The Mathematics Educator
2014 Vol. 23, No. 2, 23-39

Examining Student Work in the Preparation of Preservice Elementary School Teachers

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This study investigated preservice teachers' perceptions of their knowledge and development resulting from analyzing student work in an undergraduate mathematics education course. Participants were given opportunities to view and analyze student work examples that portrayed errors in thinking as well as alternative solution methods. Thirty-eight undergraduate students participated in the study and a thematic analysis approach was utilized for the analysis. The findings, which include four major themes of perceived knowledge development, are shared.

In response to the perceived gaps in mathematics teachers' knowledge, the National Science Foundation and the U.S. Department of Mathematics and Science Partnerships have invested approximately $1.2 billion between 2002 and 2007 in an effort to afford content-based mathematics and science experiences to both preservice and in-service teachers (Hill, 2010). However, Philipp et al. (2007) reported that elementary students in the United States are not developing the mathematical understanding and proficiency necessary for comprehensive understanding of mathematical concepts and thinking. Mathematics educators and education scholars alike have studied the issues surrounding what makes effective teachers, which include content knowledge, pedagogical content knowledge, and pedagogical knowledge. Ultimately,

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the conclusions are relatively consistent across the literature - content knowledge improvement alone is not the answer (Hill, Ball, & Schilling, 2008; Shulman, 1986, 1987). Part of being an effective teacher is the ability to interpret student responses and successfully respond in a mathematically grounded way (Fennema, Franke, Carpenter, & Carey, 1993).

According to the National Council of Teachers of Mathematics, effective teachers must possess several kinds of knowledge for teaching (NCTM, 1991). Among these are knowledge of the challenges students are likely to encounter in learning, knowledge about how ideas can be represented to teach effectively, and knowledge about how students' understanding can be assessed (NCTM, 2000). Part of being an effective teacher requires understanding mathematics content, pedagogical strategies, and their students as learners (Hill et al., 2008; NCTM, 2000; Shulman, 1987). One current issue is that teacher education programs often exclude experiences that develop this robust knowledge base necessary for effective teaching (National Research Council, 2001; Swars, Hart, Smith, Smith, & Tolar, 2007).

Preservice teachers (PSTs) must be comfortable with mathematical content, but must also be able to recognize and evaluate the multiple representations that accompany mathematical ideas (Wilson, Shulman, & Richert, 1987). They must be able to recognize which representations and pedagogical approaches are most appropriate for their learners. The experiences necessary to develop these types of understandings are not always prevalent in PST education coursework. Nevertheless, viewing, analyzing, and discussing student work can help to develop these skills and can easily be integrated into university coursework (Crespo, 2000).

Student work is defined in this study as the responses elementary students provide to a posed mathematical problem, which may include calculations, drawings, verbalizations, use of manipulatives, and videos. As Stacey, Helme, Steinle, Baturo, Irwin, and Bana (2001) have discussed, the insights unlocked by structured exposure to students' work help to improve PSTs' knowledge and skills for teaching. Kazemi and Franke (2004) and Little, Gearhart, Curry, and Kafka (2003) found that viewing student work collectively helped inservice teachers understand their students in ways that led to advances in teaching and learning. It is hypothesized that careful analysis of students' work can be influential to PSTs understanding of mathematics content, pedagogical strategies, and their students as learners (Crespo 2000; Son & Crespo, 2009). However, little is known about how PSTs perceive their experiences with student work during their undergraduate coursework.

Although many studies have examined factors that affect the development of teacher skill sets (Ball, Hill, & Bass, 2005; Carpenter, Fennema, Peterson, Chiang, & Loeff, 1989; Hill, 2010; Hill, Rowan, & Ball, 2005; Mewborn, 1999; Morris, Hiebert, & Spitzer, 2009), few have examined the experiences that come from analyzing student work. This study investigates PSTs' experiences analyzing student work and focuses on the perceptions of the participants through a thematic analysis lens (Aronson, 1994). More specifically, we were interested in PSTs' perceptions of knowledge and skill changes regarding future teaching endeavors. Our research was guided by the following research question: What are PSTs' experiences around analyzing student work?

