they now enjoy. This family has to save $9,500 yearly to reach an 85% probability of meeting this modest goal, or 9.5% of pretax income. We do not need delve into the wealth and savings statistics of Canada or the United States to say that not many families are recording savings at this rate, and even fewer 40 year olds have $100,000 in net liquid investible assets that can be set aside for retirement without being used for other purposes like children's education or support of elderly relatives.

We would like to be able to advance our work in two significant directions. We would like to be able to have a better way to estimate reasonable parameters for the annual savings amounts and wealth available for retirement goals, that is, better empirical evidence on the values of initial wealth and saving. We cannot use the standard cross-sectional averages that are readily available. We need longitudinal data be family to see what patterns actually occur over time. We suspect that this evidence would confirm our suggestion in the previous paragraph that most families are not saving enough for the retirement income they would like to enjoy. The minority who have good pension plans and have been members of them for most of their working life will be OK, but that minority is also shrinking as pension plans are converted to defined contribution and the employer contributions restrained.

The second advance in this work would be incorporation of a jump process. The pattern of annual savings for an individual family is likely not only variable, but also subject to some large discontinuities as the family moves through the financial life cycle. Some of these discontinuities are unpredictable and could not be modeled, but some significant ones can, end of financial support of children, final mortgage payment, weddings. Incorporating this capability into the SFV model may be possible, though unquestionably it will be challenging work.

Notes

1. The spreadsheet is available at http://www.yorku.ca/ntahani/Research/TargetRetirement08.xls.

Appendix

In this appendix, we define the *stochastic future value* (SFV) and compute its first two moments. We then show how we can approximate its probability distribution.

Given the cumulative return process (R_t) in Eq. (4), the future value at time T of the initial wealth w is simply wR_T. On the other hand, an investment of S_u at time u is equivalent to a present value of $\frac{S_u}{R_u}$ at time 0; or to a future value of $R_T \frac{S_u}{R_u}$ at time T. Because the savings are invested continuously, their total future value is obtained by integrating over the time interval [0, T]. The SFV is then obtained as in Eq. (1) by:

$$SFV = wR_T + \int_0^T S_u \frac{R_T}{R_u} du \tag{A.1}$$

As seen in Eq. (4), both processes (R_t) and (S_t) are Geometric Brownian motions. Therefore, the SFV can be seen as a (continuous) sum of lognormal variables. Thanks to the literature on Asian options (i.e., arithmetic average options), many moment-matching-based approximations with known distributions are available.

Combining Eq. (1) and Eq. (4), the SFV can be written as:

$$SFV = we^{\overline{\theta}T+\tilde{\sigma}B_T} + se^{\overline{\theta}T}\int_0^T e^{(\overline{\eta}-\overline{\theta})u}e^{\tilde{\sigma}(B_T-B_u)}e^{\beta Z_u}\,du \tag{A.2}$$

Using standard stochastic calculus, it can be shown that the first two moments of the SFV are given by:

$$\begin{cases} E_1 = e^{\tilde{\mu}T} \times \left(w + \dfrac{s}{A_1}\left(e^{A_1T} - 1\right)\right) \\ E_2 = e^{A_2T} \times \left(w^2 + \dfrac{2ws}{A_3}\left(e^{A_3T} - 1\right) + \dfrac{2s^2}{A_4A_5}\left(e^{A_4T} - 1\right) - \dfrac{2s^2}{A_3A_5}\left(e^{A_3T} - 1\right)\right) \end{cases} \tag{A.3}$$

where $(A_i)_{i=1,\ldots,5}$ are functions of the allocation parameter α given by:

$$\begin{cases} A_1 = \eta - (1-\alpha)r - \alpha\mu \\ A_2 = 2(1-\alpha)r + 2\alpha\mu + \alpha^2\sigma^2 \\ A_3 = \eta + \rho\alpha\beta\sigma - \alpha^2\sigma^2 - (1-\alpha)r - \alpha\mu \\ A_4 = 2\eta + \beta^2 - A_2 \\ A_5 = \eta + \beta^2 - \rho\alpha\beta\sigma - (1-\alpha)r - \alpha\mu \end{cases} \tag{A.4}$$

Matching the first two moments E_1 and E_2 with those of a lognormal distribution with parameters a and b leads to Eq. (6).

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