Literature Review

According to Hill et al. (2008), there is agreement within the field of mathematics education that effective teachers have a strong knowledge of students' mathematical ideas and thinking. In particular, effective teachers know the kinds of misconceptions students are likely to hold or develop. The traditional focus of mathematical content knowledge is not adequate in developing these skills (Hill et al., 2008; Philipp et al., 2007). Content knowledge is more usefully developed in the context of student thinking and pedagogical skills (Philipp, 2007). Analyzing student work shows promise in preparing PSTs to have strong content knowledge while also promoting understandings of how students think and providing opportunities for PSTs respond in pedagogically sound ways (Crespo, 2000; Kazemi & Franke, 2004; Son & Crespo, 2009). Studies show that teacher preparation programs that stress structured analysis of student work can preserve a focus on content knowledge while adding a focus on student thinking and pedagogical strategies (Crespo, 2000; Son & Crespo, 2009).

A common focus in students' work is errors. Error patterns are defined here as systematic and consistent mishaps occurring across students due to inaccurate or inefficient procedures or strategies (Mercer & Mercer, 1998). There have been a multitude of error patterns identified in mathematics education. For example, the standard subtraction algorithm is often misused due to misunderstandings with place value and the concept of borrowing (Riccomini, 2005). Error patterns in mathematics are too numerous to count, but it is important to note that students tend to make analogous mistakes at similar grade and topic levels (Ashlock, 2010). Because of this consistency, a focus on student work within teacher education programs remains applicable when PSTs enter the classroom. Historically, error pattern information was rarely published in teaching manuals or mathematics education literature (Babbitt, 1990), which increases the importance of purposefully including them in teacher education programs today.

Babbitt (1990) also argued that error patterns reveal underlying conceptual misunderstandings that students possess, and that teachers' knowledge about these errors promotes feedback and positive outcomes in students. Furthermore, Ashlock (2010) asserted that recognizing and being able to use student errors as teaching opportunities is a very important part of teaching mathematics. Focusing on student errors in teacher preparation programs encourages PSTs to evaluate the level of conceptual understanding in the work they are viewing as well as evaluate their own conceptual understandings (Crespo & Nicol, 2006).

Teachers of mathematics must have deep conceptual understandings of the topics they teach in order to be effective (NCTM, 2000). This effectiveness stems from being able to see the material and paths to a solution in a variety of ways. When this happens, teachers are able to predict the potential errors that students might make, and, by doing so, are better prepared to use these errors as teaching tools. However, the ability to examine students' work and determine the thinking behind the errors is not an intuitive skill. It requires guidance and practice. Without this guidance, PSTs have been shown to quickly evaluate and move past students' work rather than carefully analyze and learn from it (Crespo & Son, 2009). Prospective teachers can be better prepared to address students' errors by analyzing and discussing student work and errors during teacher education programs.

Theoretical Framework

Rogoff (1994) describes a theory of learning, called transformation through participation, based on sociocultural learning theory. In her description of this theory, Rogoff claims that learning takes place when people participate in shared endeavors. She makes a sharp distinction to other common theories of learning. Transformation through participation involves neither a sole focus on the learner or the teacher but rather a joint and collective effort of teaching and learning that is socially negotiated and inseparable. Learning is not dependent on a student's ability to passively take in information or the teacher's ability to transmit knowledge from expert to novice. Instead, transformation through participation holds it is the involvement in collective and social activities that produces true learning by allowing students to interact with others and the material being learned. Therefore, learning cannot happen in isolation or without negotiation, discussion, and reflection.

PST education is no exception. In order for PSTs to gain the skills necessary for effective teaching, they must be exposed to activities that allow them to participate, negotiate, discuss, and reflect. Student work analysis activities provide a platform for PSTs to interact with peers while remaining contextually based in authentic teaching tasks. For this study, PSTs were required to participate in actively analyzing, discussing, and reflecting on student work and error patterns. The socially negotiated participation took place during analysis activities in class as well as on group take-home projects. By allowing for this reflection and interaction, this sociocultural approach provided the opportunity for student work analysis to transform PSTs' thinking and perceptions about their own content knowledge, pedagogical skills, and students as learners.

Method

Participants

Participants for the study were 38 PSTs majoring in elementary education at a large university located in the Southeastern United States. All 38 PSTs were taking the first mathematics course in a two-course sequence required in the major. The first course in this sequence focuses on mathematical content while discussing pedagogical strategies in a peripheral role. The age range of the participants was 18–22. The participants had the following self-identified ethnic backgrounds: Hispanic (2), and Caucasian (36).

The participants had a wide variety of educational backgrounds, involvement with children, and achievement levels in the program. Some participants had spent as many as two full semesters working at local elementary schools. During this time, PSTs taught micro lessons, instructed small groups of elementary school students, evaluated peers' teaching, and tutored individual students in multiple subjects. However, others had not yet worked with students at a school setting in any capacity.

Sampling Procedures

At the end of the semester, all 38 participants were retrospectively surveyed (Ericsson & Simon, 1993) regarding their perceptions of what learning transpired from their analysis of student work (see Figure 1). After reading through the 38 open-ended surveys, reoccurring perceptions were identified. The researchers worked to identify these individually, and then discussed and negotiated in order to come to a final agreement. Twelve survey responses that highlighted the most frequently occurring perceptions were then selected for further analysis. These 12 surveys led to the final themes gleaned from the data.

Student Work Analysis Activities

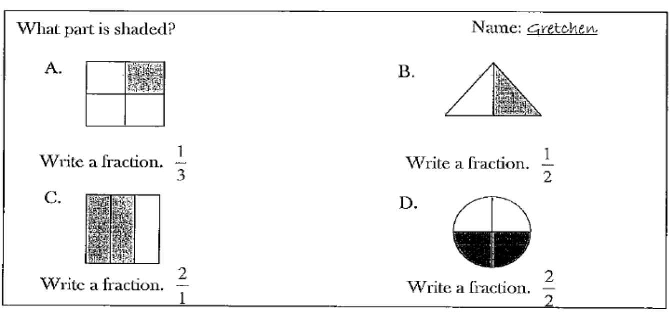
The participants analyzed elementary school students' mathematical work with the goal of identifying and explaining

Interview

1. Going into the class, how important did you think viewing student work was? Had you thought about using it?
2. Did the course change the way you will use/view student work?
3. What was the most important thing you learned about using student work to determine the levels of understanding your students have achieved?
4. How do you envision yourself using student work and/or error patterns in your classroom?

Figure 1. The open-ended survey instrument.

solutions and error patterns. An example of such an activity is included in Figure 2 (Ashlock, 2010, p. 57). All participants viewed student work in two separate capacities during the semester. First, participants analyzed student work during class on a biweekly basis. Student work from both local elementary schools and textbooks (Ashlock, 2010; Sowder, Sowder, & Nickerson, 2009) was displayed in class. PSTs were assigned the task of analyzing the work in groups and then reporting out about their findings. PSTs were asked to pay particularly close attention to error patterns during this time. The PSTs also examined and discussed unique solutions and the possible thinking elementary students might have used to arrive at their answers.



a) Describe the error pattern seen in this work.
b) This child's assignment asks "What part is shaded?" What question would need to be asked for her answers to be correct? (ignore question B, which she answered correctly)
c) Why do you think this student answered question B correctly? Explain and support why you believe this. Based on her other answers, what would you expect her answer to be?

Figure 2. Student-work analysis activity created for this study.

Second, the PSTs participated in a take-home student work analysis project as a semester long endeavor. This group project required them to analyze student work samples for errors as well as respond to prompts about student thinking, next steps, and appropriateness of procedures. The participants also gave grades and feedback to the student work they viewed. In several cases throughout the semester, participants were also asked to predict future errors the elementary students would potentially make based on the work they observed. The project presented three topics: place value, fraction concepts, and fraction operations. It was completed by topic in groups outside of class time. Ongoing class discussions about the project's completion were used to help ensure collaboration among group members and help maintain the sociocultural framework of learning that informed the study.

Data Collection and Analysis

All data for the study were collected through the open-ended survey questions asked to the 38 participants. The open-ended survey questions were designed in part to retrospectively capture the perceived learning that occurred including the effects on knowledge and skills from analyzing student work. Furthermore, the questions were designed to elicit responses that showed how the PSTs perceived the use of student work and error patterns would impact their future teaching endeavors. It is certainly possible that a combination of pre and post open-ended surveys could have produced different results, but only a retrospective survey process was chosen. We were interested in exploring how the teachers perceived their growth after they had learned what analyzing student work looked like. Individuals have been found to be more accurate and reliable in describing experiences retrospectively than when they are forced to infer about unfamiliar topics (Ericsson & Simon, 1993).

Thematic analysis (Aronson, 1994) was used to analyze the data collected for this study. The authors read through the opened-ended survey data and paraphrased as well as pulled direct quotes to highlight reoccurring patterns. These patterns led to the identification of the preliminary themes. Twelve participants who exemplified the newly identified preliminary themes were chosen for further analysis. This was done in part because fully analyzing and demarcating themes from 38 surveys is an overwhelmingly large task. The preliminary themes and supporting data were then examined in these twelve participants. The data was further reduced to sub-themes to help the researchers make sense of the numerous patterns present in the preliminary themes. These sub-themes were coded as perceptions, tools, strategies, assessments, understanding, knowledge, content, and implications. Some data was coded multiple times as they fit into more than one sub-theme. Finally, the eight sub-themes and supporting data were further reduced by identifying cross-cutting patterns to reveal the major themes being put forth by the data. The researchers agreed that four final themes were revealed by the thematic reduction and reconstruction of the data. These four final themes were then searched for in all 38 surveys. The results of this final, scaled up analysis follow.

Results

The first major theme found in the data was identified as "perceptions about the importance of viewing student work". It was discovered that within the 8 sub-themes, many participants were referencing the role of student work in their learning. For example, several mentions were made about how viewing student work can help reveal a student's level of understanding and provide a valuable tool for teachers. This first theme appeared in 26 of the 38 surveys. The majority (17 participants) of those 26 said their perceptions about the importance of viewing student work had changed. They now claimed that viewing student work to better understand children's thinking was a worthwhile endeavor. The remaining 9 also spoke about viewing student work, but reported that the class made no difference in their perspectives about viewing it. However, it is important to note that 7 of these 9 participants stated they already held viewing student work as an important teaching tool and consequently, felt no effect from this study. Ultimately, only 2 participants felt the treatment failed to change their view about the importance of viewing student work.

One PST commented on her perspective change towards understanding children's thinking by saying:

> I had thought about looking through [student work], but I hadn't really considered how important it actually was. I think this [class] has made me realize that viewing student work helps teachers determine how well the student understands the material much more than just viewing final answers.

Similarly, a second participant responded:

> Coming into class, I thought of student work in respect to math as pretty black and white with right and wrong answers. Now, I will discuss my students' work with them to discover their thought processes and evaluate their errors.

With the high rate of occurrence (26/38, 68%), exposing PSTs to analyzing student work influenced their views about the importance of the issue and the insight it provides into children's thinking.

The second major theme found in the data was identified as "implications surrounding the absence of viewing student work" (19/38, 50%). Participants were beginning to think about the classroom implications and issues that could arise if a teacher fails to focus on student work and error patterns. Viewing and discussing students' work led PSTs to ponder what might be missed if a teacher failed to view and analyze student work. Participants addressed that they now perceived it problematic for teachers to check only final answers and endorse only one strategy to solve a problem. They opined that viewing student work could allow teachers to better check for understanding and make sense of (and use) strategies that students invent. In several cases, participants shared their fear that students' lack of understanding could go unnoticed if a teacher failed to view students' work. The participants cited student-invented strategies and student errors from the treatment activities as sources for their new thinking about using student work to inform the teaching of mathematics.

In one quote that highlighted the second theme, a PST said, “Sometimes students could have their own methods and reasoning for solving problems. These methods and reasons should be valued, and could be completely missed if teachers only look at right and wrong final answers.” A second participant shared, “Sometimes students might have ways of solving problems that you might not think of. Those strategies might help you learn or other students in the class you are teaching learn.” In these and many other cases, it was discovered that the participants were grappling with the realization that without viewing and analyzing student work, they as teachers may indeed miss valuable information that their students’ performances could portray.

The third major theme found in the data was identified as “analyzing student work as a way to predict future errors or struggles.” Participants also saw using student work as a way to help them prepare lessons and be mindful of where students may hit pitfalls. The PSTs claimed viewing and analyzing student work provided them with insight about where and why these mistakes happened. Although many participants mentioned that talking with students about general errors could be helpful, they seemed to agree that tailoring these conversations around specific errors from current students would be more beneficial. One survey read, “The most important thing I can see is to study the error patterns to predict where they could potentially make an error in the future.” Another survey provided an example of the participants’ thinking about gleaning specific errors from current students:

> I think that one of the biggest things I learned from seeing the kids’ problems was the kind of mistakes they made. A lot of them were ones that I had never made myself so I probably wouldn’t have thought about them. I know some [text]books give examples of mistakes, but using ones right from your kids would be so much better.

The ability to be proactive and predict when students might make mistakes appeared in 11 (29%) of the surveys. This is a much lower percentage than the first two themes, but the extension to future planning and prediction was an unexpected outcome of the study given how novice the participants were in terms of teaching experience. This theme emphasized the perceived changes that occurred in the PSTs' ability to link mathematical content and understanding students as learners.

The fourth and final major theme was identified as "identification of tools and methods necessary to effectively and pragmatically use student work in the classroom". Some participants suggested methods such as "incorporating past student work into lessons with current students and having them work to understand error problems of others." Others suggested that teachers should use student work to better prepare themselves for teaching. This was articulated well by one participant who commented, "I will analyze student homework to judge which errors are common and need to be addressed as a class and which are individual misunderstandings that need to be discussed one on one." This theme revealed that many participants were extending student work analysis into a more advanced method for potentially altering the way they teach and the way students learn. The final theme surfaced in 14 of the 38 surveys (37%).

This theme, along with the previous three, showed that PSTs were grappling with the potential of viewing student work far beyond what was explicitly discussed and asked of them during the study. Furthermore, all four themes suggested that the participants perceived that their knowledge of pedagogy and of students as learners had been influenced by the student work analysis activities in this study.

After looking at the major themes holistically, it was determined that the analysis of student work and error patterns had positive effects on the perception of PSTs' ability to use their mathematical knowledge in relation to students' work. It was also discovered that PSTs began to think about ways to improve teaching and their understanding of students. Most importantly, this research shows that positive impacts can be made on PSTs by implementing a focus on student work analyses within a teacher education program. The initial goal of this study was to discover whether or not student work could be useful in the preparation of PSTs. The data analysis revealed more than what was sought or anticipated. PSTs looked beyond their own mathematical knowledge to discuss altered teaching strategies, pragmatic use of student work analysis, and other implications of both using and not using analysis of student work. They seemed to realize the potential of this activity and saw the type of information that can be garnered from the work students' produce.

Discussion

Student work serves as a window into the thinking, understanding, and misconceptions that students possess. Using student work in the preparation of PSTs has the ability to be a useful approach to increasing teachers' knowledge and skills. It has also revealed the potential to help PSTs realize the importance of multiple representations, which has been shown to be necessary in PST education (Wilson et al., 1987). Additionally, the results have shown that PSTs anticipate carrying the practice of analyzing student work into their classrooms.

The results and literature reviewed have generated more questions about the potential for student work analysis within teacher preparation programs. What types of teacher knowledge are being affected by these analyses? Can student work analyses lead to changes in the ways teachers are prepared to use assessment strategies – perhaps shaping the use of more formative assessment measures? Using the teacher knowledge framework set forth by Hill et al. (2008) as well as the instruments being developed by the Learning Mathematics for Teaching project (Ball, Hill, & Bass, 2005; Hill, Rowan, & Ball, 2005; Schilling & Hill, 2007) may help shine more light on these questions. Moreover, future research must be done to determine the effects of student work analysis on PSTs' future practices.

Conclusions

The major themes from the data analysis suggest that PSTs have gleaned important information from the analysis activities, and the resulting learning they perceived warrants future research aimed at discovering just how powerful analyzing student work can be. This study suggests using student work and error analysis in PST education settings certainly has potential, and, with continued effort and dissemination, student work analysis is likely to have positive and lasting effects on the preparation of PSTs. However, for this to happen, future research must begin to examine whether or not actual gains are being made in areas such as content knowledge, pedagogical skills, or knowledge of students as learners. Quantitative research, possibly with the help of the Learning Mathematics for Teaching project for assessing mathematical knowledge for teaching, can help answer if, and to what degree, PSTs are developing knowledge and skills as a result of analyzing student work. A measurement such as this is necessary to strengthen and extend what is presented here.

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